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STATIC AND QUASI-STATIC SEISMIC RESPONSE
ANALYSES OF BURIED PIPELINES

BY

EIICHI KURIBAYASHI

LEON RU-LIANG WANG

AND

TAKAYUKI HADATE

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PUBLIC WORKS RESEARCH INSTITUTE
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Tsukuba Science City, Ibaraki-Prefecture 305, Japan

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Static and Quasi-static Seismic Response Analyses of Buried Pipelines

ABSTRACT

This report describes the results and methods of seismic response analyses for the buried pipelines subjected to ground shaking. The results are consist of each result of upper bound analyses, static analyses and quasi-static analyses.

In upper bound analyses, the results show highest level of the response behavior of buried pipelines. Accordingly the bounds of the response are as a basis of normalization of the results of static analyses and quasi-static analyses.

In static analyses and quasi-static analyses including elastic and elasto-plastic behavior of the soil and joint springs, the results show lowest level of the response behavior of buried pipeline. Because the slippage between soil and pipe arises at the yielded portion in the soil.

In these analyses, assumptions, formulations and results of each analysis are derived from seismological and geological standpoints.

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Leon Ru-Liang Wang
Eiichi Kuribayashi
and
Takayuki Hadate

I. INTRODUCTION

Recent studies^(1, 2, 3) have shown that buried gas, water and sewer pipelines have been damaged heavily by earthquakes. Other than the catastrophic failures such as landslides and soil liquefactions, buried pipelines, damages caused by earthquake excitations (effects of ground wave propagation) have been found to be a major mode of failure⁽⁴⁾. Also, the axial responses of long buried pipelines has been found to be predominant during seismic ground shaking^(4, 5). The dynamic effects of buried pipelines has been observed to be small⁽⁵⁾ due to high restraints and damping from the surrounding soils and are neglected. State-of-the-art papers^(6, 7, 8) on the subject have been reported.

The analysis and design of buried pipelines, which by their nature have both temporal and spatial variations, are much different from those of buildings. Presently, there are no codified provisions for the design of buried pipelines to resist seismic loads in the United States. To evaluate the adequacy of the existing pipelines and to aid the design of future systems, this paper summarizes the currently available static and quasi-static analyses for the seismic response behavior of buried pipelines (i.e., pipe strains, relative joint displacements/rotations between pipe segments and relative displacements between the pipe and the ground due to seismic ground shaking)^(9, 10, 11, 12, 13)

The analyses reported include upper bound analyses by simplified and quasi-static approaches; static analyses based on beams on elastic foundation and beams on elastic-plastic foundation; quasi-static analyses for elastic and elasto-plastic responses for long straight (continuous or segmented) buried pipelines.

Although the discussions for some analyses have been presented and published^(14 to 15), the assumptions, formulations and results of these analyses are grouped together in the report for the purposes of comparison and completeness.

II. UPPER BOUND ANALYSES

II.1 Simplified Approach

Although there are currently no codified provisions to design underground pipelines for earthquake effects, a 'Simplified procedure'⁽¹⁶⁾ to estimate the underground pipe strains and curvatures due to seismic shaking has existed for some time. Basically, the analysis assumes no relative motion between the pipe and the ground. Thus, as upper bounds, one can take the seismic ground strains as the pipe strains and the seismic ground curvature as the pipe curvatures. This is equivalent to assuming that the pipe has no stiffness, and therefore follows the ground exactly. Details of this approach can be found in References 15 and 20.

Assuming that the seismic wave shape remains constant while traversing the pipeline, the maximum axial strain of the ground, $\epsilon_{\max}^*$, which will be taken as the upper bound of the axial strain of the pipe, $\epsilon_{p, \max}$, due to ground shaking will be:

$$\epsilon_{p, \max} = \epsilon_{\max} = V_{\max}/C_p \quad (1)$$

The maximum ground curvature, $X_{\max}$, which will be taken as the upper bound of the maximum curvature of the pipeline, $X_{p, \max}$, will be:

$$X_{p, \max} = X_{\max} = A_{\max}/C_s^2 \quad (2)$$

where $V_{\max}$ is the maximum ground velocity and $A_{\max}$ is the maximum ground acceleration during a seismic event at the site; C_p and C_s are the longitudinal (compressive) and transverse (shear) wave propagation velocities respectively of the controlling environments with respect to the pipeline.

If a continuous piping system can meet both sets of upper bound criteria (strain and curvature), the pipeline will be adequate against earthquakes producing ground velocities and accelerations less than the $V_{\max}$ and $A_{\max}$ used in the analysis.

For segmented pipelines (Fig. 1), the maximum relative joint displacements and maximum joint rotations become important design parameters in addition to the pipe strains and curvatures. If we assume that the pipeline consists of rigid segments which have their mid points move with the ground exactly, then the maximum relative motion/rotation between two points on the ground will be entirely taken up by the relative displacements and rotations of segments at the joints. Hence, the upper bounds of maximum relative joint displacement, $U_{p, \max}$ and maximum relative joint rotation, $\theta_{p, \max}$, of the pipeline shown in Fig. 2 can be expressed as:

$$U_{p, \max} = \epsilon_{\max} L \quad (3)$$

$$\theta_{p, \max} = X_{\max} L \quad (4)$$

where L is the length of the pipe segment; $\epsilon_{\max}$ and $X_{\max}$ are the maximum free field ground strain and curvature defined in Eqns. (1) and (2) respectively.

If a buried segmented piping system can meet all four sets of upper bounds (pipe strain and curvature; joint displacement and rotation) specified in Eqn. (1) to Eqn. (4) for a design earthquake, the pipeline will be conservatively safe because in the real case, the pipe strain and relative joint displacement will jointly take-up the imposed ground strain and both the pipe curvature and joint rotation will jointly take-up the imposed ground curvature.

Since these upper bounds are unique and simple to obtain, they will be used as a basis for the normalization of results presented later.

* Symbols which are consistent within a section are defined when they first appear in the section.

II.2 Quasi-static Approach

II.2.1 General Description and Assumptions

In the simplified approach for the upper bounds, pipe strains and curvatures are estimated from free field conditions. For relative joint displacements and rotations, what one needs to know about the pipeline is segment length. There is no soil-structure interaction involved.

However, in reality, the pipeline (typical of water/sewer transmission lines) is made up of pre-fabricated pipe segments. These pipe segments are connected at joints, which are sealed by a rubber gasket or caulked by cement/lead. Figure 1 shows a buried piping system schematically.

Due to the motion of the ground relative to the pipe segment during earthquakes, resistance between the pipe and the surrounding soil develops. To model such soil resistance, a uniformly distributed linear soil spring, k , is used.

The joint resistance between two pipe segments is modeled by a joint spring and a dashpot. Note that the joint spring constant from a rubber-gasket, cement-caulked or lead-caulked joint is in general very small as compared to the stiffness of pipe itself or the resultant soil spring constant along a pipe segment.

Under seismic excitation, both pipe segments and joint springs are all subjected to the imposed ground displacements/strains. However, it is anticipated that most of the ground displacements/strains will be absorbed by the movements of joint springs and very little by straining of the pipes. For simplicity and conservatism (in estimating joint extension/contraction) purposes, all pipe segments will be assumed to be infinitely rigid. Thus, the upper bound for the relative joint displacements of segmented pipelines can be obtained. Furthermore, such simplification would enable us to verify some observations of seismic response of buried pipelines by the Japanese(4, 5) initially.

An earthquake motion traveling along a pipeline resembles the problem of wave propagation in an elastic media. An incident earthquake at one end of a pipe segment will not simultaneously reach the other end some distance away. Thus, for wave propagation problems, a time lag is generally associated with the wave in the direction of propagation. Since the dissipation of seismic wave energy is negligible along a pipeline during the period of investigation, we assume that the wave form remains constant in the course of propagation. More assumptions will be made later in related sections.

II.2.2 Formulation of the Problem

A long buried piping model consisting of n -segments is shown in Fig. 3 where $M_1, M_2, \dots, M_i, \dots, M_n$ are equivalent masses of each segment of underground pipes which should include the mass of the pipe and the soil that moves with the pipe; $K_1, K_2, \dots, K_i, \dots, K_{n-1}$ and $C_1, C_2, \dots, C_i, \dots, C_{n-1}$ are spring constants and damping coefficients at joints between pipes, K_0, K_n and C_0, C_n are spring and damping constants at the end supports.

The coordinates that define the motion of the ground and the pipe during an earthquake are also shown in Fig. 3 in which $x_1, x_2, \dots, x_i, \dots, x_n$ are longitudinal displacements of mid-sections of pipe segments; $x_{G1}, x_{G2}, \dots, x_{Gn}$ are the corresponding ground displacements in the direction of the pipeline axis; x_{G0} and x_{Gn+1} are the ground movements at the ends; $L_1, L_2, \dots, L_i, \dots, L_n$ are pipe segment lengths.

Referring to Fig. 4, the dynamic equilibrium equation for pipe segment i is:

$$M_i \ddot{x}_i + F_{i-1} - F_i + f_i = 0 \quad (5)$$

in which $M_i \ddot{x}_i$ is the inertia force; F_{i-1}, F_i are joint resistant forces at both ends of the pipe and f_i is the resistance from soil surrounding the pipe.

Using the coordinates, $x_i > x_{G1}, x_i > x_{i-1}, x_{i+1} > x_i$, the above resistant forces can be expressed as:

$$\begin{aligned}
F_i &= C_i(\dot{x}_{i+1} - \dot{x}_i) + K_i(x_{i+1} - x_i) \\
F_{i-1} &= C_{i-1}(\dot{x}_i - \dot{x}_{i-1}) + K_{i-1}(x_i - x_{i-1}) \\
f_i &= d_i L_i(\dot{x}_i - \dot{x}_{Gi}) + k_i L_i(x_i - x_{Gi})
\end{aligned} \tag{6}$$

where d_i and k_i are damping and spring constants per unit length of surrounding soil.

Carrying out the equilibrium equations for all segments, the system of equations of motion for a pipeline of n -segments in matrix form will be obtained as follows:

$$\begin{aligned}
[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} \\
= [d] \{\ddot{x}_G\} + [k] \{\dot{x}_G\} + [C'] \{\dot{x}_G'\} + [K'] \{x_G'\}
\end{aligned} \tag{7}$$

where $[M]$, $[C]$ and $[K]$ are the system mass, damping and stiffness matrices; $\{\ddot{x}\}$, $\{\dot{x}\}$ and $\{x\}$ are the acceleration, velocity and displacement vectors of the pipeline system; $\{\ddot{x}_G\}$, $\{\dot{x}_G'\}$, $\{x_G\}$ and $\{x_G'\}$ are the ground and support velocity and displacement vectors; $[d]$, $[k]$, $[C']$ and $[K']$ are soil and support damping and spring matrices.

II.2.3 Earthquake Motion Inputs

In order to study the seismic response of buried pipelines governed by the equation of motion shown in Eqn. (7), the ground velocity and ground displacement time histories for every station must be known. As indicated earlier, the wave form is assumed to remain constant during the course of investigation, the earthquake motion vectors $\{\ddot{x}_G\}$ and $\{x_G\}$ can be more realistically represented by incorporating a delay time, which is the time required for the seismic wave to travel from one segment to another. Accordingly, just one earthquake time history data is needed for the analysis.

The inputs of the time-space varying ground motions starting from the first end restraint are:

$$\begin{aligned}
\ddot{x}_{Go} &= A_{\max} f(t) \\
\dot{x}_{Go} &= V_{\max} g(t) \quad \text{for all } t \geq 0 \\
x_{Go} &= \Delta_{\max} h(t)
\end{aligned} \tag{8a}$$

$$\begin{aligned}
\ddot{x}_{Gi} &= 0 \\
\dot{x}_{Gi} &= 0 \quad \text{for } t < t_{d,i} = \sum_{j=1}^{i-1} \Delta T_j + \Delta T_i / 2 \\
x_{Gi} &= 0
\end{aligned} \tag{8b}$$

$$\begin{aligned}
\ddot{x}_{Gi} &= A_{\max} f(t - t_{d,i}) \\
\dot{x}_{Gi} &= V_{\max} g(t - t_{d,i}) \quad \text{for } t \geq t_{d,i} = \sum_{j=1}^{i-1} \Delta T_j + \Delta T_i / 2 \\
x_{Gi} &= \Delta_{\max} h(t - t_{d,i})
\end{aligned} \tag{8c}$$

where $A_{\max}$, $V_{\max}$, $\Delta_{\max}$ are expected maximum values of acceleration, velocity and displacement of seismic data. $\Delta T_i = \frac{L_i}{C_{pi}}$ is the delay time for a seismic wave traveling L_i -distance of pipe; C_{pi} = wave velocity of soil surrounding i^{th} segment of the pipeline.

II.2.4 Quasi-static Model

The system of dynamic equations developed in the previous sections involved inertia and damping effects, which, together with the time delay effect in ground excitations, become an extremely tedious problem to solve. Furthermore, it is very difficult, if not impossible, at this time to obtain reasonably accurate values for the equivalent mass and damping coefficients for the system.

For general design purposes, it is desirable that the design procedure be simple and the analysis be economically feasible, yet reasonably correct. For this reason, we need a simple model which will provide results similar to the exact model for the same problem.

In recent years, several investigations^(6, 5) done in Japan, indicated that dynamic effects are not significant in the behavior of underground piping systems. In view of this information, we will convert the dynamic model into a simple quasi-static problem by neglecting the $\{\ddot{x}\}$, $\{\dot{x}\}$, $\{\ddot{x}_G\}$ and $\{\dot{x}_G\}$ terms. For simplicity, we assume equal pipe segment length in the analysis model. The system of equations becomes:

$$[K] \{x\} = [k] \{x_G\} + [K'] \{x_G'\} \quad (9)$$

in which $[K]$ is a tridiagonal-symmetrical matrix, $[k]$ and $[K']$ are diagonal matrices. Or in expanded form, the governing equations of equilibrium are as follows:

$$\begin{aligned}
 & \begin{pmatrix} \bar{K}_1 & -K_1 & & & \\ -K_1 & \bar{K}_2 & -K_2 & & \\ & \ddots & \ddots & \ddots & \\ & & -K_{i-1} & \bar{K}_i & -K_i \\ & & & \ddots & \ddots \\ & & & & -K_{n-1} & \bar{K}_n \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{Bmatrix} = \begin{pmatrix} k_1 L & & & & \\ & k_2 L & & & \\ & & \ddots & & \\ & & & k_i L & \\ & & & & \ddots \\ & & & & & k_n L \end{pmatrix} \begin{Bmatrix} x_{G1} \\ x_{G2} \\ \vdots \\ x_{Gi} \\ \vdots \\ x_{Gn} \end{Bmatrix} \\
 & \quad n \times n \qquad \qquad n \times 1 \qquad \qquad n \times n \qquad \qquad n \times 1 \\
 & + \begin{pmatrix} K_0 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ & & & & & 1 \\ & & & & & & K_n \end{pmatrix} \begin{Bmatrix} x_{G0} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ x_{Gn+1} \end{Bmatrix} \quad (10a) \\
 & \quad n \times n \qquad \qquad n \times 1
 \end{aligned}$$

where

$$\bar{K}_i = K_{i-1} + K_i + k_i L_i \quad (10b)$$

Note that the ground displacement vectors, $\{\underline{x}_G\}$ and $\{\underline{x}'_{G1}\}$ are constantly changing from one instant to another due to the delay time effect although the wave form remains unchanged.

For simplicity, we further assume a constant delay time, DT for the seismic wave traveling from one segment to another in a given system. Using the first pipe segment as the time reference, the input ground displacement vectors can be defined as follows:

$$\{\underline{x}_G\} = \begin{Bmatrix} x_{G1} \\ x_{G2} \\ \bullet \\ \bullet \\ x_{Gi} \\ \bullet \\ x_{Gn} \end{Bmatrix} = \Delta_{max} \bullet \begin{Bmatrix} h(t) \\ h(t-DT) \\ \bullet \\ \bullet \\ h(t-[i-1]DT) \\ \bullet \\ h(t-[n-1]DT) \end{Bmatrix} \quad (11a)$$

$$\{\underline{x}'_G\} = \begin{Bmatrix} x_{G0} \\ 0 \\ 0 \\ \bullet \\ \bullet \\ 0 \\ 0 \\ x_{Gn+1} \end{Bmatrix} = \Delta_{max} \bullet \begin{Bmatrix} h(t + \frac{1}{2}DT) \\ 0 \\ 0 \\ \bullet \\ \bullet \\ 0 \\ 0 \\ h(t - [n - \frac{1}{2}]DT) \end{Bmatrix} \quad (11b)$$

Now, let us define the following parametric constants for the quasi-static analysis:

1) Normalized Joint Spring Parameters

$$\alpha_i = K_i/k_i L \quad i = 1 \text{ to } n-1 \quad (12)$$

2) Normalized End Restraint Spring Parameters

$$\gamma_0 = K_0/k_1 L \quad (13a)$$

$$\gamma_n = K_n/k_n L \quad (13b)$$

3) Soil Resistant Spring Ratios

$$\beta_i = k_i/k_1; \quad i = 1 \text{ to } n \quad (14)$$

Substituting these parameters [Eqns. (12) to (14) into Eqn. (10)], the system of the governing equation becomes:

$$\begin{aligned}
& \begin{bmatrix} (\gamma_0 + \alpha_1 + 1) & & & & & \\ & -\alpha_1 & & & & \\ & & (\alpha_1 + \alpha_2 \beta_2 + \beta_2) & & & \\ & & & -\alpha_2 \beta_2 & & \\ & & & & \ddots & \\ & & & & & (\alpha_{i-1} \beta_{i-1} + \alpha_i \beta_i + \beta_i) & \\ & & -\alpha_{i-1} \beta_{i-1} & & & & -\alpha_i \beta_i \\ & & & & & & & \ddots \\ & & & & & & & & (\alpha_{n-1} \beta_{n-1} + \gamma_n \beta_n + \beta_n) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{Bmatrix} \\
& = \begin{bmatrix} 1 & & & & \\ & \beta_2 & & & \\ & & \ddots & & \\ & & & \beta_i & \\ & & & & \ddots \\ & & & & & \beta_n \end{bmatrix} \begin{Bmatrix} x_{G1} \\ x_{G2} \\ \vdots \\ x_{Gi} \\ \vdots \\ x_{Gn} \end{Bmatrix} + \begin{bmatrix} \gamma_0 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & \ddots \\ & & & & & \gamma_n \beta_n \end{bmatrix} \begin{Bmatrix} x_{G0} \\ 0 \\ \vdots \\ 0 \\ \vdots \\ x_{Gn+1} \end{Bmatrix} \quad (15)
\end{aligned}$$

in which all elements in the matrices are dimensionless quantities. It is interesting to note that this governing equation for the analysis of buried pipelines does not involve pipe segment length variable, L , since its effect has been built into the relative values of α , β and γ . Furthermore, the effects of pipeline length can be studied by varying the number of segments and the magnitude of DT used in the analysis.

For the analysis, the following special cases of α , β and γ were studied:

- 1) Pipeline buried in a uniform soil environment,

$$\beta_2 = \beta_3 = \dots \beta_i = \dots \beta_n = 1 \quad (16)$$

- 2) Pipeline without end-restraints

$$\gamma_0 = 0 \quad \text{and} \quad \gamma_n = 0 \quad (17)$$

- 3) Pipeline with uniform joint construction

$$\alpha_1 = \alpha_2 = \dots \alpha_i = \dots \alpha_{n-1} = \alpha \quad (18)$$

which is assumed to be the case in this paper. As indicated earlier, the joint resistance is much smaller than the soil resistance along a pipe segment, thus α in general will be a small quantity even though the effects of α are studied parametrically.

II.2.5 Method of Solution

Since the system of governing equations shown in Eqn. (15) for the quasi-static model involves only symmetrically tridiagonal and diagonal matrices, it is more efficient and advantageous to convert $[K]$ into an upper triangular matrix by eliminating the subdiagonal elements. The system can then be solved directly by backward substitution⁽¹⁷⁾. This scheme, which eliminates matrix inversion, reduces computer storage and has less truncation error.

The response of the pipe-segment x_i 's were computed at each time step for the whole time-history of the input earthquake record. These responses were then used to determine the following two parameters:

$$\begin{aligned} Y_i &= x_i - x_{Gi} = x_i(t) - x_G[t - (i-1)DT] & i &= 1 \text{ to } n \\ U_i &= x_i - x_{i+1} & i &= 1 \text{ to } n-1 \end{aligned} \quad (19)$$

where

Y_i : is the relative displacement between the ground and the pipe-segment i .

U_i : is the relative displacement between two adjacent pipe-segments, i.e., elongation or shortening of the joint spring.

These two parameters enable us to deduce the general response behavior of the pipeline under a certain earthquake record and other prescribed conditions.

Note that a computer program for the simplified quasi-static analysis of rigid buried pipelines has been written and reported by Cheng⁽¹⁰⁾, and results of the analysis have been reported⁽¹⁸⁾; this paper only presents the important results/conclusions without details.

Computations were made for various conditions by varying the value of different parameters, such as the delay time, the number of pipe-segments used, the joint stiffness parameters, the end restraint coefficients, and the relative soil resistant ratios. The computer program determines the maximum values of Y and U for each condition and subsequently outputs $Y_{\max}$, $U_{\max}$ and the time and location when the maximum Y and U occur.

II.2.6 Results and Discussions

In this study, El Centro May 18, 1940 S90W component displacement time history record shown in Fig. 5 was used.

To establish a basis for comparison of the seismic response behavior of buried pipelines for various parameters, the following conditions are arbitrarily set as the 'reference conditions':

Number of masses	: $M = 6$
End restraint conditions	: None, i.e. $\gamma_0 = 0$ and $\gamma_n = 0$
Soil conditions	: Uniform, i.e. $\beta = 1, 1, 1, 1, 1, 1$
Delay time	: $DT = 0.1$ Second
Seismic input	: El Centro May 18, 1940 S90W Component

Note that the seismic response behavior of buried pipelines presented in this paper are limited to the maximum relative displacements between the pipe, segment and the ground, $Y_{\max}$, and between the pipe segments themselves, $U_{\max}$ (for the rigid pipe segment assumption, $U_{\max}$ represents the joint spring elongation/shortening conservatively) for various joint spring constants.

For the evaluation of the effects of a particular parameter to the response behavior $Y_{\max}$ and $U_{\max}$, only that parameter will be varied from the above mentioned referenced conditions.

(1) Effect of Joint Stiffness

The results of the relative displacements between pipes, $U_{\max}$ and the relative displacements between pipes and their surrounding soil, $Y_{\max}$ against normalized joint spring stiffness, α are shown in Fig. 6 and Fig. 7, respectively.

One can see from Fig. 6 that the relative displacement between pipe segments, $U_{\max}$, (i.e. the indication of joint extension or contraction) is about 1.4 inches (3.56 cm) for small joint stiffness (low α value) and decrease asymptotically approaching zero as α increases toward infinity. On the other hand, Fig. 7 shows that the relative displacement between the pipe segment and ground, $Y_{\max}$ is very small when α is small and increases tremendously as α increases beyond 0.1. The low $Y_{\max}$ quantity for small α values agrees well with the observations by the investigations^(4, 6, 5) in Japan. Thus, for further discussions, the joint stiffness parameter α has been limited to less than 0.1.

(2) Effect of Delay Time

The delay time of seismic waves traveling from one pipe segment to another is determined by two variables. One is the length of the pipe segment and the other is the seismic propagation velocity

which is related to the soil stiffness. Thus, one can interpret the increase of delay time as an increase in pipe segment length or by a decrease in soil stiffness.

Just changing the delay time, DT, from the 'reference conditions', the effect of DT on U_{max} is shown in Fig. 8. Apparently, the role of DT is to cause a magnification in U_{max} . This result is quite consistent with the observations by Kachadoorian⁽¹⁾ who reported that pipelines in soft soil (longer delay time) experienced more damage during an earthquake than pipelines in firm soil.

(3) Other Effects

The effects of end restraints, pipeline length and wave forms on the response behavior of buried pipelines have been examined to be less significant than those mentioned above. Since these results can be found in the earlier report⁽¹⁰⁾, they are not repeated in this paper.

II.2.7 Discussions

The proposed upper bound quasi-static model seems capable of evaluating the general longitudinal response behavior of buried pipelines subjected to seismic shaking/vibrations. Parametric studies involving such important parameters as joint stiffness, uniformity of soil conditions, delay time, and constraints, pipeline length and wave forms have been performed. The following general concluding remarks can be made:

1. Higher joint stiffness will produce larger relative displacements between pipe segments and the ground. For low joint stiffness, the relative displacement between pipe segments and the ground is very small, which agrees with the general field observations.
2. The effects of non-uniformity of soil environments and the delay time are the two most important influential parameters on the response behavior of buried pipelines. For uniform soil, the longer the delay time or the softer the soil stiffness, the larger the response will be. Under non-uniform soil environment, the larger the differences the transition of one type of soil to another, the higher the response will be.
3. The effect of end restraints is also observed, but such an effect is not as large as those caused by non-uniformity of soil and the variation of delay time.
4. As to the effect of the seismic wave form, it is found that a wave form which has more cycles of maximum peaks seems to yield large response values. More research in this area is recommended.

III. STATIC ANALYSES⁽¹²⁾

III.1 Beam on Elastic Foundation Model

III.1.1 Preface

In a previous section, upper bound values for the maximum axial strain in a straight buried pipeline can be obtained by assuming that the pipeline is very flexible with respect to the soil and that inertia effects can be neglected. For the maximum relative displacement at joints, an upper bound is obtained by assuming that the inertia effects are negligible, the pipeline is rigid with respect to the soil. In the case of actual segmented systems, the pipe, soil and joint stiffnesses will interact to produce a combination of pipe strains and joint displacements that are individually below their respective upper bound magnitudes.

In this context, a refined analysis is proposed. This analysis uses a beam on elastic foundation approach to determine the effects of soil strain on the axial pipe deformation and relative joint displacement.

The pipeline is assumed to be straight and of prismatic cross section. The pipe, joint, and surrounding soil are assumed to behave as linear elastic materials. Although the assumption of linear elastic behavior is not a true representation of real materials, it is expected to result in conservative estimates of pipe strain and joint displacement. Nondimensional curves are presented relating pipe strains and relative joint displacement to pipe, joint, and soil stiffnesses. This refined approach is discussed below.

III.1.2 Analysis

This 'beam on elastic foundation' model is developed to obtain axial pipe strain and relative axial joint displacement of long pipelines subjected to ground displacement. This solution accounts for the longitudinal stiffness of the soil at the pipe/soil interface and the axial stiffness of the pipe and joint material. The key assumption in this approximate formulation is that the pipe is subjected to a uniform soil strain over several segments.

Figure 9 indicates the basis for the development of the pipe strain and joint displacement approximation. The longitudinal ground strain with respect to the pipe axis is assumed to be of uniform value, ϵ_s , and the pipe is assumed to be a segmented system with uniform pipe and joint properties. Due to uniformity of ground strain and assumed symmetry in pipe material and joint properties, pipe segment elongations are equal and relative motion between pipe and soil at each joint is the same. The joint spring force, S , at the end of each segment then becomes the product of joint stiffness, k_j , and relative joint displacement, Δ_j :

$$S = k_j \cdot \Delta_j = 2k_j [(u_g(L/2) - u(L/2))] \quad (20)$$

where $u_g(x)$ is the ground displacement at point x along the pipe axis and $u(x)$ is the pipe displacement at point x . (Note the origin is located at the center of the pipe segment.)

Figure 10 shows a differential segment of pipe acted upon by internal axial forces as well as an additional distributed longitudinal force due to soil-pipe interaction.

Basic force equilibrium in the axial direction results in:

$$P(x) = P(x + dx) + k_s [u_g(x) - u(x)] dx \quad (21)$$

where axial forces, $P(x)$ and $P(x + dx)$, may be expressed as follows:

$$P(x) = \sigma(x)A = \epsilon(x)EA = \frac{du}{dx} EA \quad (22)$$

$$P(x + dx) = \left[\frac{du}{dx} + \frac{d^2u}{dx^2} (dx) \right] EA \quad (23)$$

where $\sigma(x)$ is pipe stress at location x , $\epsilon(x)$ is pipe strain at location x , E is pipe Young's modulus of elasticity, A is pipe cross-sectional area, and k_s is soil stiffness [force/(length)²] in the axial direction. Substitution of Eqns. (22) and (23) into Eqn. (21) yields:

$$\frac{d^2u}{dx^2} - \frac{k_s}{EA} u(x) = -\frac{k_s [u_g(x)]}{EA} \quad (24)$$

the basic differential equation which can be solved for pipe displacement.

Assuming the ground strain is uniform over a number of segments of pipe and is equal to ϵ_m , then:

$$u_g(x) = \epsilon_m x \quad (25)$$

and the solution to Eqn. (24) has the general form:

$$u(x) = Be^{\gamma x} + Ce^{-\gamma x} + \epsilon_m x \quad (26)$$

where

$$\gamma = \sqrt{\frac{k_s}{AE}} \quad (27)$$

The pipe strain, $\epsilon(x)$, is of the form:

$$\epsilon(x) = \frac{du}{dx} = \gamma Be^{\gamma x} - \gamma Ce^{-\gamma x} + \epsilon_m \quad (28)$$

The displacement boundary condition at the center of the referenced pipe segment is zero, i.e., $x = 0$; $u(0) = 0$ which leads to

$$B + C = 0 \quad (29)$$

The strain boundary condition at the end of the pipe segment, $x = \frac{L}{2}$ is

$$\begin{aligned} \epsilon(L/2) &= \frac{S(L/2)}{AE} = \frac{2k_j}{AE} [u_g(L/2) - u(L/2)] \\ &= -\frac{2k_j}{AE} [Be^{\gamma L/2} + Ce^{-\gamma L/2}] \end{aligned} \quad (30)$$

and

$$\epsilon(L/2) = \gamma Be^{\gamma L/2} - \gamma Ce^{-\gamma L/2} + \epsilon_m \quad (31)$$

Combining Eqns. (29) to (31), one finds

$$C = -B = \frac{\epsilon_m}{\gamma(e^{-\gamma L/2} + e^{\gamma L/2}) + \frac{2k_j}{AE} (e^{\gamma L/2} - e^{-\gamma L/2})} \quad (32)$$

Substitution of C and B into Eqn. (26) yields:

$$u(x) = \epsilon_m \cdot \left[x - \frac{\sinh(\gamma x)}{\gamma \cosh\left(\frac{\gamma L}{2}\right) + 2 \frac{k_j}{AE} \sinh\left(\frac{\gamma L}{2}\right)} \right] \quad (33)$$

The above equation can be represented in nondimensional form as:

$$u(x) = \epsilon_m L \cdot \left[\frac{x}{L} - \frac{\sinh\left(\frac{x}{L} \cdot \sqrt{R_s}\right)}{\sqrt{R_s} \cosh \frac{\sqrt{R_s}}{2} + 2 R_j \sinh \frac{\sqrt{R_s}}{2}} \right] \quad (34)$$

where,

$$R_s = k_s \cdot \frac{L^2}{AE} = \text{Relative soil stiffness} \quad (35)$$

and

$$R_j = k_j \cdot \frac{L}{AE} = \text{Relative joint stiffness} \quad (36)$$

This formulation can be used to obtain dimensionless equations for the maximum pipe strain ratio, Γ_s , and the relative joint displacement ratio, Γ_j . The pipe strain ratio is defined as maximum pipe strain divided by uniform ground strain and is a maximum where $x = 0$, thus:

$$\Gamma_s = \frac{\epsilon_{pm}}{\epsilon_m} = 1 - \frac{1}{\cosh \frac{\sqrt{R_s}}{2} + \frac{2R_j}{\sqrt{R_s}} \sinh \frac{\sqrt{R_s}}{2}} \quad (37)$$

The joint displacement ratio is defined as maximum relative joint displacement divided by the product of uniform ground strain and pipe segment length, thus:

$$\Gamma_j = \frac{\Delta_j}{\epsilon_m L} = \frac{2[u_g(L/2) - u(L/2)]}{\epsilon_m L} \quad (38)$$

Substitution and simplification results in the following equation

$$\Gamma_j = \frac{\Delta_j}{\epsilon_m L} = \frac{\tanh \frac{\sqrt{R_s}}{2}}{\frac{\sqrt{R_s}}{2} + R_j \tanh \frac{\sqrt{R_s}}{2}} \quad (39)$$

Equations (37) and (39) are plotted in nondimensional form as Figures 11 and 12. Knowing the maximum soil strain, ϵ_m , these figures can be used to determine maximum pipe strain and relative joint displacement assuming the soil and joint properties can be estimated with reasonable accuracy and that no slippage occurs between the pipe and the soil. In using these graphs it is recommended

that ground strain, ϵ_m , be taken as the peak ground strain, $\frac{V_m}{C_L}$, where V_m is the maximum probable ground velocity and C_L is the longitudinal wave propagation velocity with respect to the pipeline.

III.1.3 Sample Problem

As an example of the use of the graphs in Figures 11 and 12, consider the following problem:

Pipe Properties

Cast iron, modulus of elasticity, $E = 14 \times 10^3$ ksi (96.5 GPa)

8 in. pipe: outer diameter, $d_o = 9.05$ in. (23 cm)

wall thickness = 0.41 in. (1 cm)

area, $A = 11.1$ in² (72 cm²)

segment length, $L = 20$ ft. (6.1 m)

Joint Properties

Rubber gasket joint stiffness⁽¹⁹⁾, $k_j = 8.8$ k/in (1.54 MN/m)

Soil Properties

Longitudinal soil stiffness, $k_s = 13.0$ ski (90 MPa)

Longitudinal wave propagation velocity, $C_L = 866$ ft/s (264 m/s)

Seismic Parameters

Maximum probable ground velocity, $V_m = 8.5$ in/s (21.6 cm/s)

From Eqn. (1), the maximum ground strain is:

$$\epsilon_m = 0.82 \times 10^{-3}$$

From Eqn. (3), the upper bound joint displacement is:

$$\Delta_m = \epsilon_m L = 0.20 \text{ in. (0.51 cm)}$$

Dimensionless Parameters

$$\text{Relative soil stiffness, } R_s = k_s \cdot \frac{L^2}{AE} = 4.8$$

$$\text{Relative joint stiffness, } R_j = k_j \cdot \frac{L}{AE} = 1.4 \times 10^{-2}$$

Solution

From Figures 11 and 12, the pipe strain ratio, Γ_s , is approximated as 0.41, and the joint displacement ratio, Γ_j , is estimated as 0.72. Thus, the pipe strain becomes 0.34×10^{-3} corresponding to an elastic pipe stress of 4.7 ksi (32.5 MPa) and the relative joint displacement, Δ_j , is 0.14 in. (0.37 cm).

III.1.4 Discussions

The normal range of R_s values is expected to be from 10^{-1} to 10^{-2} for segmented pipe. R_j values will be a function of joint material. For the rubber gasketed joint it is expected that R_j will generally be 10^{-1} or less. Little research has been conducted on other joint systems such as lead caulked or leadite. Bolted, flanged connection stiffness could be estimated by considering bolt area, connection length, and modulus of elasticity of bolt material. In lieu of published data, experimental tests could be run on representative samples to establish a joint stiffness value and acceptable movement until leakage occurs. Obviously, more research is required in this area.

The basic pipe model assumes all joint stiffnesses are equal and that symmetric behavior will occur under the conditions of uniform ground strain. Thus, as the R_j value increases, the model represents a situation in which the far ends of the pipe system move with the soil. Thus, the pipe strain ratio, Γ_s , approaches 1.0 and the joint displacement ratio, Γ_j , approaches 0.0 for increasing values of R_j . This condition represents a piping system with cross connections, manholes, valves, or other nonuniform characteristics that constrain portions of the system to essentially displace the same as corresponding points on the ground.

In order to model a continuous pipe with free ends, the segment length, L , should be taken as the continuous pipe length and the R_j value should be taken as 0. For an appreciable length of continuous pipe the resultant R_s value will become extremely high resulting in large pipe strains and low values of relative joint displacements. For instance, a continuous pipe of length equal to ten basic segments will result in an increase in the R_s value of 100 relative to that for the basic

segment because the relative soil stiffness varies as the length squared. See Figures 11 and 12 for the effects of this increase in R_s . It is noted that the sum of Γ_s , the pipe strain ratio, and Γ_j , the joint displacement ratio, will be greater than unity because Γ_s is based on the maximum axial strain in a segment rather than the average axial strain.

It is expected that Figures 11 and 12 will provide a useful design aid to those involved in the seismic analysis of segmented buried pipe-lines. It is to be noted that under high levels of seismic activity, slippage may occur between the pipe and soil due to high surface shear stresses. This analysis assumes slippage does not occur and thus provides a conservative estimate of expected strains.

III.2 Beam on Elastic-Plastic Foundation Model

III.2.1 Preface

The analyses shown thus far are based on the assumption of elastic behavior in the soil. In other words it is assumed that the longitudinal soil resistive force can be developed to an unlimited magnitude. A refined approach, assuming slippage between the pipe and the soil is discussed in this section. This analysis method is expected to be valid for moderate to strong earthquakes with relatively high ground strain levels (i.e., 10^{-3}).

III.2.2 Analysis

By coupling the work of Shah and Chu⁽²⁰⁾ and that of Goodling⁽²¹⁾, in the analysis of underground piping for the nuclear industry, with experimental work to determine pipe pullout forces, an inelastic (elastic-plastic) approach has been developed to approximate strains and relative joint displacements in a linear (straight) pipeline.

Figure 13 illustrates the basic assumptions for this development as applied to a segmented piping system. The pipe segments are assumed to be in a peak uniform tensile ground strain field as shown in Figure 13a. For a bell ended pipe, as commonly used in buried pipe installations, soil forces acting on the protruding bell are assumed to force this end of the pipe segment and the soil to move as a unit. In other words, the pipe and the soil experience the same absolute displacement at the bell ends. (This is in contrast with the beam on elastic foundation model where the center of the pipe segment was assumed to displace the same as the ground.) The total ground displacement, Δ_s , developed over a pipe segment length will be the product of peak uniform ground strain, ϵ_m , and pipe segment length, L . Since each bell end is assumed to displace the same as the soil, the relative displacement, Δ_s , between bell ends will be a combination of relative joint motion, Δ_j , and pipe elongation, Δ_p , (see Figure 13b),

$$\Delta_s = \Delta_p + \Delta_j \quad (40)$$

The pipe elongation will be due to joint force, F_j , (the product of joint stiffness, k_j , and relative joint displacement, Δ_j) and the soil resistive force distribution.

The analysis procedures assume that the soil resistive force function is not ideally elastic as assumed in the previous analyses but is either ideally elastic-plastic or ideally rigid-plastic. The use of the different assumptions is dependent on the strain condition near the bell end of the pipe. Consider first the strain distribution as noted in Figure 13d where the peak pipe strain is equal to the uniform ground strain over a region from the bell to point Ω . For the elastic-plastic soil resistive force model, in the region where pipe strain, ϵ_p , is equal to ground strain, ϵ_m , no surface resistive forces develop since no relative motion exists between pipe and soil. This is followed by a transition region where relative pipe and ground motion is greater than zero but less than that required to cause slippage. In this region the soil resistive force, f , is the product of longitudinal soil stiffness, k_s , times the relative motion between pipe and soil. Beyond this elastic transition region the soil resistive force reaches its limiting value, f_s . When pipe strains match the uniform ground strains, overall force equilibrium of the pipe segment will not be changed if the total force developed in the elastic transition region is replaced by an extension of the uniform slippage force length sufficient to produce the same soil resistive force. Then, $f_s \ell_1$ will be equivalent in magnitude to the force obtained by integrating the actual soil resistive force distribution (elastic transition and slippage

force region) over the pipe segment length (see Fig. 13c).

Although force equilibrium is satisfied, if the elastic transition region is relatively long the overall displacement compatibility could be affected by the "equivalent" slippage force approximation (i.e., the value of pipe elongation, Δ_p , could be incorrectly estimated). If it is additionally assumed that this elastic transition region is short as compared to the pipe segment length, then an expression for the pipe elongation can be directly obtained as:

$$\Delta_p = \epsilon_m(L - \ell_1) + \frac{f_s \ell_1^2}{2AE} + \frac{k_j \Delta_j \ell_1}{AE} \quad (41)$$

where A represents the cross sectional area and E is the pipe Young's modulus.

Substituting expressions for Δ_s and Δ_p into Eqn. (40), an equation for Δ_j becomes:

$$\Delta_j = \frac{\epsilon_m \ell_1 - \frac{f_s \ell_1^2}{2AE}}{1 + \frac{k_j \ell_1}{AE}} \quad (42)$$

The value of ℓ_1 can be obtained by first writing the expression for force equilibrium on the segment,

$$f_s \ell_1 = \epsilon_m AE - k_j \Delta_j \quad (43)$$

and then substituting for Δ_j from Eqn. (42) and solving for ℓ_1 ,

$$\ell_1 = \frac{2AE}{k_j} \left[\sqrt{1 + \frac{2\epsilon_m k_j}{f_s}} - 1 \right] \quad (44)$$

when ($k_j > 0$).

Alternately, for very small values of k_j ,

$$\ell_1 = \frac{\epsilon_m AE}{f_s} \quad (k_j \approx 0) \quad (45)$$

If ℓ_1 is less than segment length, L , the assumption of peak pipe strain is verified. The relative joint displacement, Δ_j , is then calculated from Eqn. (42).

For practical applications, k_j will be sufficiently small in gasketed connections to be ignored with little loss of accuracy. When the influence of joint stiffness, k_j , is in doubt use the smaller value of ℓ_1 from Eqn. (44) and Eqn. (45).

The above analysis assumed elastic-plastic soil behavior but simplified the elastic soil force transition zone as a statically equivalent extension of the slippage force region.

It was possible to maintain force equilibrium because the peak pipe strain was assumed to be equal to the ground strain and hence was known. It was additionally assumed that the elastic transition took place over a relatively short length such that idealization of this region would not significantly affect displacement compatibility.

When the value of ℓ_1 as calculated above exceeds, L , the pipe segment length, the pipe strain is less than ground strain. Hence, the value of pipe strain is not known and force equilibrium cannot be established to determine the slippage length, ℓ_1 . For this situation the assumption of rigid-plastic soil behavior is employed. Again it is assumed the elastic transition soil force region is relatively short. For the rigid-plastic assumption, the entire elastic transition force region is modeled as a uniform slippage force distribution. The assumption is that the transition zone is so short that the

additional increase in axial force, representing the difference between elastic and plastic resistive soil force behavior, is not significant. For these assumptions the pipe strain, ϵ_p , and relative joint displacement, Δ_j , are:

$$\epsilon_p = \frac{f_s \ell + k_j \Delta_j}{AE} \quad (\ell_1 > L) \quad (46)$$

$$\Delta_j = \frac{[\epsilon_m \ell - \frac{f_s \ell^2}{2AE}]}{1 + \frac{k_j \ell}{AE}} \quad (\ell_1 > L) \quad (47)$$

III.2.3 Sample Problem

In order to make a comparison with the elastic solution based on Beam on Elastic Foundation Model, consider the same pipe problem:

Nominal pipe diameter of 8 in., outer diameter, $d_o = 9.05$ in. (23 cm)

Wall thickness = 0.41 in. (1 cm)

Pipe area, $A = 11.1$ in² (72 cm²)

Pipe modulus of elasticity, $E = 14 \times 10^3$ ksi (96.5 GPa)

Segment length, $L = 20$ ft (6.1 m)

Joint stiffness, $k_j = 8.8$ k/in (1.54 MN/m)

Maximum ground strain, $\epsilon_m = 0.82 \times 10^{-3}$

In addition, the slippage force, f_s , is assumed approximately to be 25 lb/in (4.4 kN/m) for the 8 in. pipe.

By Eqn. (44), $\ell_1 = 754$ ft (230 m)

and by Eqn. (45), $\ell_1 = 425$ ft (129.5 m).

Both values are noted to be much greater than the segment length, thus, by Eqn. (47), the relative joint displacement is:

$$\Delta_j = 0.19 \text{ in (0.48 cm)}$$

and compared to 0.14 in (0.37 cm) from the elastic analysis.

The peak pipe strain, by Eqn. (46), is:

$$\epsilon_p = 5 \times 10^{-5}$$

corresponding to a stress of 0.7 ksi (4.8 MPa) as compared to a peak strain of 34×10^{-5} from the elastic analysis.

Thus, with the assumption of slippage, the peak strain has been reduced to approximately 15 % of the value obtained from the elastic analysis and the relative joint displacement has increased 36 % above that obtained by the elastic analysis to 95 % of that predicted by the upper bound criteria.

III.2.4 Discussions

The inelastic approach, assuming slippage between the pipe and the soil, appears to be justifiable for high ground strains associated with moderate earthquakes. The technique provides a simple approximation of peak pipe strain and relative joint displacement once the slippage force has been determined. As noted in the sample problem, the effect of assuming slippage is to increase the relative joint displacement. For low values of slippage force, when the full segment length is not sufficient to develop a strain equal to the ground strain, the peak pipe strain will be less than that predicted by assuming elastic behavior.

This technique is sensitive to the assumption of the slippage force magnitude. It is expected that use of this lower bound value could underestimate peak pipe strains by a factor of 2-3. Relative joint displacements calculated by this technique will of course be conservative for low slippage force values.

IV. QUASI-STATIC ELASTIC ANALYSIS

IV.1 Preface

In the 'simplified approach,' upper bounds for pipe strains and relative joint displacements are obtained by assuming that the pipeline is continuous and very flexible or the pipeline consists of isolated rigid segments. Static analyses assume uniform ground displacements. In reality, a buried pipeline has elasticity and reacts to the seismic shaking through the media of the surrounding environments. Thus, the response behavior of the buried pipeline will be influenced by a number of physical, geotechnical and seismological parameters. The physical parameters are the geometrical and mechanical pipe properties such as diameter, thickness, segment length, and Young's modulus. The geotechnical parameters are the soil-structure interaction resistant constant, its variation along the pipeline and the wave propagation velocity. The seismological parameters are the form, duration, amplitude and the slope of the ground displacement time history.

In earlier investigations^(10, 18), a preliminary quasi-static model consisting of rigid pipe segments connected by elastic joint springs was used to study conservatively the relative joint motions of segmented pipelines due to seismic ground shaking. Based on the general formulation⁽⁹⁾, a more rigorous quasi-static analysis model has been developed to study parametrically the response of elastic buried pipelines, segmented or continuous, subjected to earthquake motion in the axial direction^(11, 21, 15). Note that in this analysis, pipe materials, joint springs as well as soil resistance are all assumed to be linearly elastic through the entire time domain.

IV.2 Formulation of the Problem

IV.2.1 Description of the General Model

A long buried piping system consisting of n -segments is shown in Fig. 14 where $K_1, K_2, \dots, K_i, \dots, K_{n-1}$ are spring constants at joints between pipe segments; K_0, K_n are spring constants at the end supports; $X_1, X_2, \dots, X_{2i-1}, X_{2i}, \dots, X_{2n-1}, X_{2n}$ are longitudinal displacements at the ends of pipe segments which are different from those of quasi-static upper bound analysis; $X_{G1}, X_{G2}, \dots, X_{Gn-1}$ are the corresponding ground displacements at the segment intersections in the same direction as the pipeline axis; X_{G0} and X_{Gn} are the ground movements at the ends; $L_1, L_2, \dots, L_i, \dots, L_n$ are pipe lengths; and $k_1, k_2, \dots, k_i, \dots, k_n$ are soil resistant spring constants per unit length along the pipe segments.

IV.2.2 Strain Energy in A Pipe Segment

Referring to Fig. 15, for a pipe segment subjected to two end forces and a linearly distributed soil resistance force, the displacement function within a segment is a cubical variation as:

$$\phi_i(x) = \left[\left(1 - \frac{x^3}{L_i^3}\right) - \frac{x^3}{L_i^3} \right] \begin{Bmatrix} X_{2i-1} \\ X_{2i} \end{Bmatrix} \quad (48)$$

The strain function is:

$$\epsilon_i(x) = \frac{d\phi_i(x)}{dx} = \left[-\frac{3x^2}{L_i^3} - \frac{3x^2}{L_i^3} \right] \begin{Bmatrix} X_{2i-1} \\ X_{2i} \end{Bmatrix} \quad (49)$$

and the stress function is:

$$\sigma_i(x) = E_i \epsilon(x) = E_i \left[\frac{-3x^2}{L_i^3} \quad \frac{3x^2}{L_i^3} \right] \begin{Bmatrix} X_{2i-1} \\ X_{2i} \end{Bmatrix} \quad (50)$$

The strain energy within a pipe segment i

$$U_{\text{pipe}}^i = \frac{1}{2} \int_v \sigma_i^t \epsilon_i dv = \frac{1}{2} \int_0^{L_i} \sigma_i^t \epsilon_i A_i dx \quad (51)$$

where σ_i^t is transpose of σ_i .

Substituting Eqns. (49) and (50) into Eqn. (51) and integrating, one obtains

$$U_{\text{pipe}}^i = \frac{1}{2} [X_{2i-1} \ X_{2i}] \begin{pmatrix} \frac{9E_i A_i}{5L_i} & \frac{-9E_i A_i}{5L_i} \\ \frac{-9E_i A_i}{5L_i} & \frac{9E_i A_i}{5L_i} \end{pmatrix} \begin{Bmatrix} X_{2i-1} \\ X_{2i} \end{Bmatrix} \quad (52)$$

The sum of energies of all segments in the system is:

$$\begin{aligned} U_{\text{pipe}}^{\text{Total}} &= \sum_{i=1}^n U_{\text{pipe}}^i \\ &= \frac{1}{2} \{X\}^t [K_{\text{pipe}}] \{X\} \end{aligned} \quad (53)$$

where

$$\{X\}^t = [X_1, X_2 \ \dots \ X_3 \ X_4 \ \dots \ X_{2i-1} \ X_{2i} \ \dots \ X_{2n-1} \ X_{2n}] \quad (54)$$

1x2n

and $[K_{\text{pipe}}]$ is a symmetrically tridiagonal matrix as shown below

$$[K_{\text{pipe}}] = \frac{9}{5} \begin{bmatrix} \frac{E_1 A_1}{L_1} & -\frac{E_1 A_1}{L_1} & & & & \\ -\frac{E_1 A_1}{L_1} & \frac{E_1 A_1}{L_1} & 0 & & & \\ & 0 & \frac{E_2 A_2}{L_2} & -\frac{E_2 A_2}{L_2} & & \\ & & -\frac{E_2 A_2}{L_2} & \frac{E_2 A_2}{L_2} & 0 & \\ & & & 0 & \frac{E_i A_i}{L_i} & -\frac{E_i A_i}{L_i} \\ & & & & -\frac{E_i A_i}{L_i} & \frac{E_i A_i}{L_i} & 0 \\ & & & & & 0 & \frac{E_n A_n}{L_n} & -\frac{E_n A_n}{L_n} \\ & & & & & & -\frac{E_n A_n}{L_n} & \frac{E_n A_n}{L_n} \end{bmatrix}$$

$2n \times 2n$

(55)

IV.2.3 Strain Energy in A Joint Spring

Referring to Fig. 16, the spring elongation and spring force can be expressed:

$$D_i = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \quad (56)$$

$$F_i = \begin{bmatrix} -K_i & K_i \end{bmatrix} \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \quad (57)$$

The strain energy within a spring is

$$\begin{aligned} U_{\text{spring}}^i &= \frac{1}{2} F_i^t D_i \\ &= \frac{1}{2} \begin{bmatrix} X_{2i} & X_{2i+1} \end{bmatrix} \begin{bmatrix} K_i & -K_i \\ -K_i & K_i \end{bmatrix} \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \end{aligned} \quad (58)$$

For the beginning support, $i = 0$, it is assumed that $X_0 = X_{G0}$. Thus,

For the end support, $i = n$, it is assumed that $X_{2n+1} = X_{Gn}$ and

The total strain energy contained in all joint springs is as follows:

where

and $[\bar{K}_{spring}]$ is a symmetrical tridiagonal matrix as follows:

$$(2n+2)(2n+2)$$

Using partitioning matrix manipulation, the total strain energy of all joint springs may be expressed as follows:

$$\begin{aligned}
 U_{\text{spring}}^{\text{Total}} = & \frac{1}{2} K_0 X_{G0}^2 + \frac{1}{2} K_n X_{Gn}^2 \\
 & + \{X\}^t \{\bar{K}_{ba}\} X_{G0} + \{X\}^t \{\bar{K}_{bc}\} X_{Gn} \\
 & + \frac{1}{2} \{X\}^t [K_{\text{spring}}] \{X\}
 \end{aligned} \quad (63)$$

where

$$(\bar{K}_{ba})^t = [-K_0 \ 0 \ 0 \ \dots \ 0] \quad (64)$$

$$\{\bar{K}_{bc}\}^t = \begin{bmatrix} 0 & \dots & 0 & -K_n \end{bmatrix}_{1 \times 2n} \quad (65)$$

and $[K_{\text{spring}}]$ is a symmetrically tridiagonal matrix as:

$$[K_{\text{spring}}] = \begin{bmatrix} K_0 & 0 & & & \\ 0 & K_1 & -K_1 & & \\ & -K_1 & K_1 & 0 & \\ & & 0 & K_2 & -K_2 \\ & & & -K_2 & K_2 & 0 \\ & & & & 0 & K_i & -K_i \\ & & & & & -K_i & K_i & 0 \\ & & & & & & 0 & K_{n-1} & -K_{n-1} \\ & & & & & & & -K_{n-1} & K_{n-1} & 0 \\ & & & & & & & & 0 & K_n \end{bmatrix} \quad (66)$$

$2n \times 2n$

IV.2.4 Strain Energy of Soil Springs Along A Pipe Segment

Referring to Fig. 17 for the soil resistant distribution, the soil spring displacement and resistance per unit length can be expressed:

$$y(x) = \left[\left(1 - \frac{x}{L_i}\right) \quad \frac{x}{L_i} \right] \begin{Bmatrix} (X_{2i-1} - X_{Gi-1}) \\ (X_{2i} - X_{Gi}) \end{Bmatrix}$$

$$= \left[\left(1 - \frac{x}{L_i}\right) \quad \frac{x}{L_i} \quad -\left(1 - \frac{x}{L_i}\right) \quad \frac{-x}{L_i} \right] \begin{Bmatrix} X_{2i-1} \\ X_{2i} \\ X_{Gi-1} \\ X_{Gi} \end{Bmatrix} \quad (67)$$

and

$$R(x) = \left[k_i \left(1 - \frac{x}{L_i}\right) \quad \frac{k_i x}{L_i} \quad -k_i \left(1 - \frac{x}{L_i}\right) \quad \frac{-k_i x}{L_i} \right] \begin{Bmatrix} X_{2i-1} \\ X_{2i} \\ X_{Gi-1} \\ X_{Gi} \end{Bmatrix} \quad (68)$$

The strain energy produced by the soil resistance within a pipe segment is:

$$U_{soil}^i = \frac{1}{2} \int_0^{L_i} R^t(x) y(x) dx \quad (69)$$

Substituting Eqns. (67) and (68) into Eqn. (69) and integrating, one obtains:

$$U_{soil}^i = \frac{1}{2} [X_{2i-1} \quad X_{2i} \quad X_{Gi-1} \quad X_{Gi}] \begin{bmatrix} \frac{k_i L_i}{3} & \frac{k_i L_i}{6} & -\frac{k_i L_i}{3} & -\frac{k_i L_i}{6} \\ \frac{k_i L_i}{6} & \frac{k_i L_i}{3} & -\frac{k_i L_i}{6} & -\frac{k_i L_i}{3} \\ -\frac{k_i L_i}{3} & -\frac{k_i L_i}{6} & \frac{k_i L_i}{3} & \frac{k_i L_i}{6} \\ -\frac{k_i L_i}{6} & -\frac{k_i L_i}{3} & \frac{k_i L_i}{6} & \frac{k_i L_i}{3} \end{bmatrix} \begin{Bmatrix} X_{2i-1} \\ X_{2i} \\ X_{Gi-1} \\ X_{Gi} \end{Bmatrix} \quad (70)$$

Using 2x2 partitioning matrices, Eqn. (70) can be written as follows:

$$\begin{aligned}
 U_{\text{soil}}^i = & \frac{1}{2} [X_{2i-1} \quad X_{2i}] \begin{bmatrix} \frac{k_i L_i}{3} & \frac{k_i L_i}{6} \\ \frac{k_i L_i}{6} & \frac{k_i L_i}{3} \end{bmatrix} \begin{Bmatrix} X_{2i-1} \\ X_{2i} \end{Bmatrix} \\
 & + \frac{1}{2} [X_{Gi-1} \quad X_{Gi}] \begin{bmatrix} \frac{k_i L_i}{3} & \frac{k_i L_i}{6} \\ \frac{k_i L_i}{6} & \frac{k_i L_i}{3} \end{bmatrix} \begin{Bmatrix} X_{Gi-1} \\ X_{Gi} \end{Bmatrix} \\
 & - [X_{2i-1} \quad X_{2i}] \begin{bmatrix} \frac{k_i L_i}{3} & \frac{k_i L_i}{6} \\ \frac{k_i L_i}{6} & \frac{k_i L_i}{3} \end{bmatrix} \begin{Bmatrix} X_{Gi-1} \\ X_{Gi} \end{Bmatrix}
 \end{aligned} \quad (71)$$

The sum of total strain energy of entire soil system

$$\begin{aligned}
 U_{\text{Soil}}^{\text{Total}} &= \sum_{i=1}^n U_{\text{soil}}^i \\
 &= \frac{1}{2} \{X\}^t [K_{\text{soil}}] \{X\} + \frac{1}{2} \{X_G\}^t [K_{\text{soil}}] \{X_G\} - \{X\}^t [K_{\text{soil}}] \{X_G\}
 \end{aligned} \quad (72)$$

where

$$\{X_G\}^t = [X_{G0} \quad X_{G1} \quad X_{G1} \quad \dots \quad X_{Gi} \quad X_{Gi} \quad \dots \quad X_{Gn-1} \quad X_{Gn-1} \quad X_{Gn}] \quad (73)$$

1x2n

and $[K_{\text{soil}}]$ is a symmetrically tridiagonal matrix as follows:

$$[K_{\text{soil}}] = \begin{bmatrix} \frac{k_1 L_1}{3} & \frac{k_1 L_1}{6} & & & & \\ \frac{k_1 L_1}{6} & \frac{k_1 L_1}{3} & 0 & & & \\ & 0 & \frac{k_2 L_2}{3} & \frac{k_2 L_2}{6} & & \\ & & \frac{k_2 L_2}{6} & \frac{k_2 L_2}{3} & 0 & \\ & & & 0 & \frac{k_i L_i}{3} & \frac{k_i L_i}{6} \\ & & & & \frac{k_i L_i}{6} & \frac{k_i L_i}{3} & 0 \\ & & & & & 0 & \frac{k_n L_n}{3} & \frac{k_n L_n}{6} \\ & & & & & & \frac{k_n L_n}{6} & \frac{k_n L_n}{3} \end{bmatrix} \quad (74)$$

2nx2n

IV.2.5 Total Potential Energy of the Soil-Structure Interaction System

The total potential energy of the buried piping system is the sum of strain energies of pipe segments, joint and soil resistant springs. Mathematically, it is expressed as:

$$U_{\text{system}}^{\text{Total}} = U_{\text{pipe}}^{\text{Total}} + U_{\text{spring}}^{\text{Total}} + U_{\text{soil}}^{\text{Total}} \quad (75)$$

Substituting Eqns. (53), (63) and (72) into Eqn. (75), one obtains the potential energy of the entire system as

$$\begin{aligned} U_{\text{system}}^{\text{Total}} &= \frac{1}{2} \{X\}^t [K_{\text{pipe}}] \{X\} \\ &+ \frac{1}{2} K_0 X_{G0}^2 + \frac{1}{2} K_n X_{Gn}^2 \\ &+ \{X\}^t \{\bar{K}_{ba}\} X_{G0} + \{X\}^t \{\bar{K}_{bc}\} X_{Gn} \\ &+ \frac{1}{2} \{X\}^t [K_{\text{spring}}] \{X\} \\ &+ \frac{1}{2} \{X\}^t [K_{\text{soil}}] \{X\} + \frac{1}{2} \{X_G\}^t [K_{\text{soil}}] \{X_G\} - \{X\}^t [K_{\text{soil}}] \{X_G\} \end{aligned} \quad (76)$$

IV.2.6 Governing Equilibrium Equation of Buried Pipelines

According to variational principle⁽²³⁾, the variation of total potential energy of an equilibrium system is equal to zero, i.e.

$$\frac{\partial U_{\text{system}}^{\text{Total}}}{\partial \{X\}} = 0$$

$$\text{or} \quad \frac{\partial U_{\text{pipe}}^{\text{Total}}}{\partial \{X\}} + \frac{\partial U_{\text{spring}}^{\text{Total}}}{\partial \{X\}} + \frac{\partial U_{\text{soil}}^{\text{Total}}}{\partial \{X\}} = 0 \quad (77)$$

Carrying through the derivatives⁽²⁰⁾ of $U_{\text{system}}^{\text{Total}}$ shown in Eqn. (76), one obtains the following equation of equilibrium of the system:

$$\begin{aligned} & [K_{\text{pipe}}] \{X\} + [K_{\text{spring}}] \{X\} + [K_{\text{soil}}] \{X\} \\ & = [K_{\text{soil}}] \{X_G\} - \{\bar{K}_{ba}\} X_{Go} - \{\bar{K}_{bc}\} X_{Gn} \end{aligned} \quad (78)$$

After grouping from Eqn. (78), one obtains the following simplified equation:

$$\begin{matrix} [K_{\text{system}}] & \{X\} & = & [\bar{K}_{\text{soil}}] & \{X_G\} \\ 2n \times 2n & 2n \times 1 & & 2n \times 2n & 2n \times 1 \end{matrix} \quad (79)$$

where both $[K_{\text{system}}]$ and $[\bar{K}_{\text{soil}}]$ matrices are symmetrical tridiagonal matrices as follows:

$$[K_{\text{system}}] = \begin{bmatrix} \left(\frac{9E_1 A_1}{5L_1} + \frac{k_1 L_1}{3} + K_0\right) \left(\frac{-9E_1 A_1}{5L_1} + \frac{k_1 L_1}{6}\right) & & & & \\ \left(\frac{-9E_1 A_1}{5L_1} + \frac{k_1 L_1}{6}\right) & \left(\frac{9E_1 A_1}{5L_1} + \frac{k_1 L_1}{3} + K_1\right) & -K_1 & & \\ & -K_1 & \left(\frac{9E_2 A_2}{5L_2} + \frac{k_2 L_2}{3} + K_1\right) \left(\frac{-9E_2 A_2}{5L_2} + \frac{k_2 L_2}{6}\right) & & \\ & & \ddots & \ddots & \ddots \\ & K_{2i-1, 2i-2} & K_{2i-1, 2i-1} & K_{2i-1, 2i} & \\ & & K_{2i, 2i-1} & K_{2i, 2i} & K_{2i, 2i+1} \\ & & & \ddots & \ddots \end{bmatrix} \quad (80)$$

$$K_{2i-1, 2i-1}^{\text{system}} = \frac{9E_i A_i}{5L_i} + \frac{k_i L_i}{3} + K_{i-1}; \quad 1 \leq i \leq n \quad (81a)$$

$$K_{2i-1, 2i}^{\text{system}} = K_{2i, 2i-1}^{\text{system}} = -\frac{9E_i A_i}{5L_i} + \frac{k_i L_i}{6}; \quad 1 \leq i \leq n \quad (81b)$$

$$K_{2i, 2i}^{\text{system}} = \frac{9E_i A_i}{5L_i} + \frac{k_i L_i}{3} + K_i; \quad 1 \leq i \leq n \quad (81c)$$

$$K_{2i, 2i+1}^{\text{system}} = K_{2i+1, 2i}^{\text{system}} = -K_i \quad 1 \leq i \leq n \quad (81d)$$

and

$$[\bar{K}_{\text{soil}}] = \begin{bmatrix} \frac{k_1 L_1}{3} + K_0 & \frac{k_1 L_1}{6} & & & & \\ \frac{k_1 L_1}{6} & \frac{k_1 L_1}{3} & 0 & & & \\ & 0 & \frac{k_2 L_2}{3} & \frac{k_2 L_2}{6} & & \\ & & \frac{k_2 L_2}{6} & \frac{k_2 L_2}{3} & 0 & \\ & & & \frac{k_i L_i}{3} & \frac{k_i L_i}{6} & \\ & & & \frac{k_i L_i}{6} & \frac{k_i L_i}{3} & 0 \\ & & & & 0 & \frac{k_n L_n}{3} & \frac{k_n L_n}{6} \\ & & & & & \frac{k_n L_n}{6} & \left(\frac{k_n L_n}{3} + K_n\right) \end{bmatrix} \quad (82)$$

$2n \times 2n$

In general terms, the elements of $[\bar{K}_{\text{soil}}]$ are defined as

$$\bar{K}_{2i-1, 2i-1}^{\text{soil}} = \bar{K}_{2i, 2i}^{\text{soil}} = \frac{k_i L_i}{3}; \quad 1 \leq i \leq n \quad (83a)$$

$$\bar{K}_{2i, 2i+1}^{\text{soil}} = \bar{K}_{2i+1, 2i}^{\text{soil}} = 0; \quad 1 \leq i \leq n-1 \quad (83b)$$

$$\bar{K}_{2i-1, 2i}^{\text{soil}} = \bar{K}_{2i, 2i-1}^{\text{soil}} = \frac{k_i L_i}{6}; \quad 1 \leq i \leq n \quad (83c)$$

except for the two ends which are as follows:

$$\bar{K}_{11}^{\text{soil}} = \frac{k_1 L_1}{3} + K_0 \quad (83d)$$

$$\bar{K}_{2n, 2n}^{\text{soil}} = \frac{k_n L_n}{3} + K_n \quad (83e)$$

IV.2.7 Models for Special Cases

For wider application of the quasi-static model to analyze buried lifeline systems subjected to seismic excitations, two special case models; one for a long buried pipeline system and the other for a rigid pipe segment will be developed.

(1) A Buried Continuous Pipeline System

For a long buried continuous pipeline system, there will be no joint springs. In this case, $X_{2i} = X_{2i+1} = X_{i+1}$ is observed. In a n-segment pipeline system, there will be n+1 degrees of freedom as shown below:

$$\{X_{\text{cont}}^t\} = [X_1, X_2, \dots, X_i \dots, X_n, X_{n+1}] \quad (84)$$

1x(n+1)

and

$$\{X_{G, \text{cont}}^t\} = [X_{G0}, X_{G1}, \dots, X_{Gi}, \dots, X_{Gn-1}, X_{Gn}] \quad (85)$$

1x(n+1)

Observing the $2i^{\text{th}}$ and $(2i+1)^{\text{th}}$ rows of Eqn. (79) as:

$$\begin{aligned} & \left(\frac{-9E_i A_i}{5L_i} + \frac{k_i L_i}{6} \right) X_{2i-1} + \left(\frac{9E_i A_i}{5L_i} + \frac{k_i L_i}{3} + K_i \right) X_{2i} - K_i X_{2i+1} \\ &= \frac{k_i L_i}{6} X_{Gi-1} + \frac{k_i L_i}{3} X_{Gi} \end{aligned} \quad (86)$$

and

$$\begin{aligned} & -K_i X_{2i} + \left(\frac{9E_{i+1} A_{i+1}}{5L_{i+1}} + \frac{k_{i+1} L_{i+1}}{3} + K_i \right) X_{2i+1} + \left(\frac{-9E_{i+1} A_{i+1}}{5L_{i+1}} + \frac{k_{i+1} L_{i+1}}{6} \right) X_{2i+2} \\ &= \frac{k_{i+1} L_{i+1}}{3} X_{Gi} + \frac{k_{i+1} L_{i+1}}{6} X_{Gi+1} \end{aligned} \quad (87)$$

Letting $X_{2i} = X_{2i+1} = X_{i+1}$; $X_{2i+1} = X_i$; $X_{2i+2} = X_{i+2}$ and adding Eqns. (86) and (87), one obtains the governing equilibrium equation for a long buried continuous pipeline system as:

$$\begin{aligned}
& \left(\frac{-9E_i A_i}{5L_i} + \frac{k_i L_i}{6} \right) X_i + \left(\frac{9E_{i+1} A_{i+1}}{5L_{i+1}} + \frac{k_i L_i}{3} + \frac{k_{i+1} L_{i+1}}{3} \right) X_{i+1} \\
& + \left(\frac{-9E_{i+1} A_{i+1}}{5L_{i+1}} + \frac{k_i L_{i+1}}{6} \right) X_{i+2} \\
& = \frac{k_i L_i}{6} X_{Gi-1} + \frac{1}{3} (k_i L_i + k_{i+1} L_{i+1}) X_{Gi} + \frac{k_{i+1} L_{i+1}}{6} X_{Gi+1}
\end{aligned} \tag{88}$$

Note that the above equation is good for any $1 \leq i \leq n-1$ values. The equilibrium of the two end supports are written as follows:

$$\begin{aligned}
& \left(\frac{9E_1 A_1}{5L_1} + \frac{k_1 L_1}{3} + K_0 \right) X_1 + \left(\frac{-9E_1 A_1}{5L_1} + \frac{k_1 L_1}{6} \right) X_2 \\
& = \left(\frac{k_1 L_1}{3} + K_0 \right) X_{G0} + \frac{k_1 L_1}{6} X_{G1}
\end{aligned} \tag{89}$$

and the last row

$$\begin{aligned}
& \left(\frac{-9E_n A_n}{5L_n} + \frac{k_n L_n}{6} \right) X_n + \left(\frac{9E_n A_n}{5L_n} + \frac{k_n L_n}{3} + K_n \right) X_{n+1} \\
& = \frac{k_n L_n}{6} X_{Gn-1} + \left(\frac{k_n L_n}{3} + K_n \right) X_{Gn}
\end{aligned} \tag{90}$$

For free end supports, K_0 in Eqn. (89) and K_n in Eqn. (90) should be removed.

For fixed end support condition, $X_1 = X$ and $X_{n+1} = X$, one can write $(n-1)$, equations from Eqn. (88). Equations (89) and (90) become unnecessary in this case.

(2) A Rigid Pipe Segmented System

For a rigid pipe segment system, it is observed that $X_1 = X_2 = \dots = X_n = X$. For an n -segment pipeline system, there will be n degrees of freedom as shown below:

$$\{X_{\text{rigid}}\} = [X_1, X_2, \dots, X_i, \dots, X_n] \tag{91}$$

but there will be $n+1$ ground displacements as

$$\{X_{G, \text{rigid}}^t\} = [X_{G0}, X_{G1}, \dots, X_{G1}, \dots, X_{Gn-1}, X_{Gn}] \tag{92}$$

Observing $(2i-1)$ and $2i$ rows of Eqns. (79) as

$$\begin{aligned}
& -K_{i-1} X_{2i-2} + \left(\frac{9E_i A_i}{5L_i} - \frac{k_i L_i}{3} + K_{i-1} \right) X_{2i-1} + \left(\frac{-9E_i A_i}{5L_i} - \frac{k_i L_i}{6} \right) X_{2i} \\
& = \frac{k_i L_i}{3} X_{Gi-1} + \frac{k_i L_i}{6} X_{Gi}
\end{aligned} \tag{93}$$

and

$$\begin{aligned} & \left(\frac{-9E_i A_i}{5L_i} + \frac{k_i L_i}{6} \right) X_{2i-1} + \left(\frac{9E_i A_i}{5L_i} + \frac{k_i L_i}{3} + K_i \right) X_{2i} - K_i X_{2i+1} \\ &= \frac{k_i L_i}{6} X_{Gi-1} + \frac{k_i L_i}{3} X_{Gi} \end{aligned} \quad (94)$$

Letting $X_{2i-1} = X_{2i} = X_i$; $X_{2i-2} = X_{i-1}$; $X_{2i+1} = X_{i+1}$ and adding Eqns. (93) and (94), one obtains the equilibrium equation for a rigid pipe segment system as:

$$\begin{aligned} & -K_{i-1} X_{i-1} + (k_i L_i + K_{i-1} + K_i) X_i - K_i X_{i+1} \\ &= \frac{1}{2} k_i L_i X_{Gi-1} + \frac{1}{2} k_i L_i X_{Gi} \end{aligned} \quad (95)$$

Note that the above equation is good for any $2 \leq i \leq n-1$ values. For $i = 0$, $X_0 = X_{G0}$, the first row of the equilibrium equation is

$$\begin{aligned} & (k_1 L_1 + K_0 + K_1) X_1 - K_1 X_2 \\ &= (K_0 + \frac{1}{2} k_1 L_1) X_{G0} + \frac{1}{2} k_1 L_1 X_{G1} \end{aligned} \quad (96)$$

For $i = n$, $X_{n+1} = X_{Gn}$, the last row of the equilibrium equation is

$$\begin{aligned} & -K_{n-1} X_{n-1} + (k_n L_n + K_{n-1} + K_n) X_n \\ &= \frac{1}{2} k_n L_n X_{Gn-1} + \left(\frac{1}{2} k_n L_n + K_n \right) X_{Gn} \end{aligned} \quad (97)$$

IV.3 Ground Motion Input

The solution of pipe motion X shown in Eqn. (79) depends on the inputs of the ground motion X_G . Since X_G is a function of time, the solution of X is also a function of time. Thus, the method proposed is called the 'Quasi-static' model.

Assuming that the wave form of the traveling seismic excitation remains constant over the entire length of the pipeline which is divided into n -segments, the inputs of the time-space varying ground motions starting from the first support which are different from those for the quasi-static upper bound analysis are:

$$X_{G0} = \begin{cases} 0 & t < 0 \\ \Delta_{\max} h(t) & t \geq 0 \end{cases} \quad (98a)$$

$$X_{G1} = \begin{cases} 0 & t - \Delta T_1 < 0 \\ \Delta_{\max} h(t - \Delta T_1) & t - \Delta T_1 \geq 0 \end{cases} \quad (98b)$$

$$X_{Gi} = \begin{cases} 0 & t - \Delta T_i < 0 \\ \Delta_{\max} h(t - \Delta T_i) & t - \Delta T_i \geq 0 \end{cases} \quad (98c)$$

where $\Delta_{\max}$ is maximum ground displacement input in a record; $h(t)$ is the displacement time function; ΔT_i is the delay time of seismic wave traveling from the first support to the end fact of i^{th} pipe segment considered, which can be expressed as

$$\Delta T_i = \sum_{j=1}^i L_j / C_j \quad (99)$$

where C_j is the traveling wave velocity of soil surrounding the pipe segment j .

IV.4 Method of Solution

Note that the quasi-static elastic analysis model shown in Eqn. (79) can be solved easily since the equation is basically a static one. Since X_G is a function of time, the solution of X is also a function of time. If seismic inputs $X_{G1}(t_1)$ $X_{G2}(t_2)$... are given, the response of pipe segments $X_1(t)$, $X_2(t)$... are calculated numerically. With this numerical procedure, various wave forms of input and time lag can be incorporated without difficulty as long as we can define the ground displacement variation in time and in space.

The system of governing equations requires the input of ground displacement at an instant of time. Similarly to that discussed for the upper bound quasi-static model, the response of nodal displacements, X_i , are calculated by a modified Gauss-elimination procedure⁽¹⁷⁾ at each time step for the entire time-history of the earthquake input record. The resulting pipeline nodal displacements, X 's, are used to determine two design parameters and a soil resistance parameter

$$\epsilon_i = (X_{2i} - X_{2i-1}) / L_i \quad (100)$$

$$U_i = X_{2i+1} - X_{2i} \quad (101)$$

$$Y_i = X_{Gi} - X_{2i} \text{ or } Y_i = X_{Gi} - X_{2i+1} \quad (102)$$

where

ϵ_i = average strain in i^{th} pipe segment

U_i = relative displacement, extension/contraction, of i^{th} joint spring between two adjacent pipe segments

Y_i = relative displacement between the ground and the pipe.

Note that pipe stresses and joint forces can be obtained directly from pipe strains and relative joint displacements.

To prove the correctness of the formulation, Serna⁽¹³⁾ has verified the results with those by the finite element approach solved by the large computer program ANSYS⁽²⁵⁾. The results indicate that the differences are within 1 % to 3 %.

To investigate its efficiency and correctness further, Serna⁽¹³⁾ has solved Eqn. (79), the governing equation independently, by inversion of the tridiagonal system matrix instead of by Gauss elimination procedure⁽¹⁷⁾. The results are almost identical and Gauss elimination procedure seems to be a little more efficient in computation time.

After this investigation, the confidence of the quasi-static formulation and the solution procedure is confirmed.

IV.5 Results of Segmented Piping Systems

IV.5.1 The Investigation Parameters

The major independent investigation parameter in this study is the joint stiffness. It varies from very soft spring, 10^{-1} kips/in, (.175 N/cm) to very stiff spring, 10^6 kips/in, (1.75×10^6 kN/cm) which approaches to a continuous piping system.

The behavioral parameters investigated are the maximum average pipe strain, $\epsilon_{p, \max}$, maximum

joint displacement $U_{\max}$ and the maximum relative displacement between the pipe and the ground $Y_{\max}$.

A referenced system is described in the next section. The effects of other parameters such as pipeline and segment length, pipe diameter, pipe materials, etc. investigated will be compared to the referenced system.

IV.5.2 The Referenced Piping System

To establish a basis for comparison of the seismic response behavior of buried pipelines for various parameters, the following conditions are arbitrarily set as the "reference" conditions:

Pipeline Parameters

Material (Cast Iron)	$E = 14000 \text{ ksi (96500 MPa)}$
Outer Diameter	$D = 18 \text{ in (45.70 cm)}$
Thickness	$t_o = 0.54 \text{ in (1.37 cm)}$
Number of Segments	$n = 20$
Segment Length	$L_i = 20 \text{ ft. (6.1 m)}$
Free End Conditions	$K_o, K_{n+1} = 0, 0$
Joint Spring Constants	$K_i = 10^{-1} \text{ to } 10^6 \text{ kips/in}$ $(1.75 \times 10^{-1} \text{ to } 1.75 \times 10^6 \text{ kN/cm})$

Soil Parameters

Wave Propagation	$C = 800 \text{ ft/sec. (240 m/sec.)}$
Resistant Spring Constant	$k_i = 3400 \text{ lbs/in/in (2340 N/cm/cm)}$

Seismic Parameters (Fig. 5)

El Centro May 18, 1940	S90W Component
Maximum Ground Displacements	$\Delta_{\max} = 7.80 \text{ in (19.80 cm)}$
Duration	$T = 50 \text{ sec.}$
Maximum Velocity	$V_{\max} = 14.5 \text{ in/sec. (36.9 cm/sec.)}$

For an evaluation of the effects of a particular parameter upon the response behavior, only that parameter will be varied from the above mentioned 'reference' conditions.

IV.5.3 Results

The study is to investigate the interactions among the various parameters previously mentioned with the variation of the joint stiffness, from fairly flexible to extremely rigid. Results of maximum average pipe strain; maximum relative joint motion and maximum motion between the pipe and the ground of the system are presented. Note that the maximum average pipe strains and relative joint motions are the controlling factors for the design of buried pipelines. The maximum motion between the pipe and the ground will determine whether or not slippage between the pipe and the soil occurs.

(1) Effect of Segment/Pipeline Lengths

The effect of segment length on the seismic response of buried pipelines is studied by considering three pipelines with different segment lengths: 10 ft. (3.05 m), 20 ft. (6.10 m) and 40 ft. (12.19 m), all of which are 400 ft. (122 m) long. The results of maximum average strains, $\epsilon_{p, \max}$, maximum relative joint displacements, $U_{\max}$, and maximum relative motion between the pipe and the ground, $Y_{\max}$, are shown in Figs. 18a to 18c respectively. The upper bound strains and relative joint displacements are also shown. One can see from these figures that the longer the pipe segment is, the larger the pipe strain, the larger the relative joint displacement and also the larger the relative displacement between the pipe and the ground. Since the segment length is directly proportional to the delay time from one end of the pipe to the other and for a given seismic traveling velocity, the delay time directly affect the seismic response behavior of buried pipelines.

The effect of pipeline length were studied in two ways. One study was done on pipelines with 20 segments consisting segment lengths varied from 10 ft. (3.05 m), 20 ft. (6.10 m) and 40 ft. (12.19 m). These represent pipelines with 200 ft. (60.96 m), 400 ft. (121.92 m) and 800 ft. (243.84 m) long.

The results of this study was identical to the one just discussed, that is, the longer the segment length, the larger the response will be. The second study was for pipelines consisting of 10, 20 and 40 segments, all of which are 20 ft. (6.10 m) in length. The results show that the responses are also the same as those of the same segment length shown in Fig. 18. This suggests that pipeline consisting of 10 segments would behave the same as long pipelines. Thus, for analysis of a long pipeline, it is only necessary to take 10 segments to give sufficient accuracy.

(2) Effect of Diameter

Pipelines with variable diameters, i.e. 6 inches (15.24 cm), 18 inches (45.72 cm) and 36 inches (91.44 cm) have been studied. Figure 19a shows that the larger pipes have a smaller pipe strain in low joint stiffness segmented system but all approaching to the same strain in large joint stiffness to the upper bound value as a continuous piping system. Figure 19b shows that there is very little difference on the relative joint motion except in the transition zone where larger pipes have a larger relative joint motion. Figure 19c shows that larger diameter pipes have much larger relative motion between the pipe and the ground. If the surrounding soil can not take such larger relative displacement, the pipe will slip. One can conclude that for a given soil environment, the possibility of slippage for larger diameter pipes is much higher than that for the smaller diameter ones.

(3) Effect of Material

The results of maximum average pipe strains, maximum relative joint displacement and maximum relative displacement between the pipe and the ground for four 18 inches (45.70 cm) diameter water pipes of different materials (cast iron, ductile iron, reinforced concrete and steel) are shown in Figs. 20a to 20c. One can see from these figures that as joint stiffness increases (approaching to a continuous pipe) the strains become larger, the relative joint displacements become smaller and the relative displacements between the pipe and the ground become larger. However, the differences for different materials is commercially available pipes used are negligible. But, one must remember that ductile material will have more ductility to accommodate the same amount of imposed strain, and thus has less likelihood to fail.

(4) Effect of End Conditions

Pipelines with three various end conditions, free-free, fixed-fixed and free-fixed have been studied. Note that the fixed-free condition has the same behavior as the free-fixed condition but in a reversible manner and hence will not be presented. The results of maximum average pipe strains and maximum relative joint displacements are shown in Figs. 21a to 21b. One can see from these figures that pipelines with fixed end condition(s) has high strain and relative joint displacement. The free-fixed condition has the same results as that of fixed-fixed condition. In practice, a pipeline joins into a heavy structure, such as buildings or man-holes would be more critical than that of a pipeline with both ends open such as a culvert during an earthquake.

IV.6 Results of Continuous Piping Systems

IV.6.1 The Investigation Parameters

For continuous piping system, there will be no joint stiffness to be considered. This study intends to investigate the various effects on the piping system embedded within soil environments, from very 'soft' to 'hard.' Thus, the soil stiffness will serve as the controlling parameter. For this study, we assume that the axial soil stiffness and the corresponding travelling wave velocity of the surrounding soil environment are related by the following relationships:

$$V_s \sim \sqrt{G/\rho} \quad \text{and} \quad k_s \sim 0.2 G \quad (103)$$

where G is the shear modulus, ρ is mass density of the surrounding soil and k_s is the axial stiffness. Since ρ will not vary to a great extent, we vary G from 500 kip/ft² (23.94 MPa) to 50,000 kips/ft² (2394 MPa). By arbitrarily assuming that $V_s = 800$ ft/sec. (243.84 m/sec) propagation velocity is corresponding to a soil axial stiffness of 3400 lbs/in/in (2340 N/cm/cm), the axial stiffnesses and the corresponding wave propagation velocities worked from above approximate relationship are shown in Table 1 below for the parametric study. The upper bound strains based on El Centro earthquake are also given.

Table 1 Soil Stiffness and Propagation Velocity Used for Parametric Study

k_s		V_s		Upper Bound
Psi	kPa	ft/sec	m/sec	strain 10^{-4}
565	3888	330	100	36.62
1130	7791	460	140	26.27
2260	15582	650	200	18.59
3400	23440	800	245	15.10
5650	38955	1030	315	11.73
6800	46885	1100	335	10.98
11300	77910	1460	445	8.28
17000	117210	1800	550	6.71
34000	234420	2500	760	4.83
50900	350940	3100	945	3.90
68000	468840	3600	930	3.36

Other parameters studied will be the same as those for the segmented piping system.

IV.6.2 The Referenced Piping System

Material (Steel)	$E = 30000 \text{ ksi (206785 MPa)}$
Outer Diameter	$D = 18 \text{ in (45.72 cm)}$
Thickness	$t_0 = 0.25 \text{ in (0.64 cm)}$
Number of Segment	$n = 20$
Segment Length	$L_i = 20 \text{ ft (6.1 m)}$
End Condition	Free-free

Seismic parameters will be the same as those used for the segmented piping system studied.

IV.6.3 Results

(1) Effect of Pipeline Length

Piping systems with three different lengths, 200 ft. (60.96 m), 400 ft. (121.92 m) and 800 ft. (243.84 m) have been studied. The maximum average strain and the maximum motion between the pipe and the ground vs. soil stiffness are shown in Figs. 22a and 22b. One can see from these figures that longer piping system has a little higher response in weak soil region. The effect diminishes as the curves converge to the same value as the soil stiffness becomes larger. The upper bound strain is also shown for comparison purpose. In general, for continuous pipeline, upper bound strains can be used without too much inaccuracy.

(2) Effect of Diameter

Continuous steel pipelines with three different diameters, 6 in. (16.83 cm), 18 in. (45.72 cm) and 36 in. (91.44 cm) have been investigated. The maximum average pipe strain vs. soil stiffness is given in Fig. 23. One can see from this figure that pipelines do confirm with the ground motion when embedded in most of the soil regions except for very 'soft' soil, where the smaller pipes have a higher strain.

(3) Effect of End Conditions

Again pipelines with three different end conditions, namely, free-free, fixed-fixed and free-fixed have been investigated. The results indicated that for continuous piping systems, the effect of end condition is negligible.

IV.7 Concluding Remarks

The quasi-static model analysis provides an efficient quantitative analysis of buried pipelines subjected to longitudinal seismic shaking. The seismic behavior of straight buried piping system observed from the parametric study can be summarized as:

1. The pipe strains obtained from the parametric study are found to be bounded by the corresponding maximum ground strains, i.e., $\epsilon_{\text{pipe}} < \frac{V_{\text{max}}}{C}$. This upper bound strain serves as a check to the safety of the pipeline.
2. Increasing joint stiffness will increase the pipe strain and the relative motion between the pipe and the ground, but decrease relative joint displacements.
3. Larger pipes will increase the relative motion between the pipe and the ground which could serve as a criterion to measure slippage possibility. It does not affect the pipe strain of the continuous system but for the segmented piping system, the pipe strain decreases and the relative joint motion increases.
4. Longer segment length will take a longer time for the seismic wave to travel from one end of the segment to the other end and the pipe will experience a stronger excitation.
5. Pipelines do move closely with the ground except where it is embedded within very 'soft' soil.
6. Pipelines embedded within soft soil have a larger relative motion between the pipe and the ground and a larger pipe strain.
7. The effects of number of segments, boundary conditions and pipe material are negligible to the behavior of the piping system during axial seismic shaking.

V. QUASISTATIC ELASTO-PLASTIC ANALYSIS

V.1. Preface

In Previous sections, only the static analysis based on beam on elastic-plastic foundations has considered the inelastic behavior of soil resistance. The analysis considers pipelines imposed by uniform ground strain only. It does not consider ground wave variation.

The quasi-static upper bound analysis and the quasi-static elastic analysis consider the variation of seismic wave by inputting the ground displacement time function in the system. However, all resistant parameters, soil, joints, etc. are considered to be linearly elastic.

In reality, both soil and joint resistances can not be linearly elastic infinitely and the ground is not subjected to uniform strain. This quasi-static elasto-plastic analysis intends to include elasto-plastic behavior for soil and joint resistance to obtain more accurate responses. The pipe material is still considered to be linearly elastic.

V.2 Formulation of the Problem

V.2.1 General

Using energy formulation approach similarly to that for the quasi-static elastic analysis, only modifications of energy due to inelastic behavior need to be discussed. Other procedures will be the same as those discussed in Section IV:— Quasi-Static Elastic Analysis.

V.2.2 Strain Energy in A Joint Spring

A elasto-plastic joint spring for i joint and its resistance curve is shown in Fig. 24. For energy formulations are separately done for elastic and elasto-plastic conditions as follows:

(1) Elastic Condition

When the relative joint displacement δ_i is less than the plastic deformation (Fig. 24), i.e.,

$$|\delta| \leq |\delta_p| \quad \text{or} \quad |F| \leq |F_p| \quad (104)$$

where F and F_p are spring and elasto-plastic spring forces. The spring displacement and spring force can be expressed:

$$\delta_i = [-1 \quad 1] \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \quad (105)$$

$$F_i = [-K_i \quad K_i] \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \quad (106)$$

The strain energy within a spring is

$$\begin{aligned} U_{\text{elastic}}^i &= -\frac{1}{2} F_i^t \delta_i \\ &= \frac{1}{2} [X_{2i} \quad X_{2i+1}] \begin{bmatrix} K_i & -K_i \\ -K_i & K_i \end{bmatrix} \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \end{aligned} \quad (107)$$

(2) Elasto-plastic Condition

When the relative positive joint displacement δ is larger than the plastic deformations (Fig. 24), i.e.,

$$\delta_i \geq \delta_p \quad \text{or} \quad F \geq F_p \quad (108)$$

The elasto-plastic strain energy of a joint spring with positive displacement will be as follows:

$$\begin{aligned} U_{\text{el-pl}}^{it} &= \frac{1}{2} F_i \delta_i - \frac{1}{2} (F_i - F_p) (\delta_i - \delta_p) \\ &= K_i \delta_i \delta_p - \frac{1}{2} F_p \delta_p \end{aligned} \quad (109)$$

since

$$F_i = K_i \delta_i = [-K_i \quad K_i] \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \quad (110)$$

$$\delta_i = [-1 \quad 1] \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \quad (111)$$

$$F_p = K_i \delta_p = \frac{1}{2} [K_i \quad K_i] \begin{Bmatrix} \delta_p \\ \delta_p \end{Bmatrix} \quad (112)$$

$$\delta_p = \frac{1}{2} [1 \quad 1] \begin{Bmatrix} \delta_p \\ \delta_p \end{Bmatrix} \quad (113)$$

Substituting Eqns. (110) to (112) into (109), the strain energy for an elasto-plastic joint spring with positive displacement in matrix form can be expressed as:

$$\begin{aligned} U_{\text{el-pl}}^{it} &= \delta_p^t \cdot F_i - \frac{1}{2} \delta_p^t F_p \\ &= \frac{1}{2} [\delta_p \quad \delta_p] \begin{bmatrix} -K_i & K_i \\ -K_i & K_i \end{bmatrix} \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} \\ &\quad - \frac{1}{8} [\delta_p \quad \delta_p] \begin{bmatrix} K_i & K_i \\ K_i & K_i \end{bmatrix} \begin{Bmatrix} \delta_p \\ \delta_p \end{Bmatrix} \end{aligned} \quad (114)$$

Note that Eqn. (113) is good only for positive relative joint motion only, i.e., $\delta = X_{2i+1} - X_{2i} > 0$. For negative relative joint motion, the sign of the unknown displacement, $X_{2i+1} - X_{2i}$, will be adjusted by itself. However, the sign for the plastic displacement limit, δ_p , must be changed, as follows:

$$\delta_p = -\frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{Bmatrix} \delta_p \\ \delta_p \end{Bmatrix} \quad (115)$$

the strain energy for an elasto-plastic joint spring with negative motion will be:

$$U_{el-pl}^i = -\frac{1}{2} \begin{bmatrix} \delta_p & \delta_p \end{bmatrix} \begin{bmatrix} -K_i & K_i \\ -K_i & K_i \end{bmatrix} \begin{Bmatrix} X_{2i} \\ X_{2i+1} \end{Bmatrix} - \frac{1}{8} \begin{bmatrix} \delta_p & \delta_p \end{bmatrix} \begin{bmatrix} K_i & K_i \\ K_i & K_i \end{bmatrix} \begin{Bmatrix} \delta_p \\ \delta_i \end{Bmatrix} \quad (116)$$

V.2.3 Strain Energy of Soil Spring Along A Pipe Segment

The elasto-plastic soil resistance-displacement characteristics over a pipe segment are shown in Fig. 25. The relative displacements between the two nodes are assumed to be linearly varied. The soil resistance is assumed to be elasto-plastic as shown in Fig. 25C with a spring constant parameter, k_i and a plastic deformation limit, y_p . Using a space coordinate z , in the axial direction, the displacement and resistance force with elastic behavior are as follows:

$$y(z) = \left[\left(1 - \frac{z}{L_i} \right) \frac{z}{L_i} \right] \begin{Bmatrix} X_{2i-1} - X_{Gi-1} \\ X_{2i} - X_{Gi} \end{Bmatrix} \quad (117)$$

$$R(z) = \left[\left(1 - \frac{z}{L_i} \right) k_i \frac{z}{L_i} + k_i \right] \begin{Bmatrix} X_{2i-1} - X_{Gi-1} \\ X_{2i} - X_{Gi} \end{Bmatrix} \quad (118)$$

The strain energy of the soil springs along a pipe segment for the elastic and elasto-plastic conditions are derived separately as follows:

(1) Elastic Condition

When the relative soil displacement $y(z)$ is less than the plastic limit, y_p along the pipe length, there will be only elastic soil spring energy over i^{th} segment in the system as follows:

$$U_{soil, el}^i = \frac{1}{2} \int_0^L R^t(z) y(z) dz \quad * \quad (119)$$

By substitution of Eqns. (117) and (118) into Eqn. (119) and after integration, one obtains:

* Superscript 't' means 'transpose.'

$$U_{\text{soil, el}}^i = \frac{1}{2} [X_{2i-1} \quad X_{2i} \quad X_{Gi-1} \quad X_{Gi}] \begin{bmatrix} \frac{k_i L_i}{3} & \frac{k_i L_i}{6} & -\frac{k_i L_i}{3} & -\frac{k_i L_i}{6} \\ \frac{k_i L_i}{6} & \frac{k_i L_i}{3} & -\frac{k_i L_i}{6} & -\frac{k_i L_i}{3} \\ -\frac{k_i L_i}{3} & -\frac{k_i L_i}{6} & \frac{k_i L_i}{3} & \frac{k_i L_i}{6} \\ -\frac{k_i L_i}{6} & -\frac{k_i L_i}{3} & \frac{k_i L_i}{6} & \frac{k_i L_i}{3} \end{bmatrix} \begin{Bmatrix} X_{2i} \\ X_{2i} \\ X_{Gi-1} \\ X_{Gi} \end{Bmatrix} \quad (120)$$

(2) Elasto-Plastic Condition

When the relative displacements between the pipe and the ground along the pipe segment partially or entirely exceed the plastic displacement limit, y_p , the soil springs then store elasto-plastic strain energy in the system. Referring to Fig. 26 for various soil resistance conditions, their energy formulations are given below:

Case (a): When the soil resistance at the beginning of the segment exceeds positive plastic limit, y_p , while the end of the segment remains elastic (i.e., $F_{2i-1} > F_p$ and $-F_p < F_{2i} < F_p$).

The plastic zone at the beginning of the segment is

$$z_{2i-1} = \frac{F_{2i-1} - F_p}{F_{2i-1} - F_{2i}} \cdot L_i \quad (121)$$

Using Fig. 25c as a reference, the strain energy from the soil spring is

$$\begin{aligned} U_{\text{soil, el-pl}}^{(a)} &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} [R(z) - F_p]^t [y(z) - y_p] dz \\ &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} R^t(z) y(z) dz \\ &\quad + \int_0^{z_{2i-1}} F_p^t y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} F_p^t y_p dz \end{aligned} \quad (122)$$

Note that the first two terms of Eqn. (122) are the same as Eqn. (119). The fourth term is unimportant because it is a constant term and will be eliminated during the differentiation process. The third term remains to be developed.

Case (b): When the soil resistance at the beginning of the segment exceeds the negative plastic limit, $-y_p$, while the end of the segment remains elastic (i.e., $F_{2i-1} < -F_p$ and $-F_p < F_{2i} < F_p$).

In this case, the plastic zone at the beginning of the segment is the same as Eqn. (121), the strain energy of the system, with only sign change for the third term is as follows:

$$\begin{aligned} U_{\text{soil, el-pl}}^{(b)} &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} R^t(z) y(z) dz \\ &\quad - \int_0^{z_{2i-1}} F_p^t y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} F_p^t y_p dz \end{aligned} \quad (123)$$

Case (c): When the soil resistance at the end of the segment exceeds positive plastic limit, y_p ; while the beginning of the segment remains elastic (i.e., $-F_p < F_{2i-1} < F_p$ and $F_{2i} > F_p$). The plastic zone at the end of the segment is

$$z_{2i} = \frac{F_{2i} - F_p}{F_{2i} - F_{2i-1}} L_i \quad (124)$$

The strain energy for the soil spring is

$$\begin{aligned} U_{\text{soil, el-pl}}^{(c)} &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_{L_i - z_{2i}}^{L_i} [R(z) - F_p]^t [y(z) - y_p] dz \\ &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_{L_i - z_{2i}}^{L_i} R^t(z) y(z) dz \\ &\quad + \int_{L_i - z_{2i}}^{L_i} F_p^t y(z) dz - \frac{1}{2} \int_{L_i - z_{2i}}^{L_i} F_p^t y_p dz \end{aligned} \quad (125)$$

If we redefine a coordinate $\bar{z}$ measuring from the end toward the beginning of the segment, the displacement and resistance functions become

$$y(\bar{z}) = \left[\frac{\bar{z}}{L_i} \left(1 - \frac{\bar{z}}{L_i} \right) \right] \begin{Bmatrix} X_{2i-1} - X_{Gi-1} \\ X_{2i} - X_{Gi} \end{Bmatrix} \quad (126)$$

$$R(\bar{z}) = \left[\frac{\bar{z}}{L_i} k_i \left(1 - \frac{\bar{z}}{L_i} \right) k_i \right] \begin{Bmatrix} X_{2i-1} - X_{Gi-1} \\ X_{2i} - X_{Gi} \end{Bmatrix} \quad (127)$$

The strain energy of the soil spring system is as follows:

$$\begin{aligned} U_{\text{soil, el-pl}}^{(c)} &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i}} R^t(\bar{z}) y(\bar{z}) d\bar{z} \\ &\quad + \int_0^{z_{2i}} F_p^t y(z) dz - \frac{1}{2} \int_{z_{2i}}^{L_i} F_p^t y_p dz \end{aligned} \quad (128)$$

Case (d); When the soil resistance at the end of the segment exceeds negative plastic limit, $-y_p$; while the beginning of the segment remains elastic (i.e., $-F_p < F_{2i-1} < F_p$ and $F_{2i} < -F_p$). The strain energy of the system is the same as Eqn. (128) except changing sign for the third term:

$$\begin{aligned} U_{\text{soil, el-pl}}^{(d)} &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i}} R^t(\bar{z}) y(\bar{z}) d\bar{z} \\ &\quad - \int_0^{z_{2i}} F_p^t y(z) dz - \frac{1}{2} \int_{L_i - z_{2i}}^{L_i} F_p^t y_p dz \end{aligned} \quad (129)$$

Case (e): When the soil resistances at both ends of the segment exceeds the positive plastic limit, y_p , (i.e., $F_{2i-1} > F_p$ and $F_{2i} > F_p$), the strain energy of soil system can be found as:

$$\begin{aligned}
U_{\text{soil, el-pl}}^{(e)} &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{L_i} [R(z) - F_p] [y(z) - y_p] dz \\
&= \int_0^{L_i} F_p^t y(z) dz - \frac{1}{2} \int_0^{L_i} F_p^t y_p dz
\end{aligned} \tag{130}$$

Case (f): When the soil resistances at both ends of the segment exceed the negative plastic limit, $-y_p$, (i.e., $F_{2i-1} < -F_p$ and $F_{2i} < -F_p$), the strain energy of the soil system becomes

$$U_{\text{soil, el-pl}}^{(f)} = - \int_0^{L_i} F_p^t y(z) dz - \frac{1}{2} \int_0^{L_i} F_p^t y_p dz \tag{131}$$

Case (g): When the soil resistance at the beginning of the segment exceeds positive plastic limit, y_p , and at the end of the segment exceeds negative plastic limit, $-y_p$, (i.e., $F_{2i-1} > F_p$ and $F_{2i} < -F_p$) the plastic zones at the two ends are as follows:

$$z_{2i-1} = \frac{F_{2i-1} - F_p}{F_{2i-1} - F_{2i}} L_i \tag{132}$$

$$z_{2i} = \frac{F_{2i} - F_p}{F_{2i} - F_{2i-1}} L_i \tag{133}$$

The strain energy for the soil spring system is:

$$\begin{aligned}
U_{\text{soil, el-pl}}^{(g)} &= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} [R(z) - F_p]^t [y(z) - y_p] dz \\
&\quad - \frac{1}{2} \int_0^{z_{2i}} [R(z) + F_p]^t [y(z) + y_p] dz \\
&= \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} R^t(z) y(z) dz \\
&\quad + \int_0^{z_{2i-1}} F_p^t y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} F_p^t y_p dz \\
&\quad - \frac{1}{2} \int_0^{z_{2i}} R^t(z) y(z) dz - \int_0^{z_{2i}} F_p^t y(z) dz \\
&\quad - \frac{1}{2} \int_0^{z_{2i}} F_p^t y_p dz
\end{aligned} \tag{134}$$

Case (h): When the soil resistance at the beginning of the segments exceeds negative plastic limit, $-y_p$ and the end of the segment exceeds positive plastic limit, y_p , the strain energy of the system becomes:

$$\begin{aligned}
U_{\text{soil, el-pl}}^{(h)} = & \frac{1}{2} \int_0^{L_i} R^t(z) y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} R^t(z) y(z) dz \\
& - \frac{1}{2} \int_0^{z_{2i-1}} F_p^t y(z) dz - \frac{1}{2} \int_0^{z_{2i-1}} F_p^t y_p dz \\
& - \frac{1}{2} \int_0^{z_{2i}} R^t(\bar{z}) y(\bar{z}) d\bar{z} + \int_0^{z_{2i}} F_p^t y_p(\bar{z}) d\bar{z} \\
& - \frac{1}{2} \int_0^{z_{2i}} F_p^t y_p d\bar{z}
\end{aligned} \tag{135}$$

V.2.4 Solutions and Preliminary Results

Based on the energy formulation discussed, Serna⁽¹³⁾ has obtained the governing equations for the seismic elasto-plastic analysis of buried pipelines. Numerical procedures similarly to that of ANSYS⁽²⁵⁾ have been programmed and preliminary results have been obtained. Since the details of these development will be presented by Serna⁽¹³⁾ in his report, this section only presents the preliminary results of one example as indicated in Table 2 for the case permitting plastic soil slippage.

Table 2-1 shows the basic control parameters of a pipeline with 20 segments subjected to El Centro Earthquake between time 2.30 second to 13.00 second. The pipeline properties, joint and soil properties are given in Tables 2-2 to 2-4.

The outputs of relative displacements between pipe and ground for the new and far ends of the pipeline are given in Tables 2-5 and 2-6. One can see from these tables that the relative displacements between the pipe and the ground are large and in the plastic range (slippage).

The outputs of pipe strains and relative joint displacements are shown in Tables 2-7 and 2-8. In general, under the slippage condition, the maximum pipe strains and relative joint displacement are a little less than the elastic analysis (Section III) as anticipated.

VI. SUMMARY AND CONCLUSIONS

This report summarizes the seismic analyses for the responses of buried pipelines subjected to seismic ground shaking. The analyses covered are as followed:

- Upper Bound Analyses
 - Simplified Approach
 - Quasi-Static Approach
- Static Analyses
 - Beam on Elastic Foundation Model
 - Beam on Elastic-Plastic Foundation Model
- Quasi-static Elastic Analysis
- Quasi-static Elasto-plastic Analysis

The upper bound analyses are simple but conservative and the bounds are used as a basis of normalization of results.

Static analyses are a higher level of analysis for the response behavior of buried pipelines. It accounts for the elasticity of the pipe material, stiffness of joint springs as well as soil resistant characteristics. However, the static analysis assumes uniform ground strains. No earthquake time-displacement inputs are necessary.

Quasi-static analysis, either elastic or elasto-plastic, are the highest level of analysis in this report. The analyses not only take the elasticity of pipe material, joint stiffness and soil resistant characteristics into account, they also take the ground displacement variations into account by inputting actual earthquake displacement-time history in the analysis. Since at each time step, the equilibrium equation is essentially a static one. Since ground displacement varies with time, the responses are also varied with time. Therefore, the analysis is called quasi-static analysis. The developments of these analyses are based on the fact that the observed earthquake responses of buried pipeline are essentially static and the effects of inertia force are negligible.

Based on these analyses, general conclusions for the seismic responses of buried pipelines can be summarized as follows:

- The behavior of seismic response of buried pipeline is controlled by the ground displacement characteristics, not by ground accelerations.
- Both pipe strain and relative joint displacements are jointly taken up the imposed earthquake ground displacement. Thus, highest pipe strain will be achieved when the pipe is continuous and longest relative joint displacement will be relieved when pipe segments are infinitely rigid.
- The responses are larger under softer soil condition. This is because softer soil has lower wave propagation speed and thus requires longer delay time for one end of the pipe segment to the other end.
- The responses are higher when there is no slippage between the soil and the pipe. Slippage reduces both pipe strains and relative joint displacements.
- For segmented pipelines, longer pipe segment will produce high pipe strain and larger relative joint displacement.

VII. REFERENCES

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Table 2-1

```

*****
*
*   ELASTO-PLASTIC PIPELINE ANALYSIS
*
*****

*****
*
*   NUMBER OF SEGMENTS           20
*   NUMBER OF JOINTS             21
*   INITIAL TIME                  2.30
*   FINAL TIME                   13.00
*   TIME INTERVAL                 0.10
*   NUMBER OF ITERATIONS          5.
*   INITIAL PERCENTAGE           0.0100
*
*****

*****
*
*   PLASTIC
*   SLIPPAGE PERMITTED
*   NORMAL STIFFNESS
*   NEW FORMULATION
*
*****

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Table 2-2

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*****
*
*   PIPELINE PROPERTIES
*
*****

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PIPE	I. DIAMETER	EX. DIAMETER	AREA	M. ELASTICITY	LENGTH
1	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
2	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
3	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
4	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
5	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
6	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
7	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
8	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
9	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
10	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
11	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
12	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
13	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
14	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
15	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
16	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
17	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
18	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
19	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03
20	0.1750E+02	0.1800E+02	0.1394E+02	0.3000E+08	0.2400E+03

Table 2-3

```

*****
*
*   JOINT PROPERTIES
*
*****

```

JOINT	STIFFNESS	MAXIMUM ELASTIC DISPLACEMENT
1	0.1000E+04	0.1000E+10
2	0.1000E+04	0.1000E+02
3	0.1000E+04	0.1000E+02
4	0.1000E+04	0.1000E+02
5	0.1000E+04	0.1000E+02
6	0.1000E+04	0.1000E+02
7	0.1000E+04	0.1000E+02
8	0.1000E+04	0.1000E+02
9	0.1000E+04	0.1000E+02
10	0.1000E+04	0.1000E+02
11	0.1000E+04	0.1000E+02
12	0.1000E+04	0.1000E+02
13	0.1000E+04	0.1000E+02
14	0.1000E+04	0.1000E+02
15	0.1000E+04	0.1000E+02
16	0.1000E+04	0.1000E+02
17	0.1000E+04	0.1000E+02
18	0.1000E+04	0.1000E+02
19	0.1000E+04	0.1000E+02
20	0.1000E+04	0.1000E+02
21	0.1000E+04	0.1000E+10

Table 2.4

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*****
*
*   SOIL PROPERTIES
*
*****

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SEGMENT	SOIL STIFFNESS	PROPAGATION VELOCITY	MAX. ELASTIC DISP.
1	0.5650000E+03	0.3960000E+04	0.3000000E+00
2	0.5650000E+03	0.3960000E+04	0.3000000E+00
3	0.5650000E+03	0.3960000E+04	0.3000000E+00
4	0.5650000E+03	0.3960000E+04	0.3000000E+00
5	0.5650000E+03	0.3960000E+04	0.3000000E+00
6	0.5650000E+03	0.3960000E+04	0.3000000E+00
7	0.5650000E+03	0.3960000E+04	0.3000000E+00
8	0.5650000E+03	0.3960000E+04	0.3000000E+00
9	0.5650000E+03	0.3960000E+04	0.3000000E+00
10	0.5650000E+03	0.3960000E+04	0.3000000E+00
11	0.5650000E+03	0.3960000E+04	0.3000000E+00
12	0.5650000E+03	0.3960000E+04	0.3000000E+00
13	0.5650000E+03	0.3960000E+04	0.3000000E+00
14	0.5650000E+03	0.3960000E+04	0.3000000E+00
15	0.5650000E+03	0.3960000E+04	0.3000000E+00
16	0.5650000E+03	0.3960000E+04	0.3000000E+00
17	0.5650000E+03	0.3960000E+04	0.3000000E+00
18	0.5650000E+03	0.3960000E+04	0.3000000E+00
19	0.5650000E+03	0.3960000E+04	0.3000000E+00
20	0.5650000E+03	0.3960000E+04	0.3000000E+00

Table 2-5

 * RELATIVE DISPLACEMENTS BETWEEN PIPE AND SOIL *
 * NEAR END *

PIPE	POSITIVE	TIME	NEGATIVE	TIME
1	0.328621630E+00	11.8	-.429268122E+00	2.3
2	0.300511658E+00	11.8	-.399650872E+00	2.5
3	0.326350093E+00	11.9	-.424964249E+00	2.4
4	0.298811759E+00	12.0	-.410663009E+00	2.5
5	0.322245777E+00	12.0	-.420976818E+00	2.5
6	0.326759875E+00	12.1	-.425335824E+00	2.6
7	0.296908319E+00	12.1	-.397724152E+00	2.8
8	0.326281250E+00	12.2	-.424901068E+00	2.7
9	0.301824749E+00	12.3	-.413121402E+00	2.8
10	0.318615913E+00	12.3	-.417359531E+00	2.8
11	0.326691628E+00	12.4	-.425274014E+00	2.9
12	0.293315709E+00	12.4	-.398239434E+00	3.0
13	0.326212466E+00	12.5	-.424827993E+00	3.0
14	0.306846499E+00	12.6	-.415579557E+00	3.1
15	0.314987242E+00	12.6	-.413751185E+00	3.1
16	0.326623321E+00	12.7	-.425201535E+00	3.2
17	0.289717376E+00	12.7	-.400722086E+00	3.3
18	0.326143742E+00	12.8	-.424775839E+00	3.3
19	0.311851621E+00	12.9	-.418001652E+00	3.4
20	0.309469998E+00	12.9	-.406695485E+00	3.4

 * MAXIMUM VALUE 0.32862E+00 *
 * MINIMUM VALUE -0.42927E+00 *

Table 2-6

 * RELATIVE DISPLACEMENTS BETWEEN PIPE AND SOIL *
 * FAR END *

PIPE	POSITIVE	TIME	NEGATIVE	TIME
1	0.421090245E+00	2.3	-.323876977E+00	11.8
2	0.400724232E+00	2.3	-.301543772E+00	11.8
3	0.425334275E+00	2.4	-.326110542E+00	11.9
4	0.409759402E+00	2.5	-.295397758E+00	12.0
5	0.422143400E+00	2.5	-.322902024E+00	12.0
6	0.424962103E+00	2.6	-.325702012E+00	12.1
7	0.398439348E+00	2.8	-.297964156E+00	12.1
8	0.425397575E+00	2.7	-.326179147E+00	12.2
9	0.412283540E+00	2.8	-.300457895E+00	12.3
10	0.418567657E+00	2.8	-.319343686E+00	12.3
11	0.425024211E+00	2.9	-.325770199E+00	12.4
12	0.397292614E+00	3.0	-.294373095E+00	12.4
13	0.425449789E+00	3.0	-.326247892E+00	12.5
14	0.414808095E+00	3.1	-.305510521E+00	12.6
15	0.415000379E+00	3.1	-.315784812E+00	12.6
16	0.425075412E+00	3.2	-.325838268E+00	12.7
17	0.399769723E+00	3.3	-.290776432E+00	12.7
18	0.425523520E+00	3.3	-.326316178E+00	12.8
19	0.417366922E+00	3.4	-.310576877E+00	12.9
20	0.414958537E+00	3.4	-.314159274E+00	12.9

 * MAXIMUM VALUE 0.42552E+00 *
 * MINIMUM VALUE -0.32632E+00 *

Table 2-7

 * MAXIMUM PIPE STRAINS *

PIPE	TENSILE	TIME	COMPRESSIVE	TIME
1	0.120964087E-04	2.3	-.103340644E-04	11.8
2	0.120286932E-04	2.5	-.982008805E-05	11.8
3	0.123411210E-04	2.4	-.104903393E-04	11.9
4	0.121747480E-04	2.5	-.964617448E-05	12.0
5	0.122950496E-04	2.5	-.104075089E-04	12.0
6	0.123456130E-04	2.6	-.104845985E-04	12.1
7	0.120043169E-04	2.8	-.970887504E-05	12.1
8	0.123403706E-04	2.7	-.104912579E-04	12.2
9	0.122046185E-04	2.8	-.980137338E-05	12.3
10	0.122525125E-04	2.8	-.103177963E-04	12.3
11	0.123448754E-04	2.9	-.104855717E-04	12.4
12	0.120078339E-04	3.0	-.959629051E-05	12.4
13	0.123394793E-04	3.0	-.104921692E-04	12.5
14	0.122339798E-04	3.1	-.995148093E-05	12.6
15	0.122090432E-04	3.1	-.102250524E-04	12.6
16	0.123440213E-04	3.2	-.104865358E-04	12.7
17	0.120427067E-04	3.3	-.948352772E-05	12.7
18	0.123388127E-04	3.3	-.104930723E-04	12.8
19	0.122605325E-04	3.4	-.100932621E-04	12.9
20	0.119297865E-04	3.4	-.996083782E-05	12.9

 * MAXIMUM VALUE 0.12346E-04 *
 * MINIMUM VALUE -0.10493E-04 *

Table 2-8

 * RELATIVE DISPLACEMENTS BETWEEN JOINTS *

JOINT	EXTENSION	TIME	CONTRACTION	TIME
1	0.429268122E+00	2.3	-.328621030E+00	11.8
2	0.820483387E+00	2.3	-.624388635E+00	11.8
3	0.817246377E+00	2.4	-.586472452E+00	11.9
4	0.799544990E+00	2.4	-.601408124E+00	11.9
5	0.830736220E+00	2.5	-.617643535E+00	12.0
6	0.802838504E+00	2.6	-.576265812E+00	12.0
7	0.820717394E+00	2.6	-.622610390E+00	12.1
8	0.819705784E+00	2.7	-.591451049E+00	12.2
9	0.797635615E+00	2.9	-.597815871E+00	12.2
10	0.829643130E+00	2.8	-.619073868E+00	12.3
11	0.805247545E+00	2.9	-.572664499E+00	12.3
12	0.817197919E+00	2.9	-.619085908E+00	12.4
13	0.822120607E+00	3.0	-.596418738E+00	12.5
14	0.795826614E+00	3.2	-.594223559E+00	12.5
15	0.828559339E+00	3.1	-.620497823E+00	12.6
16	0.807645917E+00	3.2	-.569062591E+00	12.6
17	0.813678503E+00	3.2	-.615555644E+00	12.7
18	0.824545562E+00	3.3	-.601378381E+00	12.8
19	0.793963015E+00	3.5	-.590630174E+00	12.8
20	0.824062467E+00	3.4	-.620046675E+00	12.9
21	0.414958537E+00	3.4	-.314159274E+00	12.9

 * MAXIMUM VALUE 0.83074E+00 *
 * MINIMUM VALUE -0.62439E+00 *

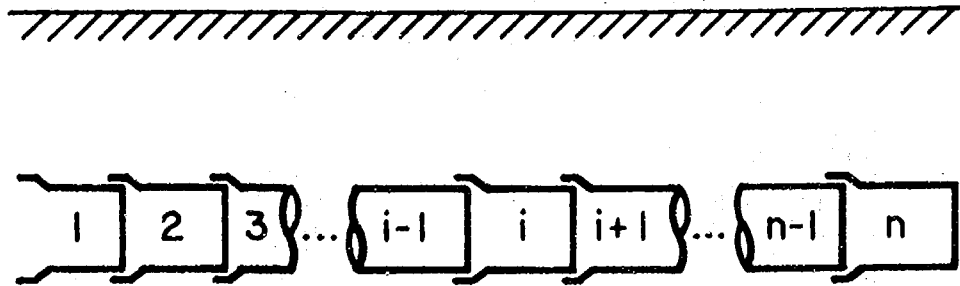


Fig. 1 Schematic of a Buried Pipeline

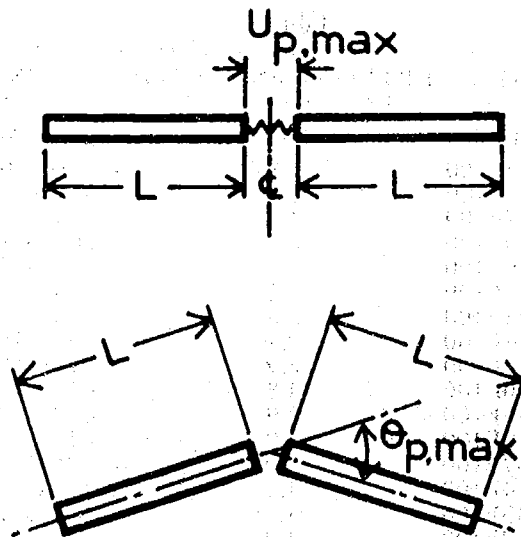


Fig. 2 Maximum Relative Joint Displacement/Rotation of Segmented Pipeline

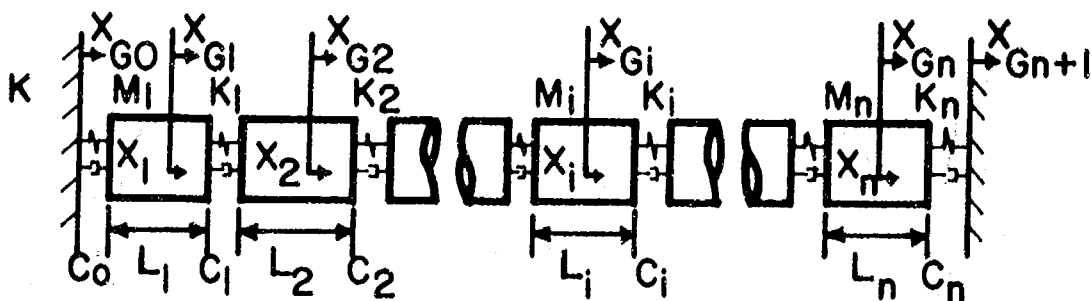


Fig. 3 A Buried Pipeline Model

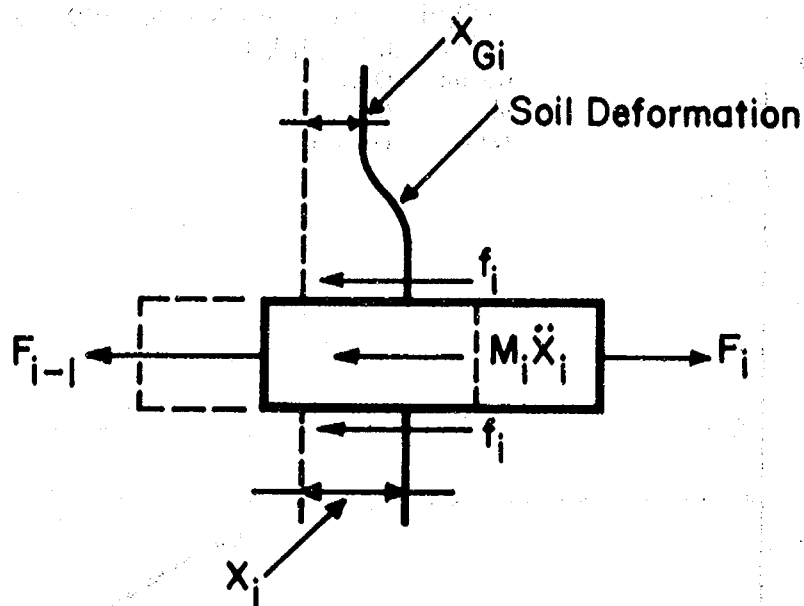


Fig. 4 Dynamic Equilibrium of a Pipe Segment

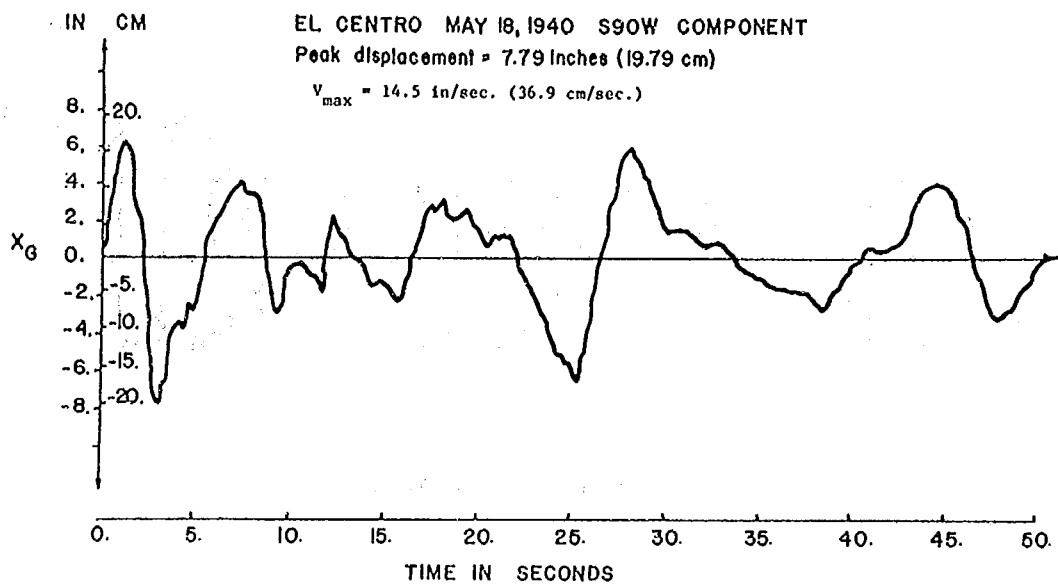


Fig. 5 Referenced Ground Displacement Input

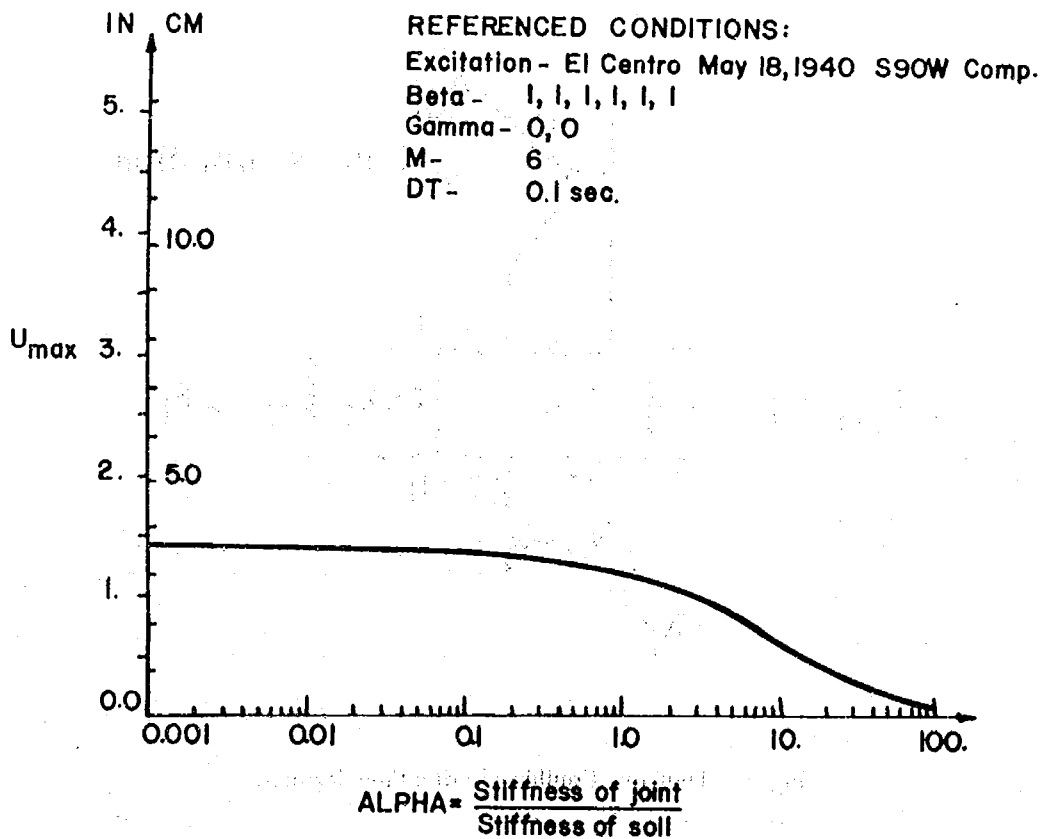


Fig. 6 U_{max} vs α under Referenced Conditions

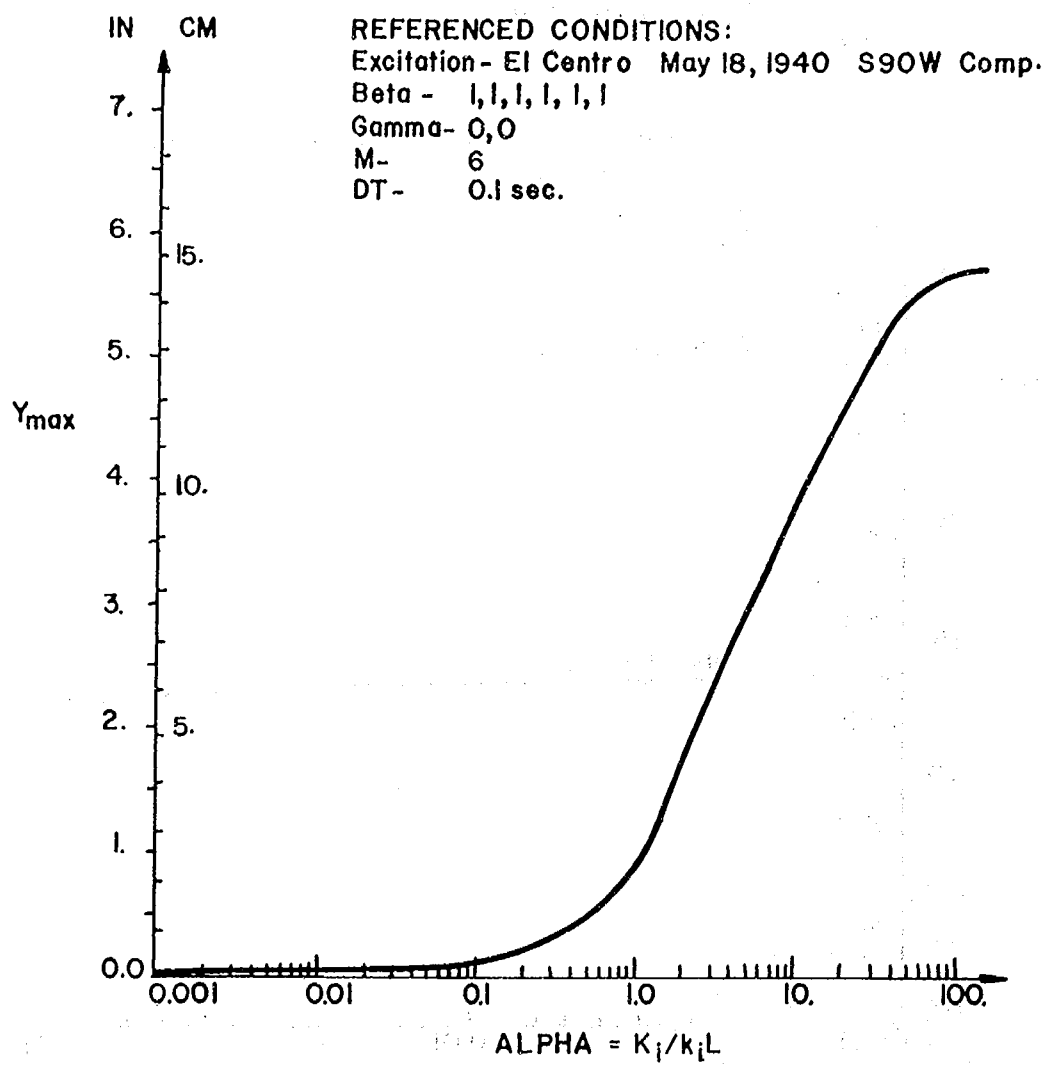


Fig. 7 Y_{max} vs α under Referenced Conditions

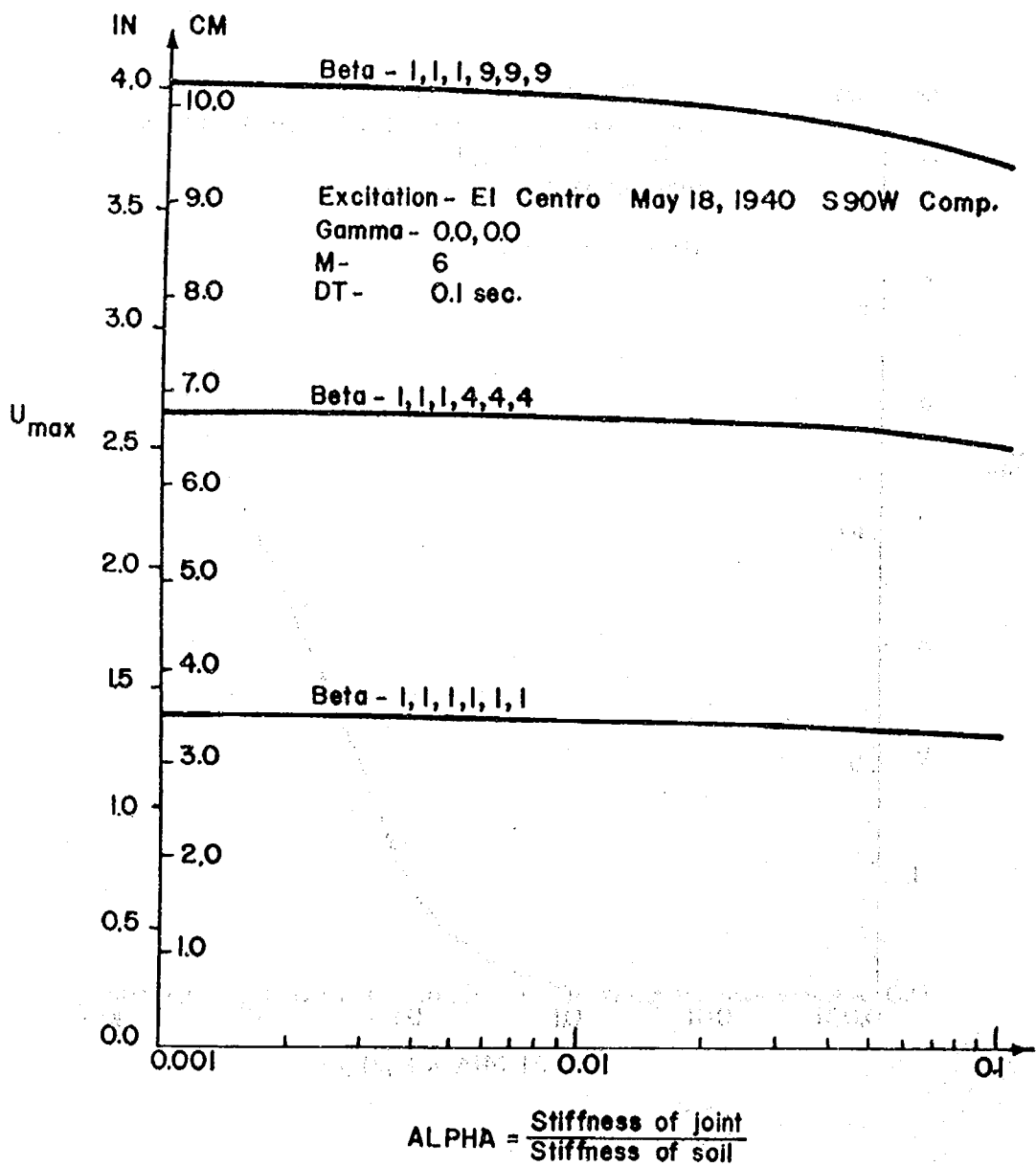


Fig. 8 Effect of Nonuniformity of Soil on U_{max}

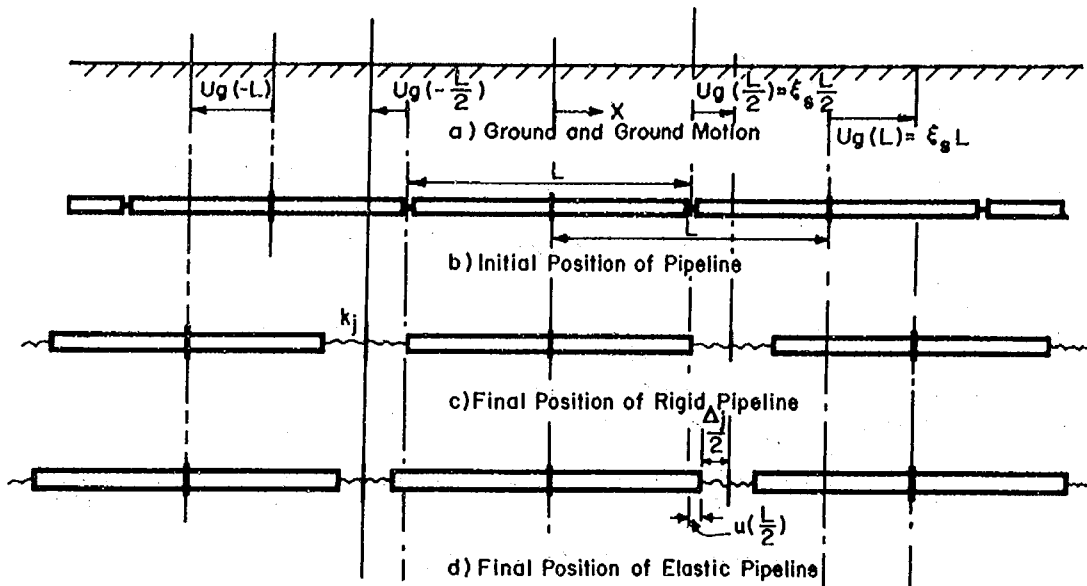


Fig. 9 Schematics of Pipe/Joint Displacements within a Region of Uniform Tensile Ground Strain

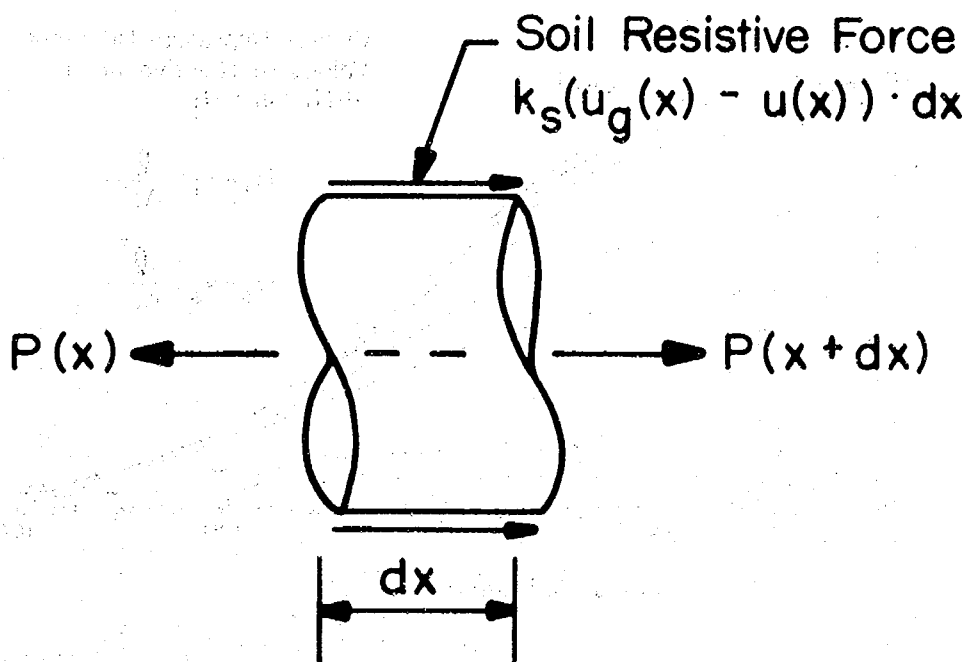


Fig. 10 Force Equilibrium on a Differential Length of Pipe

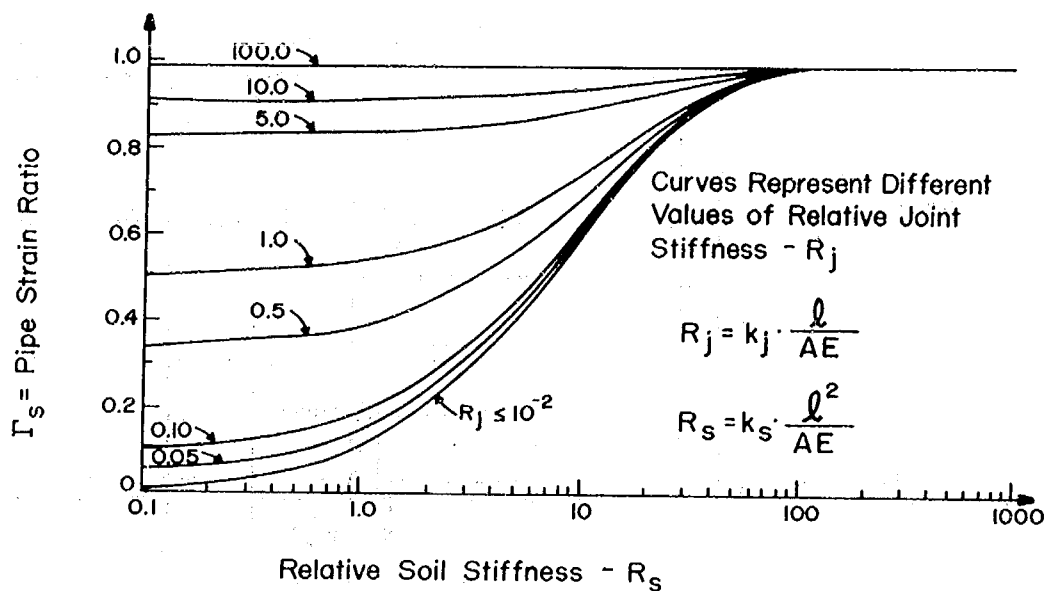


Fig. 11 Pipe Strain Ratio - Γ_s

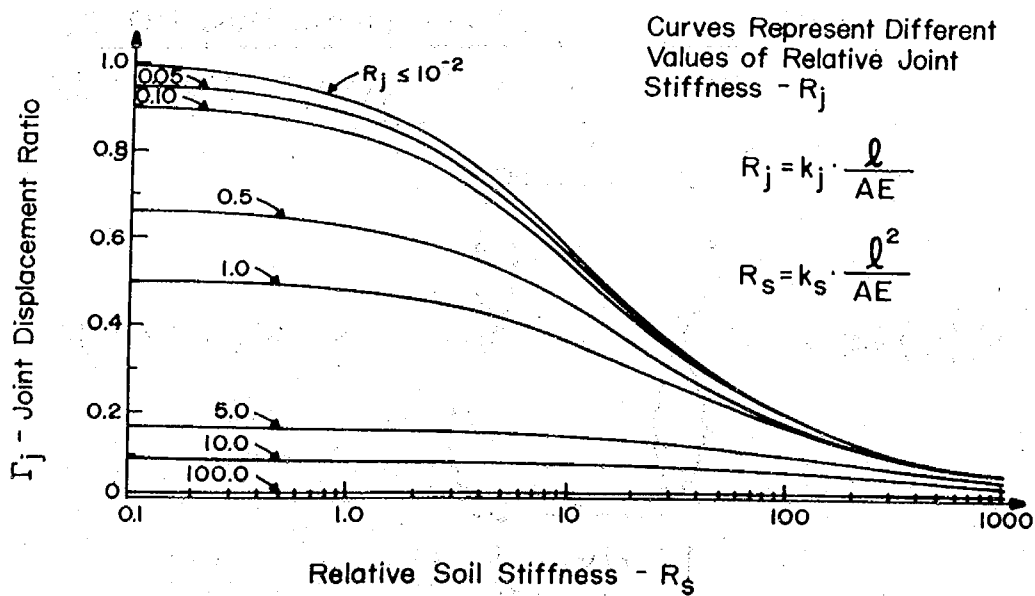


Fig. 12 Joint Displacement Ratio - Γ_j

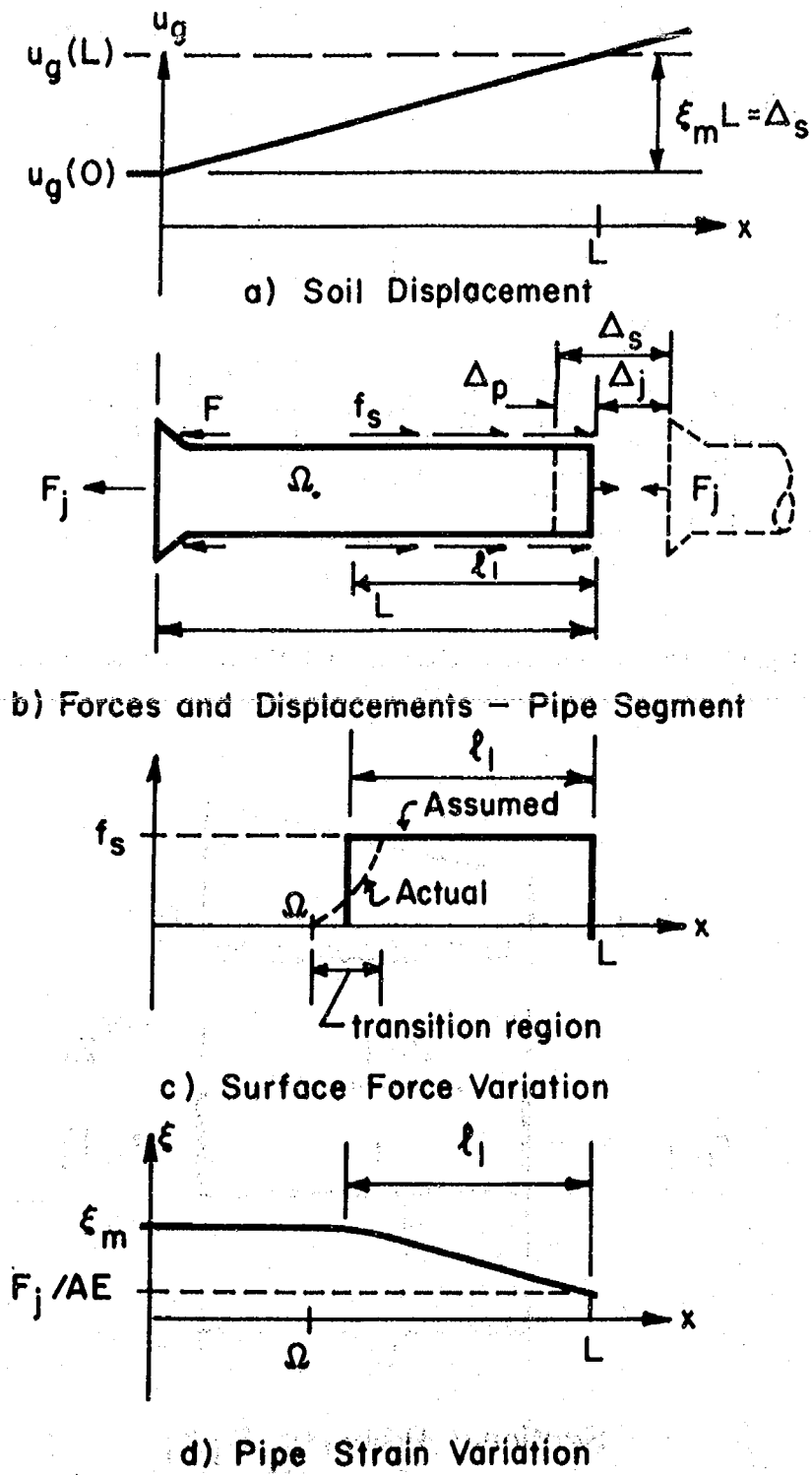


Fig. 13 Slippage Effect on Segmented Pipe

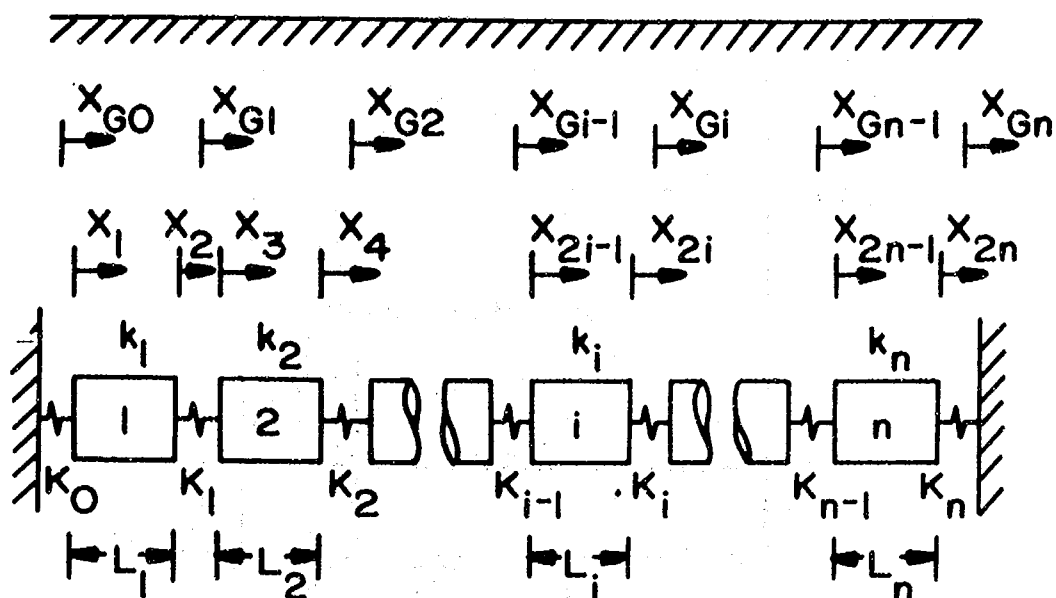


Fig. 14 Quasi-Static Elastic Seismic Analysis Model for Buried Pipeline

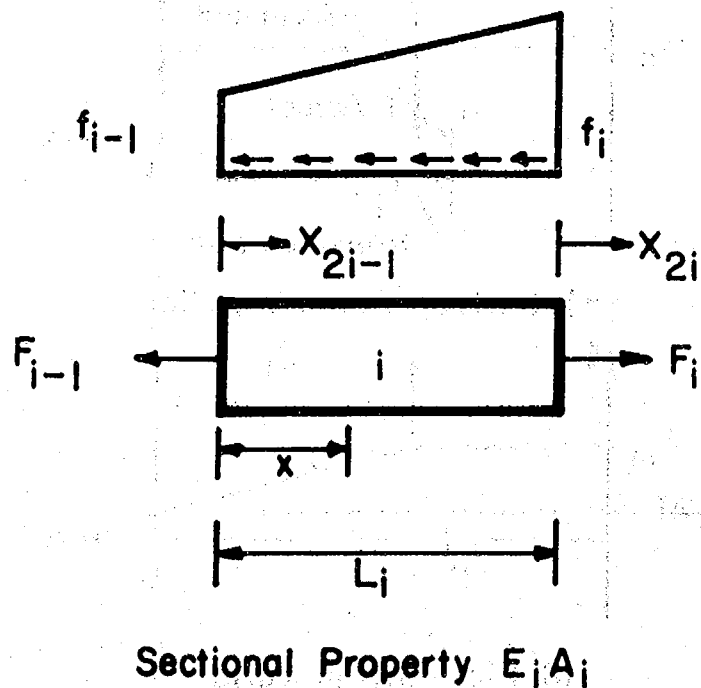


Fig. 15 A Pipe Segment with Linear Surface Resistance Force

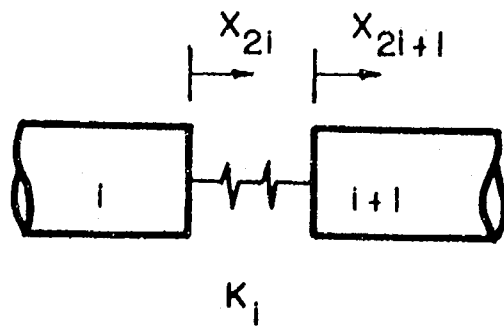


Fig. 16 A Joint Spring

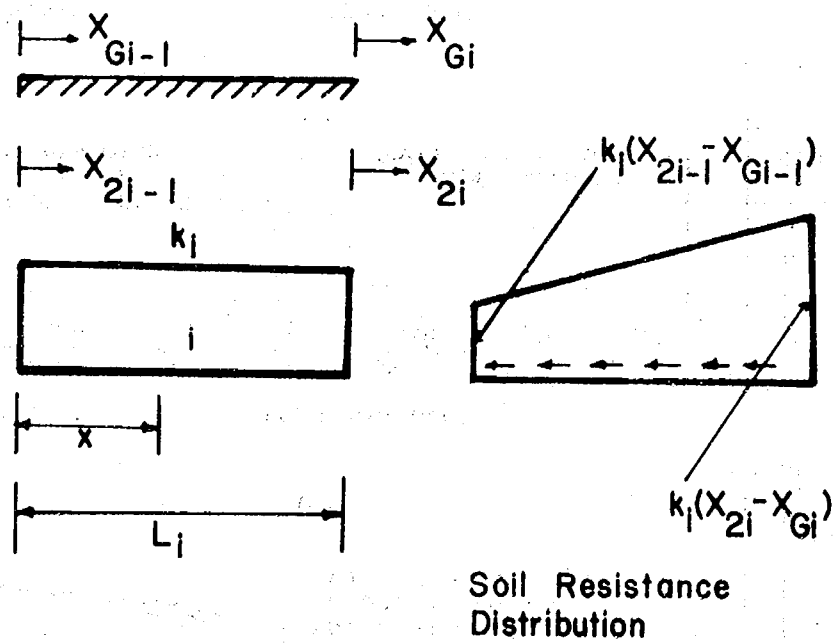


Fig. 17 Soil Resistant Distribution along a Pipe Segment

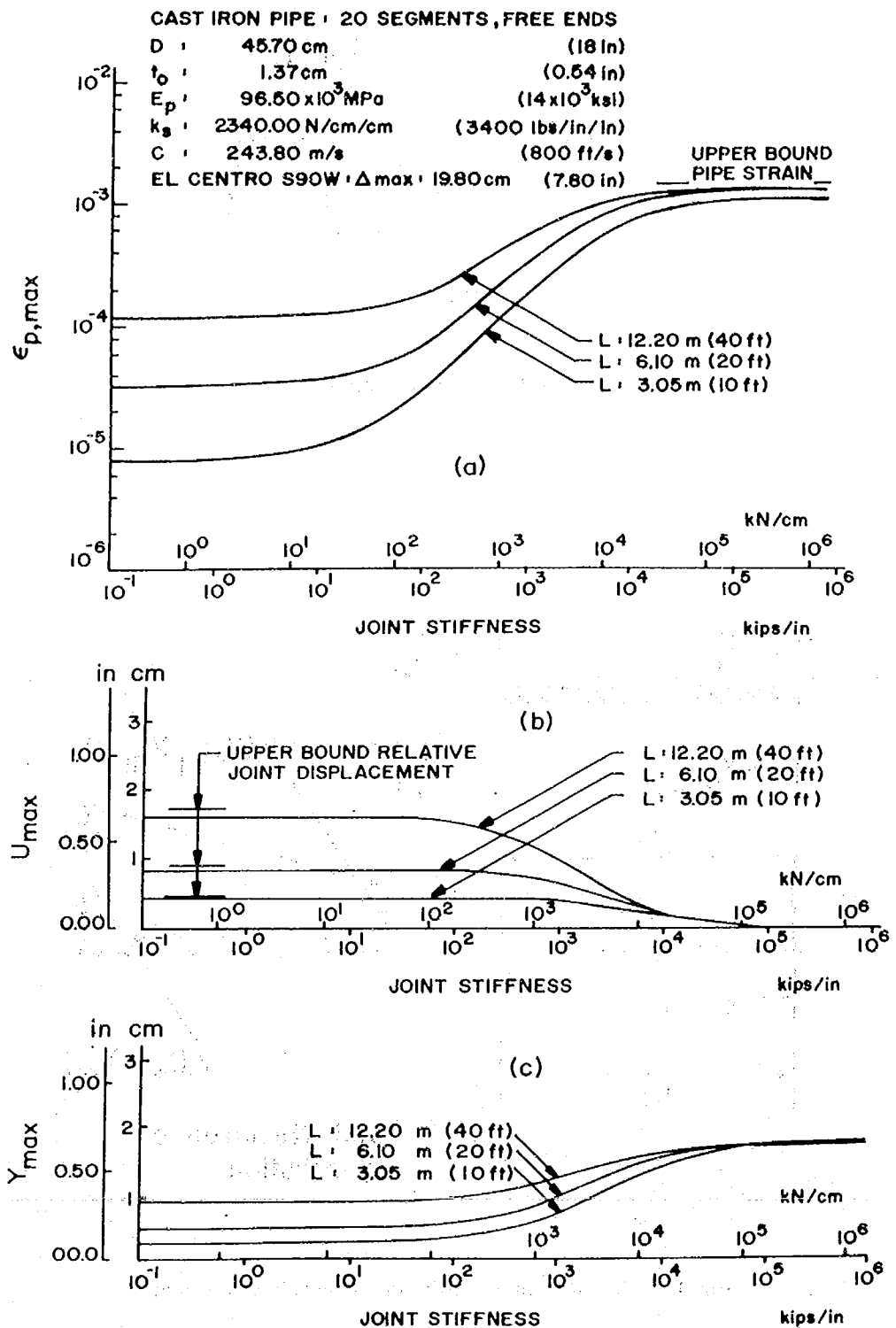


Fig. 18 Effect of Segment Length (Segmented Piping System)

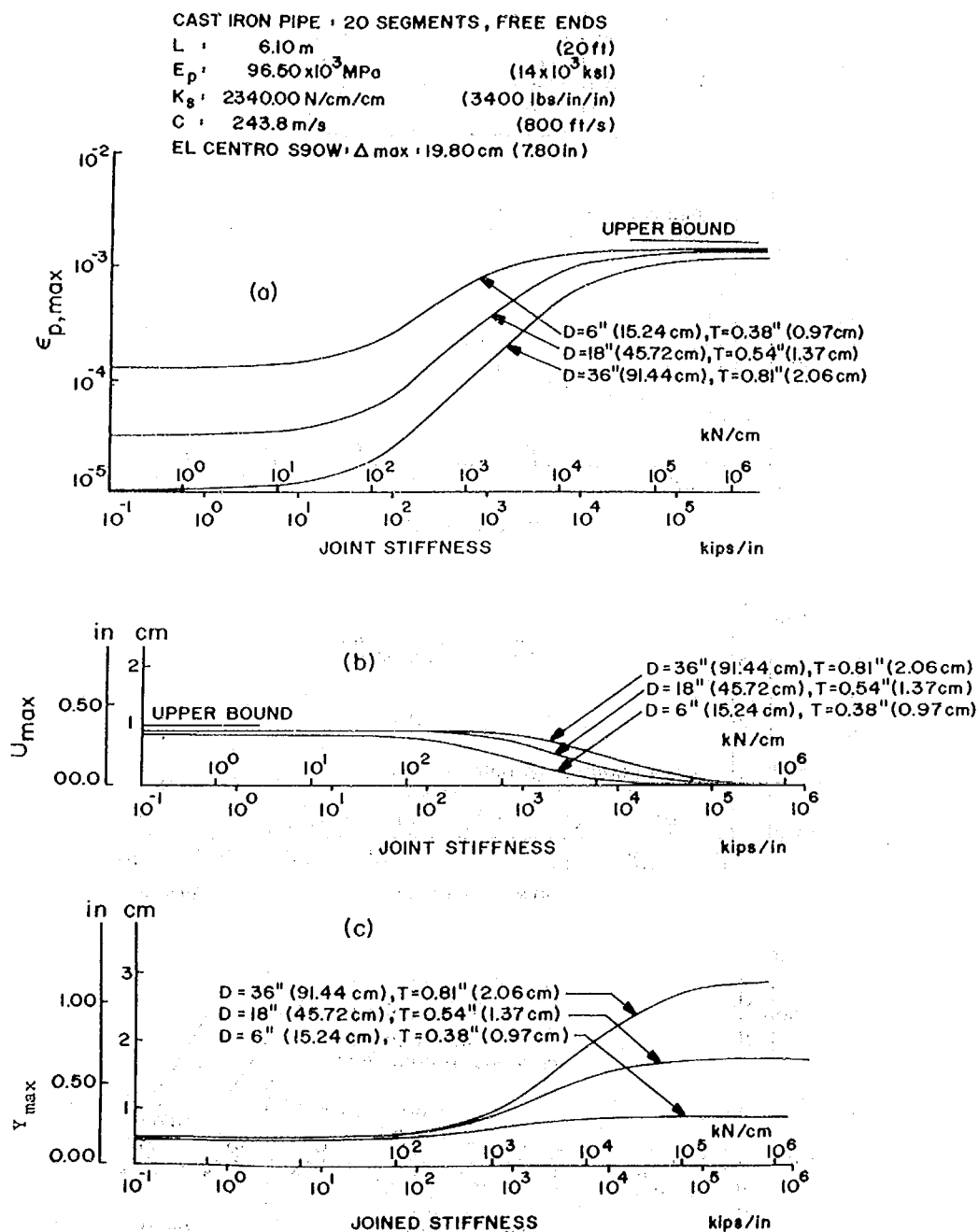


Fig. 19 Effect of Diameter (Segmented Piping System)

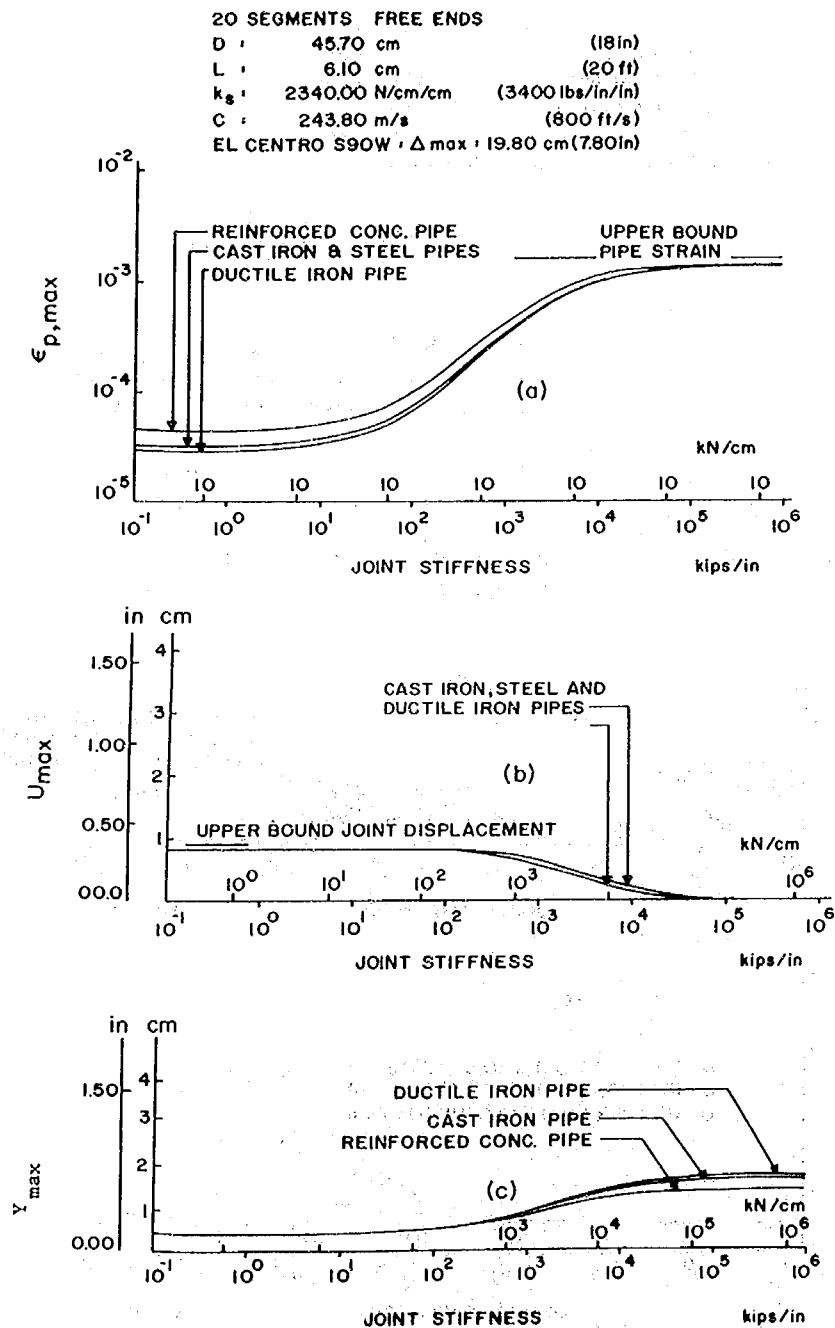


Fig. 20 Effect of Material (Segmented Piping System)

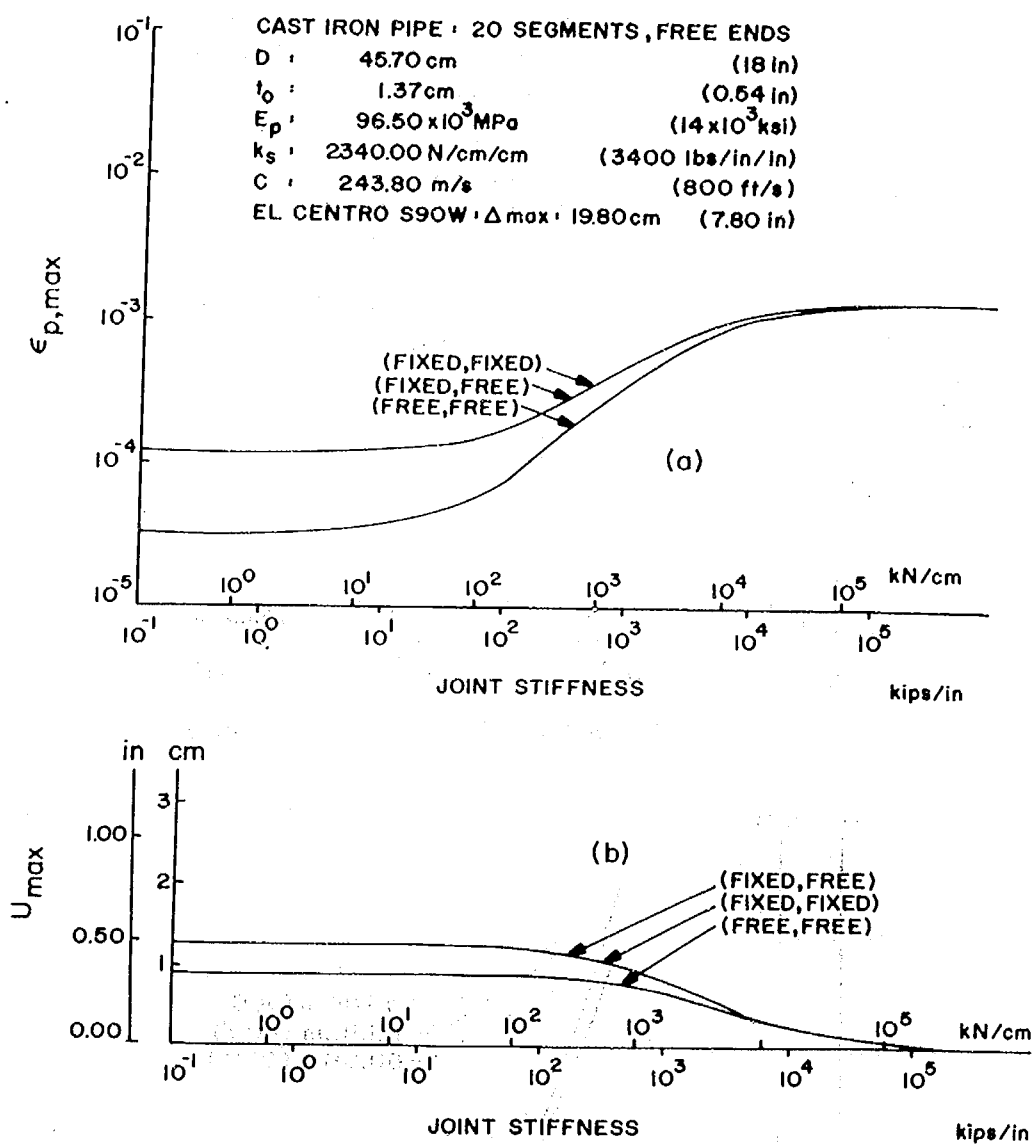


Fig. 21 Effect of Boundary Conditions (Segmented Piping System)

STEEL PIPE, FREE ENDS

D : 45.70 cm (18 in)

t_o : 0.64 cm (0.25 in)

E_s : 206.80×10^3 MPa (30×10^3 ksi)

EL CENTRO S90W : Δ_{max} : 19.80 cm (7.80 in)

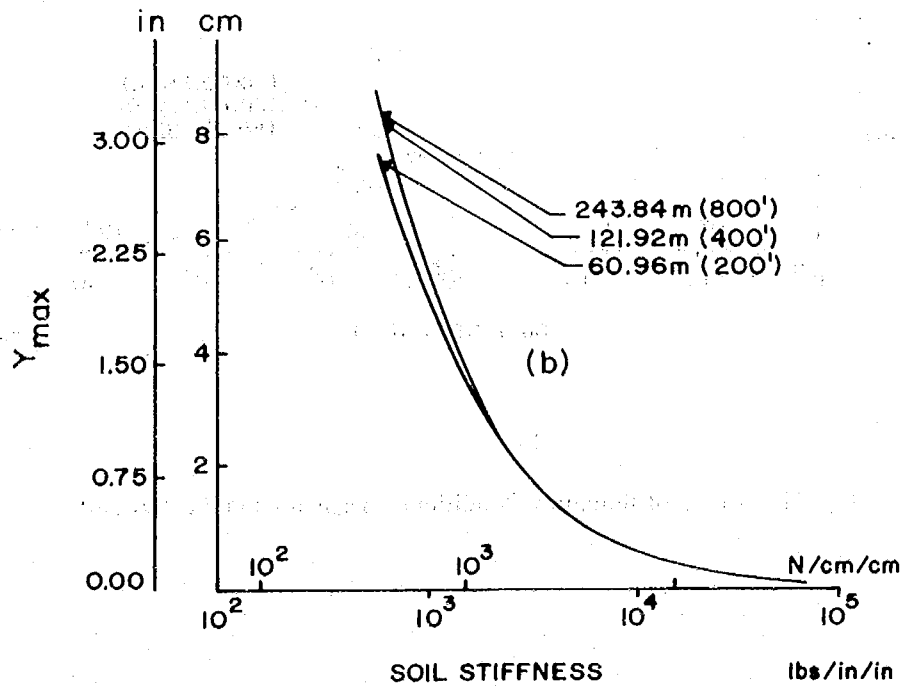
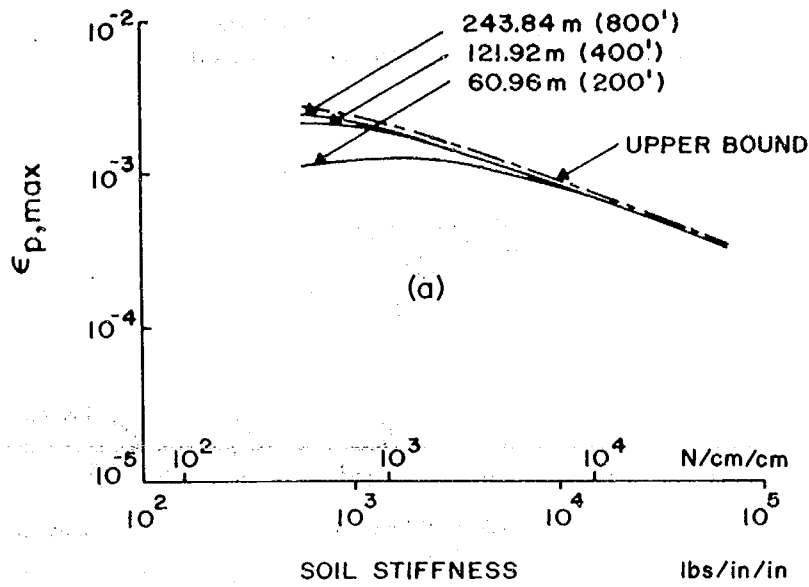


Fig. 22 Effect of Pipeline Length (Continuous Piping System)

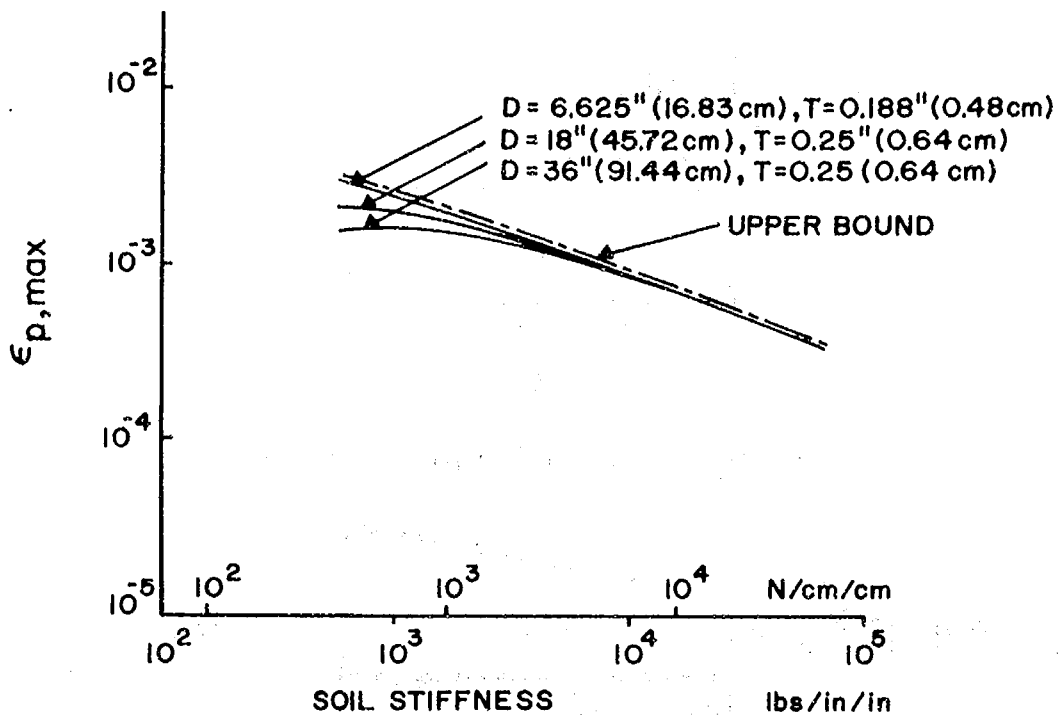


Fig. 23 Effect of Diameter (Continuous Piping System)

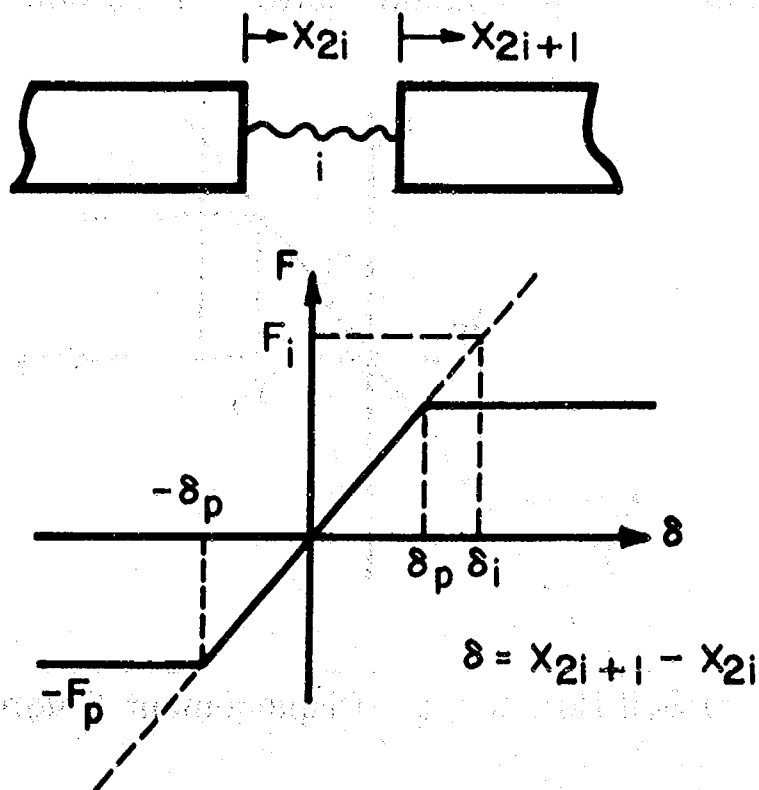
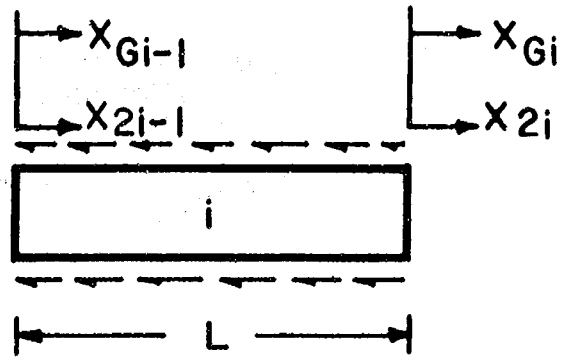
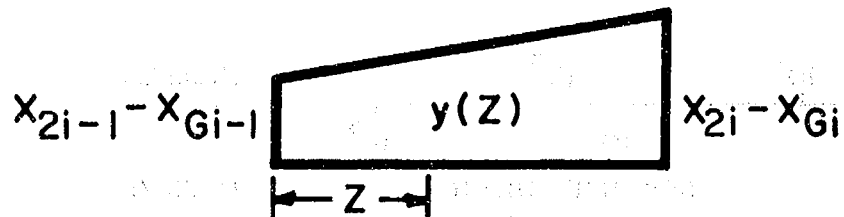


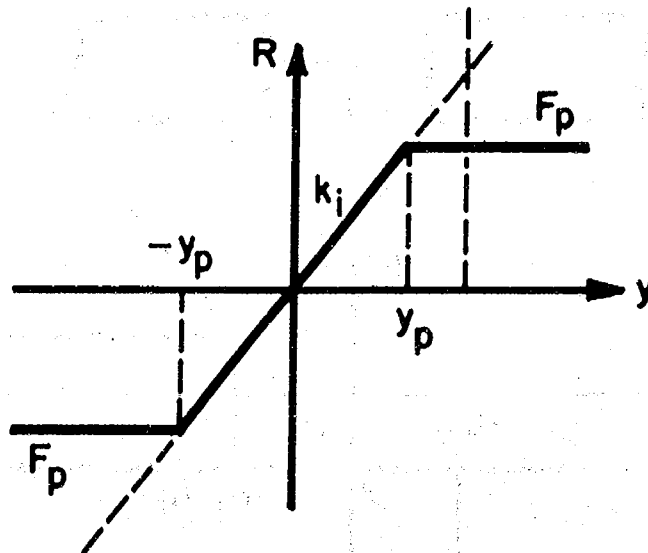
Fig. 24 i^{th} Elasto-Plastic Joint Spring and Its Resistance Curve



a) Pipe Segment and Coordinates

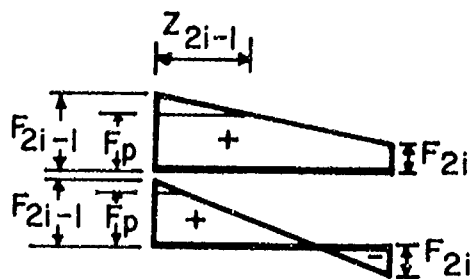


b) Relative Displacement Between Pipe and Ground

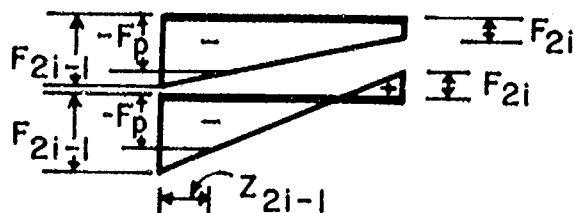


c) Soil Resistance - Displacement Diagram

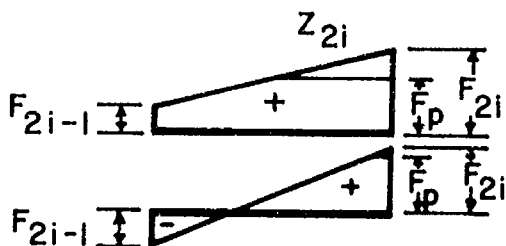
Fig. 25 Elasto-Plastic Soil Resistance-Displacement Characteristics over a Pipe Segment



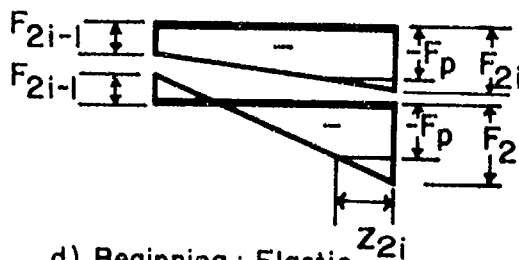
a) Beginning: Positive Plastic
End: Elastic



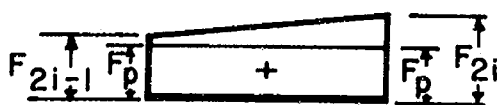
b) Beginning: Negative Plastic
End: Elastic



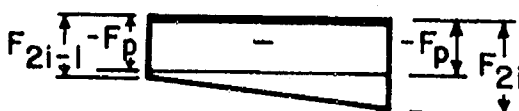
c) Beginning: Elastic
End: Positive Plastic



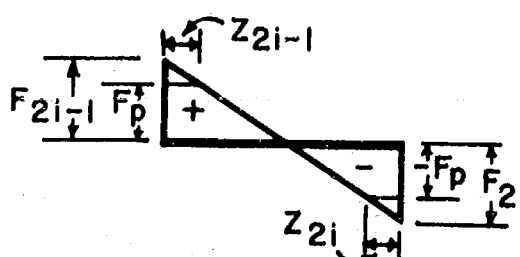
d) Beginning: Elastic
End: Negative Plastic



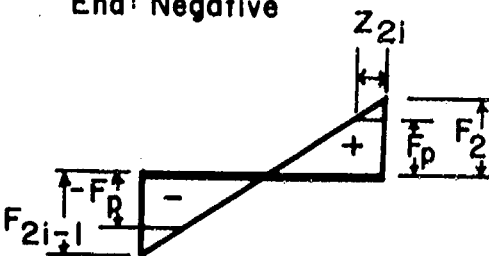
e) Beginning: Positive Plastic
End: Positive Plastic



f) Beginning: Negative Plastic
End: Negative



g) Beginning: Positive Plastic
End: Negative Plastic



h) Beginning: Negative Plastic
End: Positive Plastic

Fig. 26 Various Soil Resistances over a Pipe Segment