

# EXPERIMENTAL AND ANALYTICAL STUDY ON RC DEEP BEAM BEHAVIOR UNDER MONOTONIC LOAD

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## SYNOPSIS

This study deals with experimental and analytical investigation on reinforced concrete deep beams behavior under monotonic static load. The objective of this study is first to investigate whether the equations and design method of RC deep beams given in JRA (Japan Road Association) code can be enriched with proper modification by means of the new evidences of tested beams results or is already consistence and reliable in predicting RC deep beams behavior with a very small shear span to depth ratio. Second to reevaluate scale or element size effect on shear strength of concrete elements addressed in the mentioned codes by new test results. On the other hand reliability and applicability of numerical simulation of such members with Finite Element Method is also examined in order to generalize this by means of parametric study on RC deep beams without necessities of new experimental evidences particularly test on very large scale members.

Despite wide range of RC deep beams application in civil engineering project and prolonged research works on this subject, the behavior of these members have not yet been well clarified in various aspects nor well codified for design engineer and professionals who engage in design of such members in practice.

To investigate some aspects of RC deep beams behavior and lateral reinforcement effects in improving shear behavior of those beams, a joint project was proposed by Hanshin Expressway Public Corporation (HEPC), Public Works research Institute (PWRI) and Kyushu Institute of Technology (KIT) to conduct experimental investigation on RC deep beams behavior to be extended to numerical simulation of those members. A number of reports and technical papers are already published by respective institutes but here only codified and analytical study on the experiments is reported. Two Japanese design codes which are employed for code-based investigation are that of JRA (Japan Road Association) and JSCE (Japan Society of Civil Engineers). Due to the obtained results of the first part JRA code however showed better prediction with a consistence procedure than that of JSCE. The results and evaluation with experiment are discussed in detail. On the other hand both codes include size effect in design procedure adequately.

In order to conduct analytical study a specialized finite element package is employed for nonlinear finite element analysis. The program has capability of accepting user defined subroutine therefore different material models have been possible to be applied in this investigation. There have been a number of

materials and crack models proposed by researchers so far but no consensus on what model best results on finite elements calculation. On the other hand the response differ quite considerably in each model and in some sensitive case such as shear failure analysis, they are suffering from inconsistency and scatter distribution of obtained results in terms of employed model in both strength and crack pattern prediction. To circumvent such difficulties a sensitivity analysis also has been conducted here to pick up the best model as well as numerical solution scheme for further parametric study on wider range of member size, material properties and loading condition.

The behavior of concrete structures as a softening material in either tension or compression is characterized by a reduction of the load carrying capacity with increasing deformation after reaching a certain limit load. This global behavior is a result of stress-strain behavior in constitutive level where deformation tends to localize in a certain part while other part unloaded elastically. Moreover to sensitivity of the results to material and crack models, due to the nature of strain localization in loaded part finite element response show a mesh or geometry dependent behavior leads to nonobjective results. The deficiency of mesh dependent results can be solved by either enrichment of finite element formulation in element level or relating stress-strain relationship to concrete fracture energy via a length parameter as a function of mesh size to represent geometry property. The latter method needs however an accurate estimation of concrete fracture energy in tension and compression. In this study the latter method is adopted although other method also examined to study level of sensitivity of the results to the finite element size and employed mesh discretization. To have an adequate estimation of fracture energy of concrete JSCE design code recommendation is adopted here. The code's fracture energy expression is however evaluated by a quite large number of experimental results on concrete fracture energy to ensure reliability of the applied parameters in analysis.

Finally some important remarks and conclusions are drawn end of each chapter and also end of the report which summarizes the important finding of this study and future direction of research on RC deep beams behavior which deserve to receive more attention and work by respective research institutes and laboratories.

**Key Words:** *RC deep beam, underground structures, shear failure, design code, finite element, nonlinear analysis, size effect,*

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## 1. INTRODUCTION

RC deep beam are very useful members widely used in buildings, bridges and infrastructures. Deep beams are the beam with a depth comparable with their span length. To consider a beam as a deep beam, depth to span length (or depth to shear span length in case of point load) should be less than a certain value. This ratio is the most frequently used parameter by researchers and engineers. Despite wide application of those members, they are not adequately covered by design codes yet. In spite of many other codes, Japanese design codes have definition for RC deep beams particularly JRA (Japan Road Association) [1.1, 1.2] design code, which introduces adequate procedure for design practice. To evolve and enrich design method procedure of RC deep beams investigation and research on those members remains an attractive topic which receives attention of research institutes, universities or even the companies dealing with RC members design in real practice. Indeed behavior of reinforced concrete members is very complex and versatile and needs a comprehensive multi disciplinary research to be conducted.

RC Deep beams, in particular, show much complicated response to any sort of loads subjected to such members than conventional beams. In this regard, and also to study possibility of development and proper modification in JRA design code, a study on RC deep beams behavior and lateral reinforcement effects in improving shear behavior of those beam is carried out in Public Works research Institute (PWRI) following the experiments conducted during the year 2003 and 2004. Three sets of specimens comprise of nineteen RC beams including the experiments carried out on a joint research basis with Kyushu Institute of Technology (KIT) and Hanshin Expressway Public Corporation (HEPC) are investigated in this study. The beams have shear span to depth ratio between 0.5 and 1.5 and effective depth size from 400 mm to 1400 mm. The longitudinal tensile reinforcement ratio is kept almost constant in about 2% for all specimens while lateral reinforcement (stirrups) ratio varies by 0.0%, 0.4% and 0.8% in shear span. Since the study aims to investigate shear failure of RC deep beams, the beams are so designed ensure failure in shear mode therefore enough longitudinal reinforcement is provided to avoid failure in alternative mode where in 2% beams' effective cross sections for tensile steel ratio is found adequate. Normal strength concrete with a compressive strength ranges from 24 to about 38 MPa is used here in test specimens therefore High Strength High Performance concrete if left for future investigation.

The results of experiment compared with Japanese design codes, Japan Road Association (JRA) and Japan Society of Civil Engineers (JSCE) [1.3] prediction. Employed design codes are examined for reliability of load capacity prediction, consistency of the results in terms of shear span to depth ratio ( $a/d$  ratio) which is a key parameter in determination of failure mode and location of failure zone in different type of shear failure mode. Moreover influence of the existence and variation of lateral reinforcement in shear span region as well as size effect on shear strength of concrete in RC members are also examined. It is found however that JRA design code fits best to the test results with an acceptable safety margin with a consistence prediction in terms of  $a/d$  ratio. JSCE on the other hand results very conservative load capacity for entire specimens but utilizing recommendations of "Complementary for Standard Specification for Concrete Structures-2002" [1.4]; another publication of Japan Society for Civil Engineers which is assigned as a supplementary document for JSCE; could however improve the results but still have larger safety margin comparing to JRA. Details of the results are discussed in the following chapter.

On the other hand numerical simulation of those specimens are also performed by means of a number of material constitutive models; including that of JSCE, as well as proposed crack models and different numerical solution schemes. Non linear finite elements method is employed for numerical investigation by means of DIANA code, a general purpose finite elements package, with advance modeling features particularly for reinforced concrete structures. In order to trace compressive force path in RC beams, numbers of acrylic bars are located in the tested specimens in between loading plates and supports to measure strain in concrete in the designated path. The objective of using this method in experiment is to verify the validation of strut-tie model and also characterize and measure actual strain in the location with highest possibility of crushing or cracking in any kind of shear failure occurs in RC beams with low  $a/d$  ratio which is also used to evaluate analytical prediction. The results presented in this report are part of a larger study on shear behavior of RC deep beams in the aforementioned institutes but emphasizes is given to specimens tested in Public Works Research Institute. The analyses relatively showed reliable prediction in terms of ultimate load capacity, failure mode as well as crack patterns in certain load steps.

### *Why numerical simulation?*

As mentioned in the preceding section we need numerical simulation to adopt parametric study and

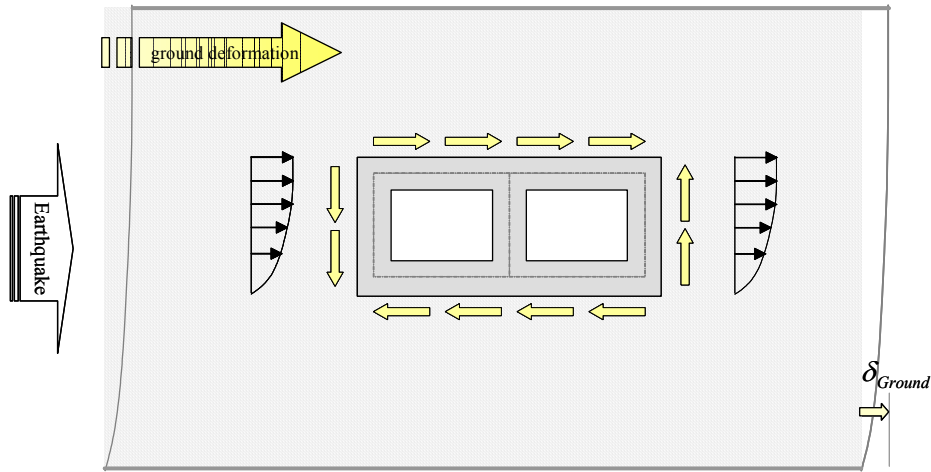


**Fig.1.1.** Underground structures under seismic load

improve design formulation for wider application. Moreover there are some other situations stipulate numerical simulation as well. In real construction, there have been many occasions that constructed structures need performance assessment due to changes in design codes, specification, and application as well as occurrence of damage or decay in under service structures. This happens quite often specially after each earthquake, which arises questions on reliability of structures in new condition. Another case is when a constructed building face new condition and needs to be retrofitted or repaired. In addition large and important structures such as inshore or offshore platforms, infrastructures such as bridges, lifelines and nuclear containments need continuous and accurate assessments of their performance in their service life.

Numerical simulation is one of the answers to all above problems. In this aspect nonlinear finite element method has dominant role in numerical assessments of structures. It is however some advantage and disadvantage here to be mentioned that there are plenty of models have been proposed so far for concrete member modeling. But there is no consensus on one model to cover entire condition of RC members and yield accurate results in all respect. Each model has its own merit and demerit. In essence for each specific problem, specific model should be employed. In other words some preliminary analyses with different constitutive and crack models ought to be performed first and then pick one model best results fit to the experimental observation then perform the final analysis for entire structure or conduct parametric study. This is a rational way in numerical simulation although there are some recommendations to use certain model for certain problem.

### ***Project Definition***

The motivation behind study on RC deep beams behavior here as well as evaluation of design code prediction is mainly due to an assigned project to investigate behavior of underground structures such as tunnels and box culverts subjected to the seismic excitation (Fig.1.1). Those structures can be modeled through a single-span or multi-span RC frames to simplify analytical process for practical purpose. Earth deformation subjected to seismic load produces shear deformation in underground structures as shown in Fig.1.1. Soil-structure interaction which cause such shear deformation is significant and should be taken into consideration in order to analyze real behavior of such structures. It is noted however that the response will be highly nonlinear and a nonlinear path dependent constitutive model should be employed. Here in this study the emphasize is given only to RC member behavior not entire system under earthquake excitation. The representative RC member chosen for this investigation is a simply supported beam under four-point load pattern. Real analysis of entire system under earth deformation results rather large internal force particularly shear force acting on resisting members and design of those members comes up with a relatively large section with small span to depth ratio. It means that the traditional principals of stress analysis are neither suitable nor adequate to determine the strength of such members. It is however found that those members are likely much closer to deep beam classification than any other types of members. Hence test on RC deep beam specimens is performed to investigate their response and evaluate current design codes prediction as well as the result of finite element analysis. It is noted that, however, behavior of the examined structures may differ if loading condition

is changed, e.g. under earthquake loading. The authors believe if we clarify response of structures, such as RC deep beams, under static monotonic load, behavior under dynamic load will be likely in similar trend although some parameters related to inherent characteristic of structures subjected to dynamic load should be taken into consideration. For instance, the effect of loading rate, as an important parameter in among other parameters, is discussed in chapter 5. Other parameters, ought to be considered in case study basis for future investigation.

### ***Literatures Review and History of RC Deep Beams investigation***

One of the earliest experimental studies on deep beam behavior and wall type beam behavior are that of de Paiva *et al* [1.5] in 1965 and Leonhardt and Walther [1.6] in 1966. The latter one follows the earlier study on shear resistance of RC beams which includes some deep beam as well [1.7]. Summary of their work in Stuttgart during 1961 to 1963 is given in reference [1.8]. The results of their study however played an important role in developing numerical method in recent decades. So far quite large numbers of experimental studies have been reported on RC deep beam behavior with or without flange, with or without opening in the web, beams under different load condition as well as simple or continuous deep beams [1.9]. Reference [1.10] brought all together over mentioned issues on RC deep beams in a book as an international reference for practicing civil and structural engineers, research institutes and universities.

Moreover to experimental studies on RC deep beams behavior, numerical simulation with finite element method is another important issue here to discuss about. Since 1962 which finite element method was first applied in analysis of concrete structures, it was rapidly developed in both theory and application. One of the earliest attempts was that of Ngo and Scordelis [1.11] who analyzed a reinforced concrete beam by means of discrete crack model. In this approach the crack location as well as the inclination is already defined and structure will be separated to two parts once crack takes place in the designated location. In this respect new element and new nodes should be generated to satisfy new situation of finite elements after cracking. Despite a number of advantages of the model in analyzing RC structures by finite element method, the method encountered several difficulties such as pre definition of crack location and direction and re-meshing of the structure while analysis is performing simultaneously. Nevertheless the most important problem was that the model as a whole

was not fit to the nature of finite element method, which is based on continuum mechanics. In order to circumvent the problem of geometry discontinuity, Rashid [1.12] proposed another method so-called smeared crack model, which deals with material discontinuity in constitutive level and fits well with finite element nature. Since then this model has achieved great popularity among researchers and has been developed enormously.

However with the advent of those methods in modeling RC structures and computer rapid progress as a tool to analyze them engineers and researchers use to be facing certain problems in numerical modeling of concrete structures. In a way concrete structures usually do not behave as continuum as those we assume in continuum mechanics. A recent attempt in response to that problem was to take advantages of fracture mechanics in modeling concrete members. Fortunately finite element method is sufficiently general to model fracture nature of concrete as discrete or continuum phenomena. Since then this method has enjoyed enormous development to minimize fracture zone size effect in analytical results and several robust theories such as crack band model [1.13], non-local theory [1.14] and embedded crack model [1.15] have been proposed and successfully applied in numerical simulation of concrete structures.

Smeared crack model basically is based on average strain approach which is smeared out in entire element surface and divided to two main categories called 'the fixed crack model' and 'the rotating crack model'. In the fixed crack theory the crack forms once principal tensile stress of concrete violates tensile strength of concrete. The crack direction will be fixed constant for the following steps. Consequent to this basic assumption a shear component will be produced due to rotation of the principal stress direction. In other words principal stress and principal strain direction need not to be coincide and shear and normal stress transfer will be modeled independently. The early version of smeared crack theory is based on this approach. On the other hand in the rotating crack approach the crack direction coincides with the principal direction of average strain therefore crack direction rotates following stress condition in each loading step. In this theory no shear transfer component will be produced and only normal stress-strain is needed to be modeled. Modified compression field theory [1.16] and softened truss theory [1.17] use this concept. Both methods have their own advantageous and disadvantageous which will be explained in the following chapters.

According to inherent nature of concrete material, theory of fracture mechanics is believed to adequately explain its behavior. The idea of using fracture mechanics in analyzing concrete structures initiated after realizing softening nature of concrete in tension by some researchers such as Lin and Scordelis [1.18]. They found that better and realistic results can be obtained if concrete stress reduces gradually instead of sudden drop to zero as Rashid assumed in his analysis. This leads to the problem of mesh dependent results where in the results of calculation will be significantly changed if the mesh is refined (non objective results). Bazant [1.19], Bazant and Oh [1.13], Bazant and Cedolin [1.20] and Rots *et al* [1.21] are in among of those who reported this problem in their calculation. ACI Committee 446 in its latest approved report [1.22] states five reasons that fracture mechanics need to be taken into account in concrete structures analysis including finite element method. Two reasons best fit to the objective of this study are 'objectivity of calculation' and 'size effect consideration'. For these reasons fracture energy concept is applied in analytical investigation of this study. It noted however that Fracture Energy concept can be applied in either abovementioned crack theories properly.

Another issue which can be well studied through fracture mechanics theory is Scale or Size Effect on shear strength of concrete structures. This might be the most compelling reason for design engineers to use fracture mechanics. It is however well-known that shear strength of concrete decrease as shear resisting size of the member increases. Fracture mechanics theory is found to be capable of explaining the phenomenon and predict variation of shear strength in terms of member size. The works of Bazant *et al* [1.23] and the sources of main body of the book) are among the most promising references which addressed size effect in detail.

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## 2. EXPERIMENTAL INVESTIGATION

In this chapter tested beams and experimentally observed results are explained and studied in detail. Totally three sets of specimens are tested during the year 2003 to 2004. Specimens are selected to cover a rather large range of size, shear span to depth ratios ( $a/d$ ) as well as reinforcement arrangement and lateral reinforcement assembled in the beams' shear span. Tables 2.1 and 2.2 introduce the specimens' material and geometric properties. The first set; Beams B2 to B12 including B10.3-1 are tested in Kyushu Institute of Technology (KIT) in the year 2003. Larger specimens B13 to B18 are tested in Public Works Research Institute (PWRI) in the consequent year of 2004. Last set comprises of three specimens B10, B10.3 and B13 which are retested in PWRI to evaluate reliability of previously tested specimens. In total taking into account the retested specimens we have nineteen specimens to study on deep beam behavior and also to evaluate our numerical simulation. The beams have shear span to depth ratio between 0.5 and 1.5 and effective depth size from 400 mm to 1400 mm. The longitudinal tensile reinforcement ratio is kept almost constant in about 2% for all specimens while lateral reinforcement (stirrups) ratio varies by 0.0%, 0.4% and 0.8% in shear span. The results of experiment reported elsewhere [2.1, 2.2 and 2.3] by means of experimental and codified study on RC deep beams behavior.

### 2.1 Test Set-up

A 30MN jack is used to load in two-point statically the larger specimens tested in PWRI. Due to the size of specimens tested in KIT, smaller jack is

employed for experiment. Test set up used in PWRI is shown in Fig.2.1.

### 2.2 Specimen's Geometries and Material Properties

Geometry characteristic and material properties are given in Fig.2.2, Table 2.1 and Table 2.2.

In Table 2.1,  $\rho_w$ ,  $\rho_{st}$ ,  $f_y$ ,  $A_{st}$  and  $A_{sc}$  are stirrups ratio, longitudinal tensile reinforcements ratio and their yield stresses, type and number of tensile and compressive longitudinal reinforcement respectively. All specimens, with or without stirrups in shear span, have a minimum lateral reinforcement in mid-span and out of span. Due to absence of shear stress in this part, shear reinforcement is naturally not necessary. But on the other hand it is evident by some experimental and analytical investigation that they may delay or in some cases prevent the propagation of diagonal crack to the compression zone. It is believed even that reinforcements in mid-span sometimes are more efficient than those in shear span due to the reason stated above [2.4]. Further study is however necessary to confirm the effect of mid-span stirrups experimentally. In Table 2,  $b$  is specimen width,  $a/d$  and  $f'_c$  are shear span to depth ratio and compressive stress respectively. Maximum load capacity and related deflection as well as shear crack initiation load are denoted as  $P_{max}$ ,  $P_{cr}^{sh}$  and  $\delta_{peak}$  respectively. Other geometrical parameters of Table.2.2 are schematically determined in Fig.2.2. All specimens are subjected to four points monotonic static load condition. Experimental data acquisition is mainly focused on mode of failure; crack patterns, load-displacement relationship as



Fig.2.1. Test Set-up at PWRI

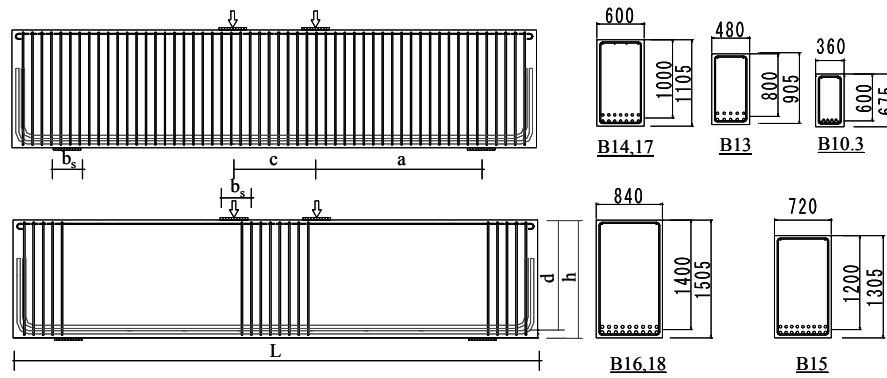


**Fig.2.2.** Detail of specimens with and without stirrups in shear span (unit: mm)

well as steel and concrete strain in some designated locations to evaluate analytical results.

All results of three sets of specimens have been used in this study in either experimental investigation or analytical verification. In order to trace compressive force path in RC beams, numbers of acrylic bars are located in between loading plates and supports to measure strain in concrete in the designated path. The objective of using this method in experiment is to verify the validation of strut-tie model and also characterize and measure actual strain in the location with highest possibility of crushing or cracking in any kind of shear failure occurs in RC beams with low  $a/d$  ratio. Figure 2.3 shows assembled acrylic bars in the specimens before casting concrete.



**Fig.2.3.** Details of Acrylic bars

### 2.3 Shear Failure Mechanism of RC beams

Failure modes can be classified in three main categories. First mode (Fig.2.4a) is tensile failure which usually occurs in members failed in direct tension or flexural failure. It is called brittle failure

of concrete as well though in flexural mode (Mode I), overall failure will be in ductile form as a result of yielding of reinforcing bars. Second failure pattern (Mode II) is called In-plane sliding failure or shear failure (Fig.2.4b). In this case overall failure of members takes place before any sign of tensile bar yielding. Despite Mode I which usually present a ductile form of failure, this mode is classified as brittle failure which in many cases a sudden drop of specimens load capacity will be observed in the vicinity of the ultimate capacity of the member. This failure mode is of interest of this study and will be explained much in detail later.

Table 2.1. Steel Properties of specimens

| Beam    | $\rho_w$<br>% | $\rho_{st}$<br>% | $f_y$<br>MPa | Ast<br>Asc       | Stirrups |
|---------|---------------|------------------|--------------|------------------|----------|
| B2      | 0.0           | 2.02             | 376          | 5D22<br><br>2D10 |          |
| B3      | 0.4           |                  |              |                  | D6@65    |
| B4      | 0.8           |                  |              |                  | D10@75   |
| B6      | 0.0           |                  |              |                  |          |
| B7      | 0.4           |                  |              |                  | D6@65    |
| B8      | 0.8           |                  |              |                  | D10@75   |
| B10-1   | 0.0           |                  |              |                  |          |
| B10-2   |               |                  |              |                  |          |
| B11     | 0.4           |                  |              |                  | D6@65    |
| B12     | 0.8           |                  |              |                  | D10@75   |
| B10.3-1 | 0.0           | 2.11             | 388          | 9D25             |          |
| B10.3-2 |               |                  | 372          | 2D16             |          |
| B13-1   | 0.0           | 2.07             | 398          | 10D32            |          |
| B13-2   |               |                  |              | 2D13             |          |
| B14     | 0.0           | 2.04             | 398          | 14D32            |          |
| B17     | 0.4           |                  |              | 4D13             | D13@100  |
| B15     | 0.0           | 1.99             | 402          | 18D35<br>2D13    |          |
| B16     | 0.0           | 2.05             | 394          | 18D41            |          |
| B18     | 0.4           |                  | 397.5        | 2D13             |          |

Table 2.2. Geometric and material Properties of specimens

| Beam    | $a/d$ | Geometry size (mm) |     |     |     |     |     |       | $f'_c$<br>MPa | $P_u$<br>KN | $P_{cr}^{sh}$<br>KN | $\delta_{peak}$<br>(mm) |
|---------|-------|--------------------|-----|-----|-----|-----|-----|-------|---------------|-------------|---------------------|-------------------------|
|         |       | $L$                | $c$ | $a$ | $d$ | $h$ | $b$ | $b_s$ |               |             |                     |                         |
| B2      | 0.5   | 700                |     | 200 |     |     |     |       | 36.2          | 1550        | 525                 | 3.16                    |
| B3      |       |                    |     |     |     |     |     |       |               | 1536        | 625                 | 4.78                    |
| B4      |       |                    |     |     |     |     |     |       |               | 1951        | 700                 | 1.85                    |
| B6      | 1.0   | 1100               | 300 | 400 | 400 | 475 | 240 | 100   | 31.3          | 1050        | 400                 | 2.77                    |
| B7      |       |                    |     |     |     |     |     |       |               | 1181        | 400                 | 2.58                    |
| B8      |       |                    |     |     |     |     |     |       |               | 1501        | 600                 | 3.26                    |
| B10-1   |       | 1500               |     | 600 |     |     |     |       | 29.2          | 616         | 325                 | 3.82                    |
| B10-2   |       |                    |     |     |     |     |     |       | 23            | 703         | 278                 | 5.28                    |
| B11     |       |                    |     |     |     |     |     |       | 29.2          | 1025        | 350                 | 15.96                   |
| B12     | 1.5   |                    |     |     |     |     |     |       | 31.3          | 1161        | 300                 | 7.05                    |
| B10.3-1 |       |                    |     |     |     |     |     |       | 37.8          | 1960        | 700                 | 6.62                    |
| B10.3-2 |       |                    |     |     |     |     |     |       | 31.15         | 1787        | 527                 | 8.62                    |
| B13-1   |       |                    |     |     |     |     |     |       | 31.63         | 2985        | 500                 | 11.87                   |
| B13-2   |       |                    |     |     |     |     |     |       | 24            | 2257        | 807                 | 9.33                    |
| B14     |       |                    |     |     |     |     |     |       | 31            | 3969        | 1100                | 9.27                    |
| B17     |       |                    |     |     |     |     |     |       | 28.7          | 5214        | 1600                | 11.92                   |
| B15     |       |                    |     |     |     |     |     |       | 27            | 5390        | 1500                | 11.91                   |
| B16     |       |                    |     |     |     |     |     |       | 27.3          | 5975        | 1900                | 10.57                   |
| B18     |       |                    |     |     |     |     |     |       | 23.5          | 8396        | 2400                | 15.79                   |

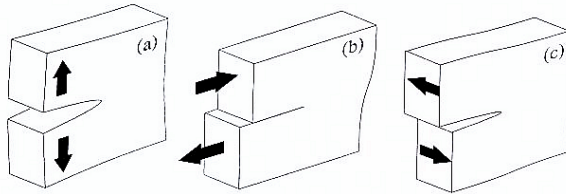


Fig.2.4. Failure modes in concrete members

Finally the third mode (Mode III) is tearing failure or out of plane sliding failure which should be modeled by three dimensional analysis ((Fig.2.4c). This mode can be observed in members subjected to torsion or out of plane loading condition.

Simulation of shear failure in concrete structures has been a long-standing key problem that for many years, tremendous attempts have been made to explain this phenomenon from mechanical as well as numerical points of views. Shear failure has been classified in several recent investigations into the following three categories; diagonal tension failure, shear compression and shear proper. In these three

types of shear failures, a diagonal tension crack usually propagates from the tensile zone to compressive zone of the beam in the shear span and eventually develops to mid-span in most cases.

**Mode II-1:** Diagonal tension failure, which in the line of thrust becomes so eccentric which in the subsequent arch mechanism is not capable of sustaining the cracking load. Diagonal tension failures generally occur in beams having large  $a/d$  ratios roughly between 3 and 6 dependent to tensile bars ratio. Beams with larger  $a/d$  ratio usually fail in flexural without the formation of the diagonal tension crack.

**Mode II-2:** Shear compression failure where RC beam fails due to the development of diagonal crack into the compressive zone and reduces the area of resisting region excessively and beam crushes once generated compressive stress exceed compressive strength of concrete. Beams failing by shear compression generally have small  $a/d$  ratios.

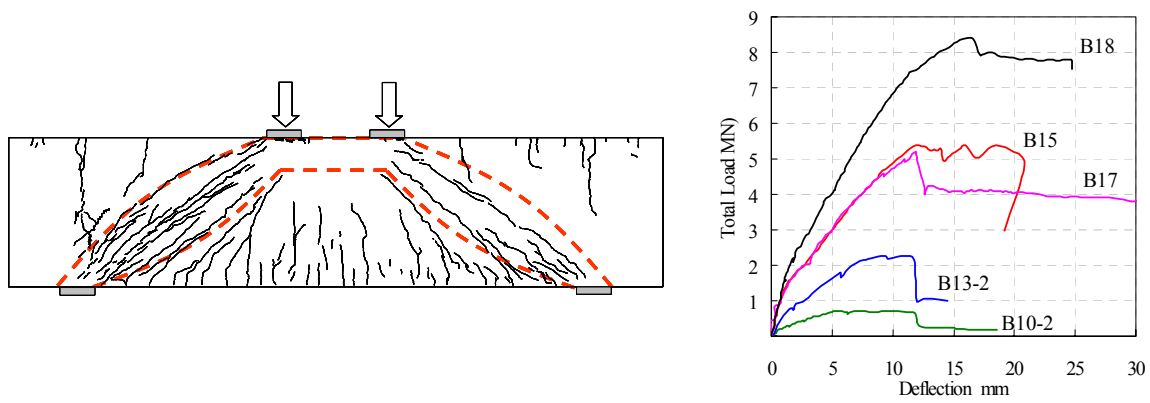


**Fig. 2.5.** Crack pattern of B18 (left) and load reserve of some specimens after the peak (right)

**Mode II-3:** Shear proper or compressive failure of struts is often observed in beams with very small shear span to depth ratio  $a/d$  (about  $a/d < 1.5$ ). In this case due to the small  $a/d$  ratio, the line of thrust will be so steep and arch action not only reserve flexural capacity in most cases but also efficiently sustains required shear force. Arch formation is clearly observed in those beams and finally beams fail due to either sudden tensile crack formation parallel to the strut axes gives rise to shearing off of the compressive zone of concrete along the line of the diagonal tension crack or compressive crush in normal direction to the strut axes. The latter case shows more reserved load after crushing (for instance Beams 13, 10.3 and B18). Figure 2.5 depicts crack pattern of B-18 at the last stage of loading where the beam failed as a result of strut compressive failure. Thrust zone is schematically shown in this figure. Despite compressive failure in strut the beam sustained almost 80% of peak load and a plateau formed after a small drop in load-displacement curve.

This phenomenon happened in some other beams such as B17 and B15, which is in contrary to what shear failure naturally implies as a sudden failure. Some results for specimens with large reserving load capacity after peak load are shown in Fig.2.5. Note however that concrete is a rate dependent material so loading rate definitely has considerable effect on RC member behaviors and should be taken into account properly in numerical simulation if experiment loading rate differs from that of constitutive models of concrete is based on. Some discussion on loading rate effect on concrete behavior is given in Chapter 5.

It is also worth to note that arch action requires a substantial horizontal reaction at the support to be formed. To satisfy this condition main bars in all specimens are well anchored with a rather long hook beyond support region. In this study only the effect of shear span's stirrups have been considered

though it is suggested to include the effect of the stirrups in mid-span on preventing diagonal crack extension to compressive zone for larger  $a/d$  ratios in further investigation.

## 2.4 Load-Deflection Responses to Four-Point Static Loading

Test results in terms of load-deflection response and crack patterns at failure stage are presented in Figs.2.6 and 2.7. Peak load for each case along with respective mid-span deflections are also noted in the figures. As can be seen some of the specimens have been tested in Kyushu Institute of Technology are subjected to a kind of cyclic type of loading which the envelope curve for better comparison are also drawn based on peak point of each cycle. The reason for doing such load processing; as mentioned by test performers, has been the danger of sudden failure while manually crack drawing on the beam's body and data collection is under process. Due to the nature of loading in those beams, the results ought to be compared by a cyclic analysis rather than monotonically loaded specimens. In cyclic form of loading it is supposed that the overall load capacity of specimens reduces. It is however to be noted here also that in first three beams with  $a/d=0.5$ , initial stiffness of member is smaller than consequent stiffness. This might be due to improper test set up at the onset of initial loading or loading plates or some other reasons which gave rise to such response. It will be shown in following section that the worst matches between experiment and analyses also belong to these three specimens. For this reason except in some limit cases, the results of these beams are omitted from further investigation.

Looking at, for instance, specimens B10-2 and B15, although beams failed in shear but the behavior after peak load is a kind of ductile behavior with a rather long plateau formed in post peak response. This might be due to continuous resistance of the formed

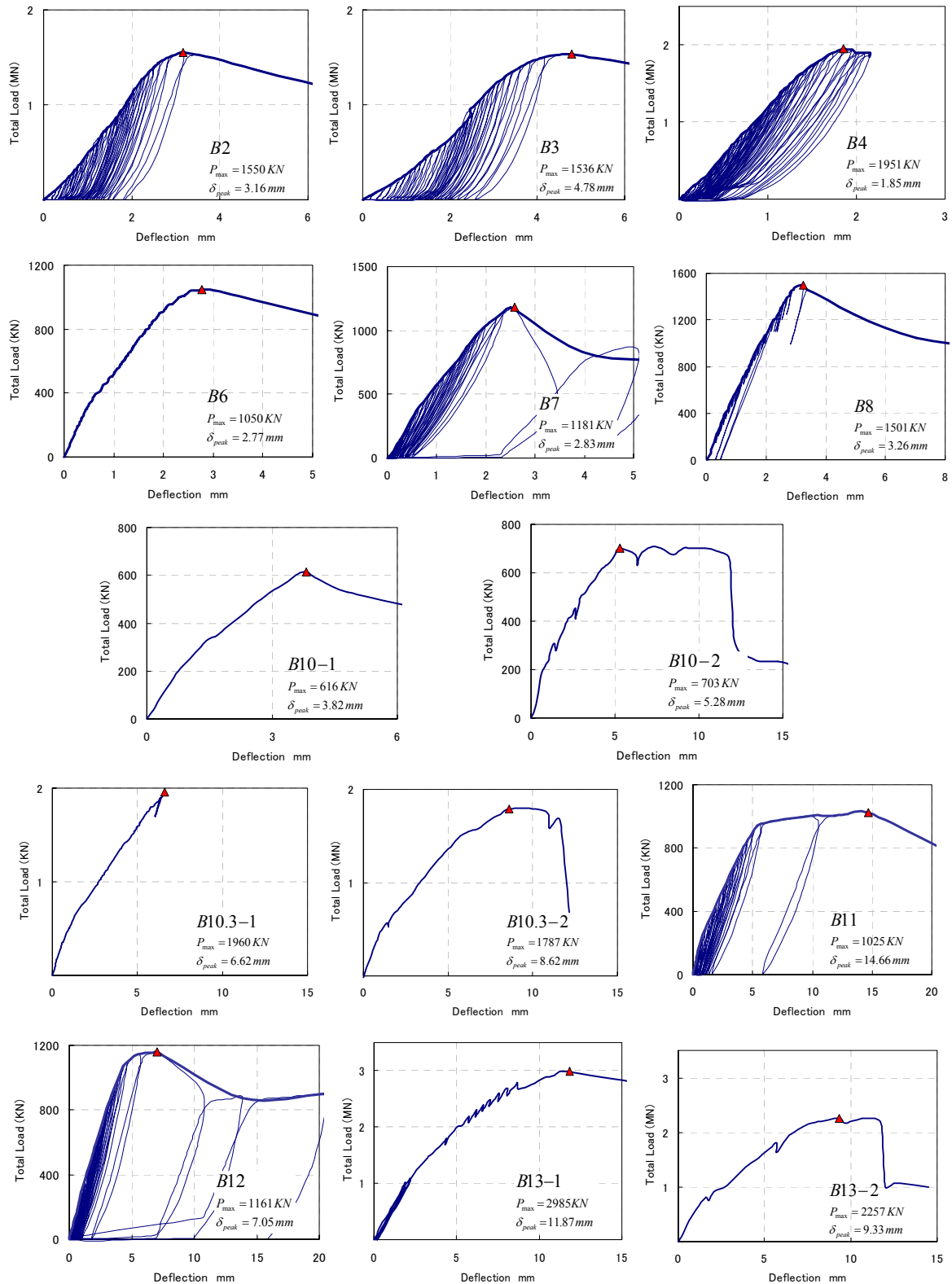


**Fig.2.6-1.** Load versus Mid-span deflection response for B2 to B13-2

arch up to compressive failure of struts. At the final stage of loading, in some parts of arch plastic hinge will be formed due to excessive compressive stress, which goes beyond compressive strength of struts and give rise to unstable resisting mechanism. Afterward the beam overall failure occur. Despite

having such failure in a number of the beams but many of them still show brittle failure. In other words, there is no criterion has yet been suggested in which load capacity of the member is maintained for larger deformation. Therefore, in order to prevent consequence of sudden drop after the peak,

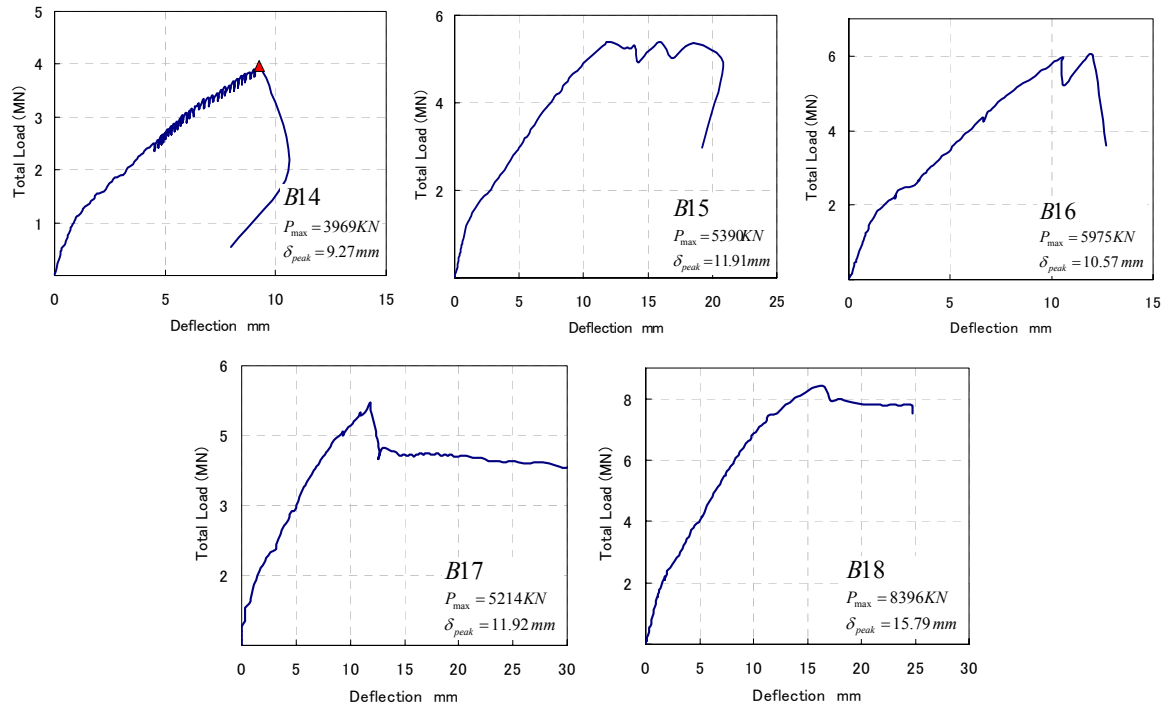


**Fig.2.6-2.** Load versus Mid-span deflection response for B14 to B18

shear failure still is categorized as non-ductile brittle failure mode.

Test results of some specimens, *e.g.* B10-2, B13-1, B14, are not so smooth in terms of load-displacement relationship but they are serrated with a number of drops in load capacity. This behavior is attributed to cracking process during loading but since the crack is not propagated, load capacity is sustained and increased again. Two types of failure mechanism have been observed in the specimens as:

- Sudden drop of load capacity after the peak is reached in a very brittle form which a large part of load capacity is lost. Specimens B10.3, B14 and B16 fall in this category.

- The behavior is not so brittle where in after the peak load, with or without a small drop, a plateau is formed or a smooth softening branch is observed before overall failure. Specimens B11, B12, B17 and B18 are in this category.

Load drop and increase process have been observed in either type of the failure in pre-peak regime in similar fashion. On the other hand, post peak regime is only formed for the second category “b” where in similar load drop and increase is repeated. This process can be either in very local form such as B17 or in more global manner such as B11 and B12. In specimen B12, an overall load deficiency is observed just after the peak due to generation of large cracks but resisting mechanism in form of strut and tie, which is also similar to the previous case, is triggered again after this softening branch and leads to small improving in load capacity by means of

increasing of the load capacity (hardening behavior). Such hardened response can be also attributed to interaction of shear reinforcement, which for this specimen is the maximum of its group, when resisting system is extensively damaged *e.g.* in B10, without stirrups, we can see only softened post-peak regime with no increase in load capacity. Due to the severe damage occurred in the preceding loading process, however, load capacity does not keep raising much, and fails after a few steps of loading. Large cracks occurred in this stage in localized form are shown in Fig.2.7-2.

## 2.5 Crack Patterns at Final Load Stage

In this section mainly crack patterns of specimens tested in PWRI are presented along with B11 and B12 as reference for the preceding section. All pictures given in Figure 2.7 show beams after failure and unloaded apparently. It is obvious however that some flexural cracks will be closed during unloading process but shear crack will remain almost same as loaded specimen. Specimen B13-1 as shown in figure has received very severe damage after the peak load which in shear span is crushed due to excessive shear stress in this part. To ensure the reliability of obtained results for this specimen, test is repeated with similar characteristic. It is however found that pre-peak regime in both specimens were acceptable while post-peak behavior in case of B13-1 was doubtful particularly the way specimens failed.

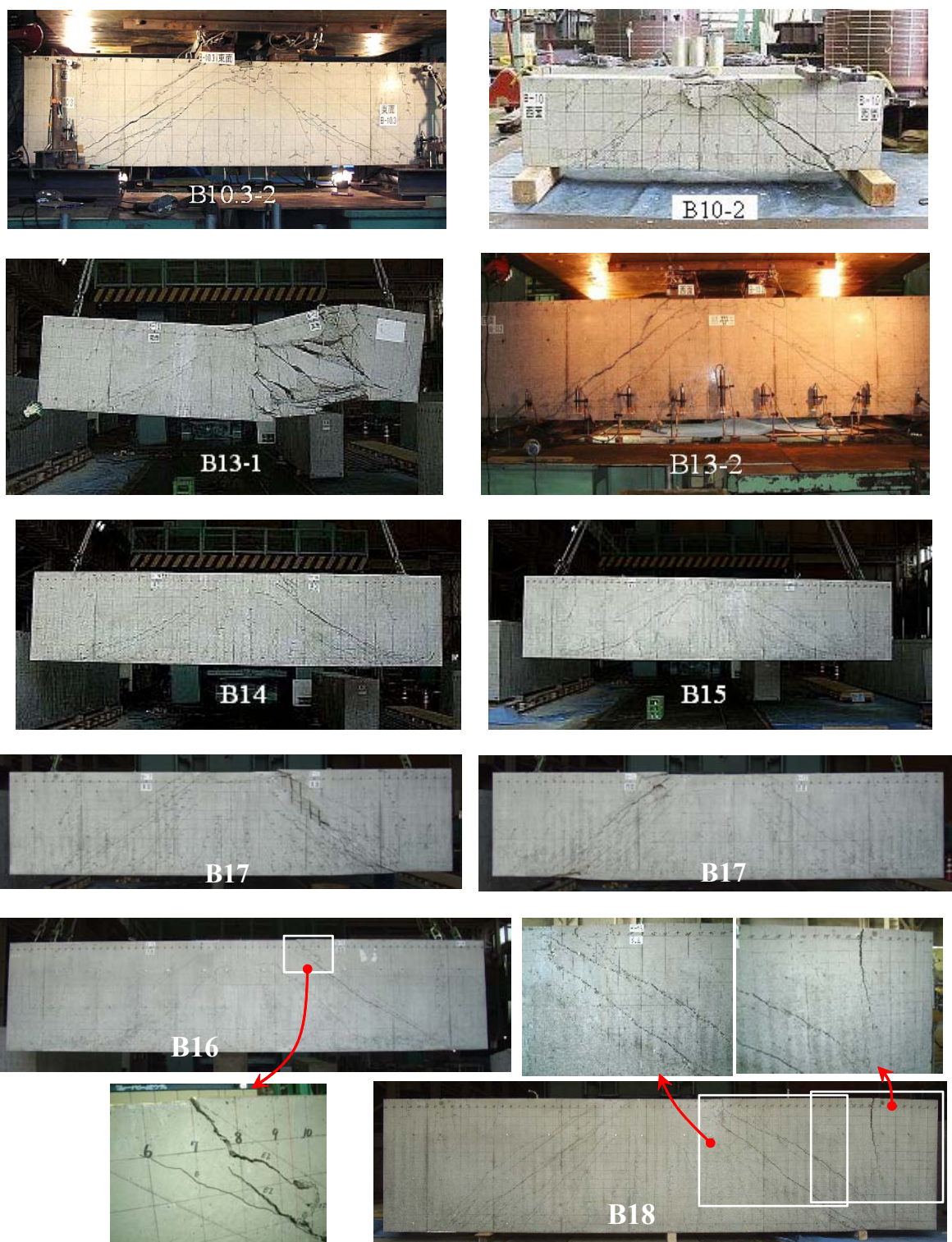


**Fig.2.7-1.** Crack patterns of beams tested in PWRI during the year 2003 and 2004

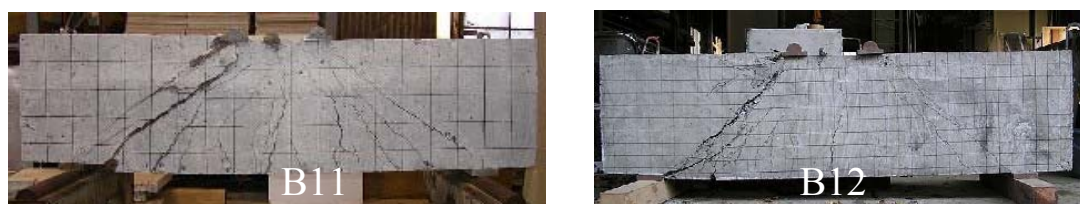


**Fig.2.7-2.** Crack patterns of two beams tested in Kyushu Institute of Technology

It is also evident that the larger the specimens are the highest the possibility of having out of balance crack (B17 and 18) in upper side of the beam above supporting plates is resulted. Finite element analysis also could capture these cracks in a certain level of load which confirms existence of such cracks in reality irrelevant to test methods and possible uncertainty of test process.

Shear failure in all beams has been observed in Mode II-2 and Mode II-3. In both cases shear sliding in main shear cracks around strut could be clearly observed with bare eyes (e.g. close up view of B16 in below right side of Fig.2.7). It is noted however that beams failed in Mode II-3 could sustain load capacity after the peak more than those failed in Mode II-2. This phenomenon might be a result of very efficient arch action as well as ductile behavior of the crushed portion of strut confined by surrounding concrete. In Mode II-2 failure crack development in compressive zone in mid-span between loading points gave rise to excessive stress in compressive concrete beyond its compressive strength. As a result the area of resisting concrete is reduced gradually and eventually stress goes beyond concrete compressive strength and gives rise to a brittle failure with a sharp drop in load carrying capacity of the beam just after the peak consequently.

## 2.6 Codified Study Based on JRA and JSCE Design Codes

Design codes JRA [2.5, 2.6] and JSCE [2.7, 2.8] are employed here to predict tested beams' load capacities. Comparisons have been made by means of shear span to depth ratio as well as effective depth size effect.

**a) JRA code:** This code present mainly tabular definitions of effective parameters on shear capacity of concrete in RC members as stated below. The parameters are determined through Part-V [2.5] as general shear strength of RC members under seismic load. The process of calculation of shear strength  $P_s$  reads:

$$P_s = S_c + S_s \quad (2.1)$$

where in  $S_c$  and  $S_s$  are concrete and steel contribution to shear strength of the member and determined through following definitions.

$$S_c = C_c C_e C_{pt} \tau_c b d \quad (2.2)$$

$$S_s = \frac{A_w \sigma_{sy} d (\sin \theta + \cos \theta)}{1.15 s} \quad (2.3)$$

Shear stress  $\tau_c$ , depth (or size) effect parameter  $C_e$  and longitudinal reinforcement effect  $C_{pt}$  are defined through tables 2.3, 2.4 and 2.5.

Influence of loading type such as repeating load and dynamic load which deteriorate shear strength of concrete is included by means of a reduction factor  $C_c$ ; (0.6 and 0.8) to shear strength of concrete.

In this study however since specimens are all subjected to monotonic loads  $C_c = 1$  is adopted.

Other parameters are  $b$  and  $d$  for beam width and effective depth,  $\sigma_{sy}$  and  $A_w$  for yield stress and total cross section area of shear steel,  $\theta$  for inclined angle of shear reinforcement and  $s$  as space between stirrups. Figures 2.8 show graphical interpretation of

Table 2.3. Definition of parameter  $C_e$  according to effective depth size

| Effective depth size (mm) | <1000 | 3000 | 5000 | >10000 |
|---------------------------|-------|------|------|--------|
| $C_e$                     | 1     | 0.7  | 0.6  | 0.5    |

Table 2.4. Definition of concrete shear strength  $\tau_c$  parameter according to  $\sigma_{ck}$

| Concrete design compressive strength $\sigma_{ck}$ ( $N/mm^2$ ) | 21   | 24   | 27   | 30   | 40   |
|-----------------------------------------------------------------|------|------|------|------|------|
| Concrete shear strength $\tau_c$ ( $N/mm^2$ )                   | 0.33 | 0.35 | 0.36 | 0.37 | 0.41 |

Table 2.5. Definition of parameter  $C_{pt}$  according to steel ratio

| Percentage of longitudinal reinforcement (%) | 0.2 | 0.3 | 0.5 | >1.0 |
|----------------------------------------------|-----|-----|-----|------|
| $C_{pt}$                                     | 0.9 | 1.0 | 1.2 | 1.5  |

Table 2.6 Definition of  $C_{dc}$  according to a/d ratio

| $a/d$    | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
|----------|-----|-----|-----|-----|-----|
| $C_{dc}$ | 6.4 | 4.0 | 2.5 | 1.6 | 1.0 |

effective parameters.

In order to take into account deep beam effect JRA (Part-IV) is employed where in deep beams are designed based on recommended process for RC foundation. This is particularly applicable here since the subject of this study is also relevant to underground structures. Based on this definition beam is classified as deep beam if shear span to depth ratio  $a/d < 2.5$ . Accordingly two new parameters are introduced as  $C_{dc} \geq 1$  (Table 2.6) for concrete part and  $C_{ds} \leq 1$  (Eq.2.5) for stirrups contribution. Experiments on very deep beams such as wall type beams evident that cracks do not cross the stirrups [2.9] therefore those members normally do not need stirrups and a minimum reinforcement of 0.2% in the form of small diameter placed in both direction, as in reinforced concrete walls, is sufficient. To this end we may say the larger the a/d ratio is, but still in deep beam range, the more effective the stirrups are in improving shear strength of the members. To utilize the effect of stirrups ratio on RC deep beams a less than one coefficient according to a/d ratio is applied to the classic shear strength estimation of lateral reinforcement e.g. shear strength based on truss theory. This is why in JRA design code assigns a reduction factor  $C_{ds}$  to shear strength produced by stirrups in normal

beams. The process of calculation of shear strength  $P_s$  therefore is reformulated as:

$$P_s = C_{dc} S_c + C_{ds} S_s \quad (2.4)$$

$$C_{ds} = \frac{1}{2.5} (a/d) \quad (2.5)$$

$$S_c = C_c C_e C_{pt} C_N \tau_c b d \quad (2.6)$$

$$S_s = \frac{A_w \sigma_{sy} d (\sin \theta + \cos \theta)}{1.15 s} \quad (2.7)$$

Shear span to depth ratio a/d is associated in  $C_{ds}$  definition. Coefficient factor to include axial force on shear capacity is indicated by  $C_N$  where in this case  $C_N = 1$  is adopted for entire calculation. Other effective parameters are similar to those defined previously.

**b) JSCE code:** Shear strength of RC members  $V_{yd}$  comprise of two terms in the absence of pre-stressing force as follow:

$$V_{yd} = V_{cd} + V_{sd} \quad (2.8)$$

where in similar to JRA definition,  $V_{cd}$  and  $V_{sd}$  are concrete and steel contribution to shear strength of the member with following definitions. Concrete contribution  $V_{cd}$  is defined by the following equations with L and h as beam's span

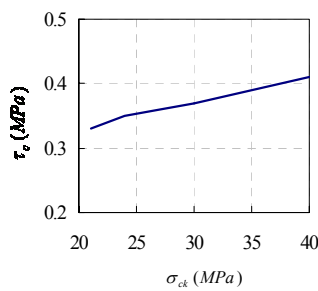


Fig.2.8.1 Shear stress  $\tau_c$  versus  $\sigma_{ck}$

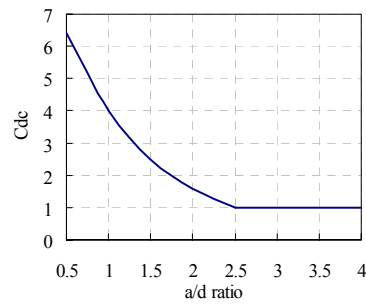


Fig.2.8.2  $C_{dc}$  variation by a/d ratio

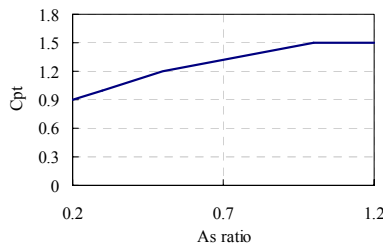


Fig.2.8.3  $C_{pt}$  variation by  $A_s$  ratio

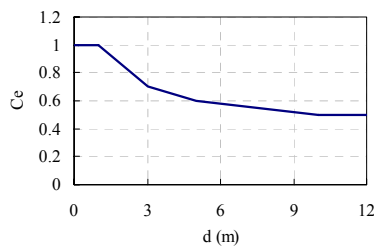


Fig.2.8.4  $C_e$  variation by effective depth  $d$

and height.

In case of  $\frac{L}{h} \geq 2$  for simple beams:

$$V_{cd} = \frac{\beta_d \beta_p \beta_n f_{vcd} b d}{\gamma_b} \quad (2.9)$$

$$\beta_d = \sqrt[4]{\frac{1000}{d}} \leq 1.5 \quad (2.10)$$

$$\beta_p = \sqrt[3]{100 \rho_s} \leq 1.5 \quad (2.11)$$

$$\rho_s = A_s / (b d) \quad (2.12)$$

$$f_{vcd} = 0.2 \sqrt[3]{f'_{cd}} \leq 0.72 \quad (N/mm^2) \quad (2.13)$$

Steel contribution  $V_{sd}$  on the other hand is very similar to JRA definition and calculated by following equation:

$$V_{sd} = \frac{A_w f_{wyd} (\sin \theta + \cos \theta) / s}{\gamma_s} . z \quad (2.14)$$

Parameters  $A_s$ ,  $f'_{cd}$ ,  $\gamma_b$ ,  $b$  and  $d$  are longitudinal tensile reinforcements area, concrete ultimate compressive strength, material uncertainty parameter which in general case will be 1.3, member web width and effective depth in critical section respectively. Since the nominal shear strength is used for comparison with experiments,  $\gamma_b = 1$  is supposed to set in all calculation. However the parameter for material uncertainty is not explicitly stated in JRA code therefore to have a meaningful comparison between two codes  $\gamma_b = 1.3$  is conducted. On the other hand experimental results

are also calibrated by the same reduction factor equal to 1.3. The value  $\beta_n = 1$  is also adopted due to the code definition for simply supported beams.

In Eq.2.14  $A_w$  and  $\theta$  are total cross section area of shear steel inclined angle of shear reinforcement,  $s$  as space between stirrups, material uncertainty factor for steel  $\gamma_s = 1.1$ , the internal lever arm  $z \approx d/1.15$  and design yield strength of shear reinforcements is  $f_{wyd} \leq 400 MPa$  for normal strength concrete.

In case of  $\frac{L}{h} < 2$  for simple deep beams:

$$V_{cd} = \frac{\beta_d \beta_p \beta_a f_{dd} b d}{\gamma_b} \quad (2.15)$$

$$f_{dd} = 0.19 \sqrt{f'_{cd}} \quad (N/mm^2) \quad (2.16)$$

$$\beta_a = \frac{5}{1 + (a_v / d)^2}, \quad a_v = a - b_s / 2 \quad (2.17)$$

and shear contribution is presented as

$$V_{sdd} = \phi V_{sd} \quad (2.18)$$

$$\phi = -0.17 + 0.3(a_v / d) + 0.33 / \rho_{wb} \leq 1.0 \quad (2.19)$$

where in  $b_s$  is support length (Fig.2.2),  $\beta_a$  and  $\rho_{wb}$  are shear span to depth ratio's effect and shear reinforcement ratio respectively. Lateral steel contribution to shear capacity for this case is denoted by  $V_{sdd}$  and is calculated by means of Eq.2.19 for any values of  $a/d$  ratio.

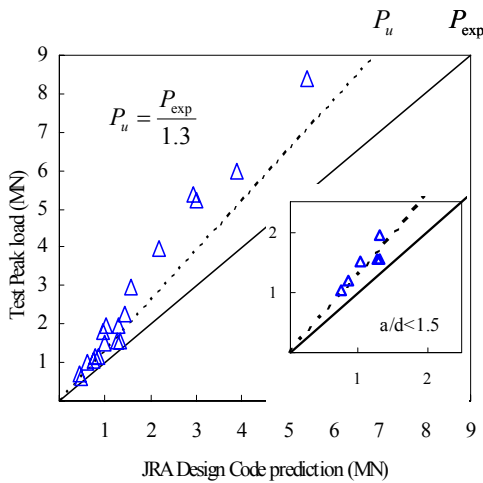


Fig. 2.9.1 JRA prediction versus test results

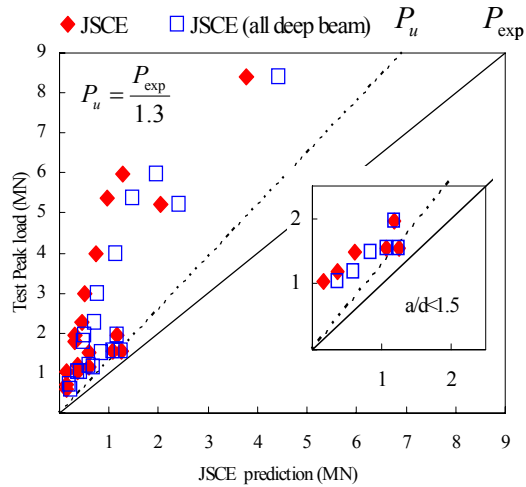


Fig. 2.9.2 JSCE prediction versus test results

### 2.6.1 Load capacity

Both codes are applied for shear load capacity calculation. The results of codified calculation and experiment are shown in figures of this section. As mentioned before, only beams with  $L/h < 2$  are considered as deep beam by JSCE design code. In JRA which recognizes beams with  $a/d < 2.5$  as deep beam, only the specimens tested in this study with  $a/d = 0.5$  are considered by JSCE code as deep beam. Nevertheless in this study, beams are designed in both cases of only following JSCE regulation as well as considering all specimens as deep beams and other words despite most design codes; including applying JSCE deep beam criteria. Figures 2.9 show ultimate loads of specimens along with those obtained by the mentioned design codes. Dotted lines in figures illustrate reduced ultimate loads by means of JSCE reduction factor  $\gamma_b = 1.3$  to calibrate test results for the sake of comparison with reduced design codes predictions. In right side of each figure, results of  $a/d < 1.5$  are represented for better comparison.

As can be seen in both figures, JRA (Part-IV) code gives much better agreement with experiment with an acceptable safety margin than that of JSCE code. JSCE code seems however is not much consistence in terms of load capacity estimation of deep beams whereby the results show a kind of scatter distribution around both solid and dotted lines. It is however observed that no data points fall significantly below either line for any of the codes. Experiment showed that shear crack initiated at about 40% of the ultimate load  $P_u$  (wherein  $P_u = P_{exp}/1.3$ ) and full shear crack will be formed approximately in  $0.5P_u$  but still beam sustained load capacity to about 80-90% of the ultimate load. Afterward shear cracks were severely widened and extended to compressive zone. Shear sliding of concrete pieces around shear crack could be clearly observed with bare eyes during this stage. This point is considered the ultimate capacity of beam in shear by a number of design codes, which the beam is in serious irreversible circumstances. Aggregate interlock, which is the backbone of current codes are almost exhausted in this stage. Figure 2.10 shows code prediction capacities in terms of above-mentioned proportions of experiment peak load.

The figure shows that JSCE code have usually estimated the load capacity of members around shear crack initiation load while JRA code yields the results near practical ultimate capacity of beams (about  $0.8P_u$ ,  $P_u = P_{exp}/1.3$ ). In other words, for beams with  $a/d > 0.5$  JRA (Part-IV) allows shear cracks occur and extend but JSCE allows only shear cracks form but not extend. It is acceptable in essence if the philosophy of JSCE code like some

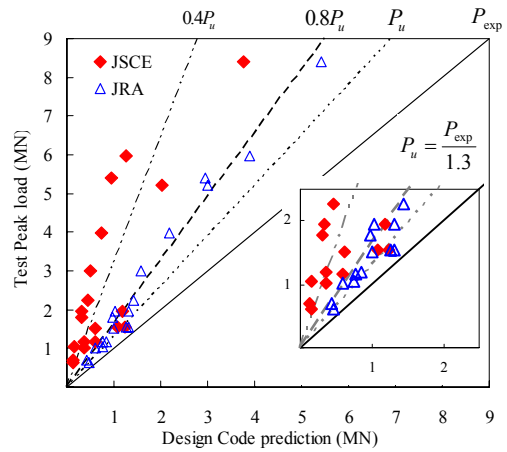


Fig.2.10 Codes ultimate load versus test

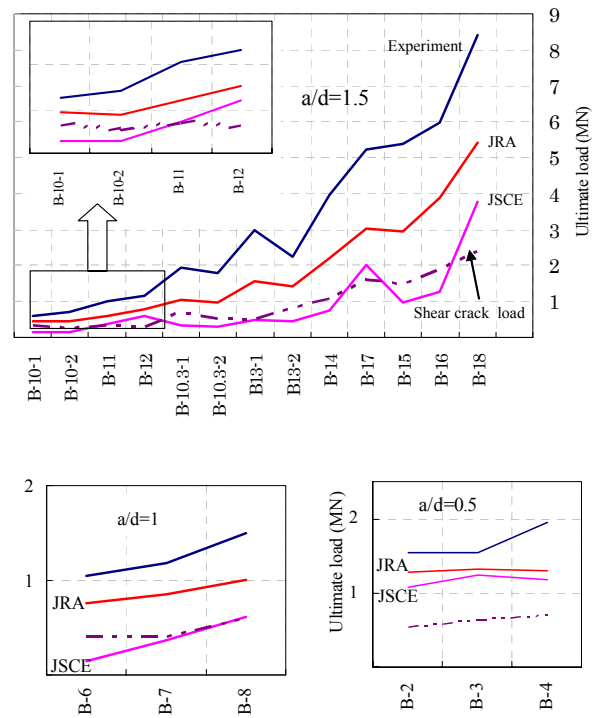


Fig.2.11 Test versus design code load (Unit=MN)

other design codes is to ensure the safety of structures before initiation of shear cracks not to reach to the ultimate load. Nevertheless the discrepancy in the results is for beams with  $a/d < 1.0$  which gave rise to a jump in predicted shear capacity of the member by JSCE and despite a big safety margin for specimens with larger  $a/d$  ratio, these beams seem to be overestimated. Figure 2.11 gives a comparison between load capacity observed in experiment and design codes. For  $a/d = 0.5$ , JSCE and JRA yield close prediction but two codes differ more as long as  $a/d$  increases. Shear crack load is also presented in this figure to clarify and follow the discussion made on figure 2.10. On the other

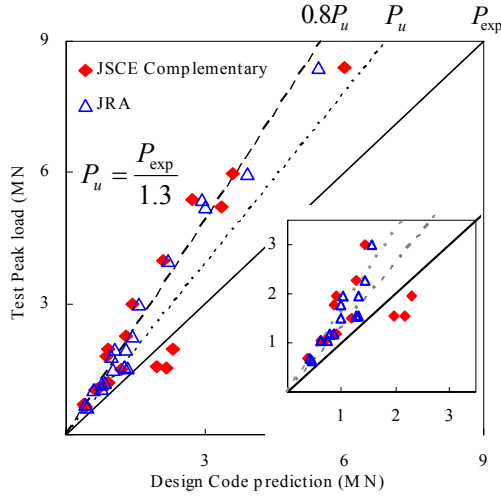


Fig.2.12 Codes ultimate load versus test

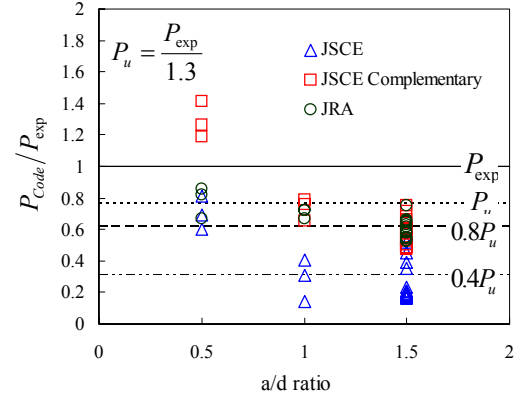


Fig.2.13  $P_{Code}/P_{exp}$  versus a/d

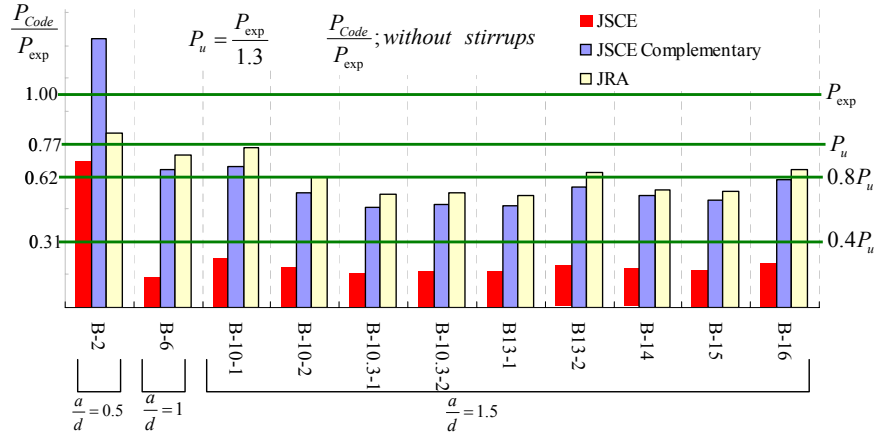


Fig.2.14.1  $P_{Code}/P_{exp}$  for specimens without stirrups

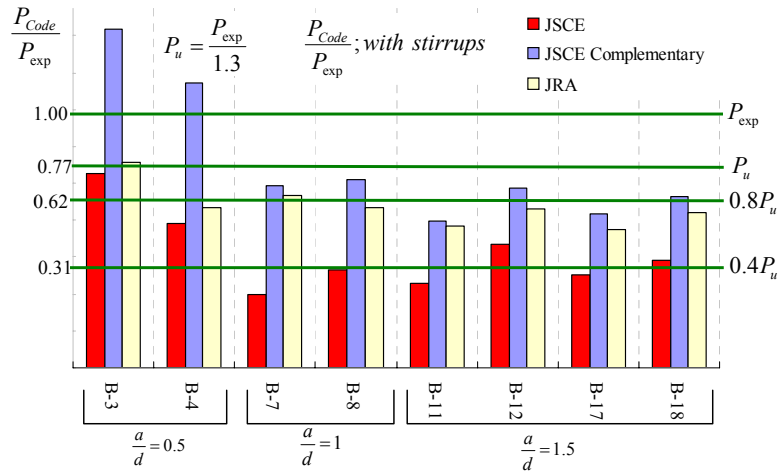


Fig.2.14.2  $P_{Code}/P_{exp}$  for specimens with stirrups

hand, complementary JSCE standard specification [2.8] presents a rigorous procedure in design of deep beams with  $a/d < 3$ . Same as design procedure for simple beams with  $\frac{L}{h} \geq 2$  (Eq.2.9 to 2.13) is

also adopted here with additional coefficient  $\beta_a$  in Eq.2.9 to include small a/d ratio effect as follow:

Table 2.7. Average prediction of codes for a/d&gt;0.5

| Design Code        | Without stirrups    |                | With stirrups       |                |
|--------------------|---------------------|----------------|---------------------|----------------|
|                    | $P_{Code}/P_{Exp.}$ | $P_{Code}/P_u$ | $P_{Code}/P_{Exp.}$ | $P_{Code}/P_u$ |
| JSCE               | 0.26                | 0.33           | 0.41                | 0.53           |
| JSCE Complementary | 0.55                | 0.71           | 0.71                | 0.92           |
| JRA                | 0.61                | 0.79           | 0.65                | 0.84           |

$$\beta_a = 0.75 + \frac{1.4}{a/d} \quad (2.20)$$

Lateral reinforcement contribution to shear capacity is calculated by means Eq.2.14, 18 and 19. Effect of  $\beta_a$  is illustrated in Figs 2.12 and 2.13. Although the results show better agreement with experiment and also close to JRA prediction but for beams with a/d=0.5 load capacity goes beyond experiment load even with no reduction factor. More clear comparisons between code and test results are presented in Figs.2.14. Disregarding first three specimens with a/d=0.5, the average results of code prediction for sixteen specimens are presented in Table 2.7. As can be seen in this table, specimens without stirrups have lower load capacity predicted by either code in general since the possibility of sudden failure is higher. As mentioned before despite JSCE with conservative prediction (0.53 in terms of  $P_u$ ) the other ones have closer results to prescribed ultimate load  $P_u$ . It is however clearly seen here as well that JRA has the best and the most consistence prediction almost in the vicinity of  $P_u$  with the least deviation. The average JRA prediction is 0.79 and 0.84 for beams without and with stirrups respectively. They are both in very reasonable range with acceptable safety margins for an economic design procedure.

### 2.6.2 Size effect study

In order to study size effect on shear capacity of beams with low a/d ratio, both codes are examined and verified with experimental results. Test specimens cover a wide range of effective depth from 400mm to 1400mm. Variation of average shear stress taking into account concrete compressive strength ( $V_u / b.d.\sqrt[3]{f'_c}$ ) in terms of effective depth is shown in Fig.2.15. To eliminate a/d effect on ultimate shear stress of the beams, only a/d=1.5 is considered here.

According to size effect investigation and theories it is evident that as the effective depth increases, the shear strength of the section decreases. The regression line is assumed to be a power function of effective depth  $d$  in order to adjust to the size effect

function proposed by JSCE and JRA. The equation is round off and rewritten in the following form

$$f(v_u) = \lambda (d^{-0.22}) \quad (2.21)$$

where in coefficient  $\lambda$  is a function of a/d ratio, reinforcement ratio and member's boundary condition. Since the three aforementioned parameters are constant for the beams used for producing Fig.2.16 consequently  $\lambda = 4.77$  is determined to best fit to the experiment data points. According to JSCE, shear stress varies in terms of  $d^{\frac{1}{4}}$  given in Eq.2.10. On the other hand JRA proposed procedure can be estimated by a function of  $d^{\frac{1}{3}}$  to take into account size of specimen. Although the foregoing expression of Fig.2.15 is a crude approximation of size effect it agrees well with that of Eq.2.16 proposed by JSCE code. There

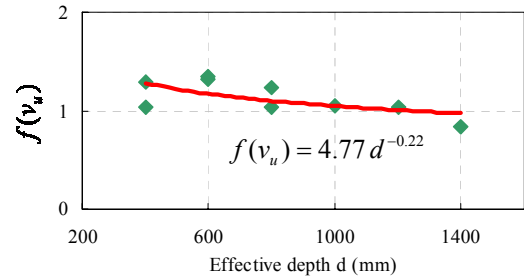


Fig.2.15 Effective depth versus shear function

$$f(v_u) = V_u / b.d.\sqrt[3]{f'_c} \text{ for } A_v=0 \text{ and } a/d=1.5$$

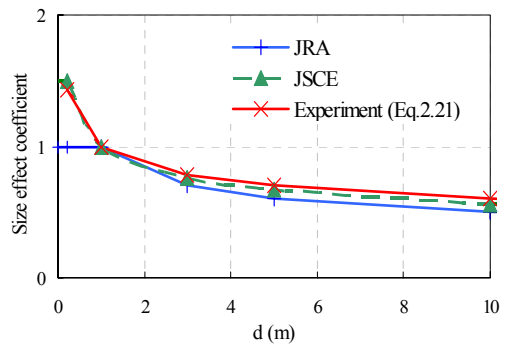


Fig.2.16 Effective depth versus size effect coefficient

is however not a significant differences between JSCE and JRA size effect expression as can be seen in Fig. 2.13 and both expressions are attributed to a reasonable estimation of member depth effect. Size effect coefficient is increase or decrease of shear strength in terms of effective depth. The maximum values for this coefficient set 1.0 and 1.5 by JRA and JSCE code respectively. In other words, in spite of JSCE code, which attributes 50% increase in shear strength capacity to size effect, JRA however does not allow any increase in shear strength for smaller effective depth. One reason for this might be the fact that JRA is usually dealing with structures with large components most of them larger than one-meter depth but JSCE should cover wider range of element size since it is to design various structures too. It is however noted that JRA also accept 40% higher shear strength for members with smaller depth ( $d \leq 300\text{ mm}$ ) where in linear design concept is applied (Part-IV, p.148) whereas in capacity design procedure no additional strength is attributed for  $d \leq 1000\text{ mm}$  as mentioned above.

## 2.7 Conclusion

A comparative study between experiment, JSCE and JRA design codes has been carried out by means of ultimate loads, shear crack loads as well as size effect issues proposed by either code. It is found that JRA code has a consistence design procedure for RC beams with low shear span to depth ratio. This code assumes beams with  $a/d < 2.5$  as deep beams while JSCE has larger limit for deep beam as  $L/h < 2$  where  $L$  and  $h$  are beam's span and height respectively (about twice bigger than JRA limit). Therefore only beams with  $a/d = 0.5$  of this experiment are designated by JSCE to follow the deep beam design procedure. Since deep beams usually have higher shear strength due to the resisting mechanism of such beams against external loads by means of compressive arch formation, there will be a discrepancy between experimental observation and estimated strength by JSCE code. Estimated shear load capacity by JSCE is around shear crack load of experiments while JRA code allows shear cracks form and extend to a certain level with higher load capacity prediction. In this sense it can be concluded that JSCE design code yields much conservative results than that of JRA except for very small  $a/d$  ratio say 0.5 where JSCE amplifies the predicted shear strength by means of a function of  $a/d$  ratio (Eq.2.17). It is noted however that no code data points go beyond experiment ultimate loads. Application of Complementary JSCE standard specification is also examined for tested beams. Despite better agreement with experiment

and closer results to JRA prediction, data points for  $a/d = 0.5$  drop below the line (Fig.2.12) or in other words, predicted load capacity goes beyond obtained load by experiment.

Concerning size effect on shear strength of RC beams with low  $a/d$  ratio, experimentally observed size effect by means of effective depth variation confirmed that both code have included this phenomenon in design procedure adequately thought JSCE equation has better agreement with experiment. The main difference between the codes lies on the beams with depth smaller than 1000mm which JRA limits the coefficient to one but JSCE goes as far as 1.5 and attributes shear strength to the size effect up to 50% higher in members with depth less than one meter. Although JRA code does not allow any increases in shear strength of member for capacity design but if linear concept is applied in design up to 40% higher shear strength for members with smaller depth ( $d \leq 300\text{ mm}$ ) is acceptable.

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### 3. MATERIAL MODELS AND SOLUTION METHODS OF FINITE ELEMENT ANALYSIS

In order to conduct parametric study on a larger number of RC beams with different geometry, reinforcement and material parameters, experimental works on such large number of specimens if not impossible but it is very much time and expense consuming task. In this regard to eliminate such hassle, a nonlinear finite element analysis by means of a general FE code (DIANA 8.1.2) is applied here with material models explained hence after. The constitutive behavior of concrete is represented by a smeared crack model, which in the damaged material is still continuum. Analytical scheme and finite element mesh discretization is presented in this section. For specimens with stirrups in shear span, lateral reinforcement will be extended to entire length of the member. According to concrete crack model, two approaches can be highlighted as the fixed crack and the rotating crack models.

In the fixed crack model, once crack initiates in a finite element, the crack direction is calculated according to the principal stress direction. The crack direction is kept constant during further load increments and considered as the material axis of orthotropy. As a general case, principal stress directions need not to be coincide with axes of orthotropy and can rotate during loading process. This assumption produces a shear stress in crack surface. In order to prevent the effects of this artificially existed shear stress in the analysis, a shear retention factor as a reduction coefficient is always applied in this model. This factor can be either of a constant coefficient or varies during analysis as a function of crack width. Complication of this model manifests itself in definition of this parameter particularly when a constant value is assigned for entire analysis procedure. It is worth to mention here that despite flexural failure with minor affect of this parameter [3.1], analytical prediction of shear failure is significantly affected by this factor.

Alternatively, the rotating crack model is presented where in the direction of the principal stress coincides to the direction of the principal strain. Since crack direction rotates according to the principal stress direction, no shear stress is generated on the crack surface and just two principal components need to be defined. However to prevent consequent effect of shear retention factor definition in analysis, only the rotating crack approach is adopted in the present study. In the analyses, perfect bond is assumed for both

smeared reinforcements and embedded bar reinforcements.

Displacement control with Newton-Raphson solution technique is adopted here to solve equilibrium equations along with Arc-length method to investigate possibility of snap-back instabilities, which sometimes occurs in shear failure analysis. Specimens are partly modeled by FEM due to the symmetric geometry and subjected to the proportional monotonic loads with 2D elements in plane-stress condition. According to constitutive model of concrete in tension based on fracture energy concept, it is supposed that softening branch of cracked concrete is sufficient to explain tension-stiffening phenomenon therefore no additional stress due to this phenomenon is included in the calculation.

#### 3.1 Material Models for Reinforcing Steels

Steel reinforcements are modeled as an elastic perfect plastic material with no hardening after yield point. This model is shown in Fig.3.1. This is basically due to the material test results which showed almost no hardening behavior for reinforcement after the yield point. It is noted that steel plates at supports and loading points are assumed to be isotropic elastic material in any stress condition.

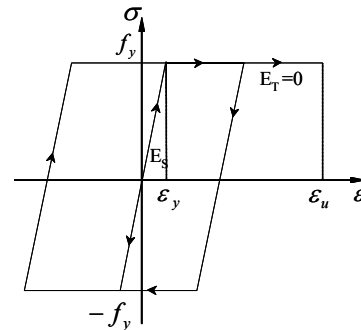


Fig.3.1. Reinforcement model

#### 3.2 Material Models for Concrete

Concrete constitutive models are assumed in a fracture type material framework with a characteristic length parameter to eliminate mesh size effect. Consequently the fracture energy in either tension or compression is set constant for a certain material as a function of material properties rather than specimen's geometry. This assumption accomplishes a mesh objective analysis particularly by taking into account energy released in fracture process irrelevant to the mesh discretization. The foregoing length parameter  $h$  is a function of element size and estimated by  $\sqrt{A}$  where in  $A$  is element area.

### 3.2.1 Concrete in tension

Two main models are applied here for the sake of comparison between the results. The first model is JSCE model and the other one is well-known exponential model which was derived experimentally by Hordijk [3.2]. In both models concrete in tension before cracking is assumed to be linear elastic. The models attributed concrete as fracture type material where fracture energy is treated as material property rather than geometric parameter.

#### a) JSCE constitutive model

The model presented by this code is intended to use for more practical purpose by design engineers. Therefore the model has received much simplification in terms of post crack regime defined by a bi-linear path. The model is so determined that again fracture energy as a constant value play a key role in the cracked concrete behavior. As a consequent to this basic assumption, tensile model will be therefore switched from the traditional form of stress-strain relationship (Fig.3.2a) to that of shown in Fig.3.2b as a function of released fracture energy. Length parameter  $h$  is cooperated in this model indirectly where average strain of crack is calculated. After some algebra the following equations can be drawn:

$$\varepsilon_0 = \frac{f_t}{E_c} \quad (3.1)$$

$$\varepsilon_1 = \varepsilon_0 + \frac{0.75 G_F}{h f_t} \quad (3.2)$$

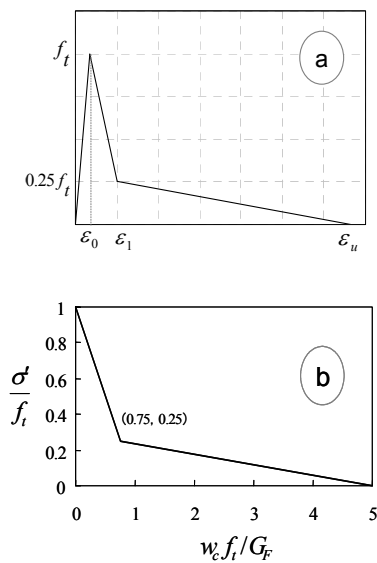


Fig.3.2. JSCE tensile (a) and fracture model (b) for concrete

$$\varepsilon_u = \varepsilon_0 + \frac{5 G_F}{h f_t} \quad (3.3)$$

Tensile strength of concrete (for those specimens with no test results) and also tensile fracture energy  $G_f$  are calculated by means of, JCSE recommendations (Eq.3.4 and 3.5).

$$f_t = 0.23 f_c'^{2/3} \quad (MPa) \quad (3.4)$$

$$G_f = 10 (d_{max})^{1/3} \cdot f_c'^{1/3} \quad (3.5)$$

where  $d_{max}$  is maximum aggregate size in mm (20mm here),  $f_c'$  is concrete compressive strength in MPa and  $G_F$  is fracture energy in N/m.

#### b) Hordijk constitutive model

In this empirical model, concrete after tensile peak stress follows an exponential softening path, is illustrated in Fig.3.3 and determined by means of the following equation.

$$\frac{\sigma}{f_t} = \left\{ 1 + \left( c_1 \frac{w}{w_c} \right)^3 \right\} \exp \left( -c_2 \frac{w}{w_c} \right) - \frac{w}{w_c} (1 + c_1^3) \exp(-c_2) \quad (3.6)$$

where  $w$  is the crack opening;  $w_c$  is the crack opening at the complete release of stress, which is a function of Mode I fracture energy  $G_f^I$  and defined by Equation 3.7,  $\sigma$  is the normal stress in crack and  $f_t$  is the tensile strength of concrete in one dimension system or effective tensile strength in two dimension system. Values of the constants are  $c_1=3$  and  $c_2=6.93$  and

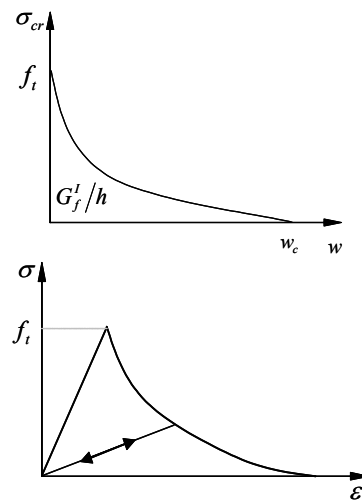


Fig.3.3. Hordijk tensile and fracture model for concrete

$$w_c = 5.14 \frac{G_f'}{f_t} \quad (3.7)$$

Tensile strength as well as fracture energy as a function of concrete maximum aggregate size and compressive strength is also defined through aforementioned Eqs. 3.4 and 3.5 respectively.

### 3.2.2 Concrete in compression

Concrete in compression is also modeled through parabolic relationship between stress and strain. First model is suggested by JSCE without association of fracture energy of concrete in compression. Second model, modified version of parabolic model represented by Feenstra [3.3], takes into account fracture energy of concrete. Both models are explained hereafter. Furthermore, the concept of Modified Compression Field Theory [3.4] is associated in analyses by means of concrete compressive strength softening due to the lateral tensile cracks (Fig.3.4). The adopted formulations presented below [3.5].

$$\sigma'_c = \beta f'_c \quad (3.8)$$

$$\beta = \frac{1}{1+K_c} \leq 1 \quad (3.9)$$

$$K_c = 0.27 \left( \frac{\varepsilon_1}{\varepsilon_0} - 0.37 \right) \quad (3.10)$$

where  $\sigma'_c$  and  $\beta$  are reduced concrete compressive strength and reduction factor respectively. Lateral strain  $\varepsilon_1$  involves in Eq.3.10 which as a result larger lateral cracks produce smaller compressive strength.

#### a) JSCE parabolic model

Before the peak concrete follows a parabola formulated in Eq 3.13 while after the peak a plateau is formed upon reaching to the ultimate strain denoted by a value less than 0.0035. Once the ultimate strain is reached stress will suddenly drops to zero. The model is illustrated in Fig.3.5 and expressed in the following formulas.

$$k_1 = 1 - 0.003 f'_c \leq 0.85 \quad (3.11)$$

$$\varepsilon'_{cu} = \frac{155 - f'_c}{30000} \leq 0.0035 \quad (3.12)$$

$$\sigma_c = k_1 f'_c \frac{\varepsilon'_c}{0.002} \left( 2 - \frac{\varepsilon'_c}{0.002} \right) \quad (3.13)$$

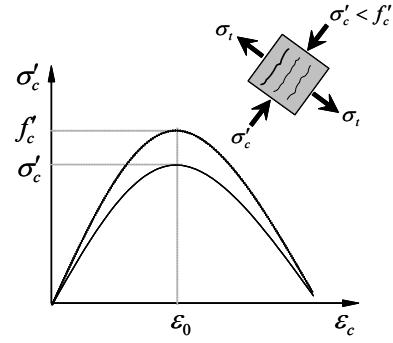


Fig.3.4. Lateral crack effect on compressive strength reduction

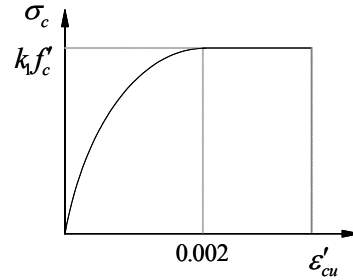


Fig.3.5. JSCE compressive model for concrete

It is however noted that in this model fracture energy of compressive concrete is not taken into consideration. Reduction factor  $k_1$  is set to one in all analyses despite code recommendation in Eq.3.8. This is due to the fact that concrete in experiment condition is quite well quality controlled therefore additional safety factor which is mainly for construction sites seems not necessary.

#### b) Feenstra fracture model

This model is a modified version of parabolic model to take into account fracture energy of concrete in compression. Concrete in this model is assumed to be elastic up to  $\frac{1}{3} f'_c$  which deformation

can be totally exhausted after unloading. Permanent deformation will be formed after this point and ultimate strain as a function of fracture energy and corresponding to the characteristic length parameter can be calculated. Figure 3.6 along with the following equations are representing this model, which in equivalent stress is determined in terms of equivalent strain. ,

$$\bar{\sigma} = \begin{cases} \frac{f'_c}{3} \left( 1 + 4 \frac{\bar{\varepsilon}}{\varepsilon_e} - 2 \frac{\bar{\varepsilon}^2}{\varepsilon_e^2} \right) & \text{if } \bar{\varepsilon} < \varepsilon_e \\ f'_c \left( 1 - \left( \frac{\bar{\varepsilon} - \varepsilon_e}{\varepsilon_u - \varepsilon_e} \right)^2 \right) & \text{if } \varepsilon_e \leq \bar{\varepsilon} < \varepsilon_u \end{cases} \quad (3.14)$$

$$\varepsilon_e = \frac{4}{3} \frac{f'_c}{E_c} \quad (3.15)$$

Consequent to the length parameter association, ultimate strain will be a function of compressive fracture energy, length parameter  $h$ ,  $f'_c$  and also  $\varepsilon_e$  as below.

$$\varepsilon_u = 1.5 \frac{G_c}{h f'_c} - \frac{11}{48} \varepsilon_e \quad (3.16)$$

Since in this model concrete is assumed to be linear elastic up to  $\frac{1}{3} f'_c$  therefore pre-peak energy will be taken into account by a correction factor  $\frac{11}{48} \varepsilon_e$  in equation 3.16. Constant value for  $G_c=50$  N/mm is adopted in analyses throughout.

In this model, stress in concrete follows a softening path to zero stress while in JSCE model, stress will suddenly drop to zero as soon as concrete strain reaches to  $\varepsilon'_{cu}$ . This will make a comb-like shape for force deflection curve by using JSCE model if possibility of concrete compressive failure is fairly high.

### 3.3 Cracking in RC Elements in Plane Stress Condition; Background Theories

In this study concrete treated as continuum material which in crack deformation is averaged or spread out over element area. This is the fundamental assumption for cracked concrete constitutive model formulation. In other word cracks and related reinforcement (except those which are treated as bar elements) are idealized such that being distributed or smeared out over the entire element body. Nonlinearity in such case is a result of concrete cracking, reinforcement plasticity or bond interaction between concrete and steel. Other types of nonlinearity such as concrete nonlinear behavior in compression, interface effect between supporting plate and concrete by means of friction or separation as well as shrinkage and creep phenomena can be added to this list but they are in the second priority of analysis though depend on the given problem any of those phenomenon can be taken into consideration.

Since the cracked concrete is an orthotropic material, axes of orthotropy ( $i$  and  $j$ ) are parallel or perpendicular to the induced smeared crack direction. Depend on the applied analytical method, axes of orthotropy may be fixed or rotated during analysis procedure.

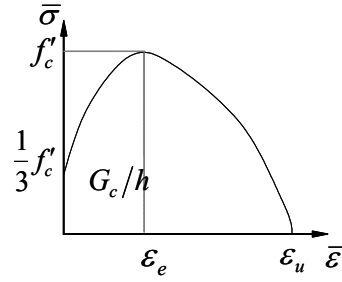


Fig.3.6. Feenstra compression model for concrete

#### 3.3.1 RC constitutive model before cracking

Reinforced concrete members are modeled with two dimensional finite elements in plane-stress condition. Before cracking concrete however is assumed to be linear elastic where element stiffness matrix can be defined through the following equations.

$$[D_c] = \frac{E_c}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (3.17)$$

where  $E_c$  and  $\nu$  are concrete elastic modulus and poisson ratio respectively.

Stiffness matrix of steel reinforcement as a one dimensional bar element is locally formulated in Eq.3.18. In order to establish RC element stiffness matrix, all bar elements are transferred to global direction with a transformation matrix  $T_s$ .

$$[D_s]_i = \begin{bmatrix} \rho_i E_{si} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.18)$$

$$[T_s] = \begin{bmatrix} \cos^2 \alpha & \sin^2 \alpha & \sin \alpha \cos \alpha \\ \sin^2 \alpha & \cos^2 \alpha & -\sin \alpha \cos \alpha \\ -2 \sin \alpha \cos \alpha & 2 \sin \alpha \cos \alpha & (\cos^2 \alpha - \sin^2 \alpha) \end{bmatrix} \quad (3.19)$$

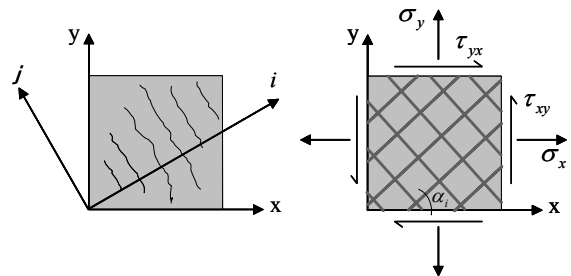


Fig.3.7 Cracked concrete axes and RC element in plane stress condition

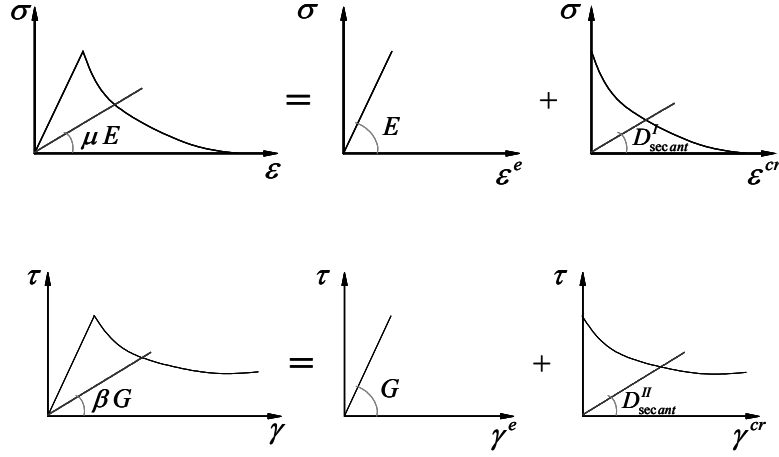


Fig.3.8. Relation between traditional and secant crack parameters

$$[D_S] = [T_s]^T [D_S]_i [T_s]_i \quad (3.20)$$

and eventually RC element stiffness matrix is derived through Eq.3.21 below.

$$[D_{RC}] = [D_C] + \sum_{i=1}^n [D_S]_i \quad (3.21)$$

Equation 3.21 associates in stiffness matrix assemblage of RC structures in finite element formulation. The stiffness matrix resulted from Eq.3.21 comprises both linear and non-linear components in each loading increment and treated as a composite material model.

### 3.3.2 RC constitutive model after cracking

Once concrete cracked, concrete element stiffness normal to the crack will be decreased drastically and element enters to nonlinear state in terms of a number of parameters. According to the assumed cracked concrete constitutive model, stiffness matrix will be defined for further load steps. Two main approaches called the fixed crack model and the rotating crack model are explained and used in this study. In either case however secant stiffness matrix is applied where in case of crack unloading, secant stiffness remains constant. For such assumption after complete unloading of a crack, no residual strain will remain. This means that upon crack closing, both normal stress and normal strain of the crack vanish. The concept of both methods in incremental form can be expressed by following equations.

$$\{\sigma\} = [D_{RC}] \{\varepsilon\} \quad (3.22)$$

Due to the applied constitutive equation we have

$$\{\dot{\sigma}\} = [\dot{D}_{RC}] \{\dot{\varepsilon}\} \quad (3.23)$$

where consequent stress and strain update is simply equate as

$$\{\sigma\}_{k+1} = \{\sigma\}_k + \{\dot{\sigma}\}_k \quad (3.24)$$

$$\{\varepsilon\}_{k+1} = \{\varepsilon\}_k + \{\dot{\varepsilon}\}_k \quad (3.25)$$

Since the crack is already initiated, concrete stiffness matrix should be transferred to the global direction through a transformation matrix similar to Eq.3.19 which in  $\alpha = \theta_{cr}$ . Matrices  $\dot{D}_S$  and  $\dot{D}_C$  are calculated based on applied constitutive models for either materials.

$$[\dot{D}_C] = [T_C]^T [\dot{D}_C] [T_C] \quad (3.26)$$

$$[\dot{D}_{RC}] = [\dot{D}_C] + \sum_{i=1}^n [\dot{D}_S]_i \quad (3.27)$$

Transformation matrix  $[T_C]$  for  $k+1$  increment is based on crack direction  $\theta$  of  $k$  increment which differs for each method.

It is noted however that in DIANA code, secant crack stiffness is adopted with similar concept explained above. The basic relationship between cracked concrete stress and strain reads;

$$\begin{Bmatrix} \sigma_{nn}^{cr} \\ \tau_{nt}^{cr} \end{Bmatrix} = \begin{bmatrix} D_{secant}^I & 0 \\ 0 & D_{secant}^{II} \end{bmatrix} \begin{Bmatrix} \varepsilon_{nn}^{cr} \\ \gamma_{nt}^{cr} \end{Bmatrix} \quad (3.28)$$

where  $\sigma_{nn}^{cr}$ ,  $\tau_{nt}^{cr}$ ,  $\varepsilon_{nn}^{cr}$  and  $\gamma_{nt}^{cr}$  are stress normal and tangent to crack with corresponding strain respectively. Secant stiffness components for normal and shear are represented by  $D_{secant}^I$  and  $D_{secant}^{II}$  by means of  $E_C$  and  $G_C$  as

concrete Young and shear modulus and parameters  $\mu$  and  $\beta$  as reduction factors depicted in Fig.3.8.

**a) Fixed Crack Approach**

In the fixed crack model, once crack initiates in a finite element, the crack direction is calculated according to the principal stress direction. The crack direction is kept constant during further load increments and considered as the material axis of orthotropy. As a general case, principal stress directions need not to be coincide with axes of orthotropy and can rotate during loading process. This assumption produces a shear stress in crack surface. In order to prevent the effects of this artificially existed shear stress in the analysis, a shear retention factor ( $\beta$  in Fig.3.8) as a reduction coefficient is always applied in this model. This factor can be either of a constant coefficient or varies during analysis as a function of crack width. Complication of this model manifests itself in definition of this parameter particularly when a constant value is assigned for entire analytical procedure. As stated before in spite of flexural failure with minor affect of this parameter, analytical prediction of shear failure is significantly affected by this factor.

In the fixed crack model both tangent stiffness and also total load procedure by means of secant stiffness are applicable. Calculation procedure by this method can be summarized as follow:

- i) Once concrete tensile strength is violated, find crack direction by means of principal stress direction in tension. This direction is fixed in whole analytical procedure.
- ii) Determine shear retention factor  $\beta$  (function of crack strain or a constant value)
- iii) Calculate  $\dot{D}_s$  and  $\dot{D}_c$  based on applied constitutive models in their local coordinates.
- iv) Transfer all calculated matrices to the global coordinate system.
- v) Construct  $\dot{D}_{RC}$  for composite element.
- vi) Calculate corresponding incremental strains for the given incremental stress.
- vii) Compute total stress and strain of the member.

It is obvious that in any steps steel and concrete stress and strain state will be checked by adopted constitutive models. Aforementioned calculation steps represent the most basic concept of this method. It is however possible to include more complex phenomenon such as multi cracks, concept of active crack and also coupling transfer

of normal stress and shear (known as shear dilatancy) by introducing related components in stiffness matrix. In this method since crack direction is fixed and all calculation of stress and strain is based on this direction, the history of analysis is kept for each steps. This is strong advantage of this method over rotating crack approach by treating concrete crack as a perfect path-dependent procedure.

**b) Rotating Crack Approach**

Alternatively, rotating crack model is presented where the direction of the principal stress coincides to the direction of the principal strain. Since crack direction rotates according to the principal stress direction, no shear stress is generated on the crack surface and just two principal components need to be defined. Due to this coaxiality assumption, rotating crack approach does not explicitly treat shear slip or shear transfer due to aggregate interlock. On the other hand this assumption greatly simplifies computation and reasonably accurate under both monotonic and cyclic loading paths where principal stress does not rotate so much [3.5].

This method represents crack direction in each moment in principal strain direction and keeps no history of already induced cracks in the previous load steps. In other words since in every step of analysis a new crack is generated due to the current principal strain directions, cracks in preceding step will be erased from the memory. Therefore the concept of active crack as well as multi direction cracks is not applicable in this method. Calculation procedure by this method is similar to that of fixed crack approach while no shear retention factor is necessary to be defined. Furthermore stiffness matrix of concrete will be calculated in current crack direction and transferred to the global coordinate system. Crack direction is updated in each step and aligned with principal stress direction. In case of plane stress condition under normal and shear stresses, principal strains of concrete are calculated with traditional equations below. All strain components used here are those of concrete element.

$$\varepsilon_{P_1, P_2} = \frac{(\varepsilon_x + \varepsilon_y)}{2} \pm \frac{1}{2} [(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2]^{1/2} \quad (3.29)$$

and corresponding local coordinate system direction  $\theta_{cr}$ ;

$$\theta_{cr} = \frac{1}{2} \tan^{-1} \left[ \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \right] \quad (3.30)$$

The so-called Modified Compression Field Theory which is one of the best available RC models is based on this concept. It is noted in reference [3.6] that in actuality, the angle of inclination of the concrete struts at failure usually lies between calculated angles through the fixed crack model and the rotating crack model. Therefore, these two theories furnish the two boundaries for the true situation. It is however found in this study rotating crack model gives rise to unstable crack direction while in the fixed crack approach due to its inherent nature, crack is stabilized during entire analysis. This might be in contrary to that of reference [3.6] since the crack directions though both theories sometimes show quite large discrepancy.

### 3.4 Solution Algorithm

Despite linear Finite Element Analysis which in relation between force vector and displacement vector can be explicitly expressed by a linear equation, displacements at current stage often depend on the displacements at earlier stages. To achieve equilibrium between internal and external forces in nonlinear equation, an *incremental-iterative* solution procedure should be preformed since the solution vector could not be calculated explicitly as we have in linear equilibrium. The nonlinearity can be a result of nonlinear elasticity, plasticity and path-dependent analysis which in the displacement is depend on load history. To this list however other type of nonlinearity such as creep and large displacement problems can be added.

#### 3.4.1 Iterative procedure

The incremental-iterative solution procedures comprise of two parts: the *increment* part and the *iteration* part. The iteration part will be discussed in this part and next part is devoted for incremental part. By using applied denotations in DIANA code documentary [3.7] equilibrium state in which the internal force vector equals the external force vector, satisfying boundary conditions.

$$\mathbf{f}_{\text{int}} = \mathbf{f}_{\text{ext}} \quad (3.31)$$

where  $i$  is prescribed. For each increment in nonlinear analysis, correction iteration is necessary to keep the error in analysis in a certain acceptable level. The exact solution leads unbalanced or residual force which is generated in each iteration converges to zero. The unbalanced force is difference between external and internal loads or in other words, difference between

applied load and resisting loads. The equation can be represented as follow.

$$\mathbf{g} = \mathbf{f}_{\text{ext}} - \mathbf{f}_{\text{int}} \leq \lambda \quad (3.32)$$

where  $\mathbf{g}$  is residual force and  $\lambda$  is convergence rate. To solve equation 3.29, stiffness matrix should be updated due to the change of displacement in each iteration. Depending on how the stiffness matrix  $\mathbf{K}$  is updated, iterative method can be classified in three broad categories with possible modification of each method: *Tangent stiffness method* (regular Newton-Raphson and modified N-R method), *Secant stiffness* or Quasi Newton-Raphson method and *initial or constant stiffness* method. Each method is schematically depicted in Fig.3.9. Regular Newton-Raphson need a few iteration to satisfy convergence criterion but it is time consuming since in every iteration the stiffness matrix need to be updated. Alternatively in modified N-R stiffness matrix is updated only in the beginning of each increment and kept constant during further iterations. Secant stiffness method on the other hand updates secant stiffness matrix in each iteration.

The method is suitable for nonlinear material with softening nature. Although any stiffness matrix can be used at the beginning of each increment but normally tangent stiffness matrix is adopted. The iterations in this method converge faster than that of initial stiffness method but slower than regular N-R procedure. The use of iterative solution requires an appropriate convergence criterion. If inappropriate criteria are used, the solution may be terminated before the adequate solution accuracy is reached or continue after the required accuracy has been reached. Detail of regular Newton-Raphson parameters are shown in Fig.3.10 and formulated below.

$${}^{t+\Delta t}\mathbf{g} = {}^{t+\Delta t}\mathbf{R} - {}^{t+\Delta t}\mathbf{F} = 0 \quad (3.33)$$

All above mentioned loads vectors can be expressed as a function of vector  $\mathbf{U}$  taking into account that this vector may also contain variables other than displacement *e.g.* rotation variables.

Assuming that  ${}^{t+\Delta t}\mathbf{U}_{i-1}$  is evaluated in the iterative solution, then a Taylor series expansion around this point gives

$$\mathbf{g}(\mathbf{U}) = \mathbf{g}({}^{t+\Delta t}\mathbf{U}_{i-1}) + \left[ \frac{\partial \mathbf{g}}{\partial \mathbf{U}} \right]_{{}^{t+\Delta t}\mathbf{U}_{i-1}} (\mathbf{U} - {}^{t+\Delta t}\mathbf{U}_{i-1}) + \text{higher - order terms} \quad (3.34)$$

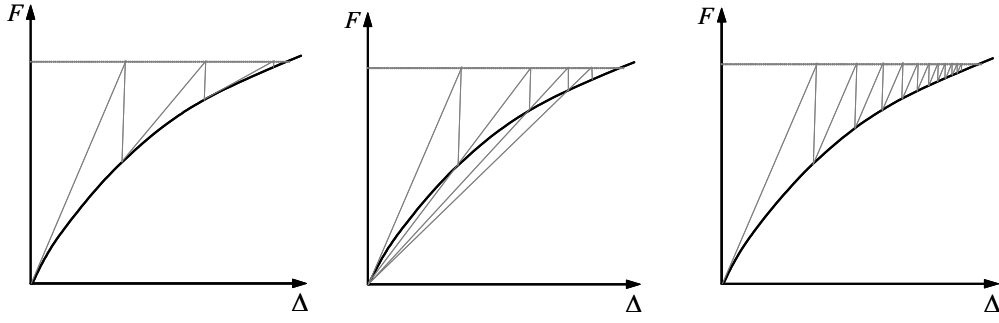


**Fig.3.9.** Left to right; Newton-Raphson, Secant and Constant stiffness method

By substituting Eq.3.33 into 3.34 and after some algebraic manipulation as well as neglecting higher order terms we obtain

$$\left[ \frac{\partial F}{\partial U} \right]_{t+\Delta t U_{i-1}} (U - {}^{t+\Delta t}U_{i-1}) = {}^{t+\Delta t}R - {}^{t+\Delta t}F_{i-1} \quad (3.35)$$

Here it is assumed that externally applied load is independent to deformation and geometric nonlinearity is not taken into account. Equation 3.32 is represented by tangent stiffness matrix in a form of

$${}^{t+\Delta t}K_{i-1} (U - {}^{t+\Delta t}U_{i-1}) = {}^{t+\Delta t}R - {}^{t+\Delta t}F_{i-1} \quad (3.36)$$

where in tangent stiffness matrix of the current stage is extracted from 3.35 as

$${}^{t+\Delta t}K_{i-1} = \left[ \frac{\partial F}{\partial U} \right]_{t+\Delta t U_{i-1}} \quad (3.37)$$

and the improved displacement solution is

$${}^{t+\Delta t}U_i = {}^{t+\Delta t}U_{i-1} + \Delta U_i \quad (3.38)$$

The relations Eq.3.35 and 3.37 construct regular Newton-Raphson solution of Eq.3.36. Since an incremental analysis is performed with time (or load) steps  $\Delta t$ , the initial conditions in this iteration are  ${}^{t+\Delta t}K_0 = {}^tK$ ,  ${}^{t+\Delta t}F_0 = {}^tF$  and  ${}^{t+\Delta t}U_0 = {}^tU$ .

The iteration is continued until equilibrium is reached up to a prescribed tolerance. The difference between several procedures is the way in which  $\Delta U$  is determined where the iterative increments are calculated by use of stiffness matrix  $\mathbf{K}$  that represents some kind of linearized form of the relation between the force vector and displacement vector. For instance in Modified Newton-Raphson method stiffness matrix is updated at the beginning of each iteration while in Constant Stiffness Method initially constructed

stiffness is used in entire analysis all over through. In situations where Regular Newton-Raphson does not converge anymore, the Modified Newton-Raphson process can sometimes still converge. On the other hand if unloading occurs, it can be advantageous to return to the linear stiffness, e.g. in a plasticity or reversal load analysis.

It is however noted that all three aforementioned methods are available in DIANA. Furthermore, two variations that can be used in combination with these procedures are considered: the *Continuation* method and the *Line Search* method. And finally, several criteria to stop the iteration loop are discussed in theoretical background of DIANA manual and syntax formats used in pre-

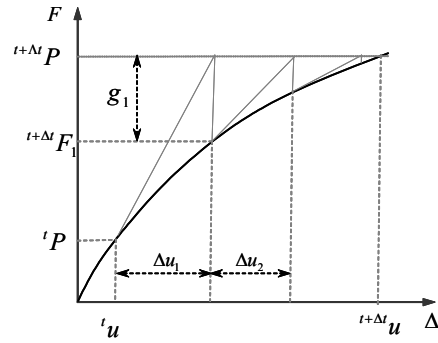


**Fig.3.10.** Newton-Raphson schematic iteration procedure

processing program. For more detail refer to appropriate references and also DIANA documentaries. Since Line Search method is applied in analytical investigation of this study, the concept is summarized here. More detail and formulation are given in DIANA user manuals and related references.

#### a) *Line Search Method*

The objective of using this method is only to increase convergence rate in iterative process. The iteration method such as those belong to Newton-

Raphson family converge to the exact solution if the prediction is reasonably close to the target value otherwise in many cases it easily fail to converge. This is true particularly for structures with strong nonlinearity such as cracking in concrete structures which is also the subject of this study. The Line Search algorithm uses minimum potential energy concept to scale increment vector  $\Delta U_i$  stated in 3.38 (e.g. displacement) in order to increase convergence rate. Therefore Eq.3.38 is modified by this scale factor  $\eta$  and represented as

$${}^{t+\Delta t}U_i = {}^{t+\Delta t}U_{i-1} + \eta \Delta U_i \quad (3.39)$$

where  $\eta$  is derived from the selected iteration method. To minimize potential energy  $\Pi$  in the line search direction derivative of  $\Pi$  to  $\eta$  must be zero. Hence  $s(\eta)$  as a target function of minimization process is determined as

$$s(\eta) = \frac{\partial \Pi}{\partial \eta} = \frac{\partial \Pi}{\partial U} \frac{\partial U}{\partial \eta} = 0 \quad (3.40)$$

First term of the equation is equal to the unbalanced force  $\mathbf{g}$  which in this case will be a function of scale parameter  $\eta$ . Second term is  $\Delta U$  which can be obtained from Eq.3.39 thus Eq.3.39 can be represented in a new form as

$$s(\eta) = \mathbf{g}(\eta) \Delta U = 0 \quad (3.41)$$

In order to optimize the solution of 3.41, the first two values for  $s(\eta)$  are derived from the original iteration process. Once the search direction is calculated, the values  $s(0)$  and  $s(1)$  are calculated by the inner product of  $\Delta U$  with respect to the out-of-balance force at the start and at the end of the iteration. As mentioned in DIANA [3.7] documentation, the Line Search method in the code is only used to 'help' the ordinary iteration processes, therefore the process does not really continue until a value  $s = 0$  is found, but the line search is terminated if the absolute value of  $s(\eta)$  has a value that is less than for instance 80% (adopted in the code) of the value  $s(0)$ . On the other hand parameter  $\eta$  is bounded between a maximum and minimum value to avoid generation of unrealistic values during iteration [Fig. 3.11].

#### b) Convergence Criteria

In any iterative analysis, the process is stopped after appropriate convergence is achieved. Here in DIANA three criteria can be set to terminate the calculation. They are load norm, displacement

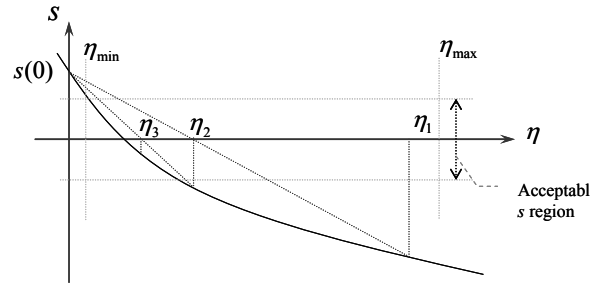


Fig.3.11. Line Search schematic iteration procedure

norm and energy norm. The vector norms used for displacement and load norms are two vector norms (also called 2-norm) known as the *Euclidean vector norms*. This norm is a geometric parameter of vector and represents vector length. For a vector  $\mathbf{V}$  with component  $v_i$ , Euclidean vector norm  $\|\mathbf{V}\|_2$ , which is a single number, is defined as

$$\|\mathbf{V}\|_2 = \left( \sum_{i=1}^n |v_i|^2 \right)^{1/2} \quad (3.42)$$

Similar concept is applied for displacement and load norm with the following definition

$$\text{Displacement norm ratio} = \frac{\|\Delta \mathbf{u}_i\|_2}{\|\Delta \mathbf{u}_1\|_2} \quad (3.43)$$

$$\text{Load norm ratio} = \frac{\|\mathbf{g}_i\|_2}{\|{}^{t+\Delta t} \mathbf{P} - {}^t \mathbf{P}\|_2} \quad (3.44)$$

Proper tolerance should be employed in either case to satisfy desired accuracy. Each of the aforementioned criteria has some drawback in terms of spurious convergence occurs in some special cases. Depends on the type of analysis proper criterion and its convergence rate should be selected. For instance if many displacement control points is used in analysis, displacement norm might be less useful. On the other hand for elasto-plastic problems with very small hardening rate when enters to the plastic zone, the out of balance load may be very small while the displacement may be much in error. In order to provide an indication of when both displacement and load are near their equilibrium, values, a third convergence criterion can be represented based on energy norm. It is usual to use the out of balance force to evaluate energy norm. However DIANA uses Line Search method in order to expedite convergence rate and also knowing that the method is based on minimum potential energy which can minimize the norm before the solution

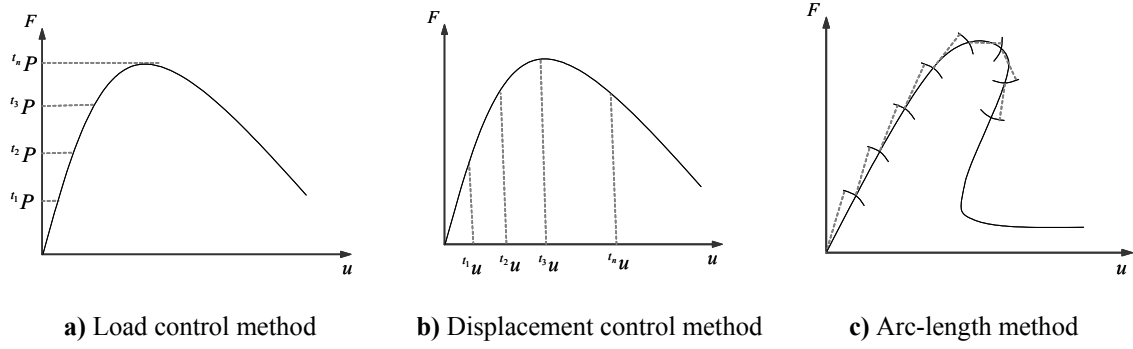


**Fig.3.12.** Incremental schemes

really converges to equilibrium, internal force is used in energy norm formulation. Energy rate is calculated as

$$\text{Energy norm ratio} = \frac{\left| \Delta \mathbf{u}_i^T ({}^{t+\Delta t} \mathbf{F}_{i+1} - {}^{t+\Delta t} \mathbf{F}_i) \right|}{\left| \Delta \mathbf{u}_i^T ({}^{t+\Delta t} \mathbf{F}_i - {}^{t+\Delta t} \mathbf{F}_0) \right|} \quad (3.45)$$

where  ${}^{t+\Delta t} \mathbf{F}_0$  is internal force at the beginning of the increment.

In this study in general, dual convergence criterion has been adopted including energy norm (rate: 0.0001) along with either displacement or load norm (rate: 0.01). It is set however when convergence rate of any of the abovementioned criteria is satisfied, calculation will be continued to the next increment.

### 3.4.2 Incremental procedure

The incremental procedure basically is divided to two types: *load control* and *displacement control*. These two methods can handle most incremental procedures depend on the required solution. For pre-peak analysis load control scheme can be the choice of solution while for analysis of pre and post peak regime, displacement control is the proper choice. The latter method can go beyond the peak and capture snap-through phenomenon particularly for softening material. However in some cases such as Snap-back even displacement control fails where alternative method called *Arc-length method* can handle this phenomenon.

In the load control method load steps are prescribed where the external load is increased at the start of the increment, by directly increasing the external force vector  ${}^t P$  (Fig.3.12.a). Alternatively in displacement control [Fig.3.12.b] displacement  $u$  is prescribed and corresponding load is calculated in each step. Both methods are to solve Eq.3.42 based on generated residual force in each iteration.

$$[K][\Delta u] = [P] - [F] = [G] \quad (3.46)$$

where  $[G]$  is unbalanced or residual force of each increment. The *Arc-length method* on the other hand uses an incremental method that can adopt the step size depending on the results in the current step. The initial choice of the step size for every increment is an important factor in the incremental-iterative process. In order to best fit the choice of step size definition, two methods are generally adopted and also two methods to choose between loading and unloading depending on the previous analysis results. This method as can be seen in Fig.3.12.c is capable to pass peak load even if snap back instability occurs in analysis. The method is applied in some analysis and will be shown in further chapters. For more detail discussion and formulation on each method interested readers are referred to DIANA documentation as well as relevant references.

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## 4. SENSITIVITY ANALYSIS

The objective of this part is to investigate response of RC deep beams to different analysis parameters and mesh discretization by means of finite element method. DIANA 8.1.2 code is employed for this sensitivity analysis. The experiments used here for verification are those explained in Chapter 2. Some parameters have been chosen for sensitivity analysis, which might have crucial or considerable effect on numerical response such as concrete constitutive laws and crack models or some others relevant to solution methods such as convergence problem and load increment size and iterative methods. For each case study, only one parameter is variable while the others are kept constant. Functionality of fracture energy method in producing mesh objective results is also examined in this part. Crack models are another issue which is also studied in this chapter. The results showed however a large discrepancy in terms of applied crack model. To limit the number of effective parameters on results only those with high probability of crucial effects have been selected in this chapter. All analyses are in smeared crack categories where other models such as discrete crack model, plasticity approach, lattice model and less popular models are disregarded in this investigation. It is however not claimed that whatever parameters or methods selected for sensitivity analysis are the only important parameters affect RC modeling with finite element analysis. Emphasizes is given to concrete material while only elasto-plastic model is considered for reinforcing bars.

Since this study is a part of larger study on RC deep beams behavior and size effect, the primary motivation on sensitivity investigation of numerical modeling is to verify reliability, consistency and applicability of finite element analysis by picking up the most suitable method for the preceding purpose. Study on size effect however requires a wide range of test specimens from very small to very big say 3000 mm height. In order to substitute experimental task with numerical modeling, this study should be primarily carried out for further analytical investigation. Experimental study on size effect was presented in the previous section.

### 4.1 Finite Element Discretization

The constitutive behavior of concrete is represented by a smeared crack model, which in the damaged material still continuum based on material models explained in the preceding section. Analytical scheme and finite element mesh discretization is

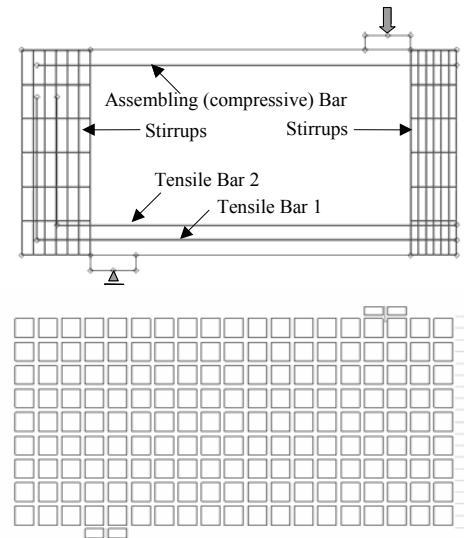


**Fig.4.1** Analytical model and FE mesh discretization (shrink mesh)

shown in figure 4.1. It is noted however that FE mesh discretization shown in this figure is just a schematic view and the number element and other characteristics of the model will different for each specimen. For specimens with stirrups in shear span, lateral reinforcement will be extended to entire length of the member. According to concrete crack model, two approaches which are explained in preceding section are applied in this chapter.

In the analyses, perfect bond is assumed for both smeared reinforcements and embedded bar reinforcements. Displacement control with Newton-Raphson solution technique is adopted here to solve equilibrium equations along with Arc-length method to investigate possibility of snap-back instabilities, which sometimes occurs in shear failure analysis. Specimens are partly modeled by FEM due to the symmetric geometry and subjected to the proportional monotonic loads with 2D elements in plane-stress condition. According to constitutive model of concrete in tension, it is supposed that softening branch of cracked concrete is sufficient to explain tension-stiffening phenomenon therefore no additional stress due to this phenomenon is included in calculation. Steel reinforcements are modeled as an elastic perfect plastic material with no hardening after yield point.

### 4.2 Sensitivity to the Crack Models

In order to find the best crack model for further finite element analyses, first this parameter is studied. Two models, the fixed crack and the rotating crack approach are utilized here. Fixed crack model is applied by means of four constant

values for shear retention factors;  $\beta = 0.01, 0.2, 0.5$  and  $1.0$ . The material model for concrete in tension and compression are Hordijk and Feenstra fracture model respectively. Sensitivity of results to the material models will be discussed in the following section. The study is carried out on several specimens but due to the similarity between the obtained results, only that of specimen B14 is presented here.

The results, as can be seen in Fig.4.2, are so scatter in terms of shear retention factor for fixed crack approach. In all calculation it is tried to have convergence by setting a combination of different load (displacement) increment size in analysis particularly before the peak. Nevertheless some calculation failed to satisfy this demand e.g.  $\beta = 0.01$ . Diverged steps are shown with yellow triangles in Fig.4.2.

On the other hand the rotating crack approach could adequately give an objective result where almost all load increments converged to the exact solution. The results here show the importance of the choice of shear retention factor in analysis of specimens with possibility of shear failure if the fixed crack approach is adopted. It is however believed that the effect of this factor is negligible if the beam fails in flexural mode [4.1]. To this end rotating crack model showed simplicity in application, robustness and accuracy in prediction of response with less numerical difficulties. For these reasons this model will be applied in further analytical investigation.

### 4.3 Sensitivity to the Material Models

Material models explained before are applied here to analyze some representative beams (B6, B14, B15, B16 and B18) to show sensitivity of analysis to the applied material models (Fig.4.3). For this case study, only the rotating crack approach is adopted in order to avoid term of shear retention factor complication in the fixed crack approach. In all analysis with JSCE material models  $k_1 = 1$  is adopted for compressive model. Although this assumption gives rise to higher prediction of failure load than that of standard JSCE material model with  $k_1 = 0.85$  but the results are still very conservative. Type1 material model however yields better prediction in finite element analysis. To investigate the reason of such premature failure strain contours are drawn in vicinities of peak load and subsequent comb like peak points (Fig.4.4). It is clearly seen that at the edge of loading plate compressive strain goes beyond ultimate strain defined by JSCE constitutive model. On the other hand since there is no softening branch stress will

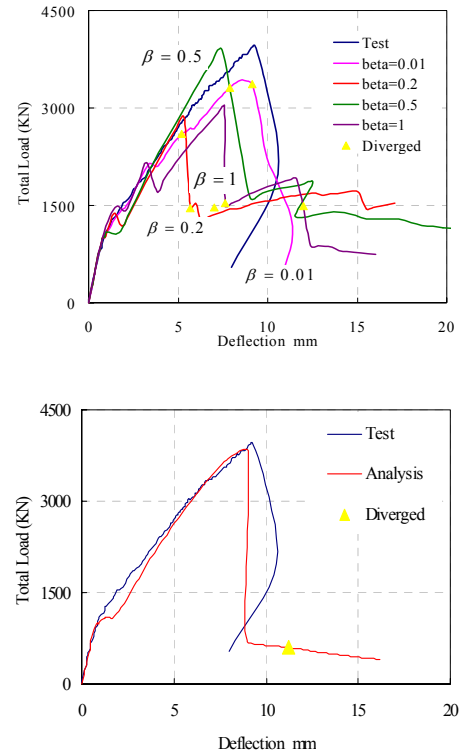


**Fig.4.2** Analytical results of B14 with fixed crack (top) and rotating crack (bottom) models

suddenly drop to zero and give rise to such comb shape. B14 is picked up for investigation on concrete compressive strain in crucial location which in the strain is possible to go beyond ultimate strain proposed by JSCE (Fig.3.5).

Principal strain in different steps of analysis with Type 2 material (JSCE) is shown in Fig.4.4. According this material mode definition, ultimate strain is set to be 0.0035 at the most. In Fig.4.4-1 (peak load), strain in adjacent of loading plate and concrete body already passed ultimate strain  $\epsilon'_{cu}$  (Fig.3.5) and compressively failed zone is formed. Hence in the following step load cannot be sustained anymore and we can see a sudden drop in load-deflection curve. In subsequent load increments failure zone will be expanded and deepened over beam depth give rise to abnormally large deformation in form of sliding one part over other part. There is a possibility for instance if loading plate have a frictional contact to concrete body; such as interface elements; instead of perfect connection between adjacent elements, less stress concentration occurs in failure regions. It is however suggested to use interface elements if dealing with such material models particularly with no softening branch. Since this element is not available in all FE codes or application of such element complicates analysis and makes numerical

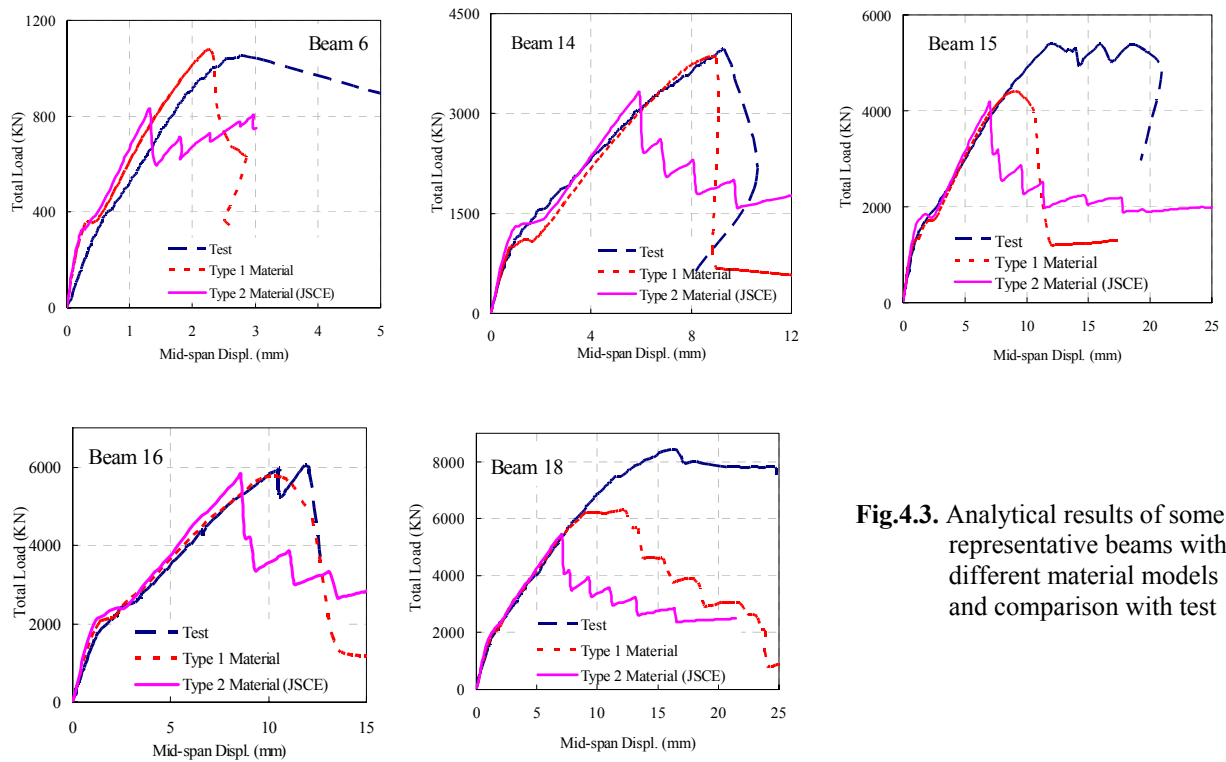


**Fig.4.3.** Analytical results of some representative beams with different material models and comparison with test

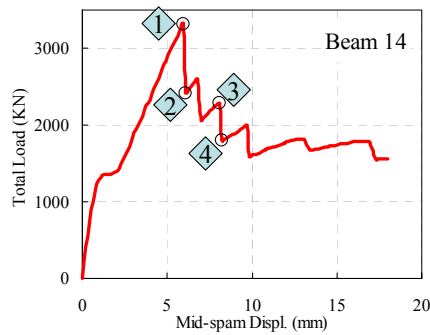
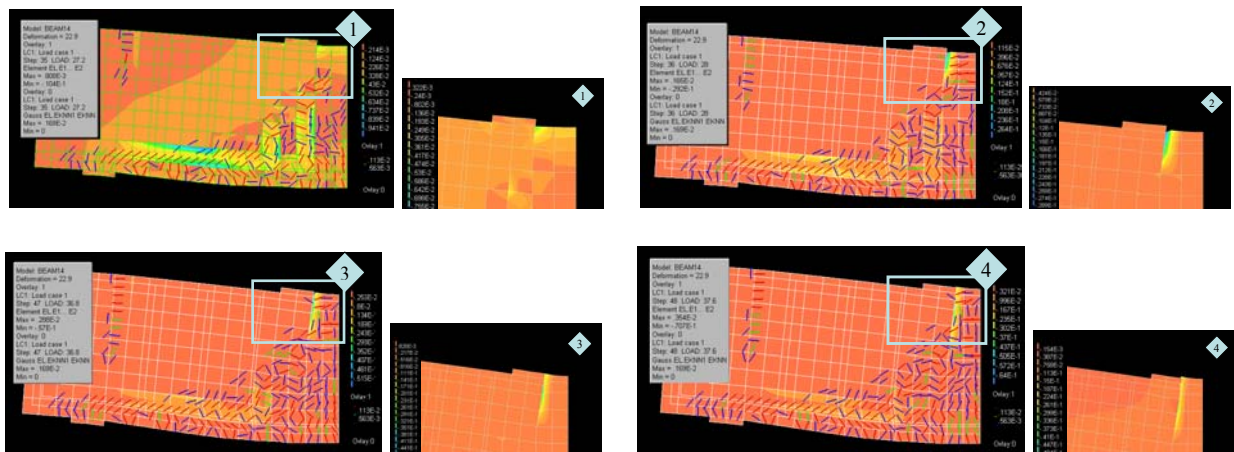


**Fig.4.4** Principal strain of Beam 14 in different load steps of analysis with Type 2 material model (JSCE). Crack patterns and strain contours are shown below.



difficulties in highly nonlinear materials such as concrete, perfect connection is used in this study. This is also in agree with the objective of this investigation to use the most practical method in analyzing RC members. Possibility of improving

analytical response by means of employing a frictional element as interface between loading plate and concrete. The applied model and obtained results will be discussed in following chapter.

#### 4.4 Sensitivity to the Finite Element Mesh Size

In order to evaluate fracture energy concept in reducing sensitivity of analysis to finite element mesh size specimen B14 is analyzed with two mesh size called here fine mesh and core mesh. Fine mesh discretization is basic mesh size used in entire analysis of this study. Core mesh on the other hand has a side size about twice larger than the fine one (finite element area will be about 4 times larger). Results in term of Load-Mid span deflection are drawn in Fig.4.5. No appreciable difference has been observed in the results and mesh size dependency is likely eliminated from the analysis. This results shows analysis from very small specimens with relatively small elements and also the largest specimens, here B18, with larger elements are all objective.

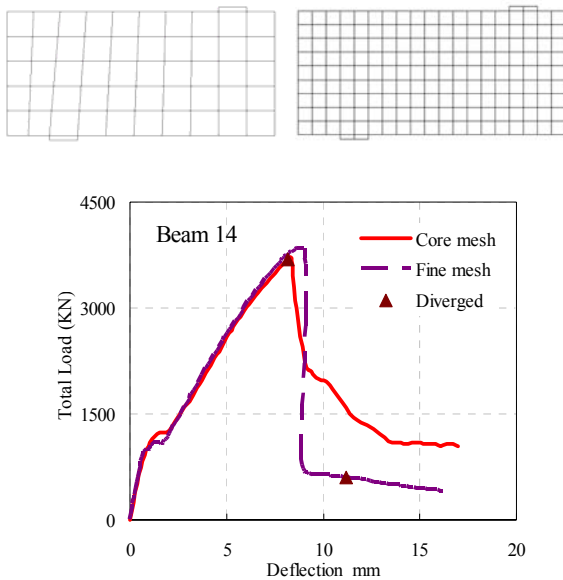


**Fig.4.5.** Core and fine finite element mesh discretization and analysis results

#### 4.5 Conclusion

Sensitivity analysis of RC members particularly RC deep beams showed that analytical results are not always objective. A number of parameters such as material and crack models as well as finite element mesh size are investigated in this section. Sensitivity in terms of element size in finite element discretization is crucial if fracture energy is not taken into account or the element is not enriched or improved to handle this problem. In this study however material models in fracture energy basis are applied and it was shown that analysis sensitivity is negligible. In case of not using fracture energy approach in FE analysis, embedded

crack model or non-local theory are alternative methods suitable for objective analysis.

According to the applied crack models, the fixed crack theory and the rotating crack theory are applied here. In contrast fixed theory lacked of giving consistence prediction in terms of different shear retention factors. Although this theory can be adequately applied in analysis of RC members failed in flexural mode, the method failed however to yield objective results for members dominated by shear such as RC deep beams. Since to our knowledge there is no consensus on the question which value should be used for shear retention factors, the author suggest rotation crack model to be applied if failure is dominated by shear. More detailed discussion will be given alone with concluding remarks for Chapter five.

#### REFERENCES

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## 5. ANALYTICAL INVESTIGATION

### 5.1 Introduction and Basic Assumptions

Following the preceding chapter, analytical investigation by means of rotating crack approach and Type 1 material model (concrete compressive and tensile model by Feenstra and Hordijk respectively) is presented in this chapter. Material models in either tension or compression are in fracture energy basis. Fracture energy for all specimens is fixed constant in compression (50 N/mm) and follows JSCE expression for tension (Eq.3.5). Mesh discretization for each specimen is proportional to the specimen size which means larger specimens have larger finite element but the elements are tried to be as structured as possible to avoid element distortion and relevant errors. In order to investigate possibility of snap-back instability, Arc-length procedure is also utilized in this chapter. The basic assumption for analysis are almost similar to those already mentioned in Chapter 4 but some case studies such as analysis with interface elements modeling between loading and supporting plates as well as bond slip model are conducted in this chapter. For effective and fast iteration process leading to convergence of calculation, line search method is adopted in entire analyses.

### 5.2 Analytical Response of the Specimens

Analytical results of all specimens are presented and discussed here and in six following sub-sections. To achieve the maximum capacity of the member by finite element analysis, several schemes for load steps are applied. The results showed sensitivity to loading scheme even though they all have converged to the exact solution with an acceptable accuracy. The differences between predicted peak loads however didn't go beyond five percent in most cases. A possible reason for such discrepancy might be attributed to very high nonlinear behavior of concrete materials where in sequence of cracking during loading process affect overall capacity of the member. The authors believe that the highest predicted peak load best represents load carrying capacity of the member if the iteration converged to the exact solution therefore the maximum values for peak loads are selected to be compared with test results. Predicted peak load to experiment peak load ratios ( $P_{FEM}/P_{EXP}$ ) are also presented in following figures in terms of  $a/d$  ratio as well as stirrups existence in the relative specimens.

#### 5.2.1 Load-deflection response

Load-deflection responses of all specimens base on above-mentioned assumptions are illustrated in Fig.5.1, 5.2 and 5.3 (A and D in the figures stand for Arc-length and Displacement control procedures respectively). Load increments which in full Iteration procedure (here 100 iterations are adopted) could not satisfy convergence criterion is noted by triangles in each graph with relative color for each method. Once calculation failed to converge in an exact solution however analysis was not terminated and further load steps are also applied to the beam consequently. In most cases once the first diverged point took place consequent calculation continued smoothly with no numerical difficulties. As shown in figures diverging of iterative solution usually occurs in peak load vicinity or just the following step after the peak. Although only one diverged point in an entire solution process may not jeopardize accuracy of the results but at least we can say the analysis is reliable before the diverged point is reached. According to experimental results of beams with  $a/d=0.5$ , load-deflection responses are not as stiff as expected. It is obvious that at the beginning of loading due to elastic behavior of the specimen and before cracking beam naturally is supposed to show stiffer behavior than after cracking. It is however observed that those beams have shown softer behavior during early steps of loading (Fig.5.1) therefore obtained load-deflection relationships are dubious and comparison between analysis and experiment, except that of peak loads in some extent, would not be reliable. On the other hand analytical predictions of other beams indicate acceptable prediction by finite element method in terms of load-deflection behavior despite conservative estimation of peak load for certain specimens.

In spite of RC beams behavior failed in flexural mode which could be predicted by FEM quite accurately, simulation of shear failure often face many difficulties in terms of constitutive laws and numerical solution. In many cases however FE analysis predicts higher load capacity for beams failed in one of shear failure mode category particularly those occur much often in RC deep beams. In contrary to analytical responses of this study, such overestimated prediction on shear capacity of RC members misleads designers to estimate real capacity of the member and come up with a weak element in shear eventually. Figure 5.4.1 on the other hand shows an about 15 percent in average underestimated prediction of real shear capacity of the members by the applied finite element method. Most of the predicted shear load capacities are located inside  $P_{FEM}/P_{EXP}=1$  circle

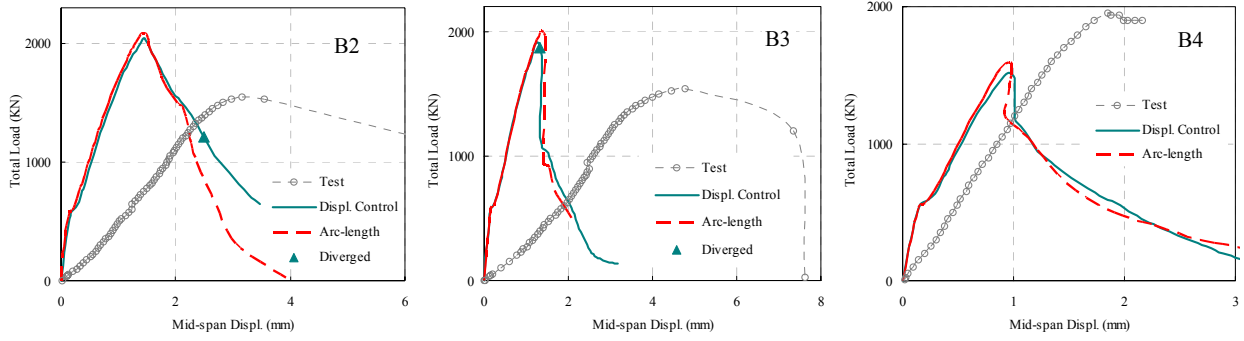


**Fig.5.1.** Analytical response of beams with  $a/d=0.5$

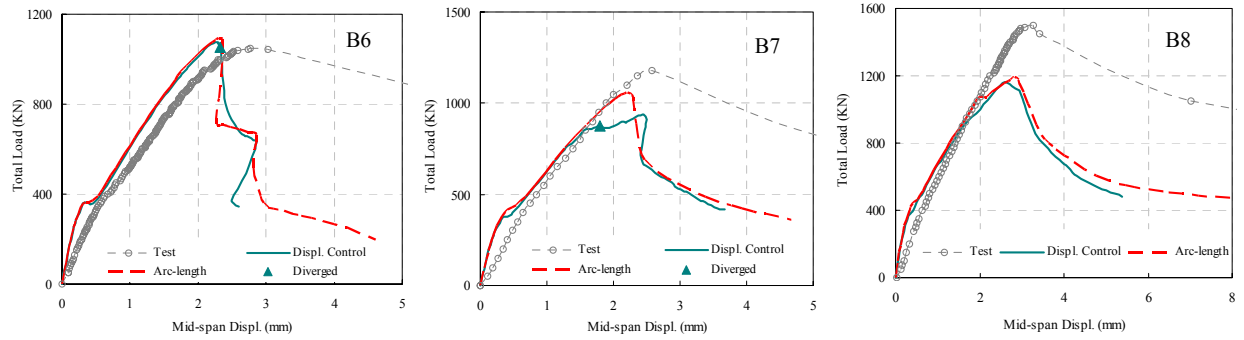


**Fig.5.2.** Analytical response of beams with  $a/d=1.0$ ,

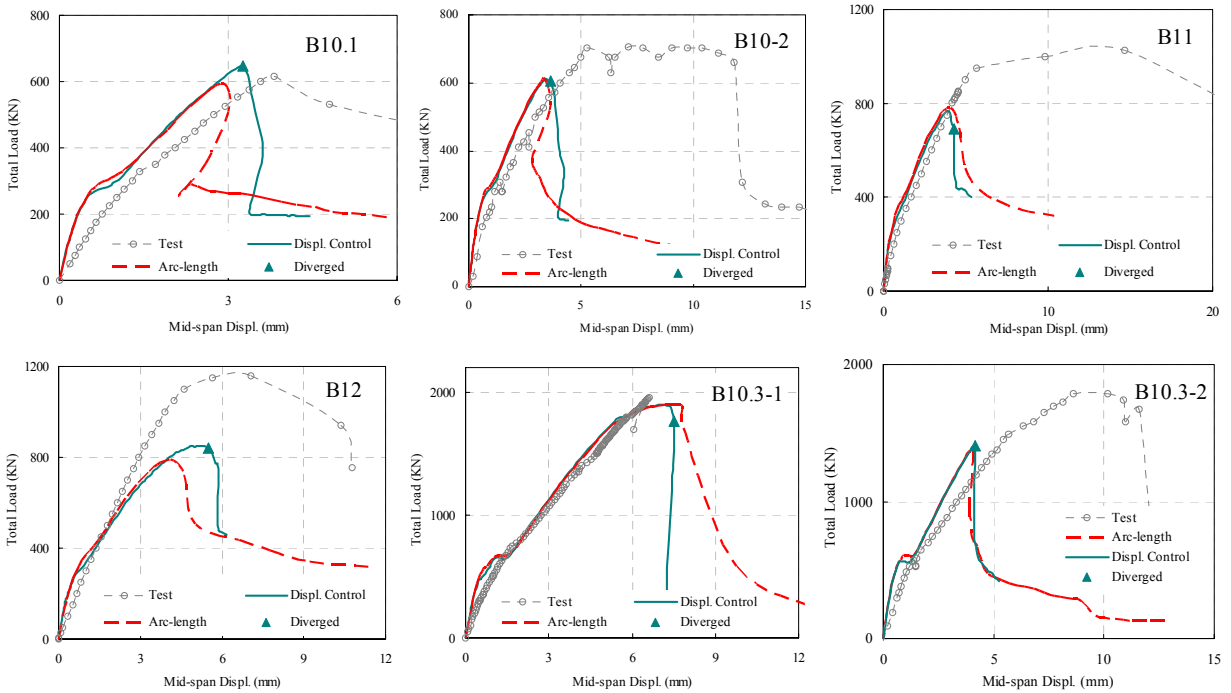


**Fig.5.3.1** Analytical response of beams with  $a/d=1.5$

which implies design safety margin is ensured. This means that there has been found a good correlation between analysis and experiment in terms of shear capacity of RC deep beams and can be utilized for

design of those members in real practice. In order to investigate more in detailed analytical prediction of beams with and without stirrups, Figs.5.4.2 show the results of two sets of specimens with  $a/d>0.5$ .

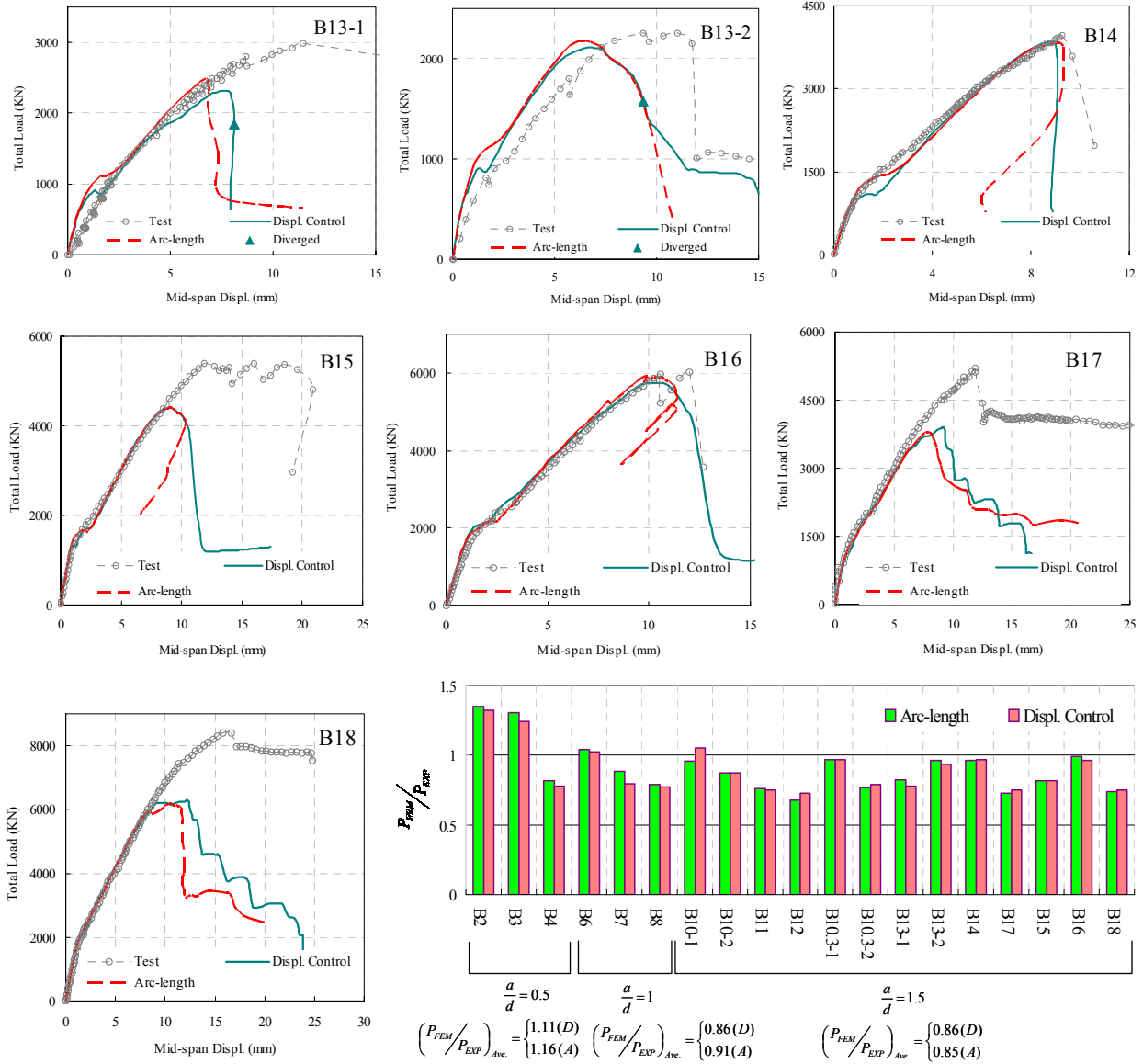


**Fig.5.3.2** Analytical response of beams with  $a/d=1.5$ , **Fig.5.3.3** Analysis peak load to test peak load ratio  $P_{FEM}/P_{EXP}$

Average  $P_{FEM}/P_{EXP}$  for the beams with stirrups is 0.76

while for without stirrups figure shows an average of 0.92. The latter one though appear to be closer to the test results but it lack uniform distribution of the results from  $P_{FEM}/P_{EXP} = 0.79$  to 1.05. Two boundaries

of the results however are in acceptable range and can be used for design of members for practice. On the other hand predictions for specimens with stirrups stand in about 0.79 with minimum deviation and very much uniform distribution of  $P_{FEM}/P_{EXP}$  ratio. Standard deviations are 0.02 and 0.1

for each set with and without stirrups respectively. This means that the data of the first set are more uniform and less dispersed than the other one.

Nevertheless the other set of data are not considered as volatile data and still in consistent range for FEM prediction. It is evident by this results also if cracking of the concrete is prevented of excessive opening by means of stirrups for instance, prediction by finite element analysis will be smother and more consistent than the concrete cracking freely. The results of numerical simulation will be more reliable therefore since we will always have a minimum percent of lateral reinforcement required by design codes. To this end however simulation of RC beams either with or without stirrups showed very good results in terms of load-deflection response as well as overall behavior of the specimens and consequence of cracking which will be discussed in the following sections.

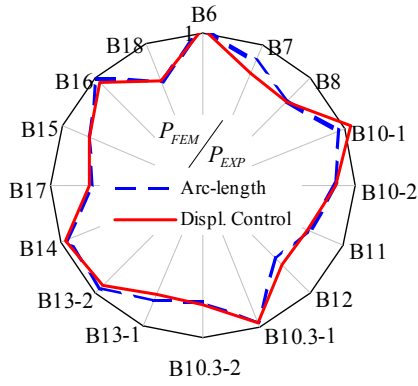


**Fig.5.4.1**  $P_{FEM}/P_{EXP}$  ratio for  $a/d > 0.5$

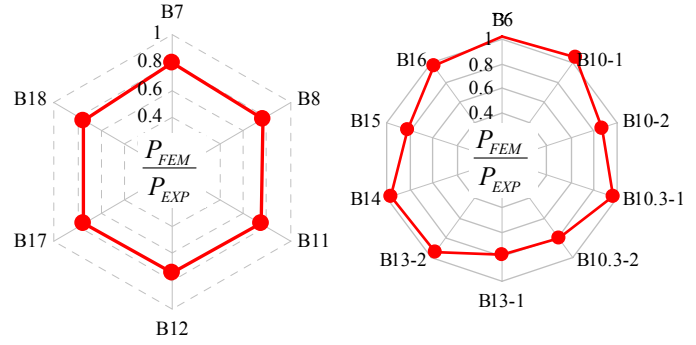


**Fig.5.4.2**  $P_{FEM}/P_{EXP}$  ratio for  $a/d > 0.5$  for beams with stirrups (left) and without stirrups (right) by Displacement Control Method

### 5.2.2 Pre-peak regime and shear crack load

Results of analysis in most cases show stiffer behavior in pre-peak regime before shear crack develops significantly. This issue can be attributed to modeling assumptions such as perfect bond assumption, fixed connection between different material for instance loading plates and concrete body as well as material properties of concrete before and after cracking. Even in case of sliding bond model, the modeling assumption itself affects the results to some extent. Those assumptions however need to be enriched by more experimental evidences. The effect of such parameters is discussed in further sections. At least in case of sliding bond model the results were much softened particularly after the shear crack extension in pre-peak regime.

It is also observed that after shear cracks extension to some part of shear span, stiffness is reduced sharply and show even softer behavior than that of experiment in some specimens. Nevertheless shear crack load is adequately captured by analysis and depicted by a change in stiffness which agrees well with test results. Shear crack load however is a milestone if shear capacity of the member is the main concern of design. Shear crack extension is well monitored by appreciable change in structure's overall stiffness rather than just some diagonal crack existence in shear span. Change in stiffness can be adequately monitored particularly if dynamic analysis is performed. Moreover change in stiffness, if dealing with dynamic analysis, results change in natural period of structure emerge a basis to evaluate damage level in RC member.

### 5.2.3 Stress in reinforcements

No yield stress is signaled either during test process or in analysis. Shear therefore triggered failure in entire specimens experimentally and numerically.

Although large deformation is associated in post-peak regime, stress in compressive as well as tensile longitudinal reinforcement decreased. This is evident by test results where in load capacity did not sharply drop to zero and a large part of load capacity is sustained during post-peak regime (B11, B15, B17 and B18). In analysis however such plateau did not form and in almost all cases load capacity is drastically decreased.

Figure 5.5 illustrates stress distribution in discrete embedded reinforcements of B10-2 during peak load along with crack pattern. Stress-jump in where the crack exists is clearly shown in this figure as well. Larger crack results larger stress concentration in reinforcement. As mentioned in Chapter 2, in order to measure concrete strain within shear span, one to three tinny reinforcements; depends on size of the beam, called here as dummy reinforcements are located along shear span. Although structural effect of those reinforcements is supposed to be negligible but large stress due to the crack opening is generated in crack location gave rise to yielding of dummy reinforcements. In order to evaluate effect of dummy reinforcement on improvement of beam's structural behavior, those reinforcements are included in finite element model by assuming discrete bar element embedded in concrete with perfect bond condition. Very small effect on peak load however observed but in some cases numerical difficulties occurred due to the interaction between steel and concrete in where stress condition is crucial. To circumvent numerical problem as well as due to the small effect of those members, dummy reinforcements have been eliminated from further FE models.

#### a) Hook stress

Length of tensile reinforcements in entire specimens extended to concrete body by a rather

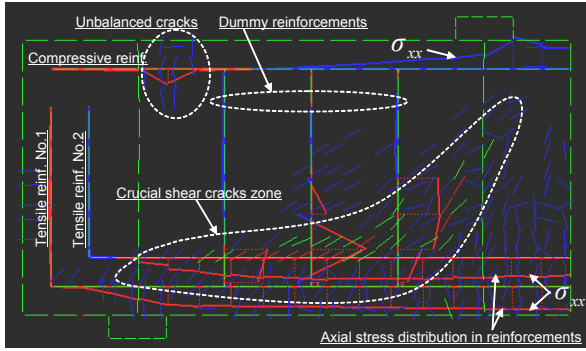


**Fig.5.5.** Stress distribution in longitudinal and dummy reinforcement along with crack propagation of B10-2 at peak load

long hook. As shown in Fig.5.5, hooks received almost no stress. This implies either the anchorage length is enough to exhaust pull out stress or bending capacity of specimen is far higher than shear capacity (over-reinforced beam). For instance in this case maximum tensile stress produced in tensile reinforcement is about 275 MPa while yield stress is about 380 MPa which means far less stress will be produced near support. The latter one however is more applicable here where in shear capacity dominants ultimate load bearing of the specimen and the beams are so designed to fail in shear intentionally in order to study shear behavior as objective of this investigation. Arch mechanism formed in almost all specimens where in line of thrusts inclined to support and sustained large capacity of beams in shear. Arch action requires substantial horizontal reaction at support which imposes heavy demands on the anchorage. This implies that arch action can only occur if concrete bond is completely deteriorated. But here since the deformed bar is used and generally no appreciable slip can take place due to mechanical interaction between concrete and steel, flexural deformation is the main source of translation displacement necessary to arch action takes place. It is believed that arch action attain flexural capacity of beams if the section is properly designed. In chapter two and also in the following sections arch action is shown by means of crack propagation over beam body.

#### **b) Tensile stress in upper reinforcement**

Upper longitudinal reinforcements are generally provided to control compressive stress of concrete not to reach its maximum capacity and fail in brittle fashion or just as assemblage bars. In simple span normal beams they are almost always under compression. In deep beams however the unbalance tensile stress in upper face of the beam results crack in concrete and tensile stress in reinforcement

whereas in normal case it is expected to be under compression due to the positive moment. The bigger the beam height is the severe cracks are expected to be produced. Many of the specimens in this study showed such cracking in where the moment is positive or zero. For this reason in some part of the reinforcement near the end of the beam tensile stress is generated. Moreover in the location where these out of balance crack observed large stress jump is also observed. Figure 5.5 depicts analytical prediction of this phenomenon and also stress jump in upper bar in crack vicinity.

#### **5.2.4 Post-peak regime and Poisson effect**

Post peak behavior of RC members can be captured by FE analysis since displacement control method or Arc-length method is adopted in analysis. The accuracy of the analysis however could not be verified by test results which in post-peak regime was not a major concern of experiment. Furthermore in order to prevent the danger of sudden failure of the specimens, after the peak is reached specimens were unloaded consequently. Therefore post- peak regime could not be recorded in general but in some special cases such as B10-2, B12, B15, B17 and B18, part of response after the peak is recorded to some extent and then unloaded eventually.

To simulate post-peak response of RC members numerical methods such as displacement control method or Arc-length method (in case where snap-back response is possible) are not the only tools required for analysis. Some material parameters such as Poisson are also need to be updated. Test results of Ronnie and Hsu [5.1] showed that during cracking process Poisson changes significantly and in some cases goes beyond one or even become as large as two. Based on conducted test in their work they concluded that current shear theories which neglect the stresses and strains due to the Poisson effect can predict pre-peak behavior but fail to correctly predict post-peak response. Although their works are mainly on membrane element but in any element which shear component dominate overall behavior such as deep beam similar phenomenon may affect post-peak response. In this study however Poisson is kept constant and effect of this phenomenon is not investigated since test post-peak results are not reliable for verification.

As shown in Figures 5.6, some of the specimens show snap-back response just after the peak. Conventional Newton-Raphson procedure however fails to follow the path and usually leads to a diverged iterative calculation. To capture this phenomenon so-called Arc-length scheme is applied in entire analysis, which in post peak

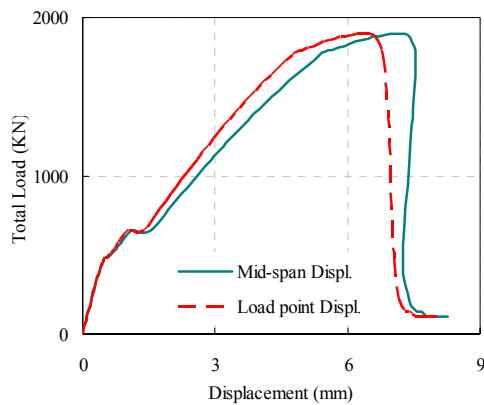


**Fig.5.6.** Mid-span and loading point deflection

regime was adequately followed by almost no convergence difficulties. It is very important to note here that snap-back instability occurs when load and displacement decrease in consequent load steps simultaneously for a certain point usually loading point. For the current specimens, load points and displacement control points are different therefore load-deflection graph itself cannot be a concrete evidence for occurrence of snap-back in analysis. Consequently further investigation is necessary even though load decrease followed by displacement decrease. Good examples for this case are B7, B10.1&2 and B10.3-1. Concerning conventional displacement control method, results are suspicious to have snap-back instability while we know Newton-Raphson procedure inherently fails to follow such path. Load-deflection relationship for loading point on the other hand evident that load path follows snap-through instability which can be appropriately captured by conventional displacement-control solution method. Figure 5.6 depicts a comparison between load versus deflection in mid-span and deflection in loading point for B10.3-1. It is clear that there is no sign of snap-back in load-deflection curve for loading point.

### 5.2.5 Effect of loading rate on results

It is well-known that concrete is a rate dependent material. Higher strain rate results higher compressive strength of concrete. This phenomenon will surely affect not only overall behavior of RC members such as load-displacement response or even mode of failure (possibly within different categories of shear failure) but also may change sequence of cracking in loading process. Although there is no consensus on how much concrete properties are affected by loading rate due to scatter test results but there is a general believe that rate effect exists therefore including this phenomenon in

material model increases accuracy of analysis. The available models for concrete and steel including those presented by design codes are generally based on quasi-static test results which in strain are applied at a slow rate of about  $5 \mu/\text{sec}$ . If the structure is subjected to the higher strain rate compressive strength of concrete increases significantly. According to test results [5.2, 5.3] an increase in strain from  $5 \mu/\text{sec}$  (static loading condition) to  $50000 \mu/\text{sec}$  (seismic load condition) can increase in average the compressive strength of concrete by 34 percent and the yield strength of steel by 27 percent. Although this will increase strength of material and also lead to higher axial force and flexural strength of RC section but it may not be necessarily beneficiary to the dynamic response and change mode of failure to brittle mode and consequently decrease energy absorption in some elements [5.2].

The loading rate on specimens tested in PWRI has been set to 100kN/min for pre-peak regime. This rate will produce a range of very low straining rate (say almost zero) in the element with low stress to as high as  $0.00002/\text{sec}$  ( $20 \mu/\text{sec}$ ) in acrylic bar placed for instance in B10.2 specimen (compressive strain of strut). It is noted that most available test results are based on uniform distribution of strain in the specimen while here since the specimens are beam subject to lateral load strain rate cannot be uniform anymore. Due to this fact that highest strain rate was about  $20 \mu/\text{sec}$  here which is very close to quasi-static load condition. Therefore analysis and experiment are loaded in almost same rate and no differences due to this factor are expected.

### 5.2.6 Crack patterns and deformation

Crack patterns during loading process are another issue to evaluate level of FE analysis accuracy. In this regard crack patterns of some representative beams tested in PWRI in their final load stage are presented and compared with experimental observation in Figs.5.7. Crack patterns of the specimen B10.3-1 in different load steps is also depicted in Fig.5.8. Analytical crack patterns are in shadow of experimental cracks for better evaluation. It is noted however that in analysis all cracks are shown in figures without any filter where crack width is presented in five colored levels. Cracks in blue are the widest which are to be compared with experiment in particular. In specimen B6 main cracks are almost captured in analysis with acceptable accuracy. Form of crush below loading plate could not be predicted in numerical analysis due to the perfect connection

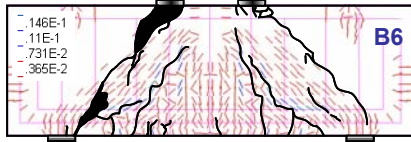


**Fig.5.7.1. B6: Pmax=1050KN**



**Fig.5.7.2 B14: Pmax=3969KN**

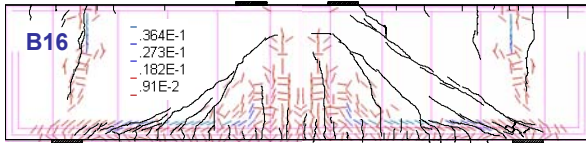


**Fig.5.7.3 B16: Pmax=3969KN**

(Test cracks are drawn with black lines)

between plate and concrete which cause spurious stiffness in the neighboring elements. In other words plate stiffness contributes at least in nearby element stiffness more than what it exists in actual structure.

On the other hand specimens B14 and B16 cracks in analysis and experiment generally show good agreement where out of balance cracks (above supports on upper side) are also captured accurately in terms of crack location and crack development. Since all specimens are half modeled by finite element method cracks of other part in Fig.5.7 and 5.8 are mirrored to cover entire length of specimen. Hence analytical crack patterns are all symmetric despite test results which in usually crucial cracks develop in only one side of specimen. This is another source of the differences between analysis assumption and experiment reality in terms of crack location and propagation where in analysis both sides are absolutely identical but in experiment they are not. Moreover perfect bond assumption gave rise to sever crack generation around longitudinal reinforcing bars while in experiment those cracks, if exist, are invisible therefore not available for verification. To this end it might be concluded that overall behavior of RC beams such as load capacity and load- deflection behavior, as well as the crack patterns in different load steps could be properly predicted by analysis.

Other important issue which is also clearly observed in the predicted cracks is crack direction

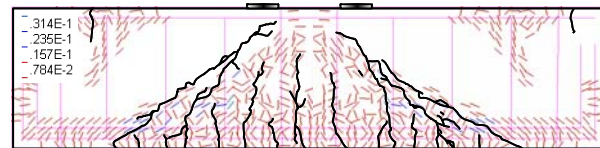
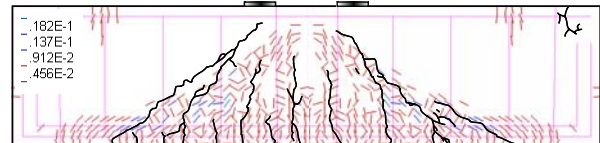
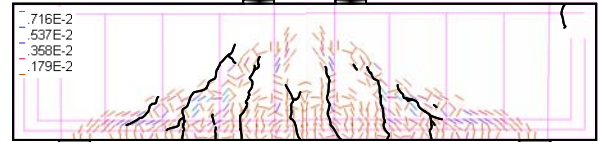
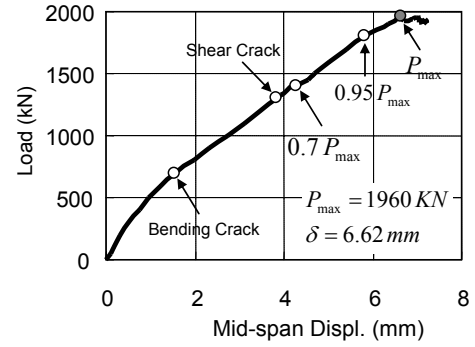


**Fig.5.8. B10.3-1 crack patterns in different load steps**  
(Test cracks are drawn with black lines).

in certain elements which are perpendicular to actual crack monitored in test. This is a drawback of rotating crack model which dismisses crack orientation in the preceding steps and permits crack rotate freely due to the current principal strain orientation. In contrast the fixed crack model keeps the history of crack data gives much consistent results accordingly in terms of crack direction.

For more detailed evaluation, cracks pattern of specimen B10.3-1 are presented in different load steps in Fig.5.8. Four load steps are selected here; after bending crack, after shear crack, 90% of ultimate load and finally crack patterns after the failure. Very good correlation between experiment

and analysis is observed in the first three steps. In final step however concrete near loading and supporting plates crushed. Such failure patterns generally cannot be captured by smeared crack modeling of concrete. Nevertheless crack propagation in other part is very well simulated by analysis.

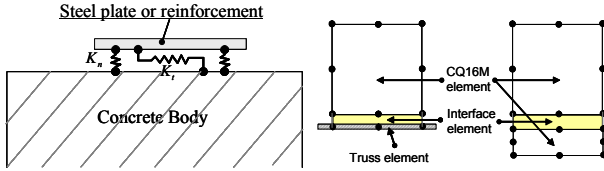


Fig.5.9. Interface element model

### 5.3 Interface Element for Bond-slip Model and Gap Element

Figure 5.9 illustrates the employed interface element used here to model contact between steel plates and concrete body as well as bond-slip between reinforcement and concrete schematically. Employed interface element consists of normal stiffness  $K_n$  and tangential stiffness  $K_t$ . Normal spring represents compressive and tensile components while tangential spring represents shear behavior of the element. Interface element describes relationship between the normal and shear traction as well as the normal and shear relative displacement. According to plane stress condition, line interface element covers both bond slip behavior and frictional contact between steel plates and concrete elements. CQ16M element here is DIANA generic name of eight-node element which in this specific problem four-point integration scheme is adopted. It is noted however that formulation in either case of line or plane interface is fully isoparametric where in line interface may be straight or curved.

#### 5.3.1 Bond-slip model

Figures 5.10 and 5.11 illustrate employed mesh discretization and applied model for bond slip

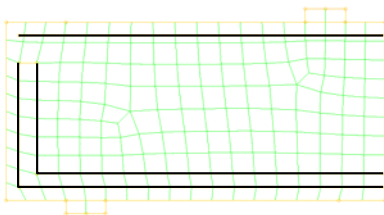


Fig.5.10 Finite element mesh discretization

simulation in this section. Despite perfect bond between concrete and reinforcement where in reinforcements are embedded in concrete elements, here in DIANA truss element should be utilized for bond slip reinforcement connecting to the element nodes. Bond element is a six-node special element comprises of two lines, one for connection to body and the other to represent one dimensional reinforcement elements. The consequence of this approach is that the element mesh should be adjusted to the reinforcement as well as bond slip elements. This is why the mesh discretization is changed here and in a way it will be more complicated to construct structure geometry than the one with embedded element. The constitutive law based on shear and normal component can be written as:

$$[D_{bond}] = \begin{bmatrix} k_n & 0 \\ 0 & k_t \end{bmatrix} \quad (5.1)$$

where  $k_t = \frac{\partial t_t}{\partial s}$  and  $k_n$  has a constant value shown in figure 5.11. Shear and normal tractions therefore will be derived as

$$t_n = k_n \Delta u_n \quad (5.2)$$

$$t_t = c \cdot s \quad (5.3)$$

The relationship between the normal traction and the normal relative displacement is assumed to be linear elastic in DIANA whereas shear traction is a nonlinear function of slip either a user defined function or predefined function. In this study Dorr polynomial model [5.4] is adopted to express shear traction in terms of relative slip. The model is shown in Fig.5.12.

The model represents a cubic function for relationship between shear traction  $t_t$  and slip  $s$  as

$$t_t = \begin{cases} c \left( 5 \left( \frac{s}{s_0} \right) - 4.5 \left( \frac{s}{s_0} \right)^2 + 1.4 \left( \frac{s}{s_0} \right)^3 \right) & \text{if } 0 \leq s < s_0 \\ 1.9c & \text{if } s \geq s_0 \end{cases} \quad (5.4)$$

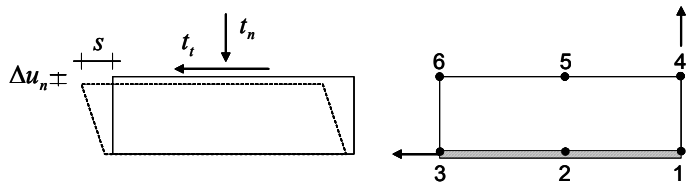


Fig.5.11 Bond slip element

where in  $c$  is a constant value and  $s_0$  is slip at maximum shear stress. Recommended values by DIANA for  $c$  and  $s_0$  which are also adopted for FE analysis here are  $c = f_t$  where  $f_t$  is concrete tensile strength and  $s_0 = 0.06 \text{ mm}$ . Secant approach is used to model unloading and reloading of interface in shear. Beside the recommended values which are used in analysis, linear tangential stiffness can be calculated by  $k_t = 5 \frac{f_t}{0.06}$  and linear

normal stiffness is set to  $k_n = 1000 \text{ N/mm}^2 / \text{mm}$  as a constant value.

Results of bond-slip model are presented in Figs.5.13 and 5.14. The pre-peak response of specimens with bond slip model is softer than that of perfect bond as expected. Except B6 where in premature failure occurred, all other specimens have been closer to experiment by means of bond slip modeling. Iterative solution did not have severe numerical obstacle in either cases and finite element mesh changes did not affect solution procedure. Triangles in Fig.5.13 indicate the points where in convergence criterion could not be satisfied during iterative process.

Tensile strain of lower longitudinal reinforcements is monitored during test process by a number of strain gauges attached to one of the reinforcement. Figure 5.14 illustrate tensile strain of reinforcement monitored during test and predicted by analysis in B103-2 specimen. Seven strain gauges are attached to the middle lower reinforcement nearby support but did not extend to hook area to evaluate analytical results in that region as well. Nevertheless tendency of test results however show that strain of end region are also well predicted by analysis. Levels of selected loads for evaluation are 300 KN; almost linear just before or vicinity of bending crack, 600 KN; around shear crack extension, 1000 KN; before peak load but shear cracks are already extended to shear span, 1380 KN; peak load in analysis with bond slip model and 1787 KN which is experiment peak load. As mentioned before stress of reinforcement with sliding bond increases as the stiffness of bond particularly in tangential direction decreases. This theory is also confirmed by this figure.

Crack patterns predicted by either perfect bond or sliding bond assumption and the one observed in experiment are also compared in Fig.5.15. It is noteworthy to mention that the load level in analysis and experiment is not identical since crack patterns compared here belongs to final stage of beam around 1787 KN whereas analysis could never reach to that point and failed in around 1380

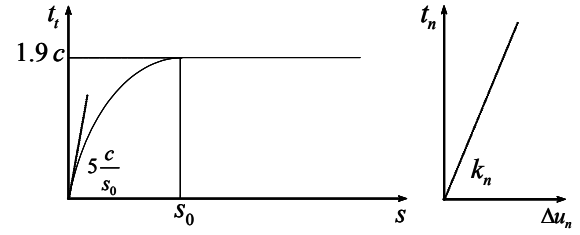


**Fig.5.12.** Dorr model for shear traction (left); linear elastic model for normal traction (right)

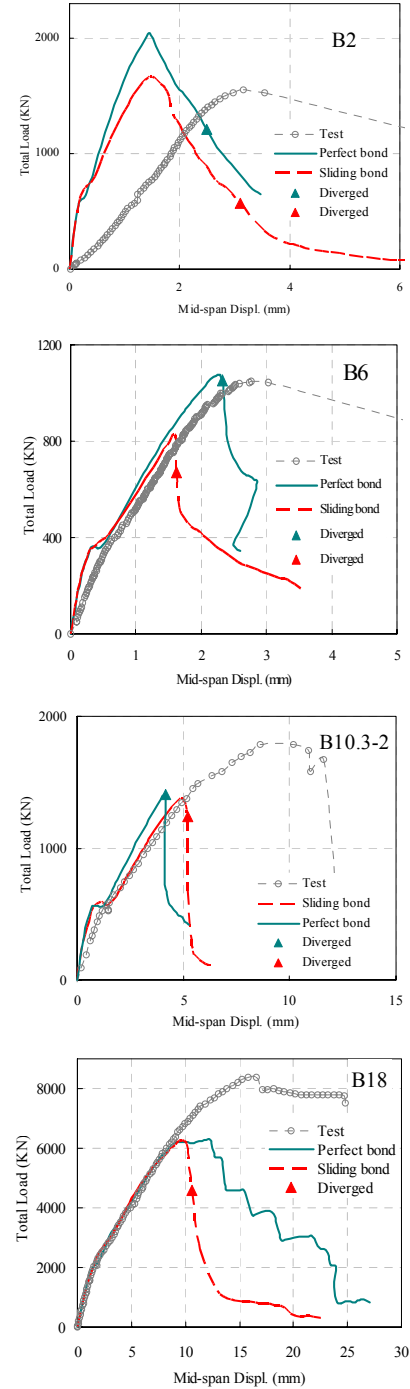


**Fig.5.13.** Comparison between response with bond-slip model and perfect bond versus experiment

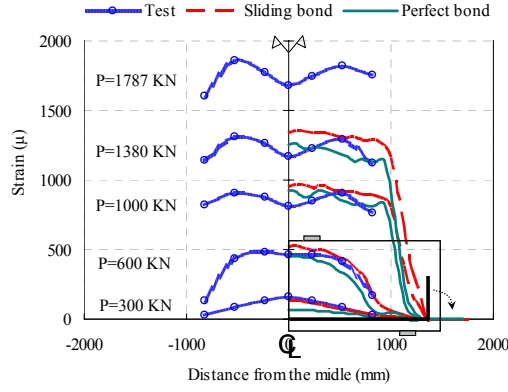


**Fig.5.14.** Strain in lower tensile reinforcement of B10.3-2 in different load level

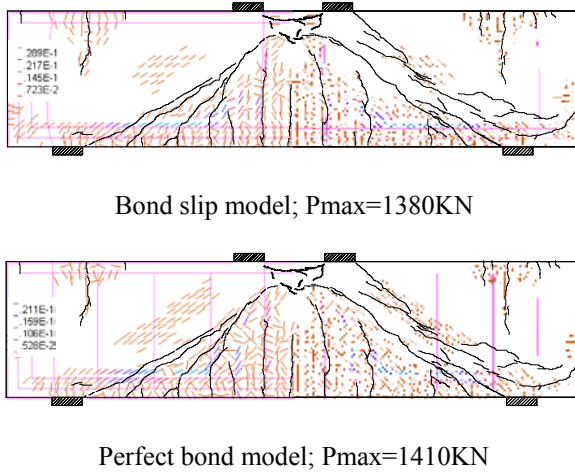


**Fig.5.15** Crack patterns of B10.3-2 at final stage (Test cracks are drawn with black lines at test maximum load Pmax=1787KN).

KN. The comparison makes sense however since all shows final crack patterns in either analysis or experiment. Both analyses could adequately predict crack propagation at least during final step. Perfect bond though produced more cracks but cracks in bond slip model are wider due to crack width modulates given on the left side of the cracked specimens (Fig.5.15). Furthermore bond failure took place in both models by means of sever cracking along upper longitudinal reinforcement. Those cracks might happen inside specimens around bar reinforcement but no evidence could be traced or measured during test process. Consequently evaluation of those cracks either the existence or the measurement could not be verified by experiment. It can be concluded however that crack patterns could be very well predicted by analysis either using bond slip model or perfect bond assumption.

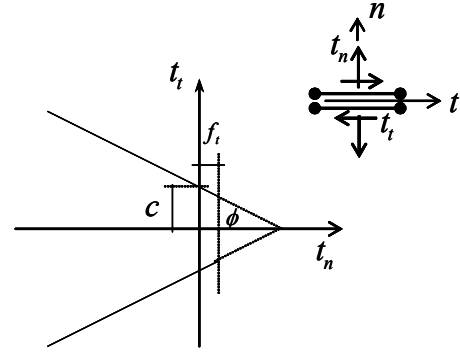


**Fig.5.16.** Coulomb friction model

### 5.3.2 Gap element to model contact between steel plates and concrete body

Basic idea of using interface element in modeling gap element is almost similar to that of explained in preceding section for bond-slip model. In this case under tension if the stress goes beyond the tensile strength of the gap element, two parts will be separated and no further interaction will be recorded. Coulomb friction model can express this phenomenon by means of the criterion shown in Fig.5.16. This model has a close resemblance with Mohr-Coulomb plasticity model as well. Traction vector and relative displacement vector are as followings:

$$t = \begin{Bmatrix} t_n \\ t_t \end{Bmatrix} \quad (5.5)$$

$$\Delta u = \begin{Bmatrix} \Delta u_n \\ \Delta u_t \end{Bmatrix} \quad (5.6)$$

Friction angle is indicated by  $\phi$  and cohesion by  $c$ . The Associated plasticity where in dilatancy angle and friction angle are equal as well as the Non-associated plasticity can be implemented in this model. Detail and formulation of the model is given in reference [5.4].

In order to investigate effect of connection between steel plates and concrete at support and loading points, depicted model is applied in analysis of B18 as a representative specimen. The objective of using interface is to study effect of steel plate stiffness as well as compressive stress accumulated at the tip of loading plate. Stress concentration in this region led to premature failure in some cases such as B18. Parameters used in the analysis are  $\tan \phi = 0.6$ ,  $c = 3 \text{ MPa}$  and elastic normal and tangential

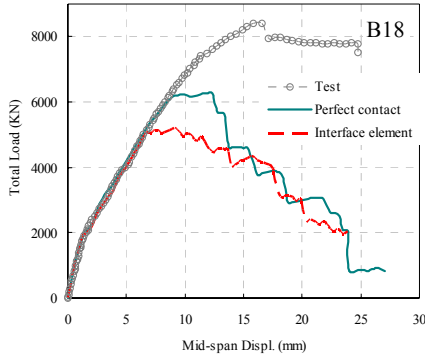


**Fig.5.17** Result with Interface element

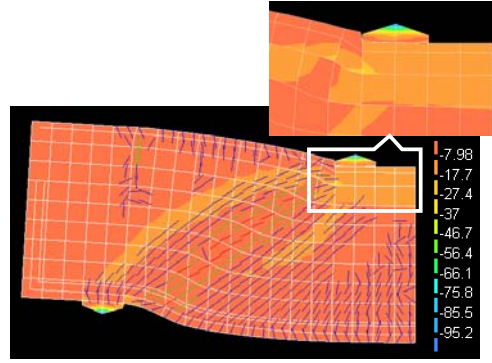
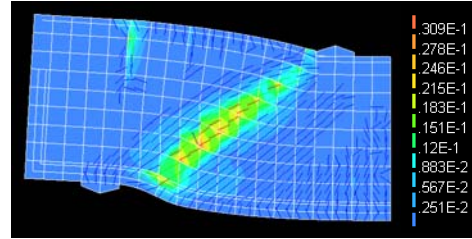
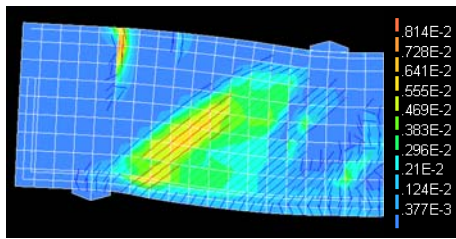
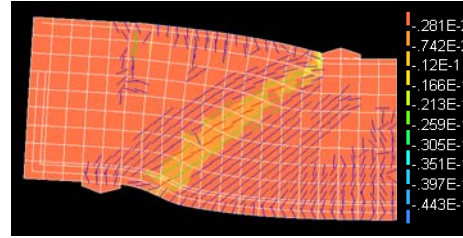
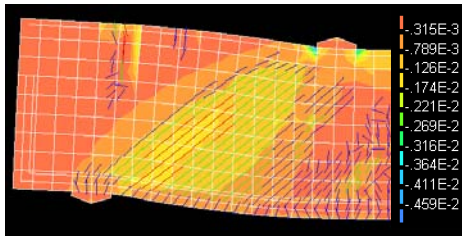


**Fig.5.18** Second principal stress after the peak. Large deformation is clearly formed behind the plate.



**a)** First principal strain at peak (left) and after the peak (right)



**b)** Second principal strain at peak (left) and after the peak (right)

**Fig.5.19** Principal strains and corresponding crack patterns using interface elements

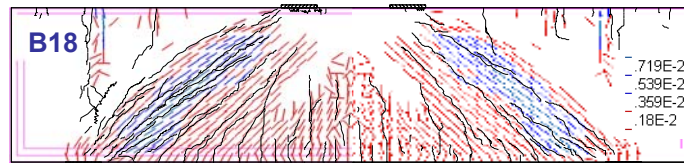


**Fig.5.20** Analytically predicted versus test crack pattern for B18 at peak load.

stiffness are set to  $k_n = k_t = 30000$ . The gap element is a brittle no-cracking element with frictional behavior. It is noted here that for this specific analysis different scheme has been used. This is because of very early instability in analytical procedure occurs when conventional Newton-Raphson method is applied. Secant method

however showed very stable procedure even after the peak with also very consistence crack patterns without undesired rotation of cracks in consequent calculation.

Another effective issue in analysis was stiffness of loading and supporting plates where due to weak tensile strength in contact steel plates, they tend to bend up by means of separation at the edges. This

behavior also yields numerical difficulty which was prevailed by using thicker size at the middle. Beam response with or without interface element modeling is depicted in Fig.5.17. The result shows that beam even failed in smaller load than the one without gap element. To investigate the reason Figure 5.18 illustrates second principal stress right after the peak, relative crack patterns as well as a close up of loading plate vicinity where strain accumulation is clearly seen. What gap element has done here is just to shift stress concentration at the tip of the plate to the other side of the plate, where in shear band is formed due to severe strain localization stress is in crucial state. However, in the following steps beam fails in shear mode with a large shear deformation in the region where shear band is formed. This does not imply that application of interface element may always give rise to such premature failure, shifting stress to the concentrated zone or etc but it might be the matter of theoretical assumptions or even selected parameters which yield such results. More investigation and verification is necessary if analysis with interface element is vital. It is noted that in analysis here Secant method is employed since Newton-Raphson faced premature numerical instability at the very early load steps. The method however could properly overcome the difficulties and very smooth solution process is achieved. First and second principal strains in vicinity of the peak as well as the peak load along with corresponding crack patterns are also shown in Fig.5.19. Beside crack distribution and development that could be predicted with excellent agreement with experimental observation, shear band formation is also adequately captured by analysis (Figs.5.19). Despite using rotating crack approach, cracks directions are very stable with minimum of variation in terms of load increment.

On the other hand selected solution procedure (Secant method), though irrelevant to application of interface element, could produce very consistence crack patterns with excellent compatibility with experiment (Fig.5.20 in particular). In spite of Newton-Raphson procedure which in every step crack would rotate freely, cracks directions here are much stable and follow the preceding step consequently. Moreover secant method faced less numerical obstacle than that of Newton-Raphson or its generic methods and almost all load steps ended with a reasonably converged iterative solution. More evidence however is necessary to examine capability of the method in different failure modes, geometries and those who might have crucial influence in analytical procedure.

## 5.4 Conclusion

In order to best simulate RC deep beams by finite element analysis, a sensitivity analysis has been conducted first in Chapter 4. The material model proposed by JSCE, Hordijk and Feenstra are applied in finite element modeling. On the other hand cracked concrete is modeled by the fixed crack theory as well as the rotating crack theory. To limit the number of comparison and prevent confusion of different parameters effect of RC member behavior, bond model as well as friction model by means of interface element between steel plates and concrete body is not included in analysis. At the first investigation Hordijk tensile model and Feenstra compressive model could predict the behavior of the specimens more reasonably than JSCE material model. The model proposed by JSCE particularly in compression lacks mainly the control of sudden drop in load capacity which might be attributed to sudden drop of strength in compressive model. For the beam with higher possibility of compressive failure in either compressive zone or struts, softening path after a certain strain in post peak regime can control numerical problems due to sudden change in stress state and also smoothened the predicted load-deflection response particularly near the peak and finally prevent premature failure of the beam. The analyses with JSCE model have encountered almost all those mentioned difficulties.

The best models selected for concrete for further investigation are Hordijk tensile model, Feenstra fracture model for compression, rotating crack model with either Newton-Raphson scheme or Arc-length procedure associated with line search to expedite convergence rate in iterative solution procedure. The suggested methods are applied for analytical investigation of this chapter along with some more detailed analyses such as bond slip modeling as well as the Gap element applied to simulate contact between concrete and steel plates.

The applied methods however could predict RC deep beams response to monotonic static load with acceptable accuracy. Predicted crack patterns are also evaluated by experiment observation. Very close correlation have been obtained which manifests the level of simulation is quite accurate and applicable in damage evaluation of RC structures. Almost all important cracks are captured by analysis properly. In order to investigate effect of bond between concrete and reinforcement, a cubic model (Dorr model with cubic relationship between shear stress and slip in bond) to simulate bond-slip phenomenon is applied. The behaviors of specimens in pre-peak regime have been improved

well toward test results. It is however noted that in at least one case, load capacity predicted by including bond slip in analysis was lower than that of analysis with perfect bond assumption. It is however important to decide at the early stage of study if a very detailed analysis is necessary or those obtained with perfect bond assumption fulfill the purpose of the analysis. This is due to the fact that modeling bond-slip makes geometry; finite element mesh construction as well as the analytical procedure more sophisticated than perfect bond modeling where in reinforcement can be simply embedded in concrete element.

Other obstacle is employing gap interface element to model connection between steel plates and concrete body. The connection associates with two components of load and displacement in 2-D space. Normal traction in tension has a limit of tensile strength where in two parts are separated if the stress goes beyond characteristic tensile strength of interface. On the other hand Coulomb friction approach has been applied to model behavior of interface in tangential direction. The result with gap element showed a shift of compressive stress concentration from the front tip of the plate to the other side of the plate where in stress has been already concentrated in that region just before shear band formation. This new stress accelerates strain localization process and eventually beam fails in smaller load capacity with a clear shear band formation. It is however no consensus on if interface gap element will improve analytical results or not since not much evidence is available. Furthermore the parameters of gap element and the model itself may results sensitivity of prediction which may require parametric study to be carried out. Brief conclusions of this chapter and preceding chapter are drawn below:

1. Fracture type material models can adequately eliminate mesh size effect in analytical responses of RC beams.
2. The model particularly in compression should follow a suitable softening path to exhaust fracture energy gradually and prevent premature failure and drop in load capacity as it was observed in JSCE compressive model.
3. The rotating crack model could predict response of RC beams failed in shear properly. The fixed crack approach lacks sensitivity to employed shear retention factor seriously.
4. If the concern of analysis is to predict post-peak behavior, displacement control will be quite adequate. On the other hand Arch-length scheme is suggested to be employed if possibility of snap-back instability is rather

high and purpose of analysis pertains to evaluate post-peak regimes as well.

5. Predicted crack patterns have been surprisingly well agreed with test observation and almost all important cracks could be captured by analysis properly.
6. Secant method however prevails over numerical difficulties better than Newton-Raphson scheme with much consistent crack patterns in terms of crack direction and development. This method is suggested for further investigation.

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## 6. CONCLUSION

Experimental and analytical studies on RC deep beams have been carried out in Earthquake Engineering Research Team of Public Works Research Institutes during year of 2004 and 2005. Experimental results are already discussed in some other publications and reports but here a codified study based on two popular Japanese design codes, JSCE and JRA as well as numerical simulation with finite element method are reported. The beams have  $a/d$  ratio (shear span to effective depth ratio) between 0.5 to 1.5 cover a wide range of RC deep beams in real application. The first part of the study pertains to evaluate code prediction with test results to investigate if the codes, particularly JRA, can adequately estimate shear load capacity of RC deep beams under monotonic load. Second to investigate size effect on shear strength of members by means of test results as well as design code prediction. The following conclusion can be drawn for this part:

1. JRA code can adequately predict shear capacity of RC deep beams with an acceptable safety margin and  $a/d$  effect on shear strength of such members are properly included in design process.
2. JSCE code on the other hand results very conservative prediction for larger  $a/d$  ratio and unsafe for  $a/d=0.5$ . This implies the results are not so consistent in terms of  $a/d$  variation. Meanwhile complementary JSCE standard specification have better prediction and more reasonable definition of deep beam in terms of  $a/d$  ratio.
3. Member size effect on shear strength of RC beams has been adequately included in either code. Although JSCE equation yields better agreement with experiment but the difference of two codes is negligible and well agree with test results. The only appreciable difference between the codes lies on the beams with depth smaller than 1000mm which JRA limits the coefficient to one if capacity concept is adopted for design. In contrast size effect coefficient in JSCE goes as far as 1.5 and attributes shear strength to the size effect up to 50% higher in members with depth less than one meter.
4. Since JRA code is mainly in tabulated form it is suggested however background formulation are also presented in the code for further investigation.
5. Continuous deep beams behavior was not the objective of this study due to lack of test evidence but it is suggested to take into account effect of moment and shear direction change in

definition of shear span, mechanism of failure as well as verification of code results with experimental evidence.

In order to establish a platform for further investigation with less dependency on test results verification finite element simulation of RC deep beams has been carried out in second part of this study. Since the number of proposed material models, crack models as well as solution process are quite large and confusing, first sensitivity analysis on effective parameters is carried out. This study showed however that analytical results may not be always objective. A number of parameters such as material and crack models as well as finite element mesh size are investigated in this section. Sensitivity in terms of element size in finite element discretization is crucial if fracture energy is not taken into account or the element is not enriched or improved to handle this problem. Higher sensitivity of the results is attributed to tensile constitutive model which in JSCE and Hordijk models both take into account the fracture energy of concrete. JSCE compressive model however does not include fracture energy concept.

According to the applied crack models, the fixed crack theory and the rotating crack theory are employed in some case study analyses. In contrast to the rotating crack theory, the fixed theory lacks of producing consistence prediction in terms of different shear retention factors. Although this theory can be adequately applied in analysis of RC members failed in flexural mode, the method failed however to produce objective results for members dominated by shear such as RC deep beams.

In Chapter 5 the results of Chapter 4 on sensitivity analysis of RC members have been utilized to analyze entire specimens and some more detailed discussions on crack occurrence, direction and development, effects of bond slip modeling as well as interface gap element between loading plate and steel plates have also been made.

The main findings of this part of study are categorized in the following items:

1. Fracture type material models best fit to the nature of concrete and adequately eliminate mesh size effect in analytical responses of RC element.
2. The compressive model proposed by JSCE gave rise to sudden drop on load capacity of the specimen while Feenstra model did not. This problem is correlated to absent of softening path in post-peak region of JSCE constitutive model.

3. Although the analysis based on Hordijk tensile model, JSCE bilinear tensile constitutive model is also supposed to produce similar results.
4. Among crack models the rotating crack approach is found much reliable if shear dominant behavior of RC members. The fixed crack model however suffers from the sensitivity of the results to the employed shear retention factor.
5. Pre-peak regime could be properly captured by using Newton-Raphson solution method under displacement control loading process. Arc-length scheme is suggested however if the analysis pertains to study post-peak regime and the snap-back instability has high possibility of occurrence.
6. Correlation between predicted cracks by analysis and monitored during test process is suitably adjusted. Almost all important cracks including those of tensile cracks on upper face of the beam near to the end region are very well captured. The predicted crack patterns therefore can be well utilized for damage evaluation of RC members under monotonic loading condition.
7. Although bond-slip model produces better results in pre-peak regime but due to its complexity and also introducing some more parameters in analysis which itself can be a source of sensitivity in analysis. Perfect bond assumption is however suggested unless a detailed analysis is vital, to be applied. The results with perfect bond have been quite acceptable and applicable in real practice.
8. It is noted here though most of analyses have been carried out by using Newton-Raphson scheme, secant method produced better results in terms of more stable and consistence crack pattern with less numerical difficulties. This method is suggested for further investigation.
9. The results confirmed that the applied finite element method can be properly used for further parametric study and analyzing new RC members though in certain cases such as totally different loading patterns or geometry, experimental verification is recommended.

Further investigation on stirrups effect on improving shear behavior of RC deep beams, repeating and dynamic load as well as multi-support continuous deep beams is recommended.

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