

**AN EXPERIMENTAL STUDY OF DYNAMIC CHARACTERISTICS
OF
THE KOMPIRA BRIDGE**

by

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概 要

二連の起振機を用いて振動実験を行ない、単径間単純トラス補剛吊橋の動特性を求めた。起振機によって吊橋を鉛直たわみ振動又は捩れ振動の状態におき、共振々動数、振動形、および減衰係数の計測を行なった。

エネルギー解法は固有振動数、振動波形の算定に十分の実用性のあることが認められた。減衰係数は振幅に無関係であるが、振動波形によって異なることが認められた。

Key Words

吊橋、振動実験、減衰係数、起振機

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1. Summary

A vibration test by the use of a twin-excitor was made to know the dynamic characteristics of a single span suspension bridge.

The bridge was put in oscillation in either vertical bending modes or in torsional ones, and resonant frequencies, modes of vibration, and damping capacities of the bridge were obtained.

Energy method calculating natural frequencies was found to be applicable to the truss stiffened single span suspension bridge. Damping capacities were found to be 0.02 - 0.04 (in logarithmic decrement) for torsional modes of oscillation and 0.022 - 0.08 for bending respectively.

2. Definition of Symbols

- δ : logarithmic decrement, δ_n corresponds to nth mode of vibration
- β : damping constant $\delta = 2 \pi \beta / \sqrt{1 - \beta^2}$
- η : vertical bending deflection
- α : torsional deflection
- ℓ : span length
- x : abscissa to the span
- y : normal ordinate
- a : component of amplitude
- m : mass per unit length
- t : time
- b, c : constant
- k : spring constant
- K : damping coefficient
- $F(t)$: force
- EI : bending rigidity
- ω : circular frequency, ω_n corresponds to nth mode of vibration
- i : unit imaginary number, $\sqrt{-1} = i$
- n : order of modes

3. Introduction

Remarkable in bridge engineering is the recent progress in theoretical and numerical technique of analysis and in the utilization of high tensile strength steel. This has brought drastic construction of long span bridges over severe nature of condition. On the contrary this trend in engineering has inevitably led to the construction of structures which are more flexible and more apt to vibrate than old ones.

Traffic loads, earthquakes, and wind are the main cause of vibration of structures, and wind, above all, plays important role, not to mention of the disaster of the Tacoma Narrows Bridge which was fallen deep in water by the wind action in 1940.

Aerodynamic Stability of a bridge can be governed by the following factors;

- a) Rigidity of the structure
- b) Aerodynamic characteristics of the cross sectional shape of the structure, and
- c) Damping capacities of the structure.

The factors above mentioned are entangled in the matter, and it is impossible to predict the aerodynamic stability characteristics of a bridge through an analytical method. Then, the full model or sectional model method of wind tunnel testing is most generally used to know the stability against wind.

In wind tunnel testing dynamic characteristics (i. e., natural frequencies, modes of oscillation, and damping capacities) as well as the cross sectional shape must be known beforehand, but only few informations are available. So, the authors have made field tests to collect data on the seven bridges tabulated in Table 1. The types of bridge tested are suspension bridge but the Onomichi Bridge.

The Kompira Bridge is the seventh of them, and the authors introduce the test results of the bridge in the followings.

4. Brief Description of the Kompira Bridge

The Kompira bridge, two hinged single span suspension bridge, was erected by the Nippon Kokan K. K. (Japan Steel and Tube Co. Ltd.) in April 1968.

The distinctive features of the bridge are as follows;

- a) Adoption of steel pipe structure for main trusses and lateral systems in order to decrease aerodynamic force on the structure.
- b) Adoption of prefabricated parallel strands of cable in order to shorten its construction period.
- c) Adoption of the center diagonal stays in order to suppress asymmetric oscillations with one node which are considered to be fatal from the viewpoint of the aerodynamic failure of structure.

(Note: In the case of this dynamic test the stays were kept slacked so that they did not work).

Details of design data are presented in Fig. 1.

5. Test Procedure

5.1 Twin-excitor

The bridge was put in oscillation by using a specially devised twin-excitor which was able to generate reciprocating motive forces either in phase or out of phase and thus able to excite the bridge in any of vertical bending motions and torsional ones. The outline of the excitor is shown in Fig. 2. The frequency of exciting force can be changed in the range of 0.2 to 10.0 cycles per second, and the maximum motive force per unit excitor is 15 tons at 10.0 cycles per second. The motive force can be kept with a constant value in spite of the change of frequency of motive force. The motive force can be cut off within 10 seconds so that it produces an accurate free vibration of structure at a resonant frequency.

The twin-excitor was set on the half, quarter, and five eighth points of the suspended structure by turns in the course of test.

5.2 Instrumentation and Recording

On the suspended structure, measurement of oscillatory amplitude in the vertical direction was made by the use of eighteen calibrated accelerometers of wire strain gauge type fitted along the tops of curbs of both sides. A diagram of complete instrumentation system is shown in Fig. 3. In the case of mode test some of accelerometers were moved about at the intermediate points of each accelerometer so as to record the motions of supplemental points.

The signals from each accelerometer were displayed on an oscillograph. In addition to the direct readings a selection of the observations was recorded on magnetic tape by which accurate resonant frequencies were measured through analogue calculation of autocorrelation functions.

6. Results

6.1 Presentation of Results

The response of structure to the exciting force shows its maximum at a resonant frequency which, strictly speaking, is not equal to the natural frequency of structure. However, these are considered to be equal when the weight of excitor is small enough compared with the weight of structure itself and the damping capacity is small. This can be true for the modes of vibration too.

6.1 (i) Resonance Curves

The structure vibrates in simple harmonic motion when it is put in oscillation by the excitor. Then, the oscillatory amplitude can easily be obtained from the acceleration signals;

$$\text{Amplitude} = \text{Maximum Acceleration} / (2\pi \times \text{frequency})^2$$

The test results are presented in Fig. 4. Real amplitude corresponds to the response against the motive force the magnitude of which is proportional to the square of its frequency. The reduced amplitude corresponds to the response against the force with constant magnitude, and this relates to the real amplitude by the following formula;

$$\text{Reduced amplitude} = \text{Real Amplitude} / (\text{frequency})^2$$

6.1 (ii) Modes of Vibration

The modes of vibration were obtained by the simultaneous measurement of vibratory amplitude at resonant frequencies. The amplitude at the intermediate point was supplemented by the signals of reset accelerometers. Test results are shown in Fig. 5.

6.1 (iii) Damping Capacities

The damping capacities are evaluated by the ratio of dissipation energy in one cycle of oscillation to the total strain energy of the structure at the time. The logarithmic decrement (δ) is defined as

$$\delta = \log (\text{nth amplitude}) - \log (\text{n+1th amplitude})$$

Amplitude may be replaced by the acceleration if the damping is viscous. The relation between the logarithmic decrement and the damping constant (β) is

$$\delta = 2 \pi \beta / \sqrt{1 - \beta^2}$$

and if δ is small it can be seen $\delta = 2 \pi \beta$.

The test results are presented in Figs. 6 and 7.

6.2 Discussion of Results

6.2 (i) Natural Frequencies

Analytical method obtaining dynamic characteristics of suspension bridges are classified into three large groups, i. e.,

differential equation method, integral equation method, and energy method.

Energy method is the most practical and simplest for analysis above all, and theoretical values have been obtained by this method derived by F. Bleich.

In the method the modes of oscillation are assumed as follows:

for symmetric vibration

$$\frac{\eta}{\alpha} \} = a_1 \sin \frac{\pi x}{\ell} + a_3 \sin \frac{3 \pi x}{\ell}$$

(number of nodes = 0 and 2)

$$\frac{\eta}{\alpha} \} = a_n \sin \frac{n \pi x}{\ell}$$

(number of nodes $n+1 = 4, 6, \dots$)

for asymmetric vibration

$$\frac{\eta}{\alpha} \} = a_n \sin \frac{n \pi x}{\ell}$$

(number of nodes $n-1 = 1, 3, 5, \dots$)

Test results are compared with calculated ones in Table 2, and it is seen that difference between calculated and measured frequencies are 7 per cent at maximum.

The main cause of the difference is considered to be that concrete slab have a little effect for vertical oscillation and ambiguity of calculating method of torsional rigidity of stiffening frames exists.

The asymmetric vertical oscillation with one node was not produced through the excitation by the excitor. The final choice was to let three young men jump with the same frequency as the natural frequency for the type of oscillation under investigation, but it was in vain.

6.2 (ii) Modes of Vibration

The ratios of a_1/a_2 have been obtained from the minimum energy principle, and the measured modes are compared with the theoretical ones with fairly good coincidence. It shows that assumption of modes by the sum of two sinusoidal waves are quite agreeable for lower modes of torsional and vertical oscillation. This holds true for higher modes too.

6.2 (iii) Damping Capacities

It is an important matter to collect data of damping capacities from the viewpoint of aerodynamic stability of suspension bridges. Nevertheless few informations are available.

Viscous damping is usually assumed for the analysis of one-degree of freedom system where the equation of motion is expressed as

$$m \frac{d^2 x}{dt^2} + C \frac{dx}{dt} + kx = F(t)$$

If we apply the similar assumption for the lateral vibration of beams, the following equation can be obtained which was derived by Inglis.

$$m \frac{\partial^2 y}{\partial t^2} + E I \frac{\partial^4 y}{\partial x^4} + 2 K \frac{\partial y}{\partial t} = f(x, t)$$

Then, logarithmic decrement δ , corresponding to the n -th natural frequency ω_n is

$$\delta_n = \frac{2 \pi K}{\omega_n}$$

Introducing $\omega_n = \sqrt{\frac{E I}{m}} \left(\frac{n \pi}{\ell} \right)^2$, we can get

$$\delta_n = 2 \pi K \sqrt{\frac{m}{E I}} \left(\frac{\ell}{n \pi} \right)^2$$

namely, logarithmic decrement is proportional to the square root of dead load of the beam, and is inversely proportional to the square root of bending rigidity and the square of vibration mode.

If we apply the complex damping in which damping force is proportional to the displacement and is in phase with velocity the equation of motion is

$$m \frac{\partial^2 y}{\partial t^2} + E I e^{(1+2b)} \frac{\partial^4 y}{\partial x^4} = f(x, t)$$

and logarithmic decrement can be derived as follows,

$$\delta = 2 \pi \tan b$$

In this case the decrement is independent from the mass and modes of vibration of beam.

The amplitudes attained by the twin-excitor were not so large i. e., 10 mm in vertical and 0.10 degree in torsional oscillation at the maximum that it was impossible to conclude which theory was applicable to the Kompira Bridge vibration. Nevertheless it is seen from the test results that logarithmic decrement is independent of the amplitude and depends on the modes of vibration only.

7. Conclusions

The following main conclusions are drawn from the results of field dynamic loading test on the Kompira Bridge.

- (i) Measured frequencies and modes are coincided with those calculated with fairly good accuracy. Energy method used is applicable to the analysis of simple spanned suspension bridges.
- (ii) Damping capacities depend upon the modes of vibration and is independent from the amplitude.
- (iii) Data of asymmetric vertical vibration with one node could not obtained, and the reasonal cause could not be derived.

8. Acknowledgements

The authors wish to acknowledge the able experimental assistance given by the members of the Structure Section of the Public Works Research Institute.

The authors also wish to acknowledge Mr. K. Sugawara, Mr. K. Mori, and Mr. M. Tsuji of Nippon Kokan K. K., for their kind help during the test.

9. References

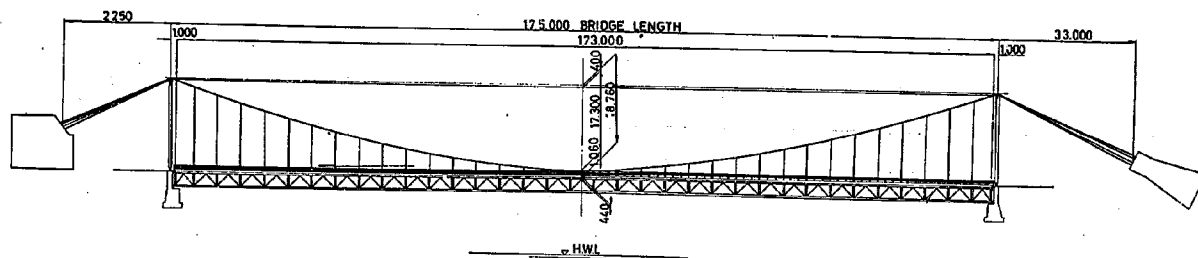
- 1) Structure Sec., P.W.R.I. March 1963
Report of Dynamic Loading Test on Wakato Bridge
(in Japanese)
- 2) Structure Sec., P.W.R.I. March 1964
Report of Dynamic Loading Test on Kawamata Bridge
(in Japanese)
- 3) Structure Sec., P.W.R.I. March 1967
Report of Dynamic Loading Test on Sonohara and
Ohmine Bridge (in Japanese)
- 4) Structure Sec., P.W.R.I. March 1968
Report of Dynamic Loading Test on Onomichi Bridge
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Report of Dynamic Loading Test on Hakogase Bridge
(in Japanese)
- 6) A. Selberg the Technical Univ. of Norway 1950
Dampening Effect in Suspension Bridges
(Report for IABSE)
- 7) M. Ito and T. Katayama May 1965
Damping of Bridge Structures
(Transaction of JSCE, in Japanese)
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The Mathematical Theory of Vibration in Suspension
Bridges

Table 1. Bridges tested

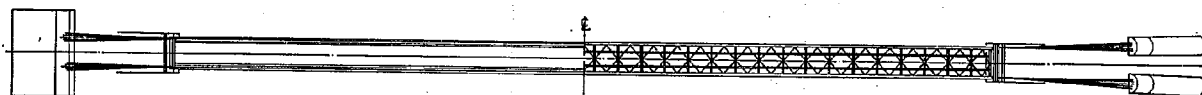
Name of Bridge	Wakato	Kawamata	Sonohara	Ohmine	Onomichi	Hakogase	Kompira
Type of Bridge	Suspension	Suspension	Suspension	Suspension	Cable stayed	Suspension	Suspension
Span (m)	85 + 361.2 + 85	158.0	136.0	120.0	85 + 215.0 + 85	204.0	173.0
Sag (m)	35.0	15.8	13.6	12.0	-	22.0	17.3
Distance of main trusses (m)	19.0	5.6	3.0	4.0	10.2	5.17	6.0
Effective width (m)	15.0	4.5	2.5	3.0	8.0	4.1	4.5
Stiffening frame	truss	truss	truss	truss	plate girder	truss	truss
Tower	flexible	rocking	rocking	rocking	rocking	rocking	rocking
Dead load (t/m/cable)	7.9	1.85	1.11	1.27	2.86	3.84	4.66
Main cable ($\times 10^2 \text{ m}^2$)	12.4	1.38	0.75	0.69	(0.8-1.35)	2.63	2.76
Rigidity of truss (m^4/cable)	0.45	0.104	0.0126	0.0125	0.427	0.148	0.0862
Height of truss (m)	4.5	2.75	2.0	2.0	2.5	3.15	2.5
Existence of storm cable	No	No	Yes	Yes	No	Yes	No
Existence of center stay	Yes	Yes	No	Yes	-	Yes	Yes
Date of test	Aug., 1962	Nov., 1963	Mar., 1965	Jul., 1965	Jan., 1968	Aug., 1968	Aug., 1969
Reference num.	1)	2)	3)	3)	4)	5)	

Table 2. Frequencies

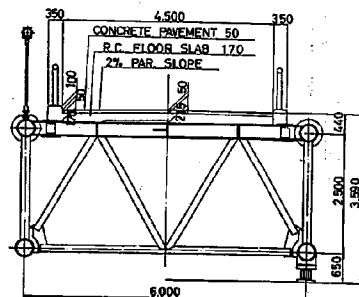
Vertical oscillation			Torsional oscillation		
Number of nodes		cyc per sec.	Number of nodes		cyc per sec.
0	E	0.617	0	E	1.67
	T	0.604		T	1.643
	E/T	1.014		E/T	1.020
2	E	1.21	1	E	2.81
	T	1.173		T	2.634
	E/T	1.031		E/T	1.068
3	E	1.85	2	E	4.17
	T	1.724		T	3.974
	E/T	1.073		E/T	1.048
4	E	2.67	E : measured freq. T : calculated freq.		
	T	2.649			
	E/T	1.007			
5	E	3.44			
	T	3.774			
	E/T	0.911			



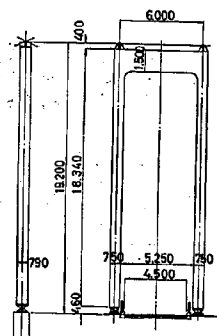
ELEVATION



PLAN



CROSS SECTION

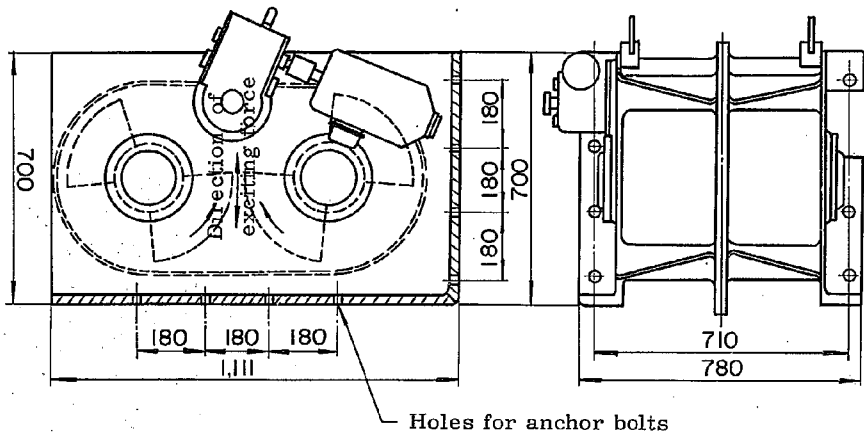


TOWER

Type	Two-hinged suspension
Class	Second class bridge
Distance tower to tower	173.000 m
Span length	173.000 m
Effective width	4.500 m
Sag ratio	$f/l = 1/10$
Total weight	401 t

Fig. - 1

(a) Unit excitor (Elevation and Section)



(b) Twin-excitor (Plan)

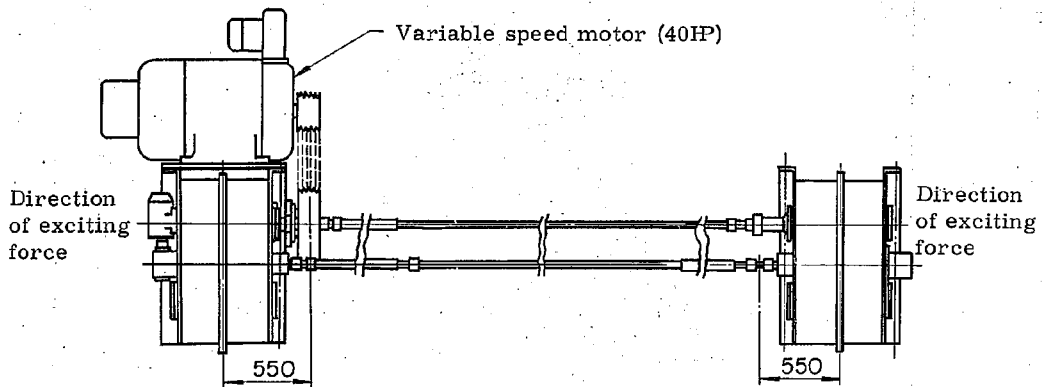


Fig. 2-1 Outline of Twin-excitor

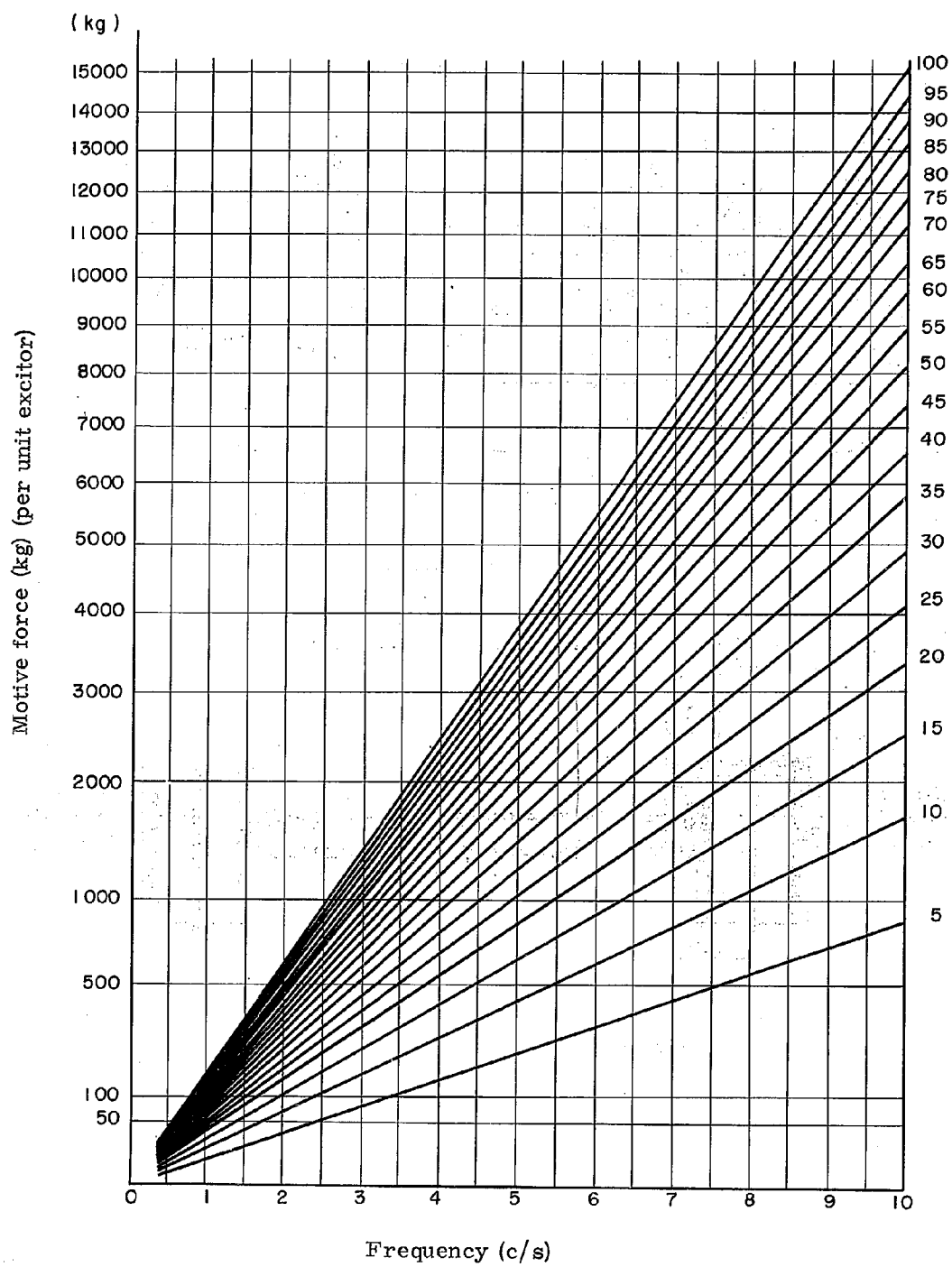
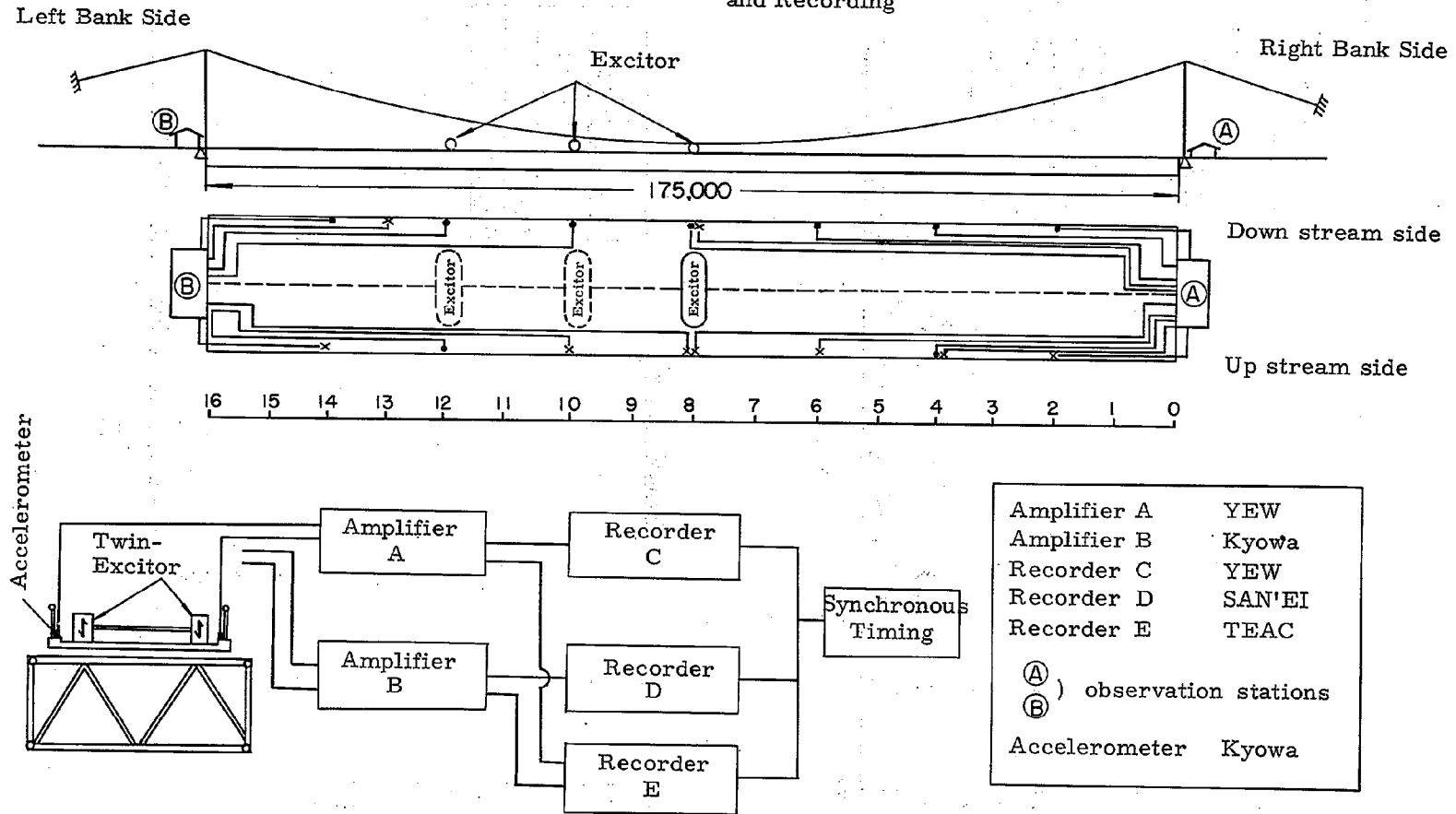


Fig. 2-2 Frequency characteristics
of Twin-excitor

Fig. 3 Block Diagram of Instrumentation and Recording



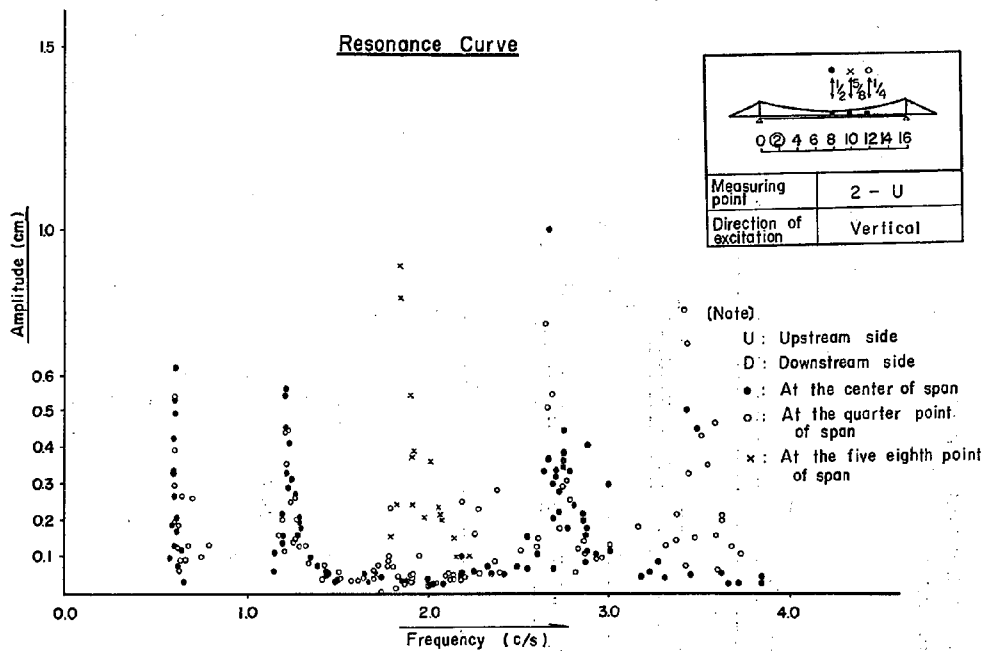


Fig 4-1(a)

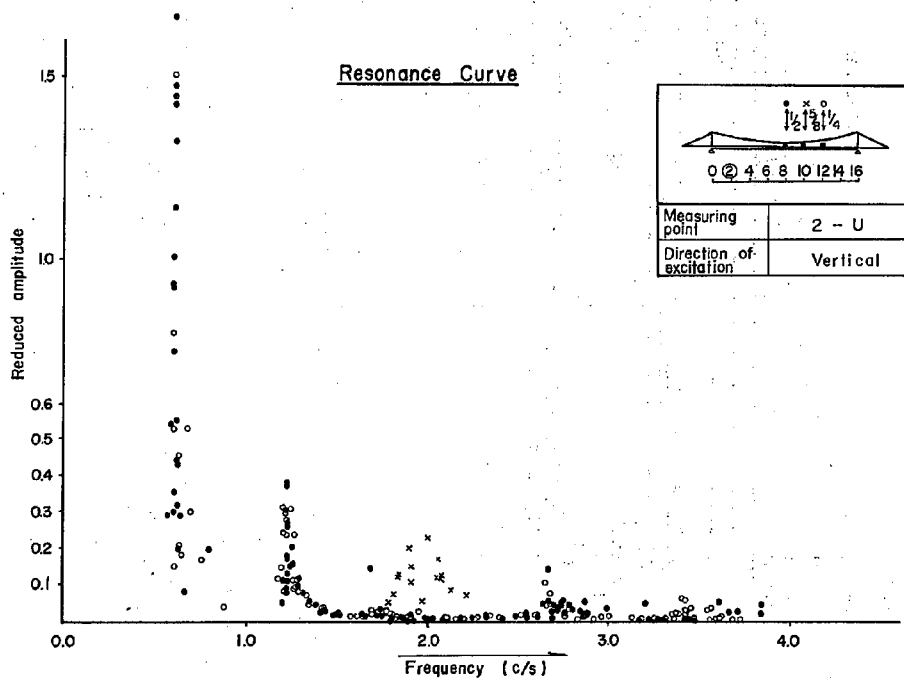


Fig 4-1(b)

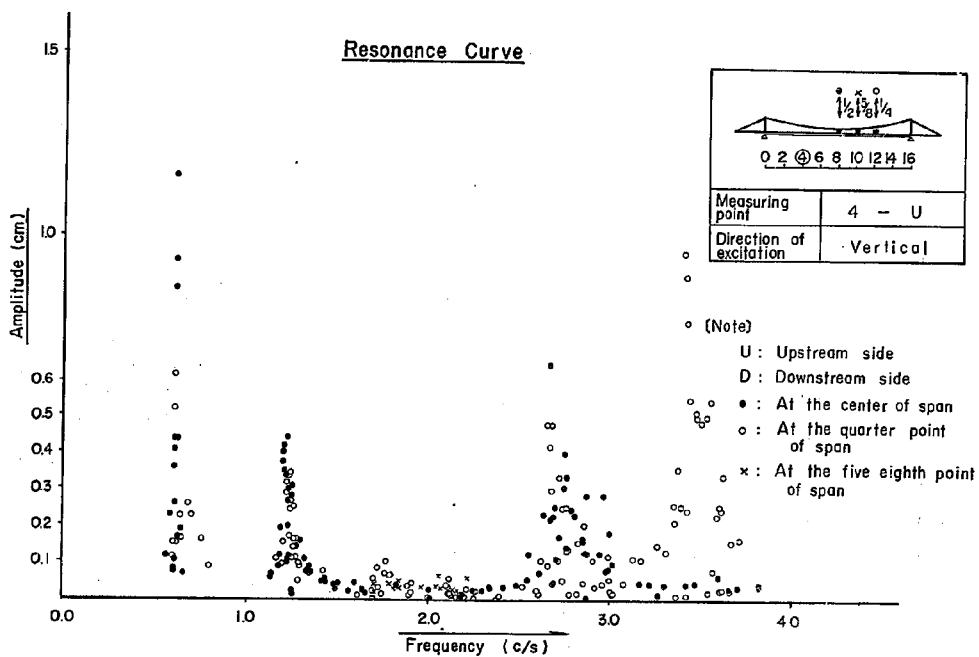


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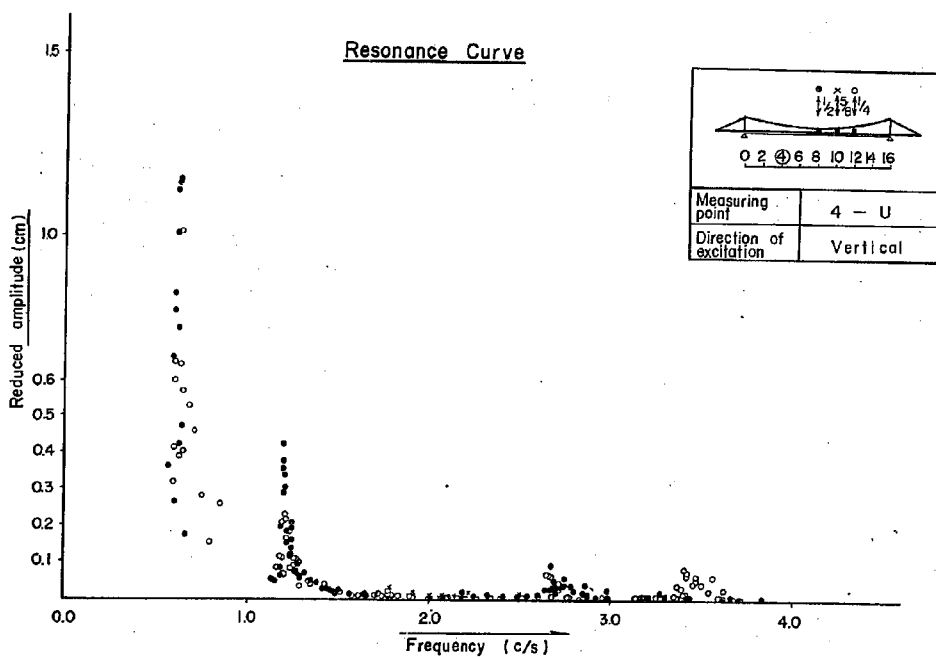


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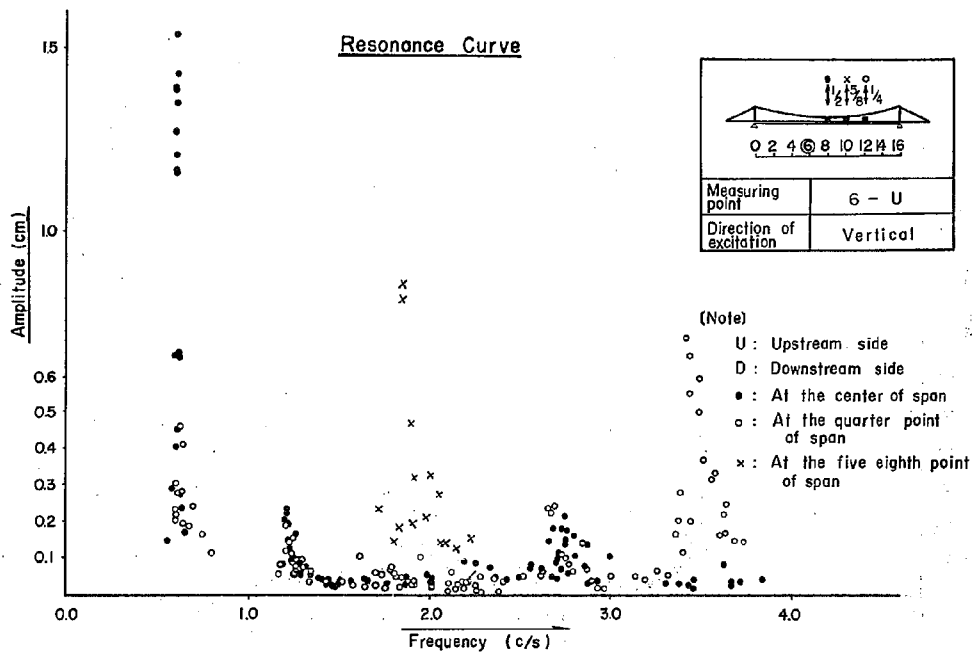


Fig 4-3(a)

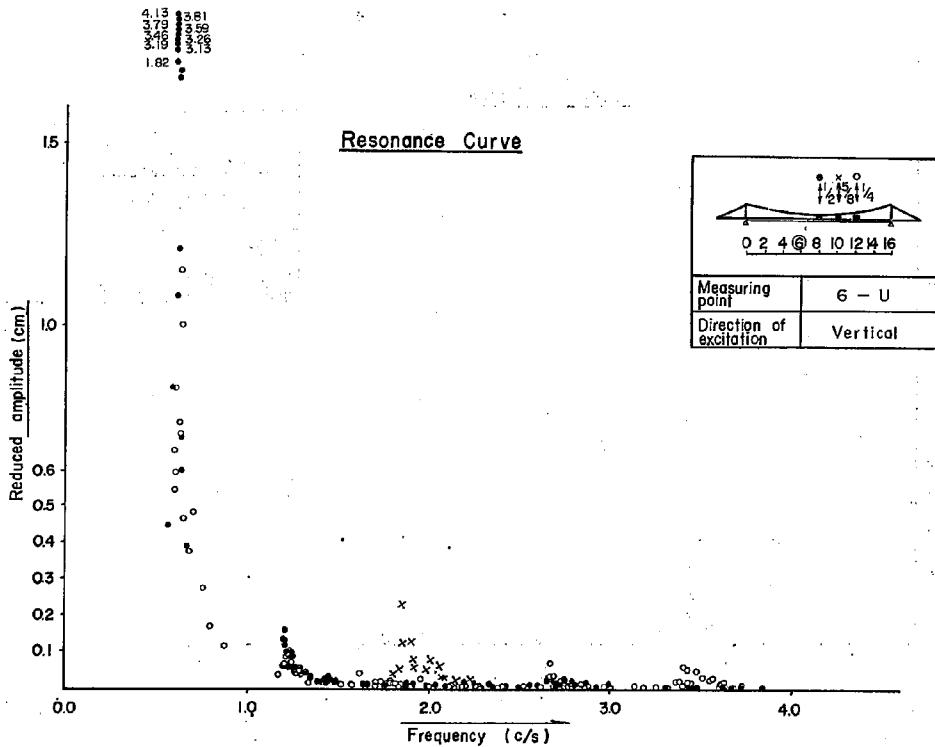


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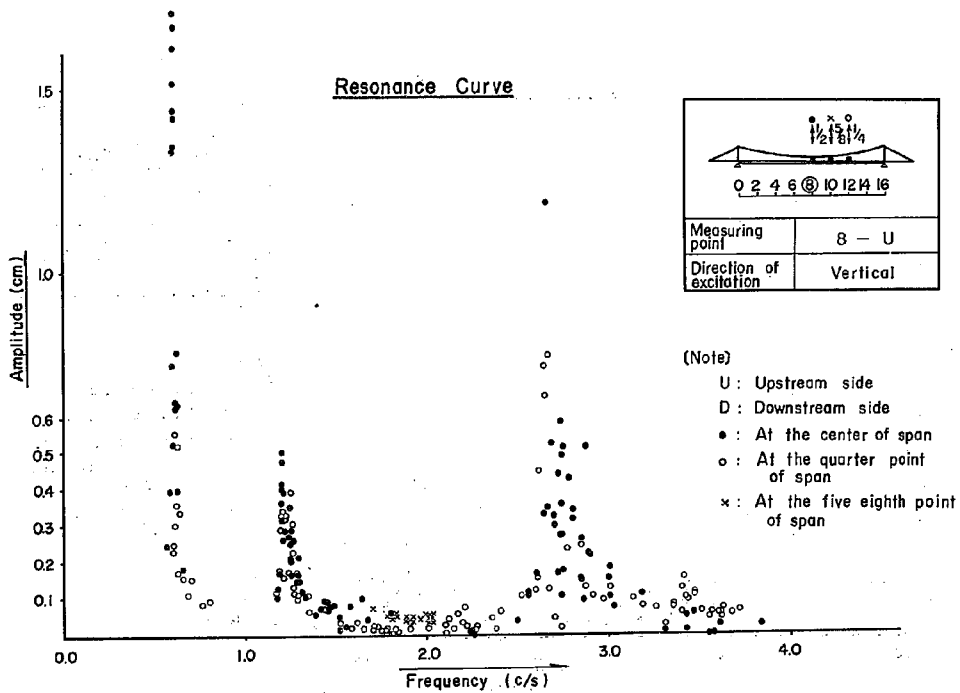


Fig 4-4(a)

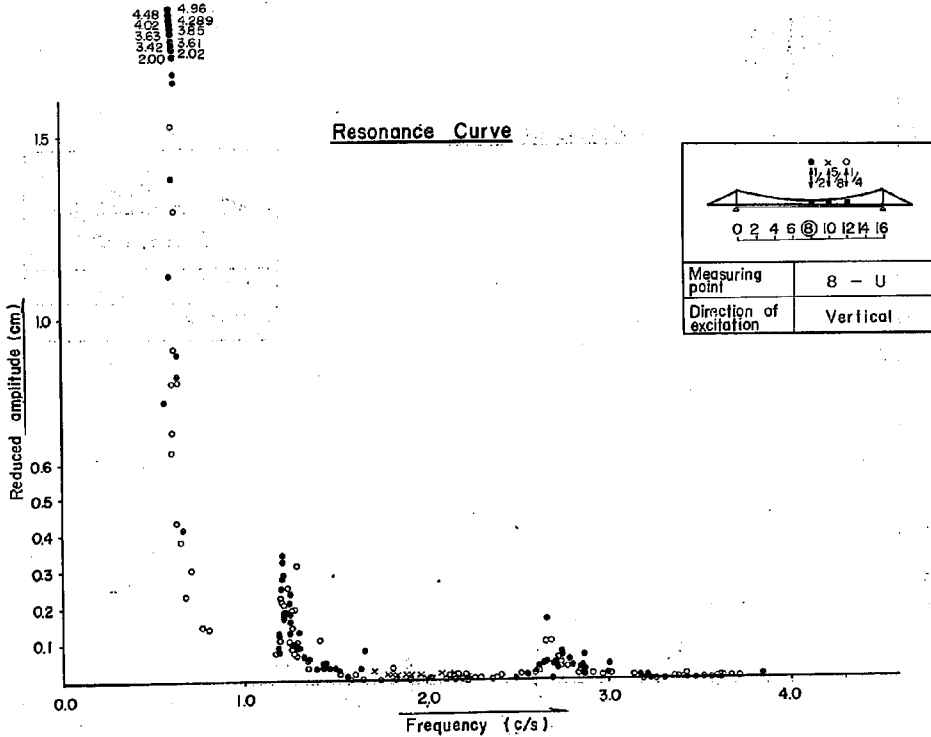


Fig 4-4(b)

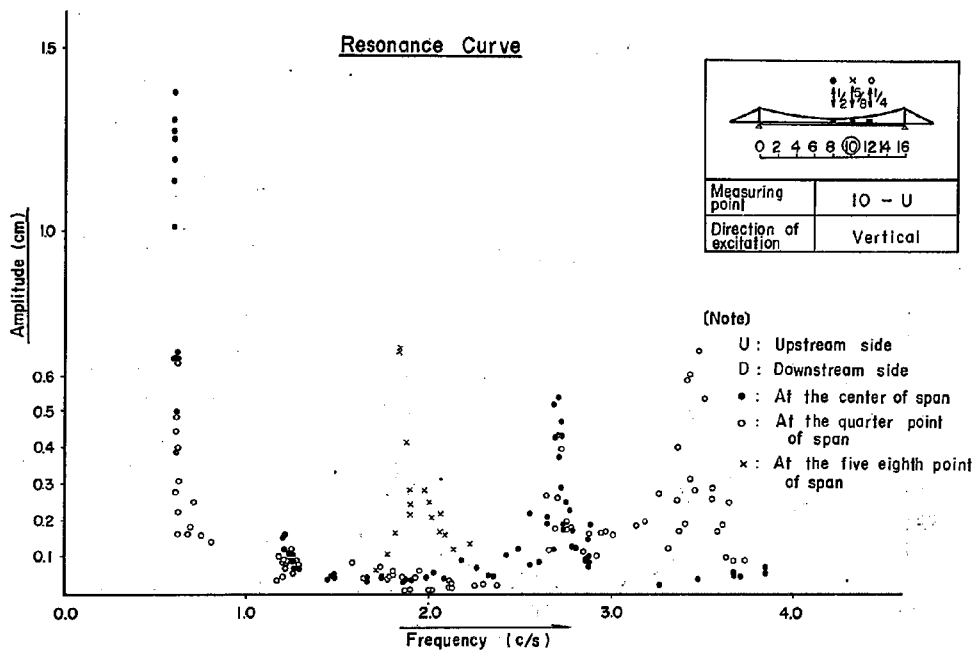


Fig 4-5(a)

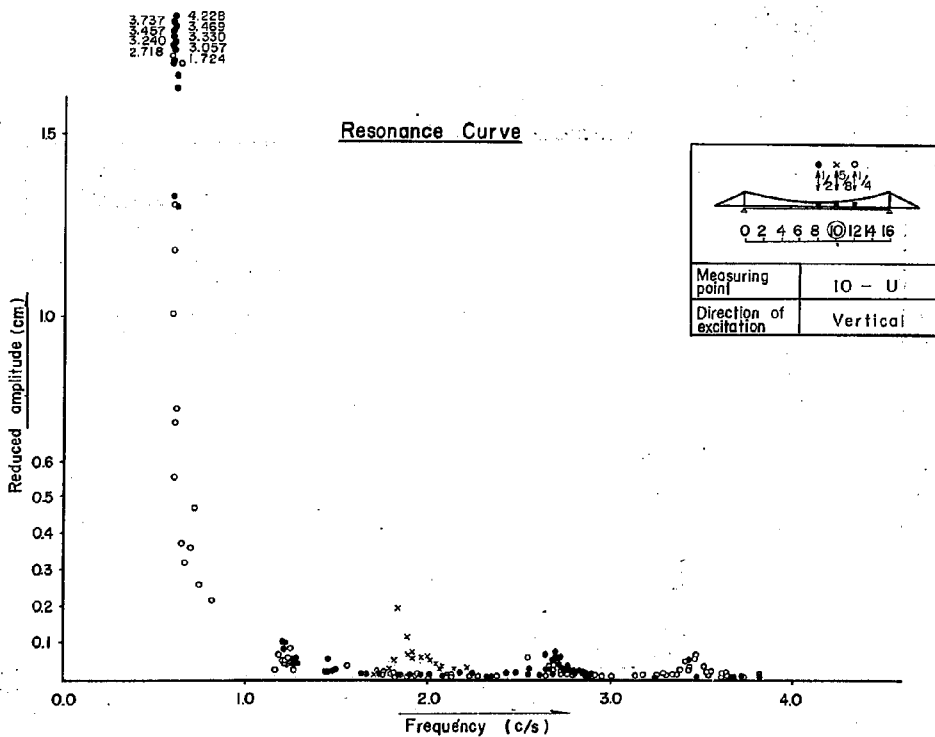


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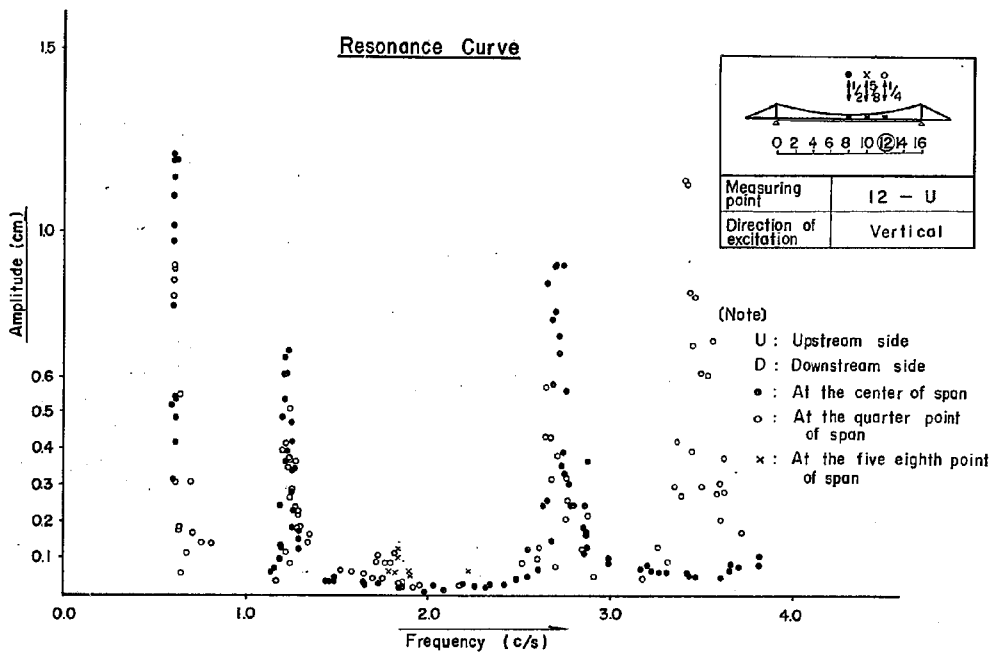


Fig 4-6(a)

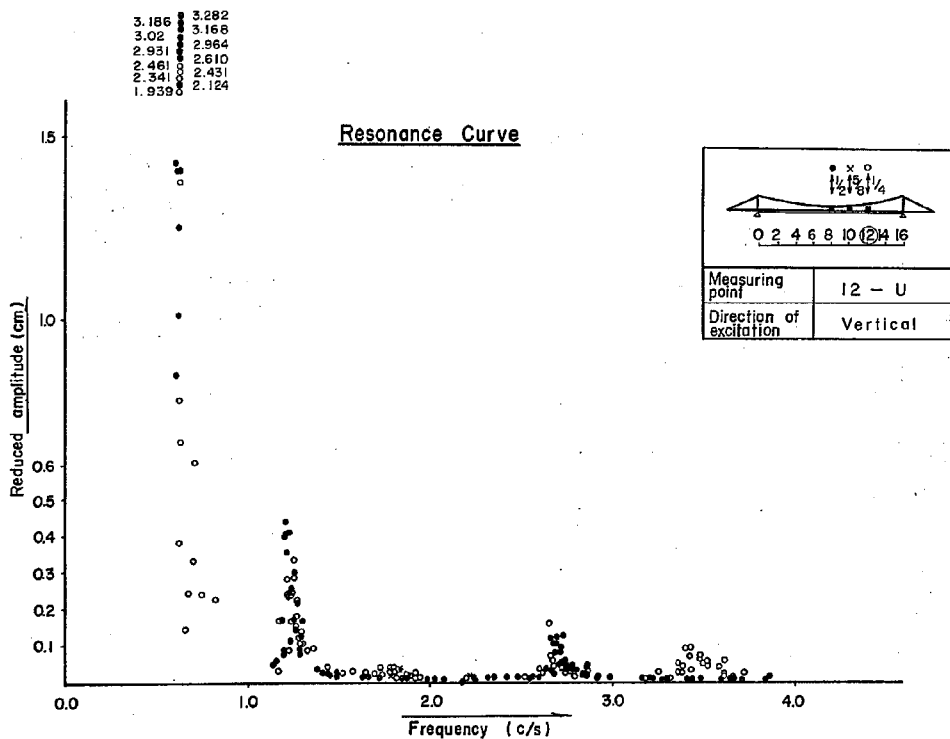


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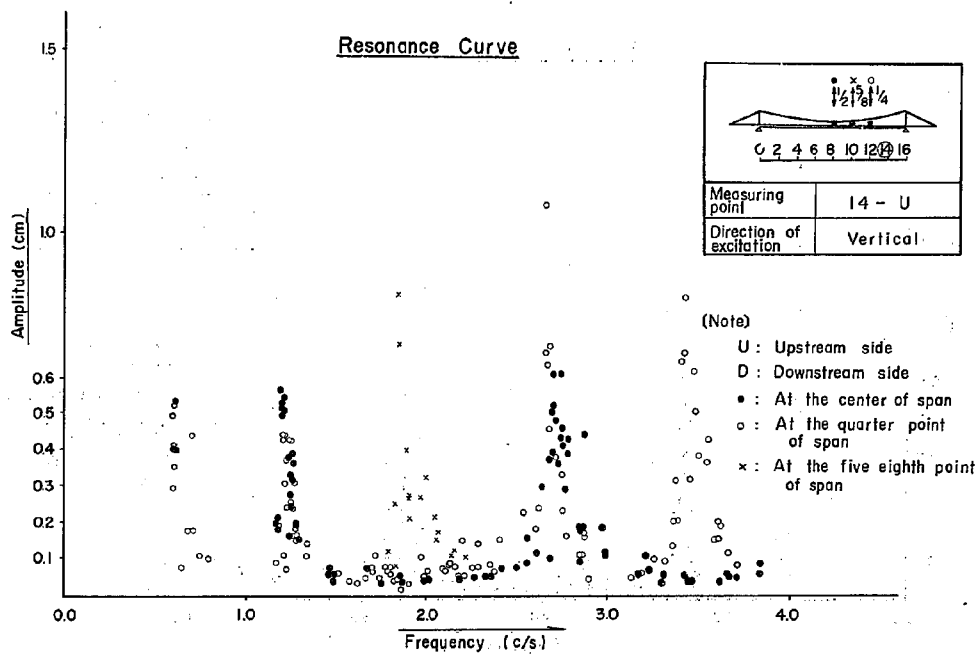


Fig 4-7(a)

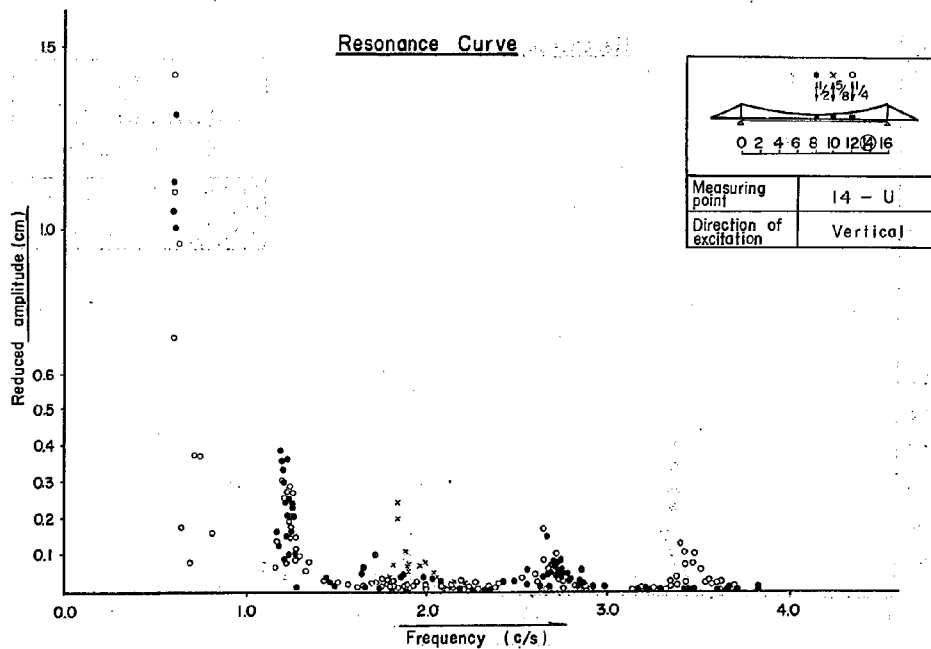


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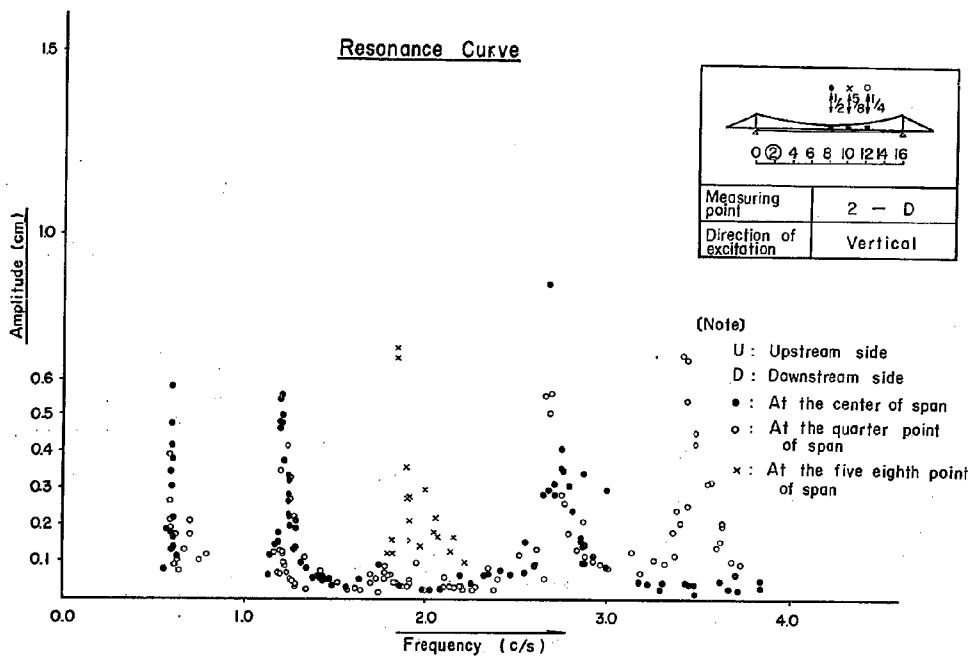


Fig 4-8(a)

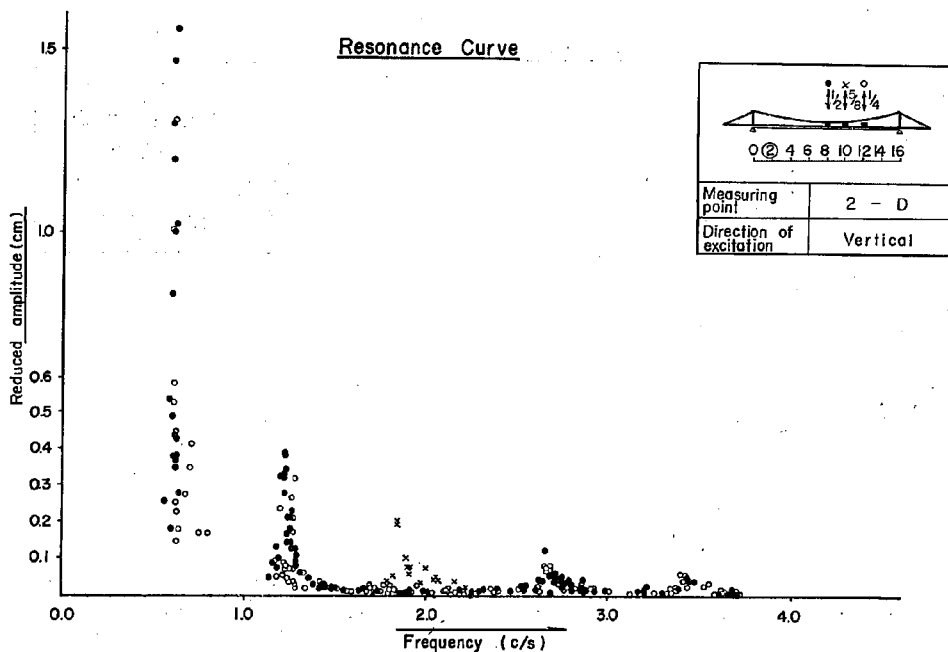


Fig 4-8(b)

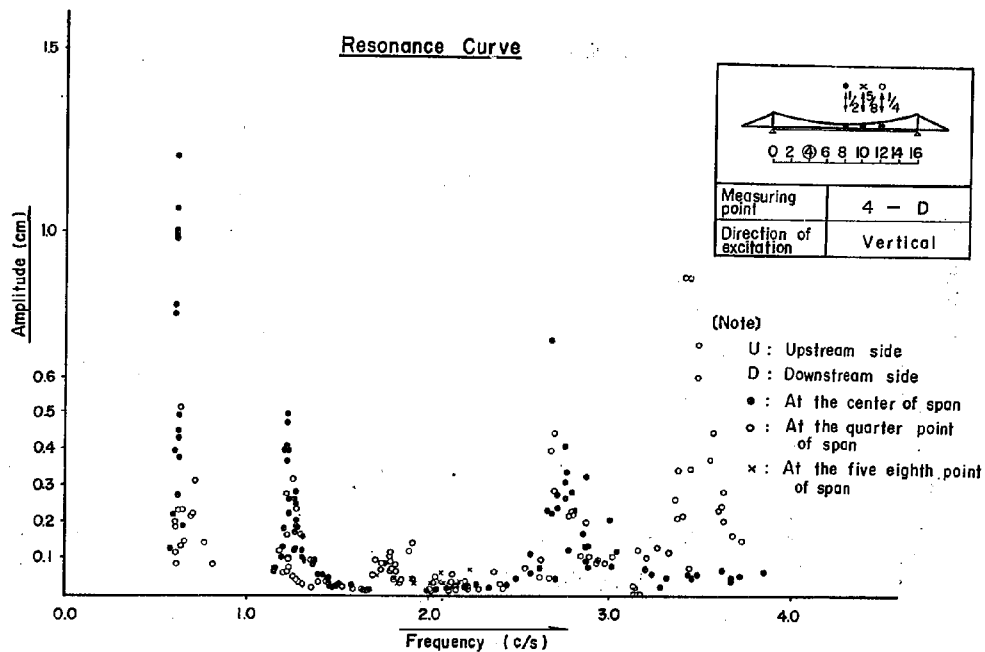


Fig 4-9(a)

3.22 ● 290
2.619 ● 261
2.603 ● 2161

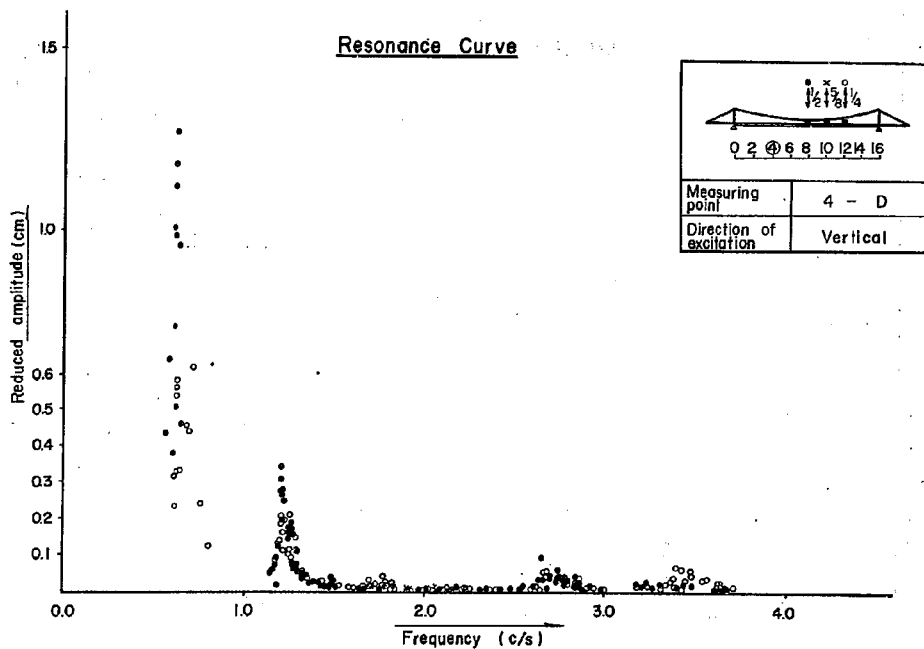


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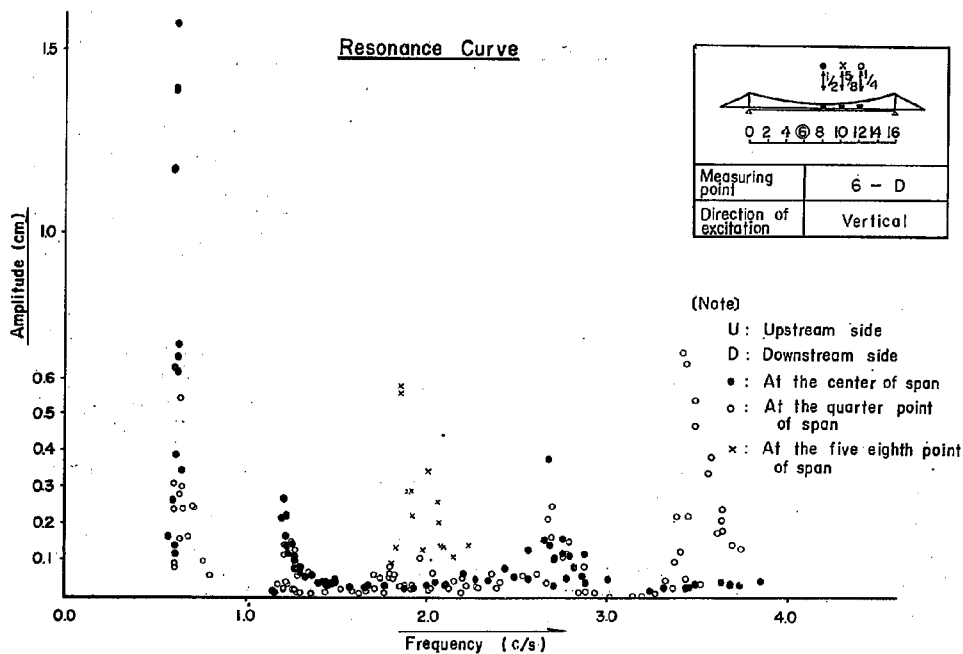


Fig 4-10(a).

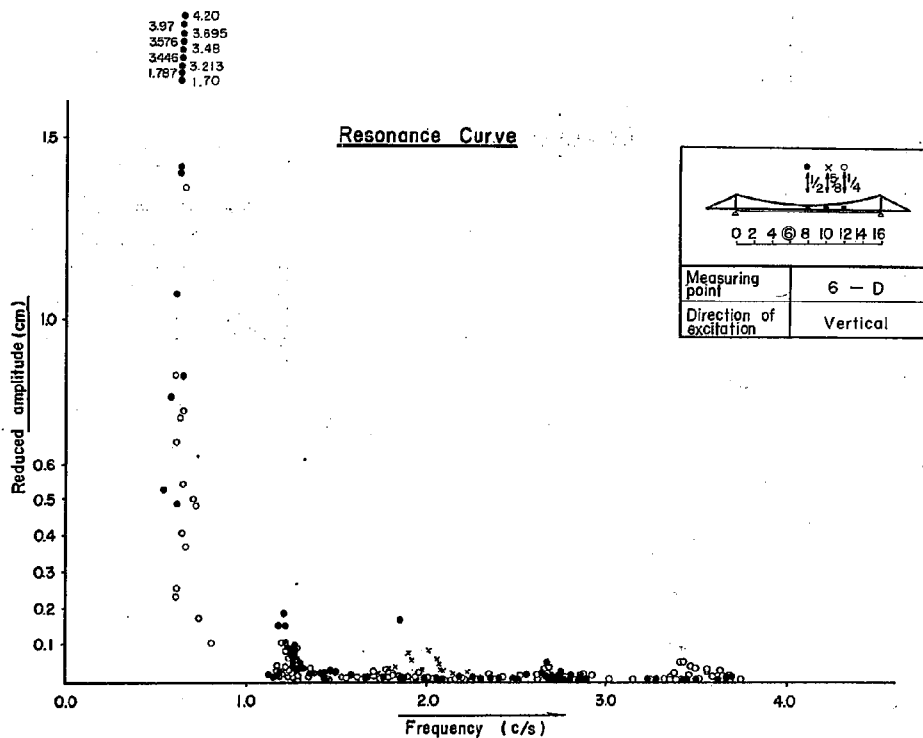


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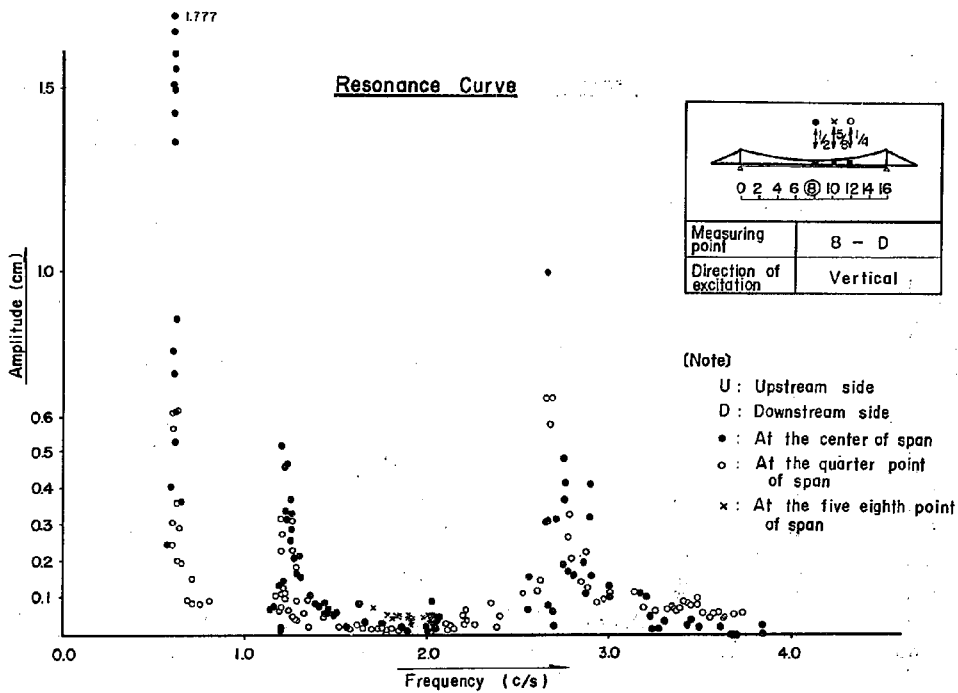


Fig 4-11(a)

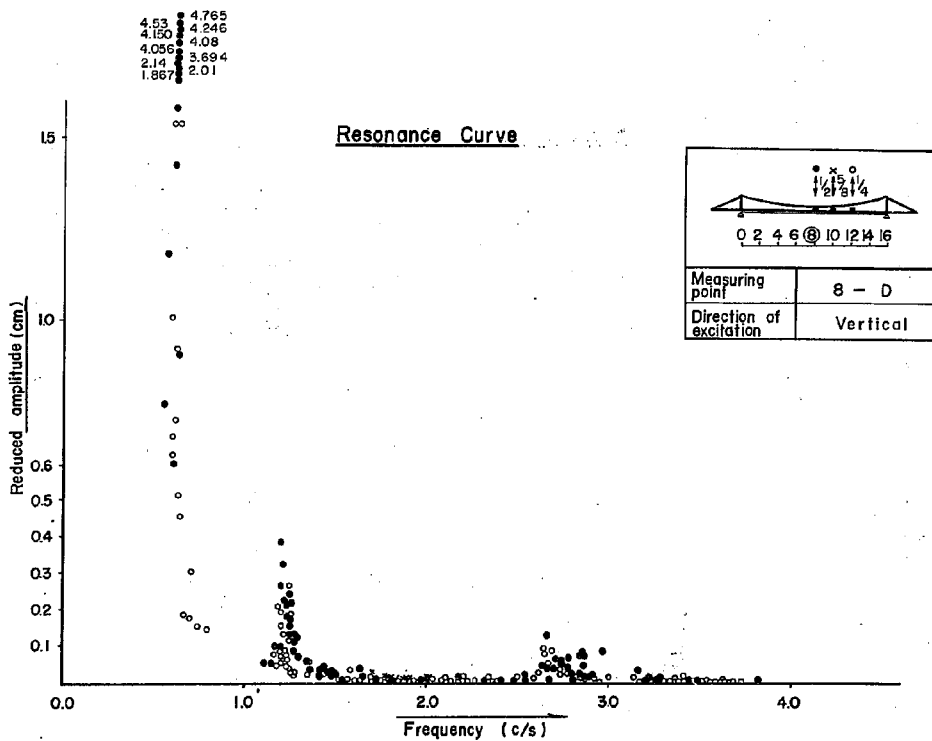
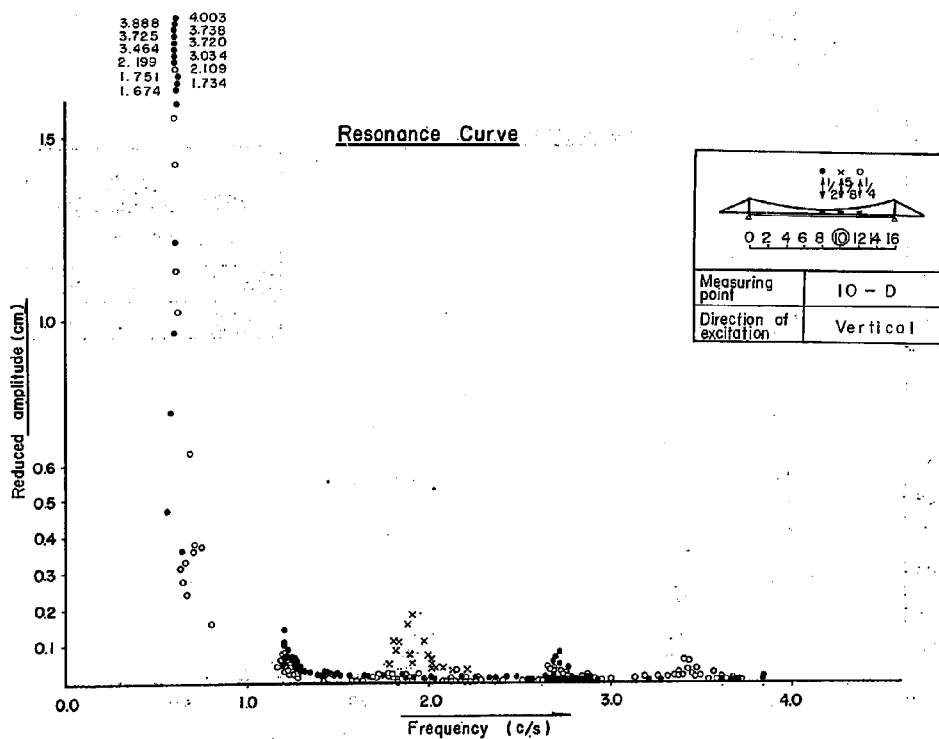
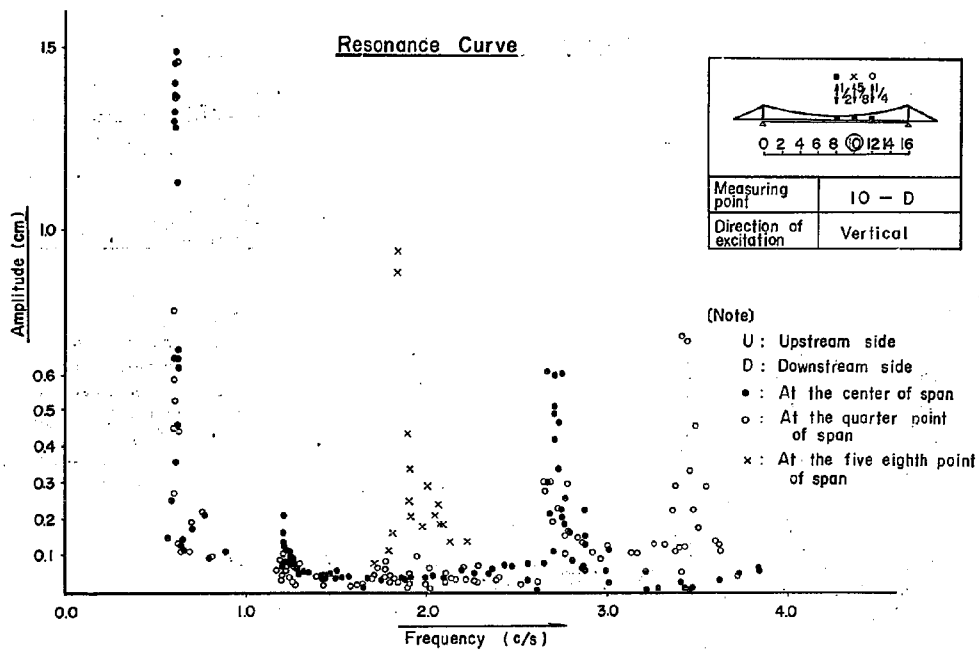


Fig 4-11(b)



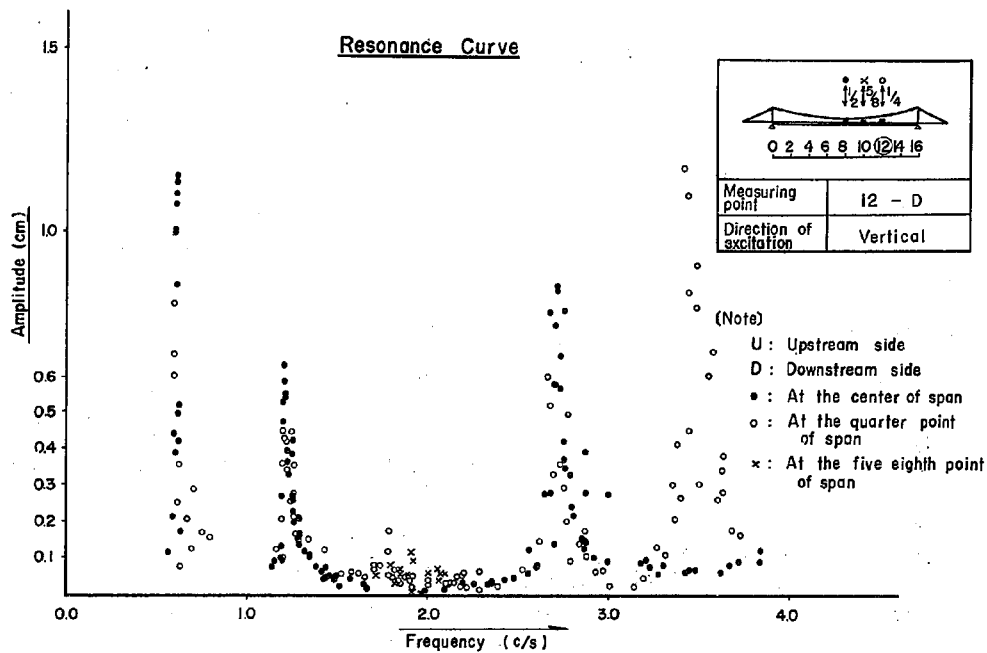


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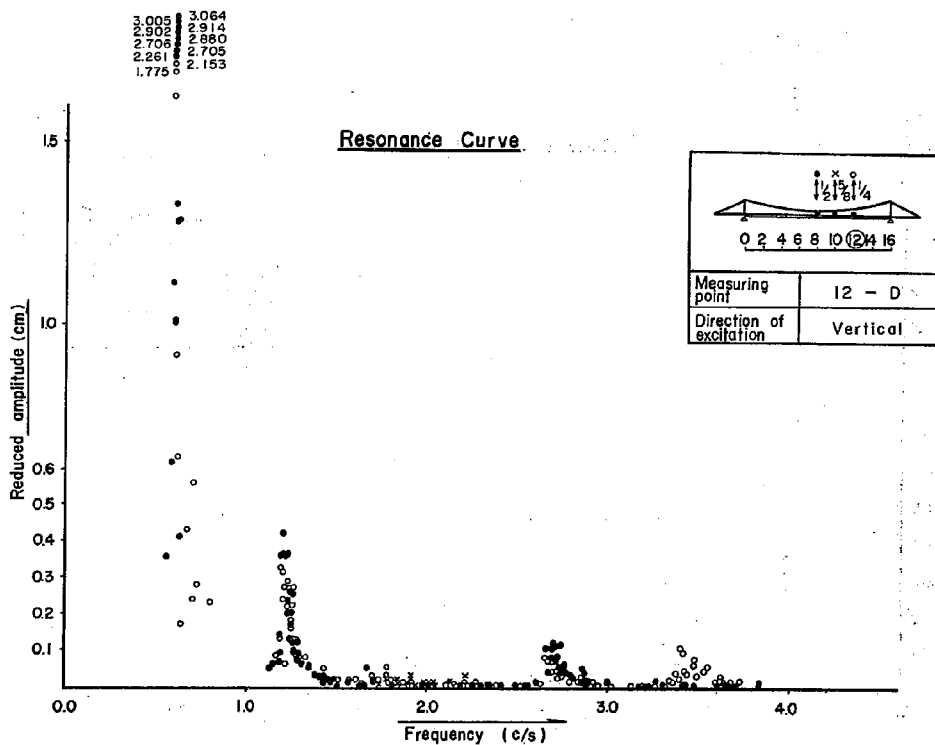


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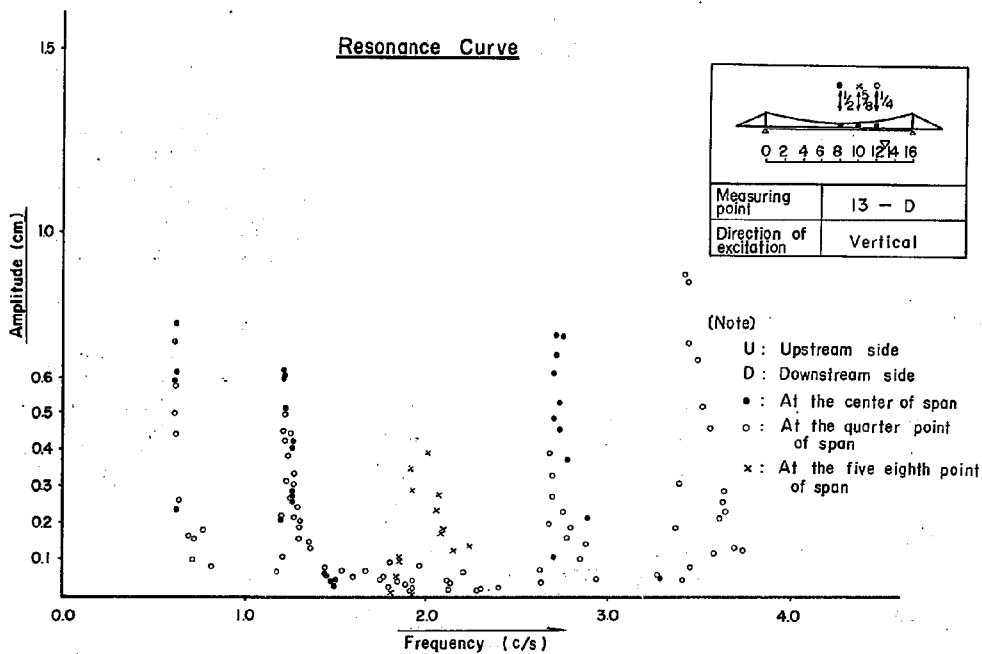


Fig 4-14(a)

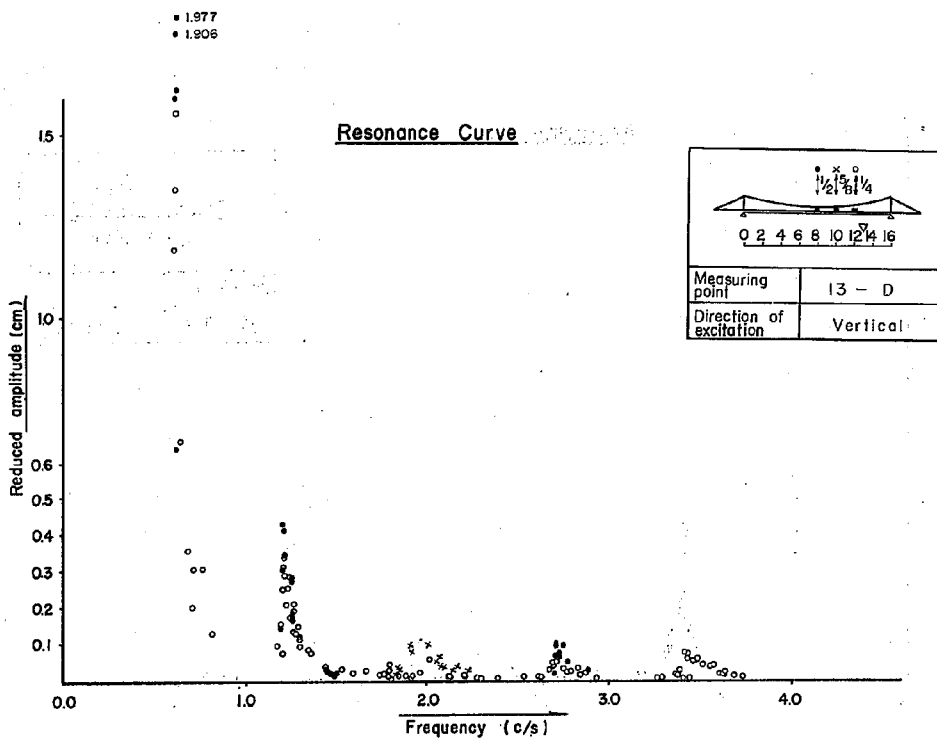


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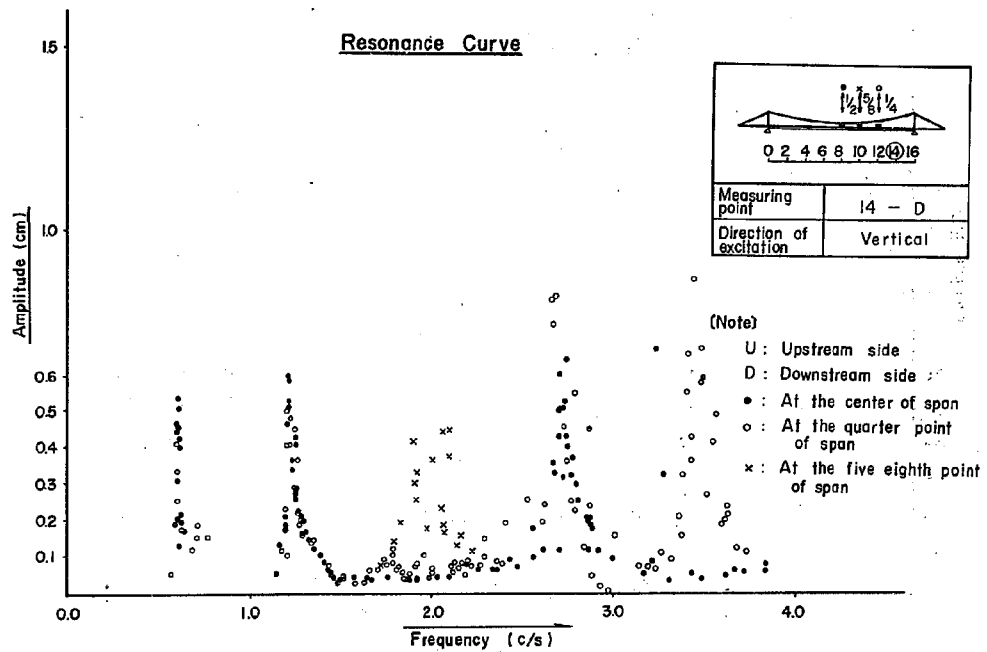


Fig 4-15(a)

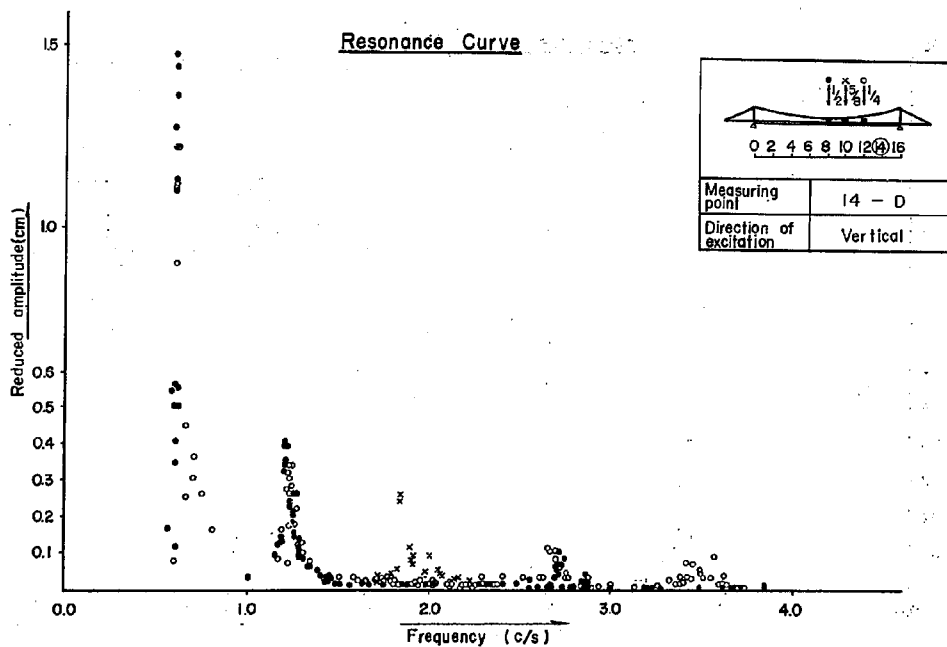


Fig 4-15(b)

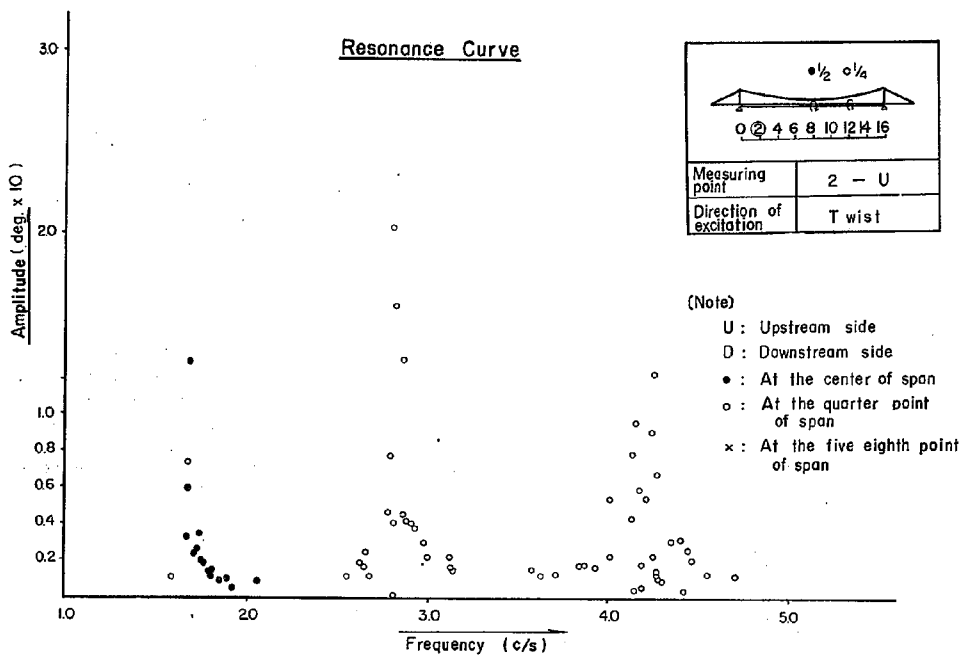


Fig 4-16(a)

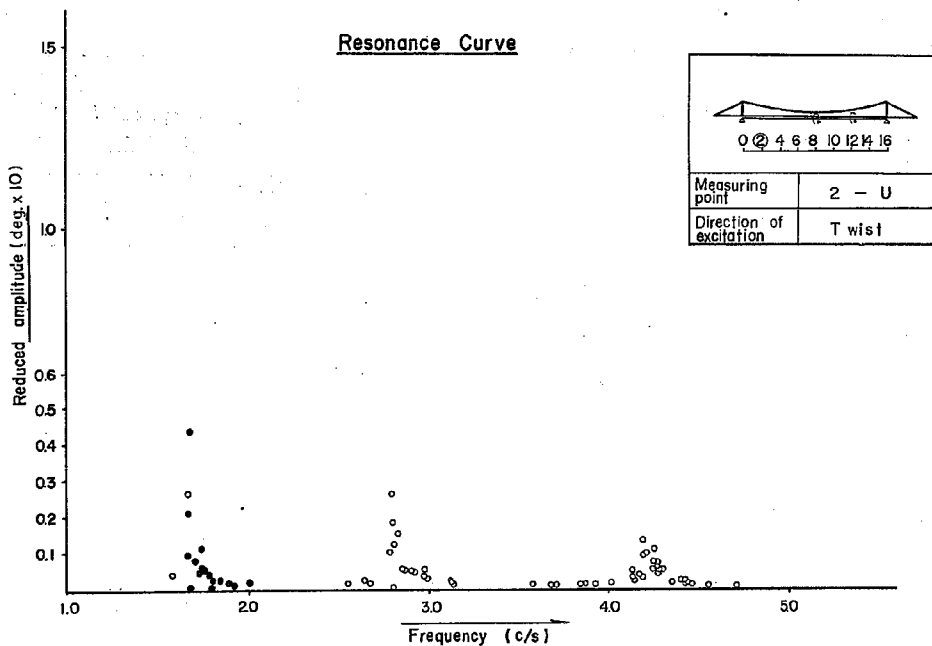


Fig 4-16(b)

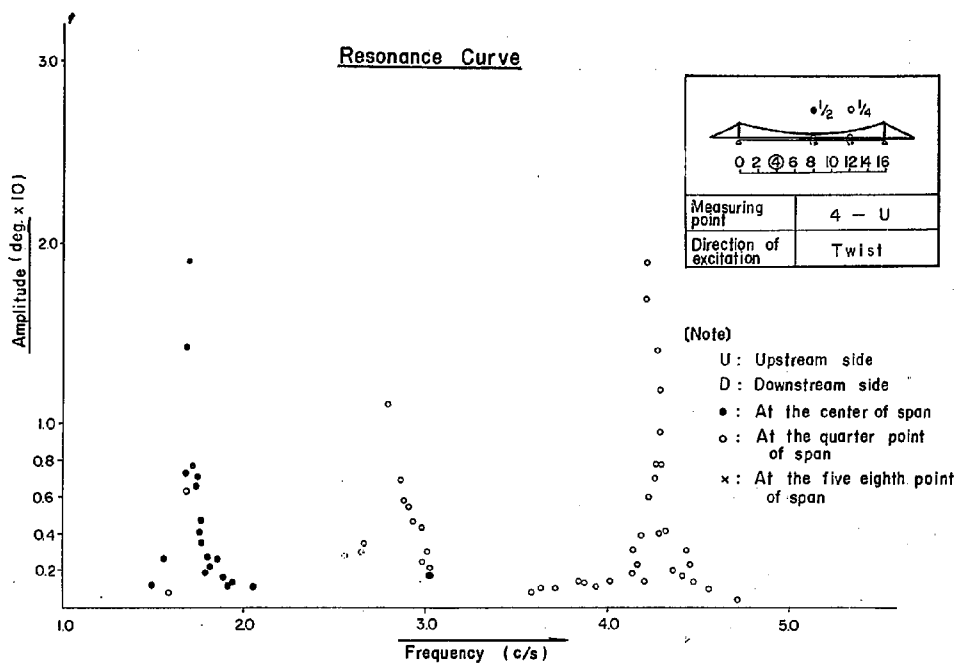


Fig 4-17(a)

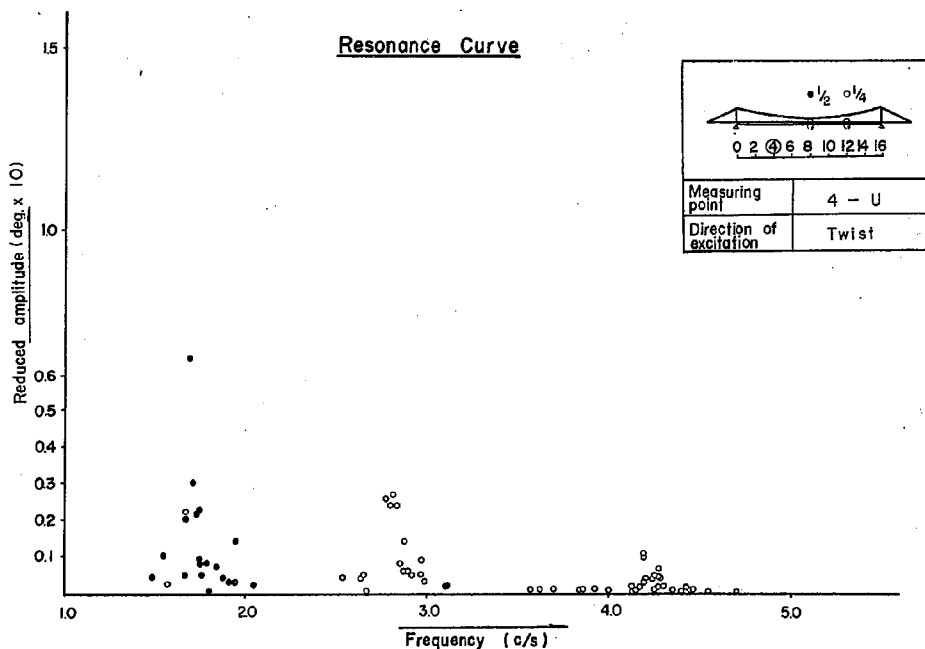


Fig 4-17(b)

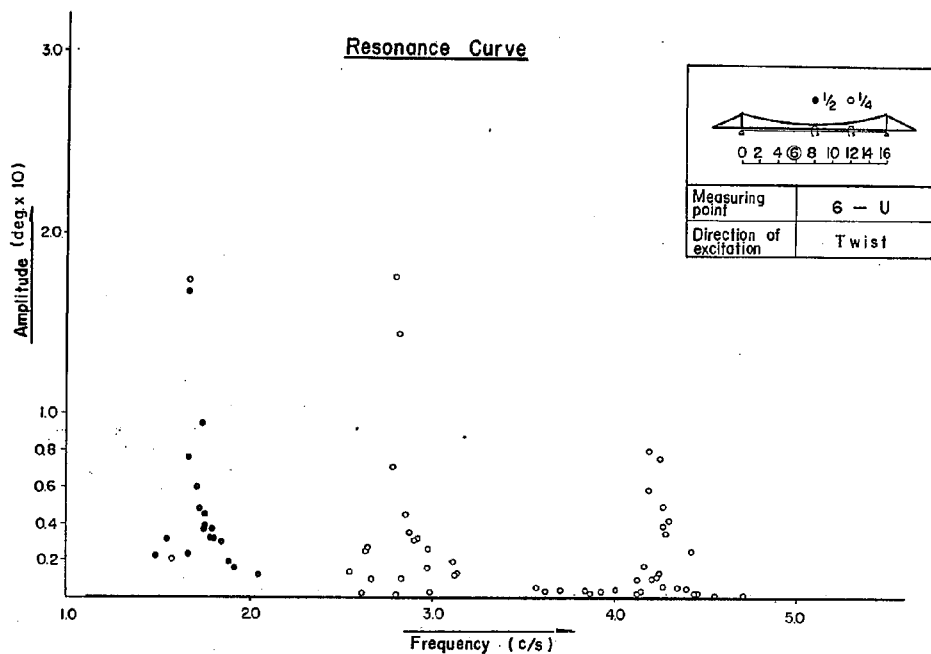


Fig 4-18(a)

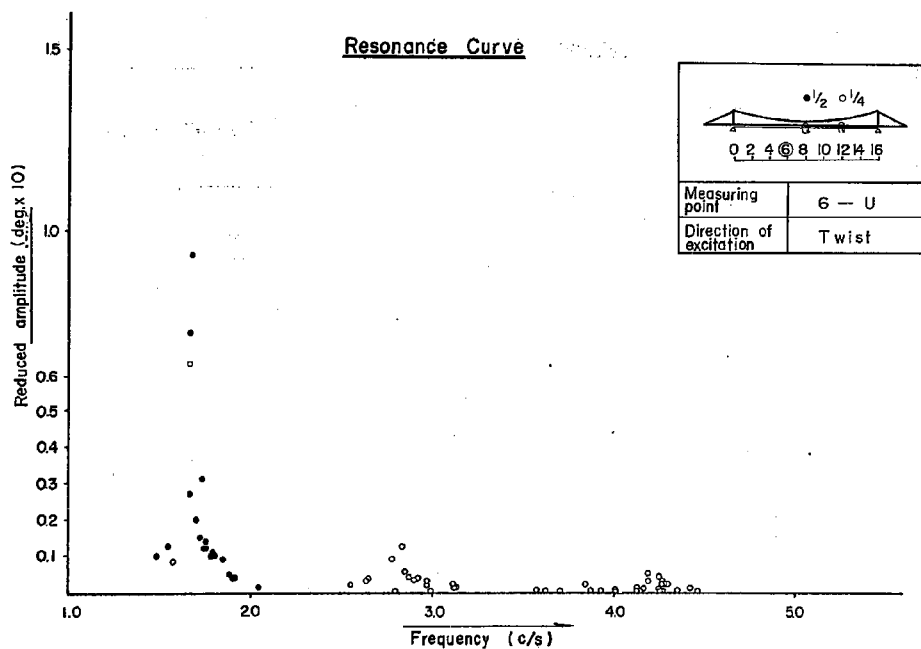


Fig 4-18(b)

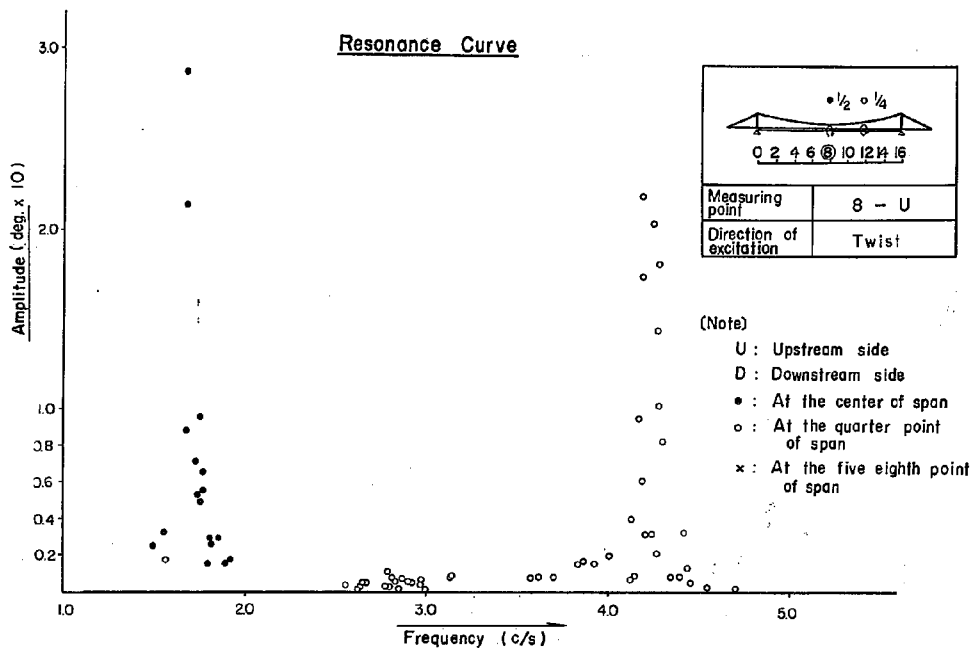


Fig 4-19(a).

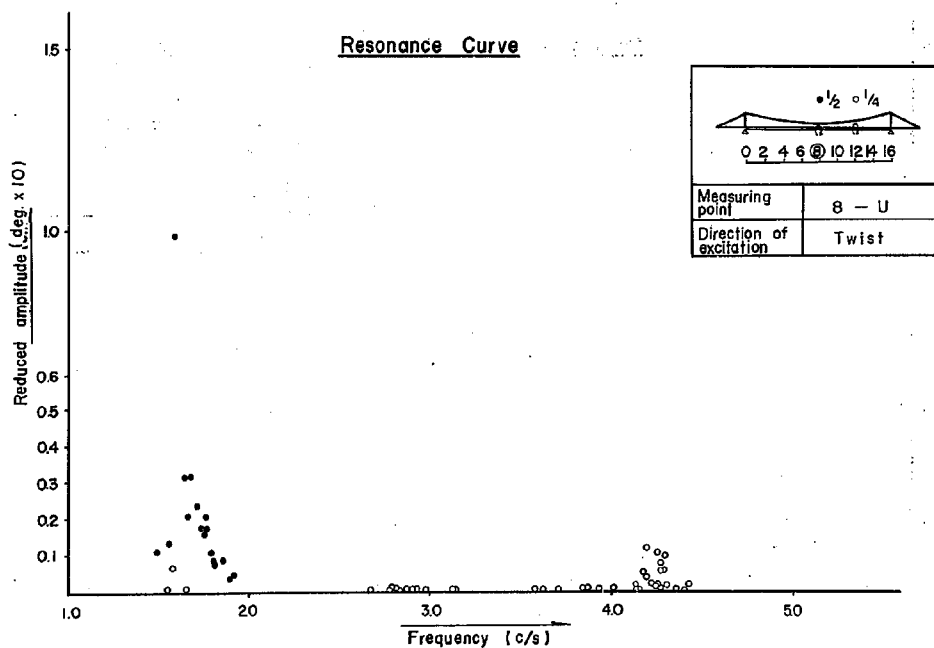


Fig 4-19(b)

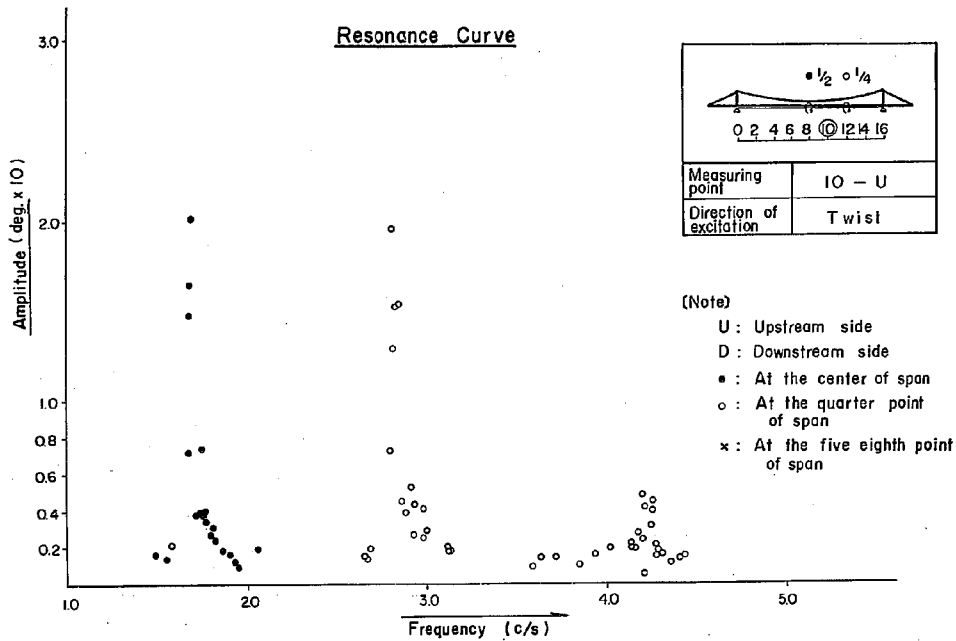


Fig 4-20(a)

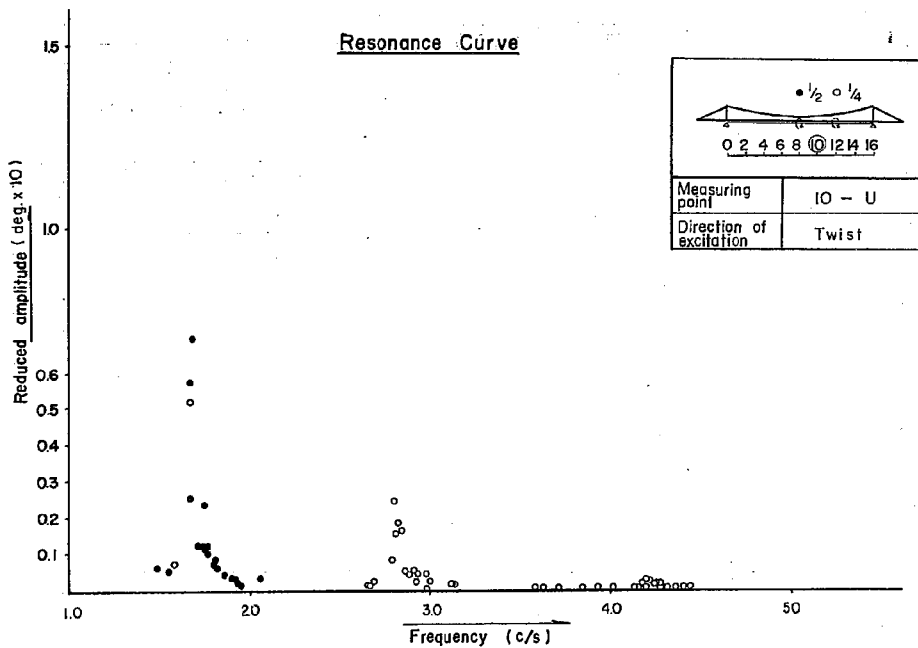


Fig 4-20(b)

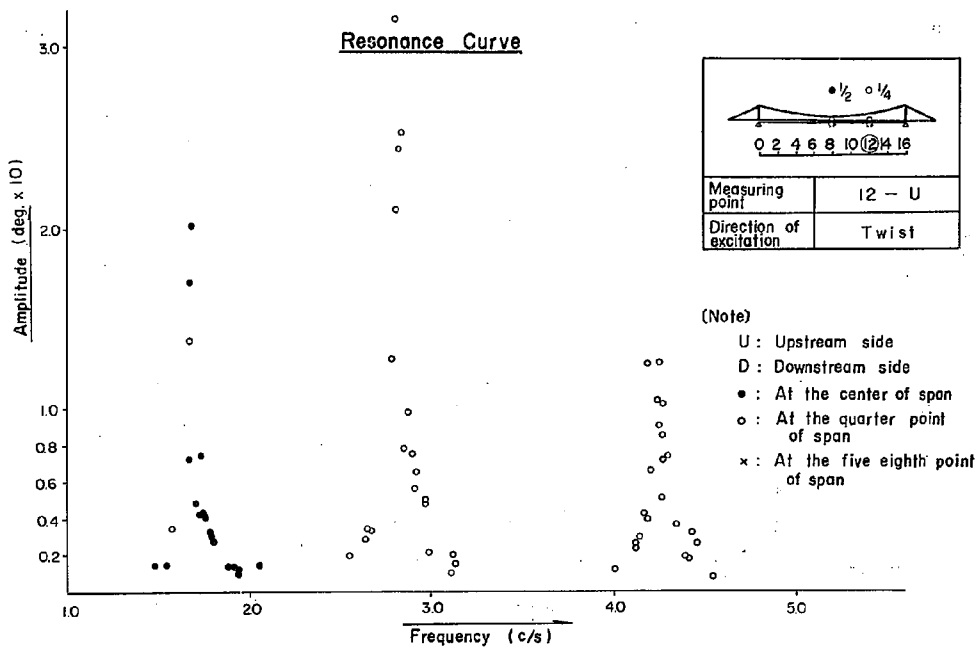


Fig 4-21(a)

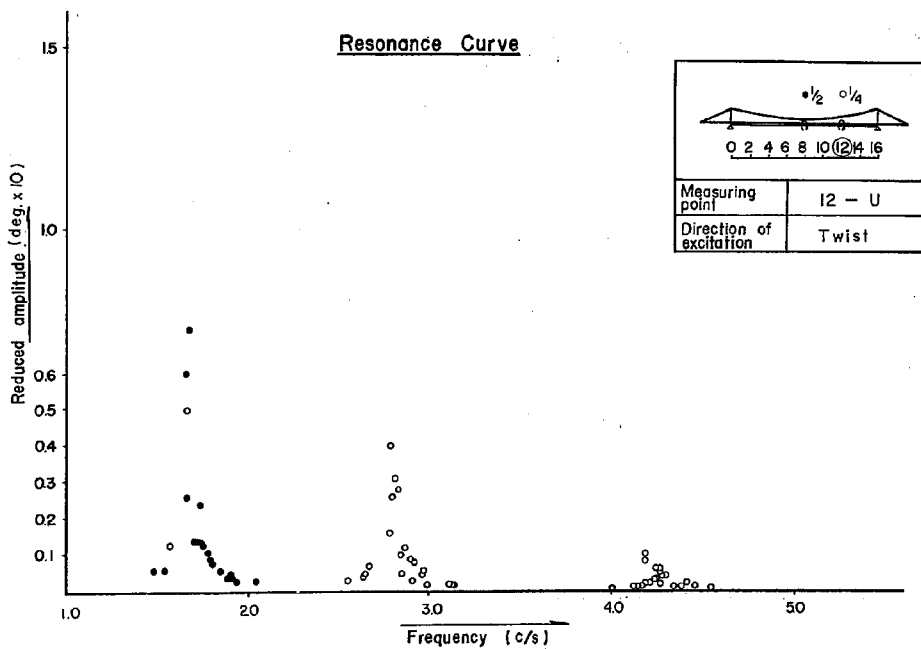


Fig 4-21(b)

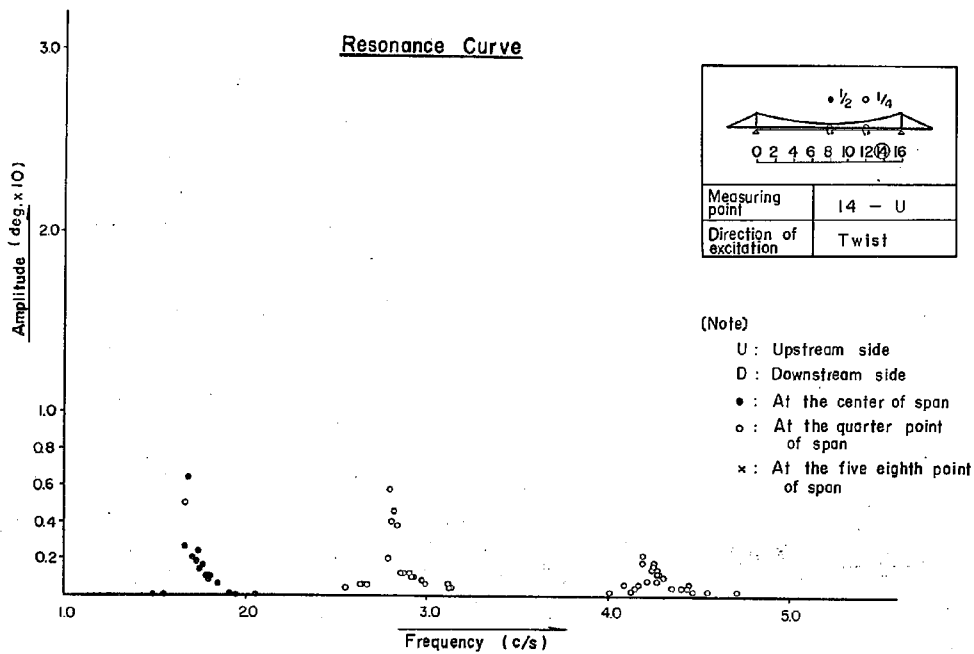


Fig 4-22(a)

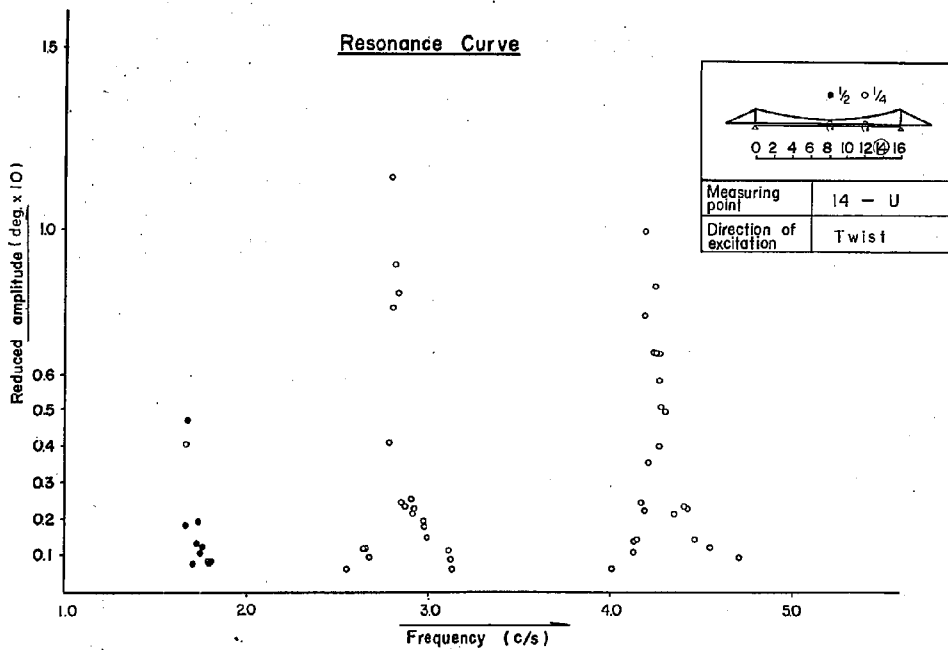


Fig 4-22(b)

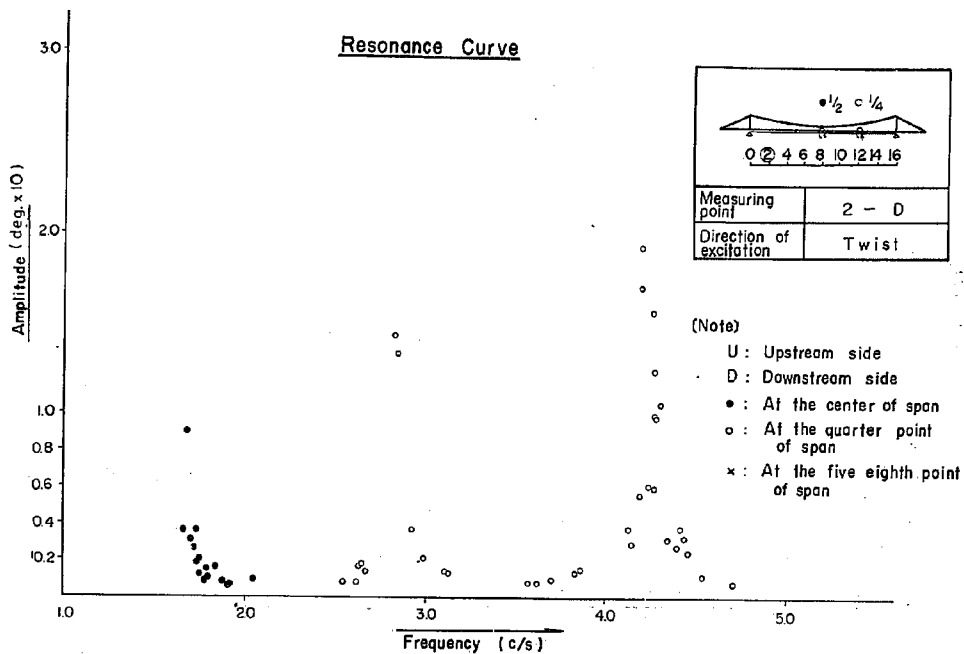


Fig 4-23(a)

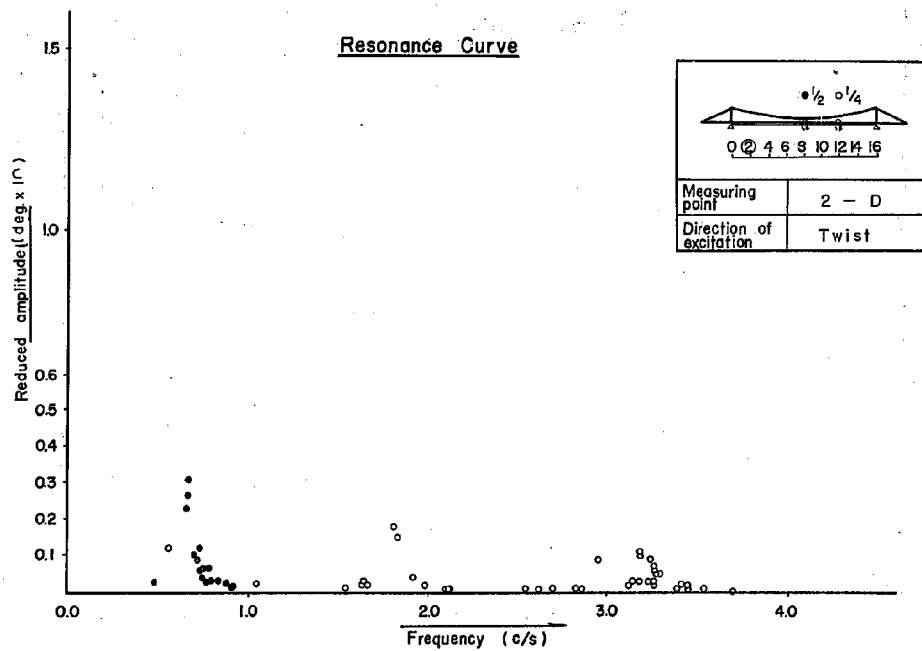


Fig 4-23(b)

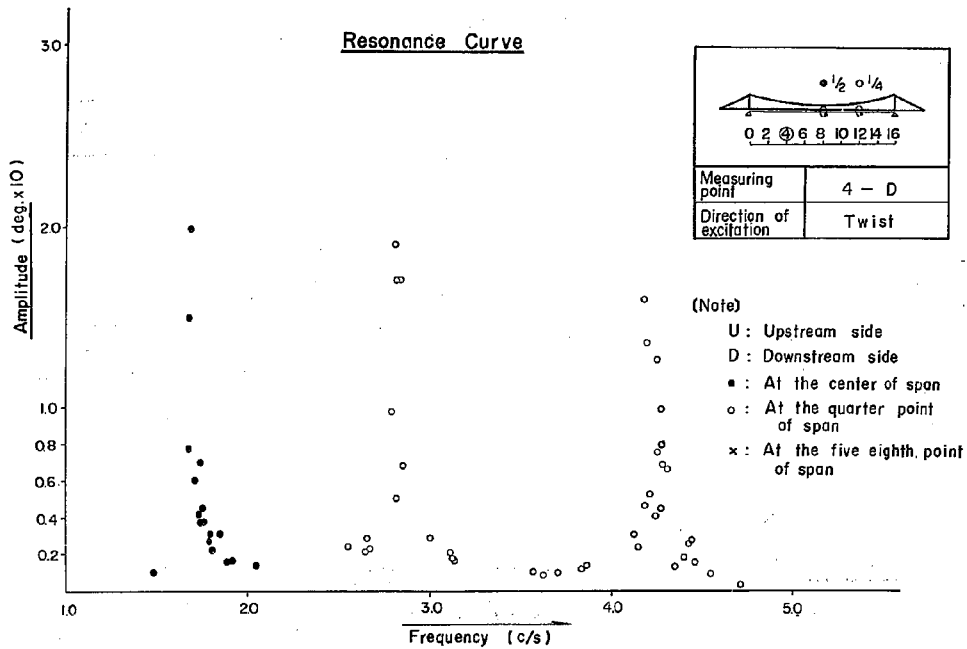


Fig 4-24(a)

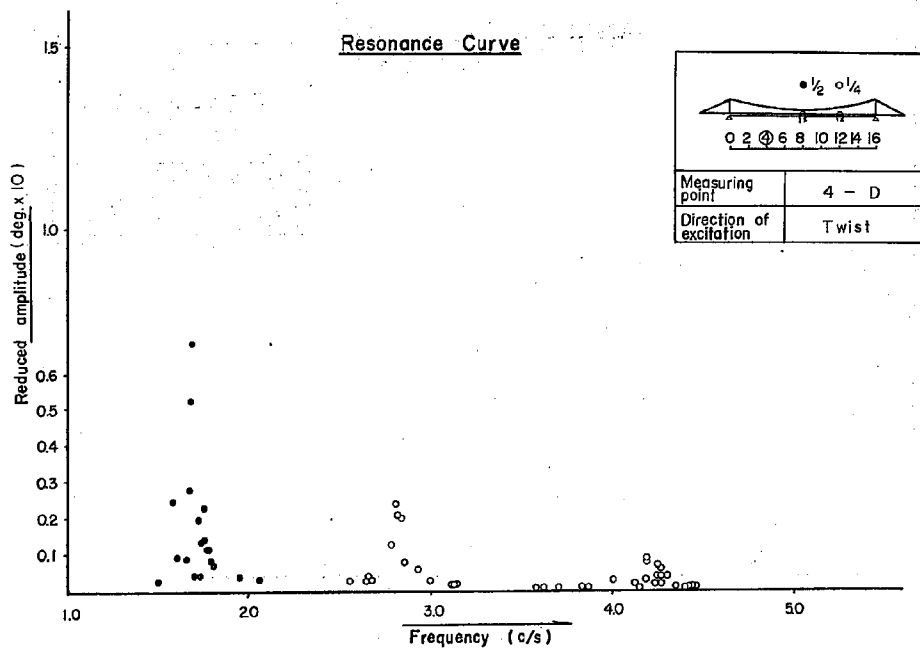


Fig 4-24(b)

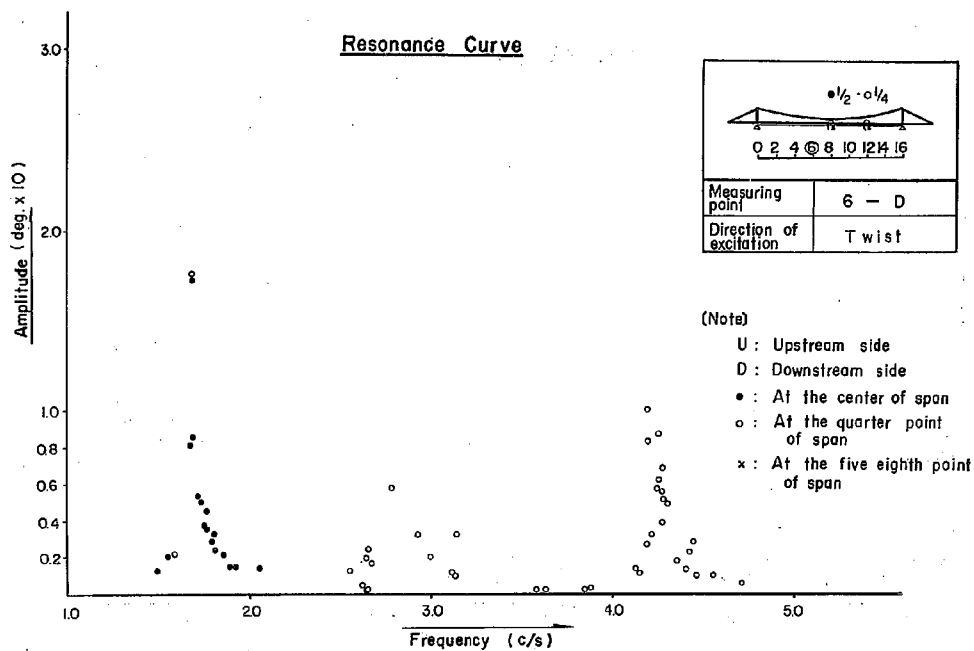


Fig 4-25(a)

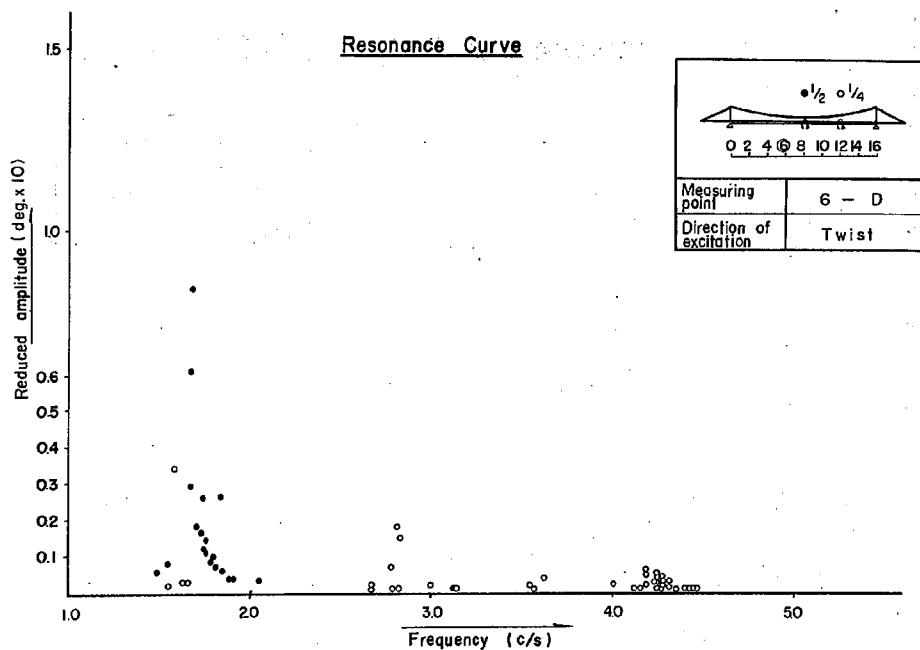


Fig 4-25(b)

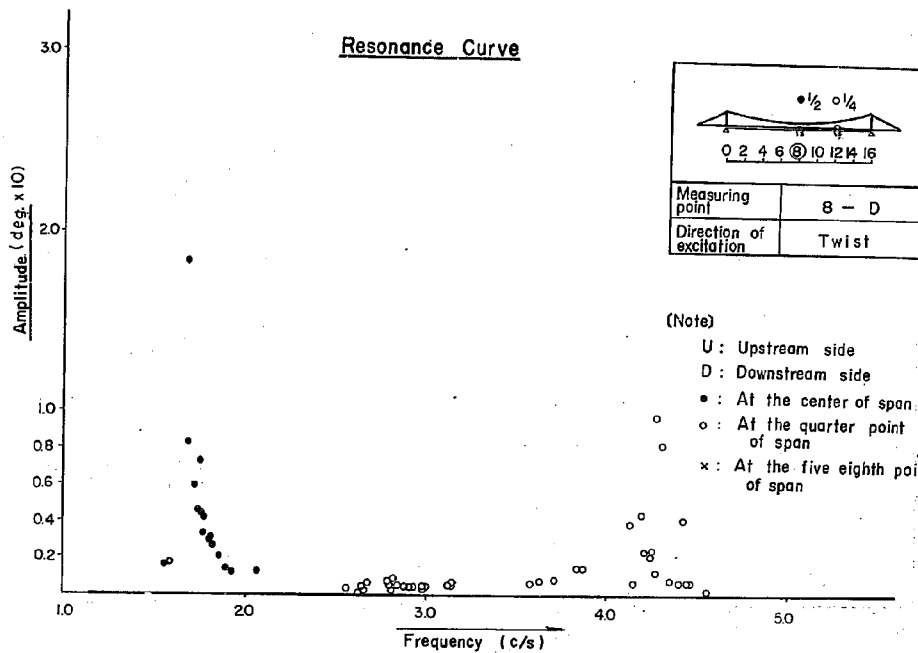


Fig 4-26(a)

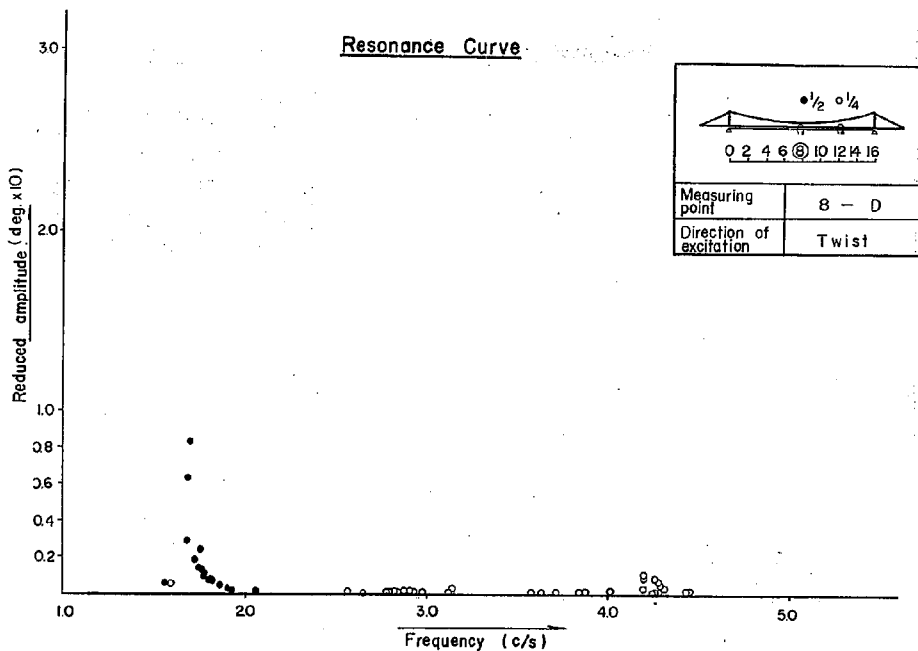


Fig 4-26(b)

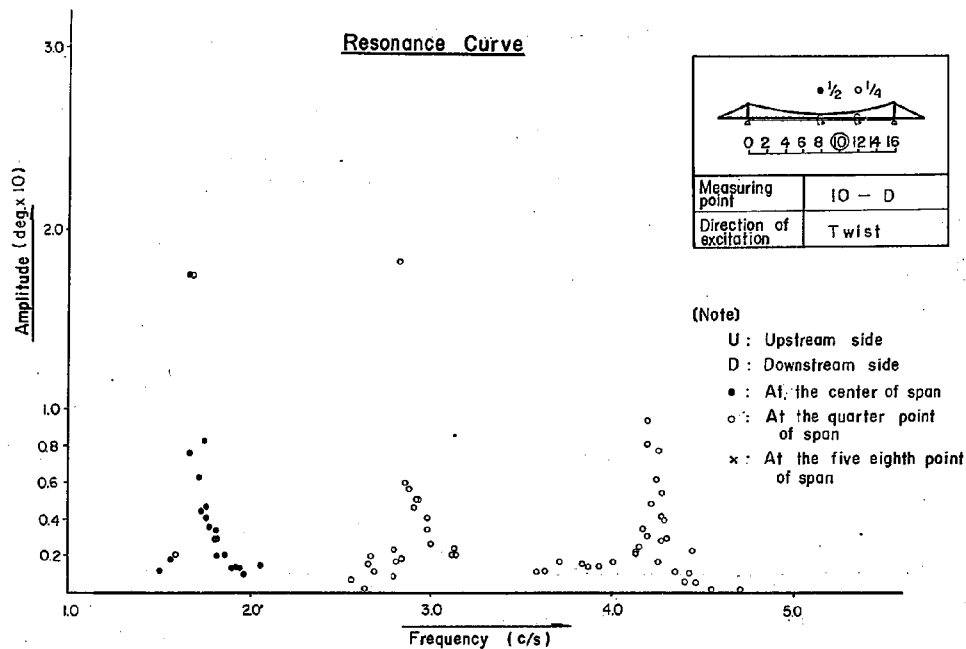


Fig 4-27(a)

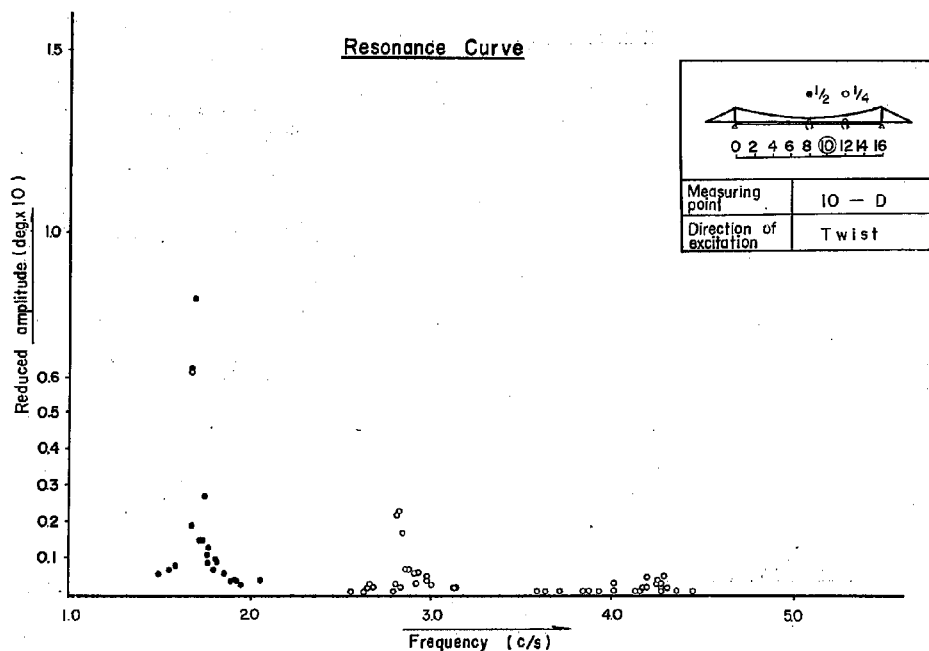


Fig 4-27(b)

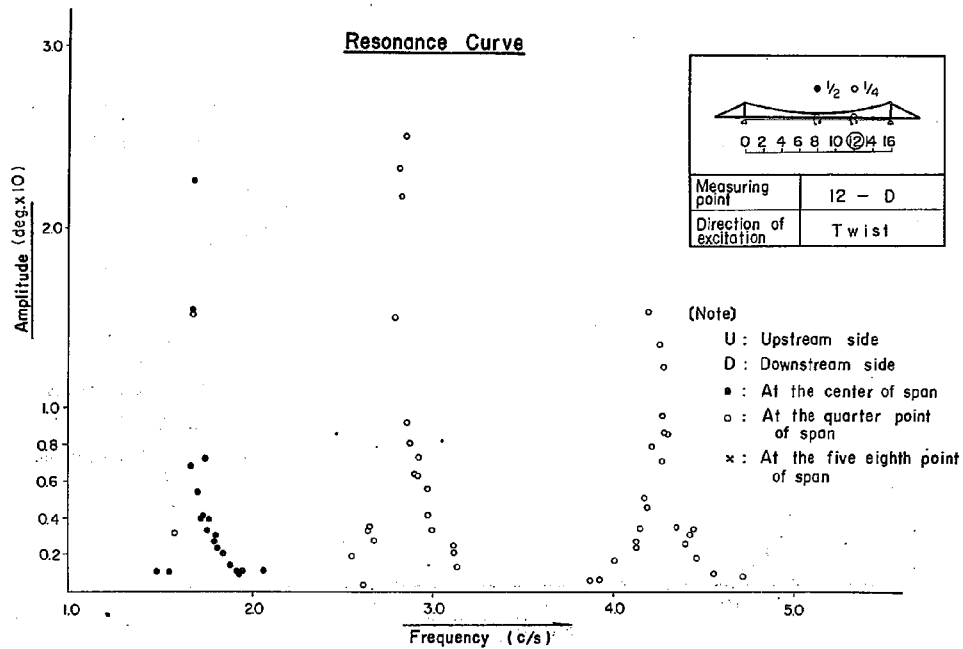


Fig 4-28(a)

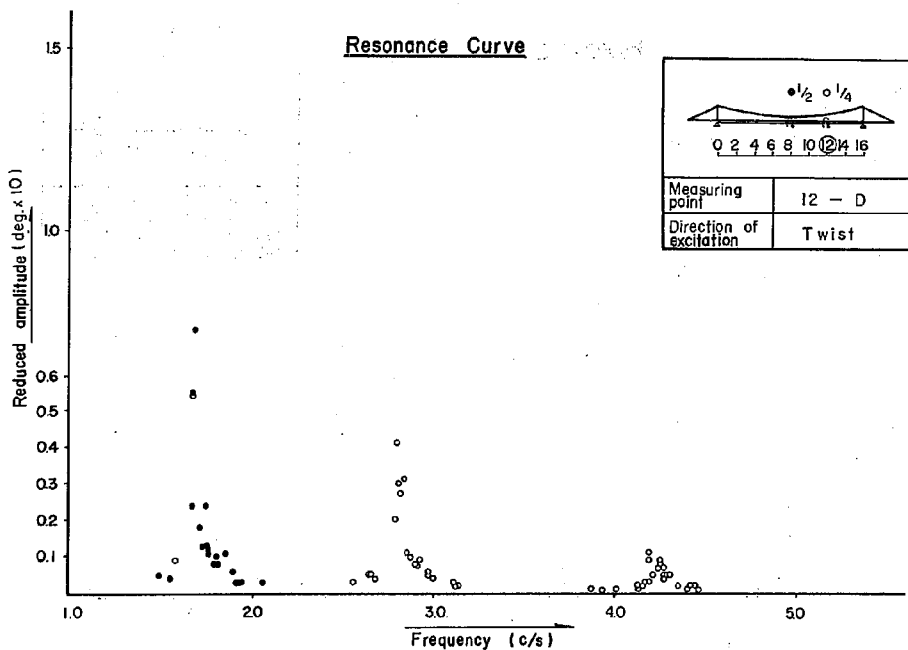


Fig 4-28(b)

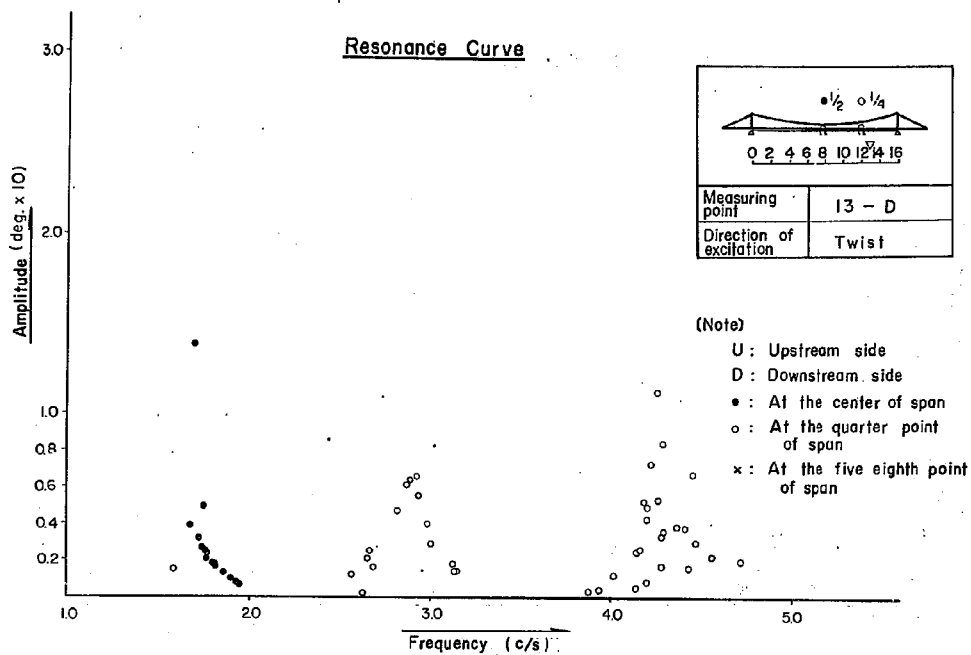


Fig 4-29(a)

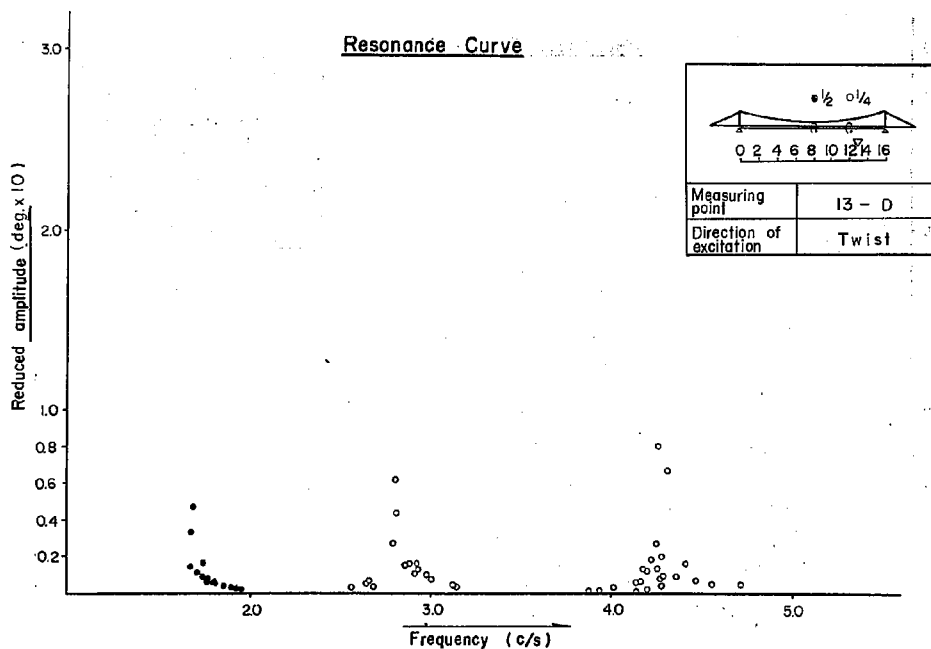


Fig 4-29(b)

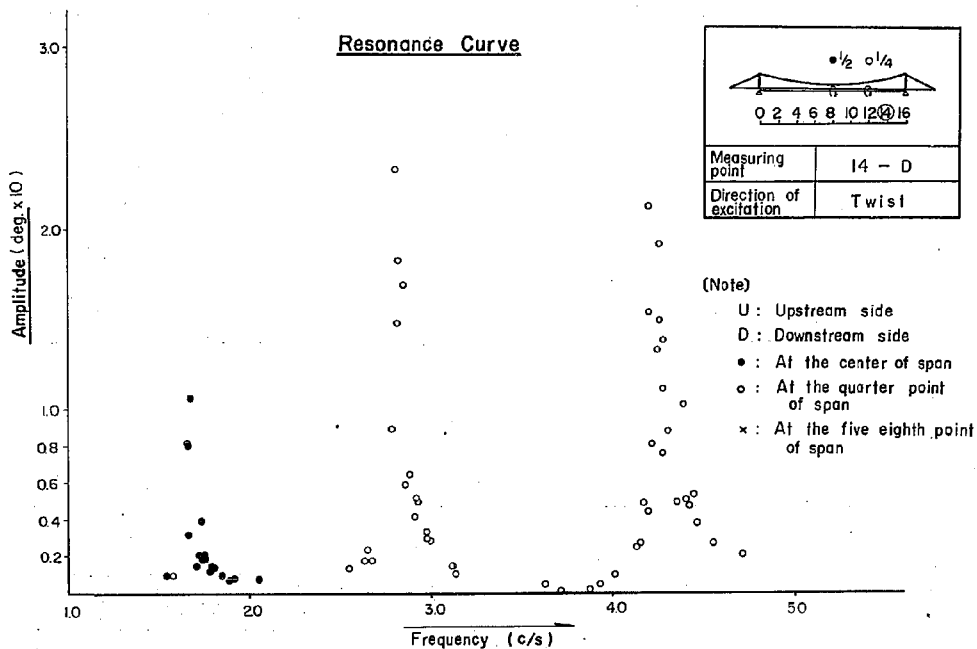


Fig 4-30(a)

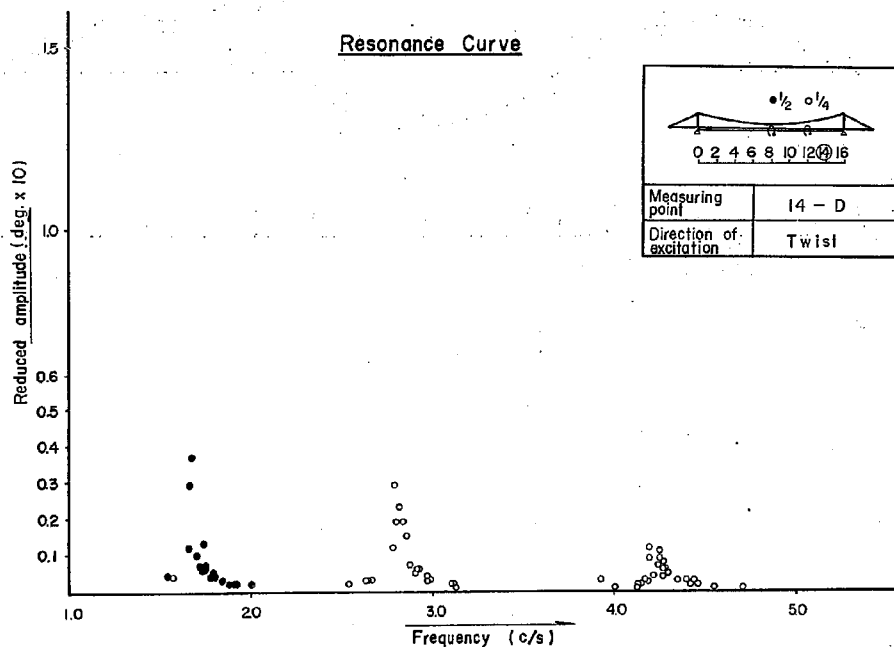


Fig 4-30(b)

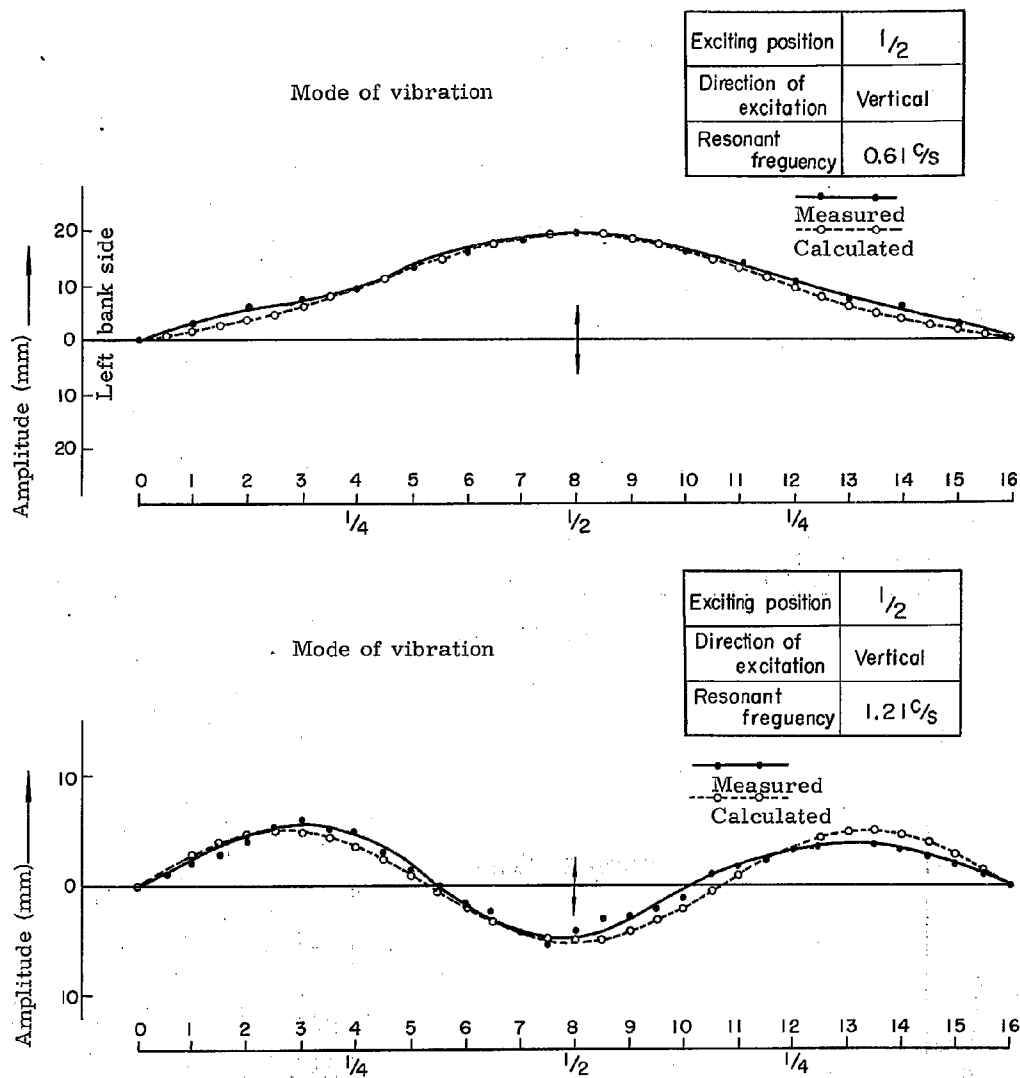


Fig. 5-1

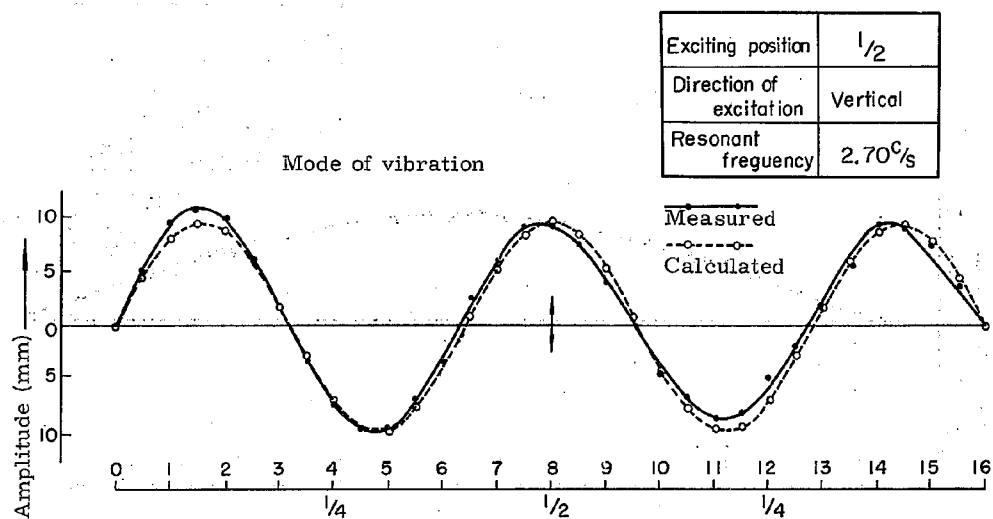
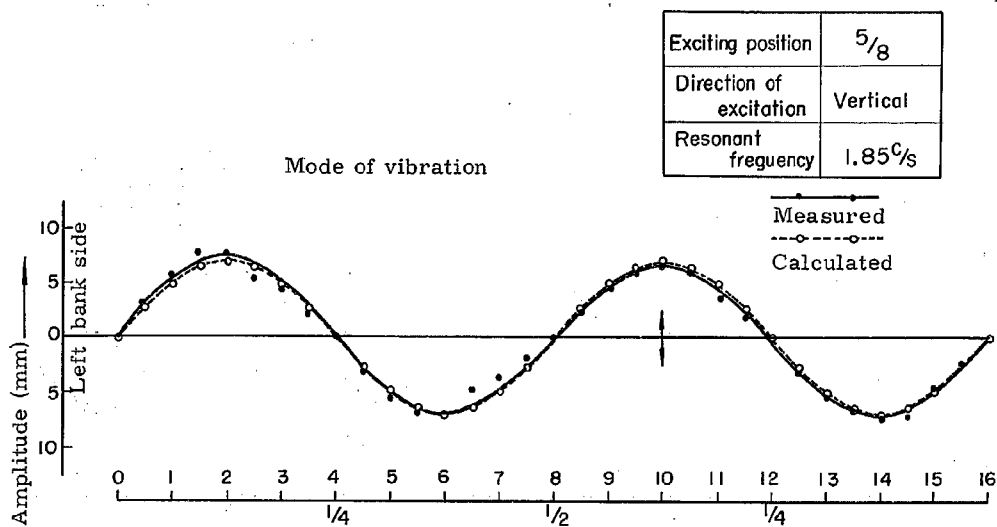


Fig 5-2

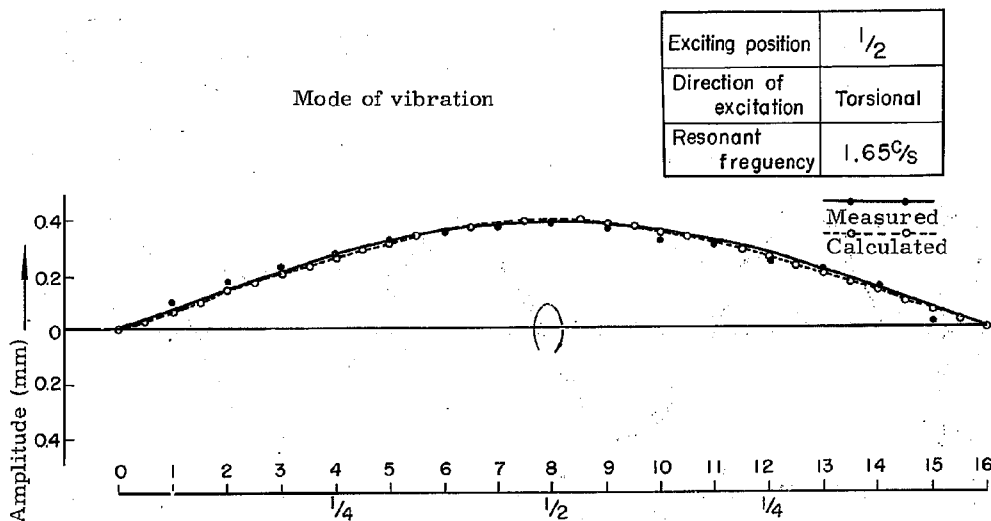
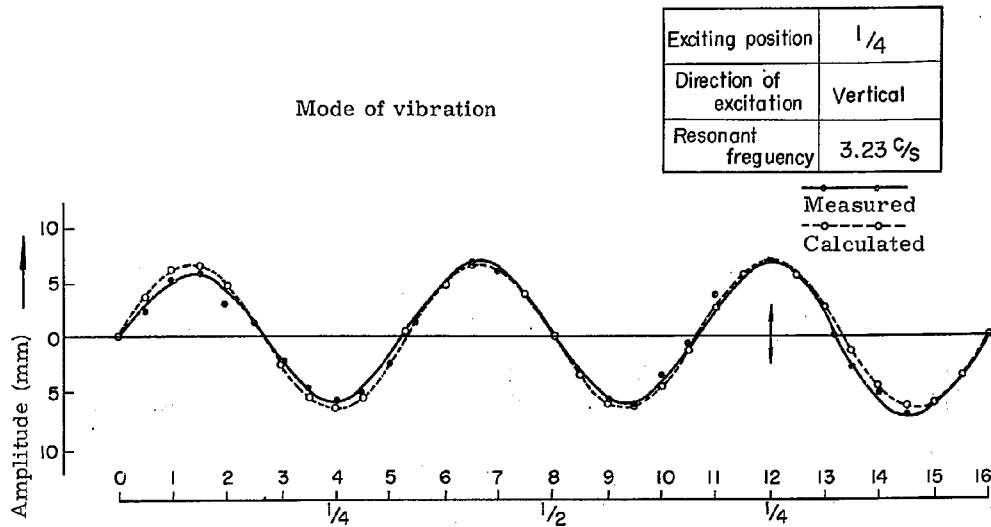


Fig 5-3

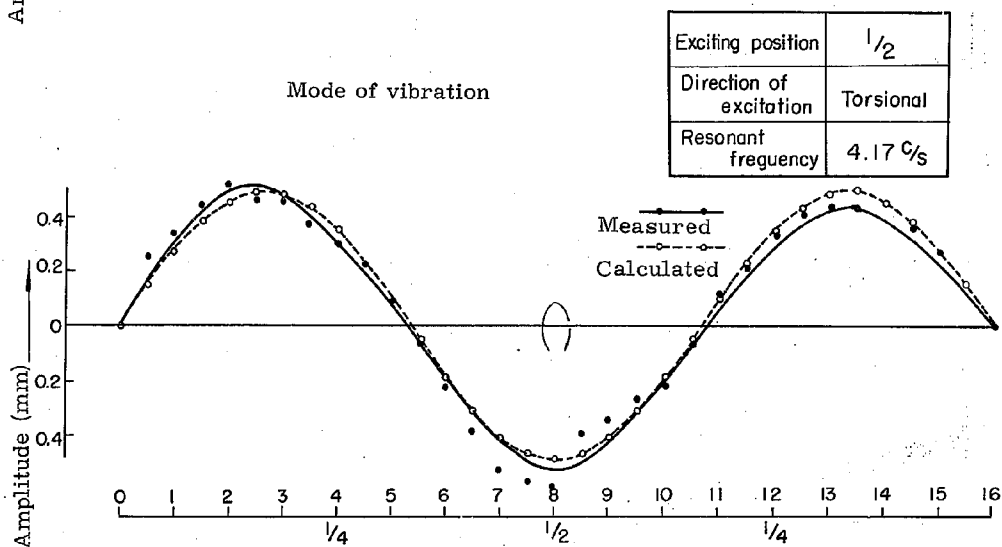
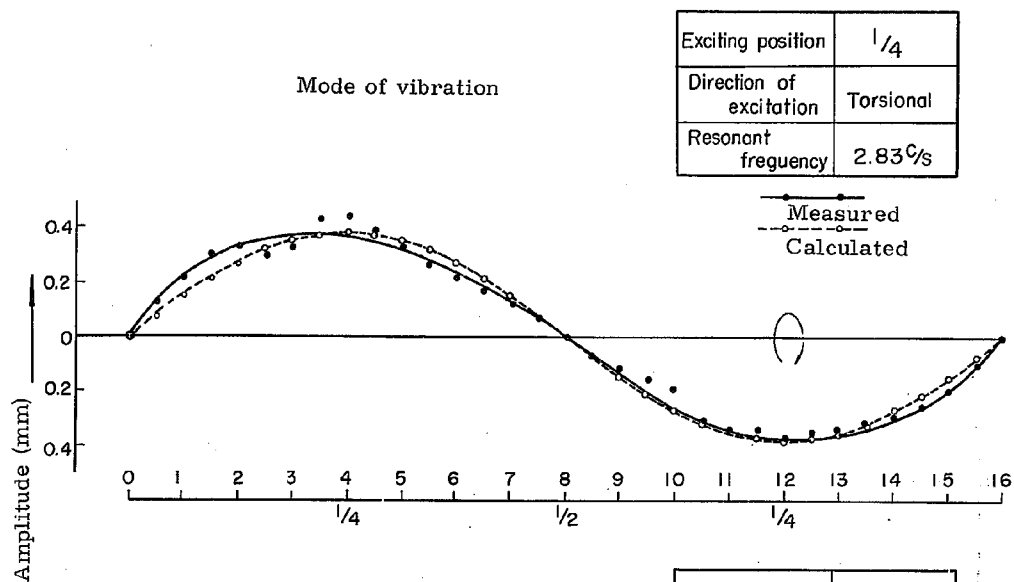
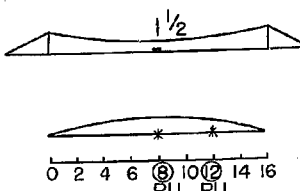


Fig. 5-4

Relation between
number of wave
and amplitude

Measuring point	8-D, 12-D	
Mode of oscillation	Vertical with zero node	
Logarithmic decrement	0.023 ~ 0.026	
Resonant frequency	0.61 c/s	

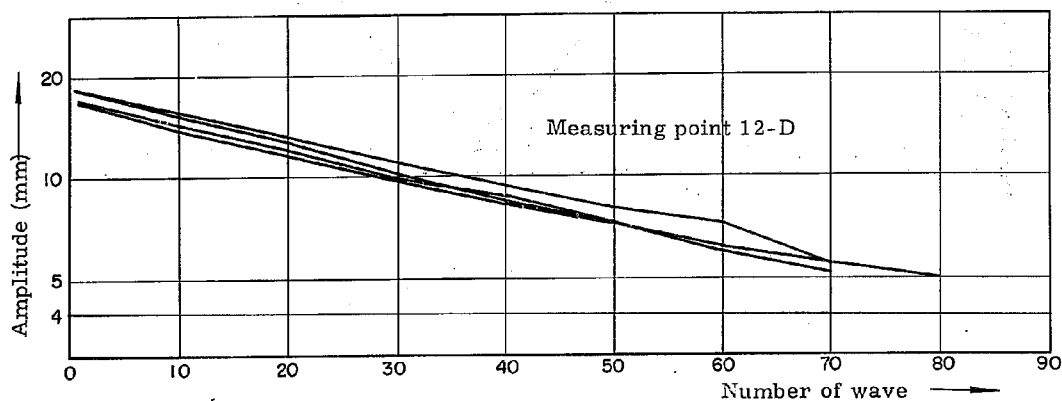
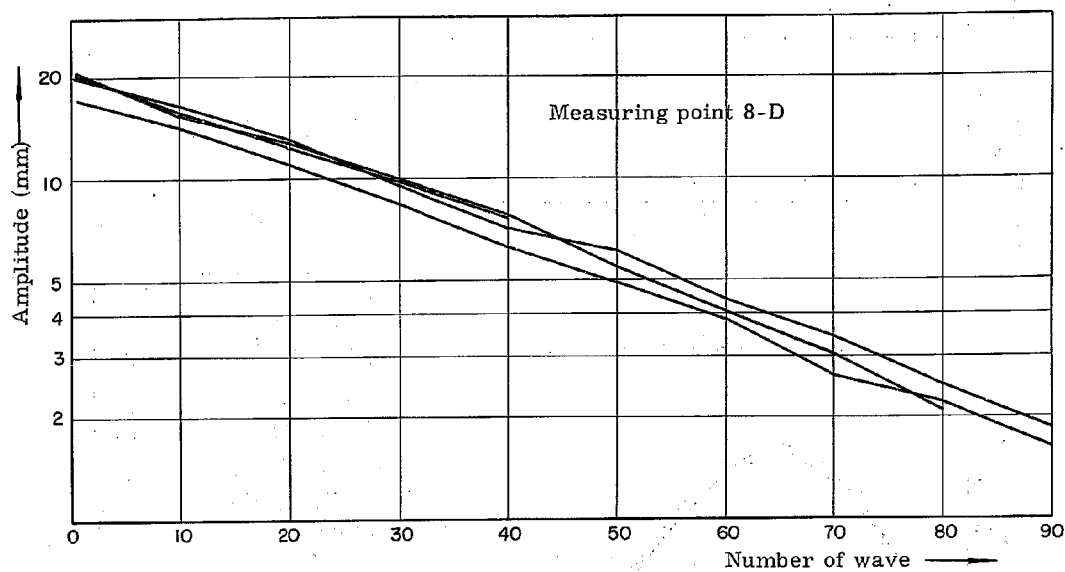
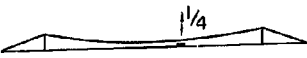
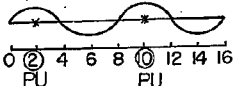


Fig. 6-1

Relation between
number of wave and
amplitude

Measuring point	8-D, 10-D	 
Mode of oscillation	vertical with three nodes	
Logarithmic decrement	0.094 ~ 0.116	
Resonant frequency	1.85 c/s	

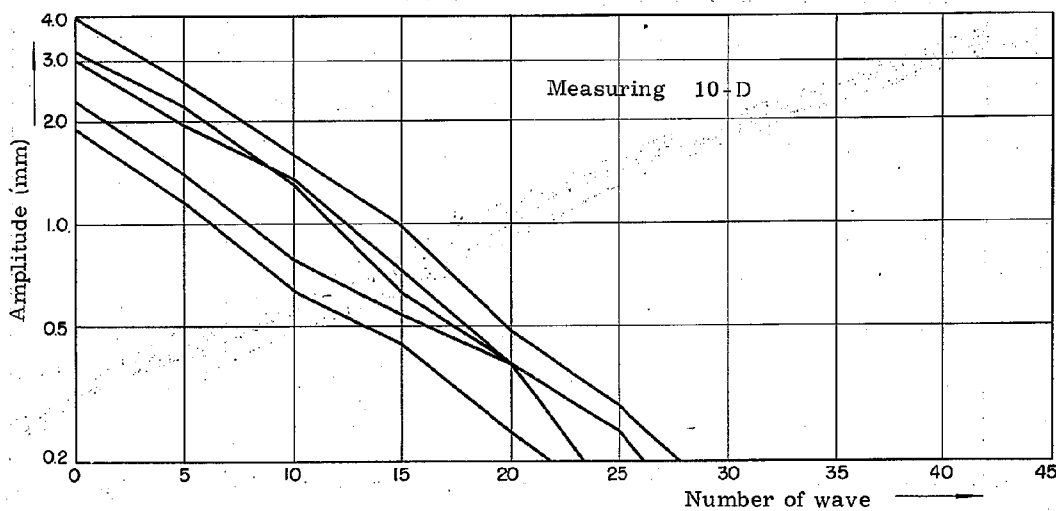
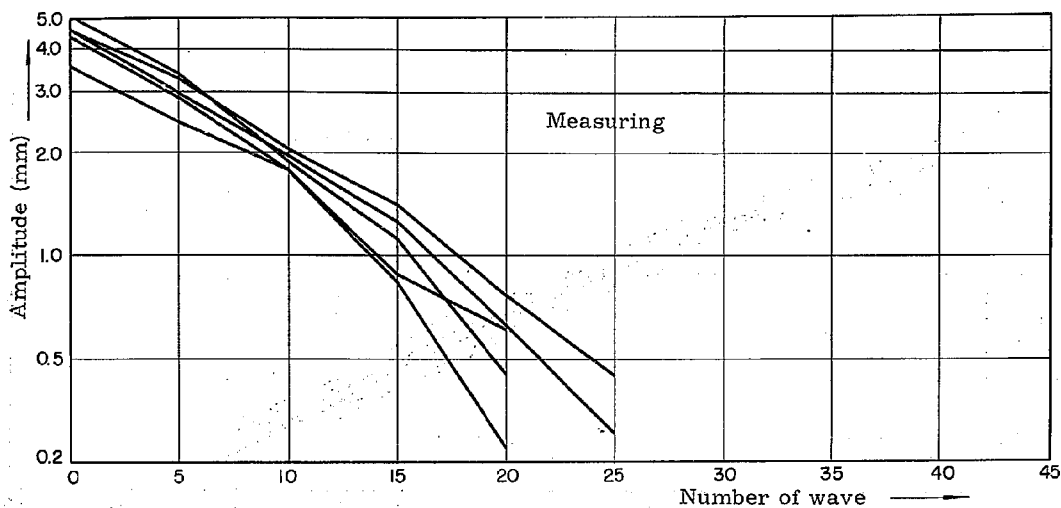
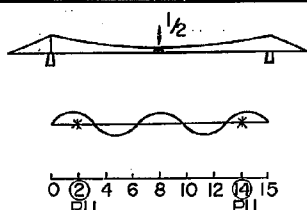


Fig. 6-2

Relation between
number of wave
and amplitude

Measuring point	2-D, 14-D	
Mode of oscillation	vertical with four nodes	
Logarithmic decrement	0.050 ~ 0.061	
Resonant frequency	2.67 c/s	

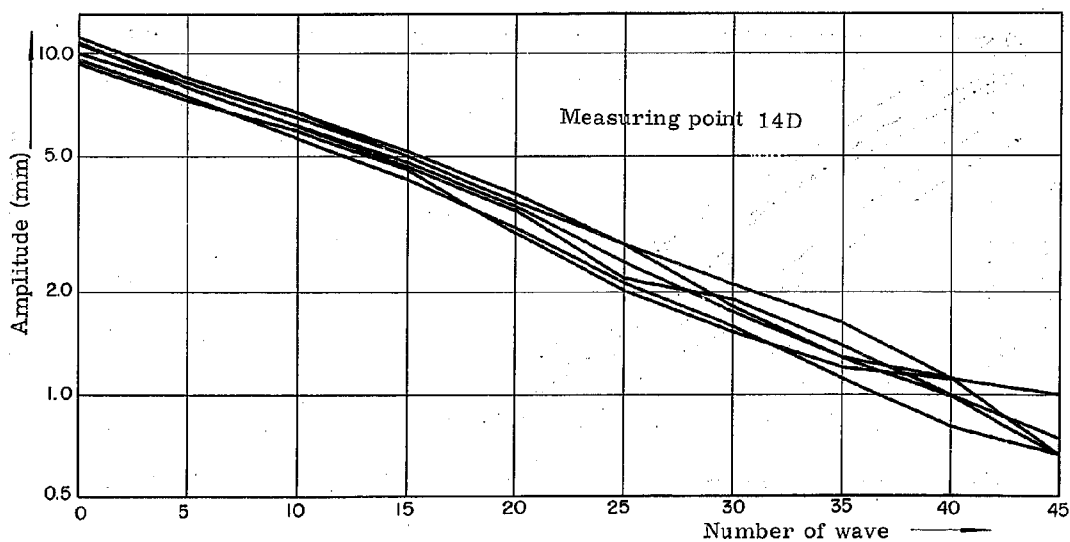
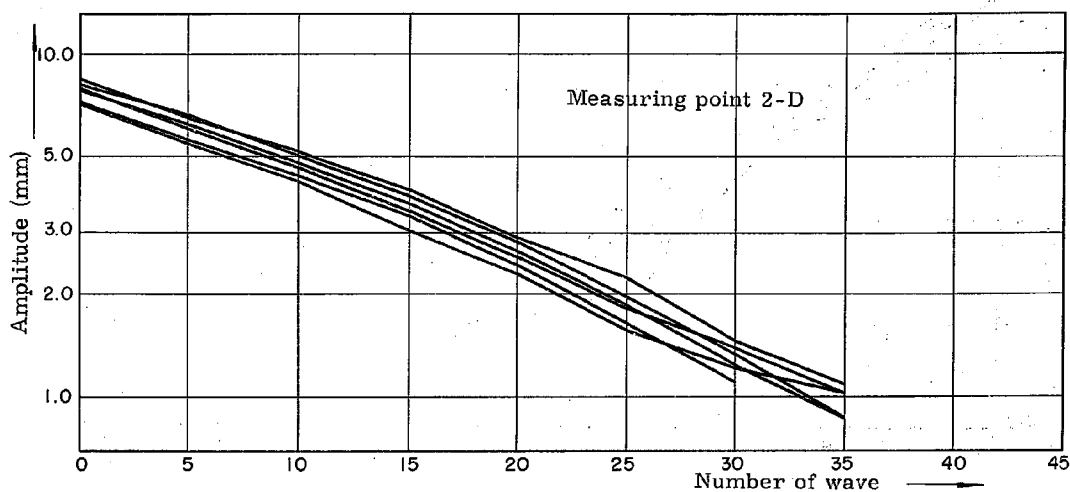
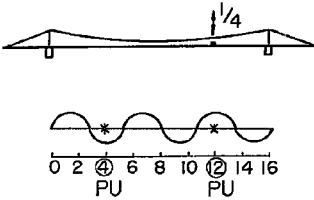


Fig. 6-3

Relation between
number of wave
and amplitude

Measuring point	4-D, 12-D	
Mode of oscillation	vertical with five nodes	
Logarithmic decrement	0,049 ~ 0,060	
Resonant frequency	3,42 c/s	

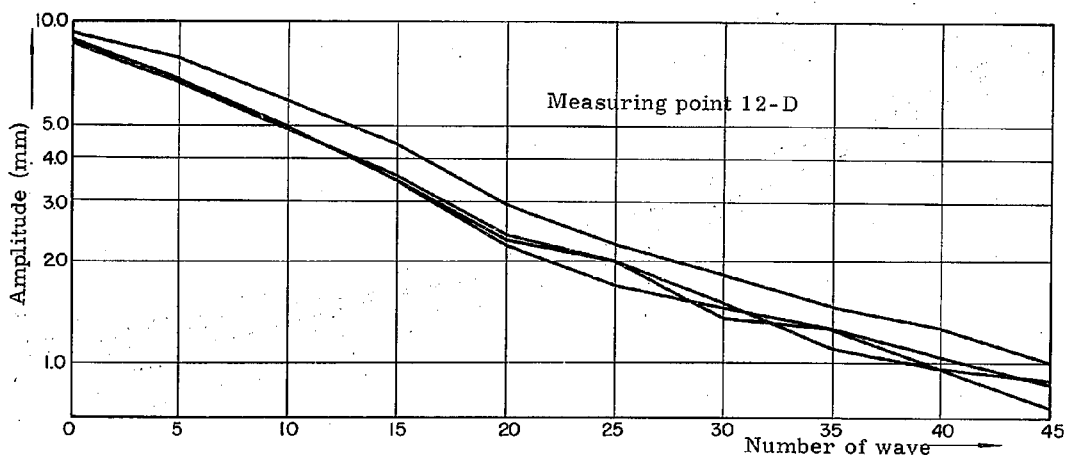
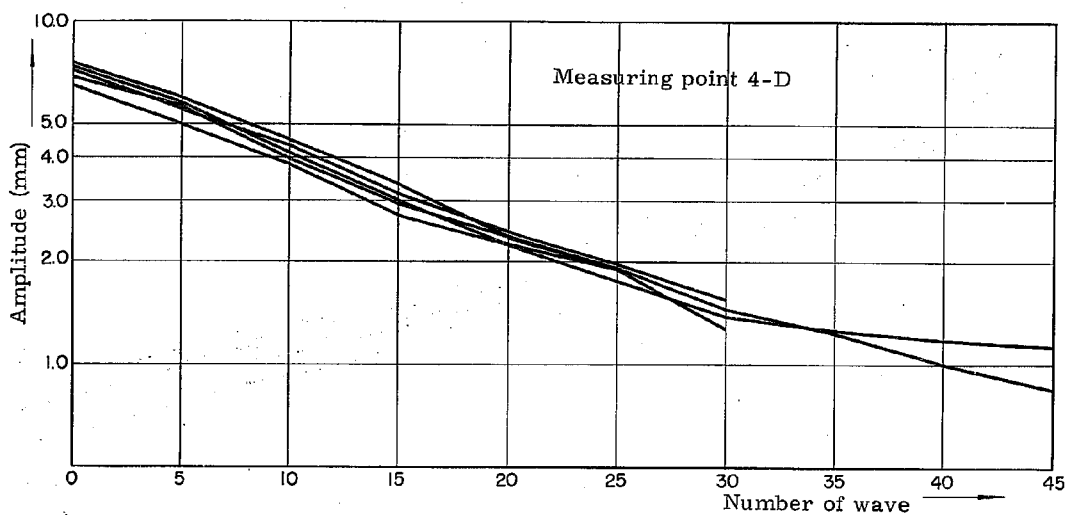
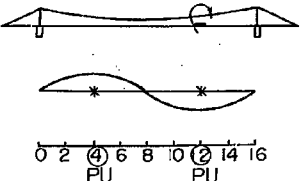


Fig. 6-4

Relation between
number of wave
and amplitude

Measuring point	4-D, 12-D	
Mode of oscillation	torsional with one node	
Logarithmic decrement	0.020 ~ 0.035	
Resonant frequency	2.81 c/s	

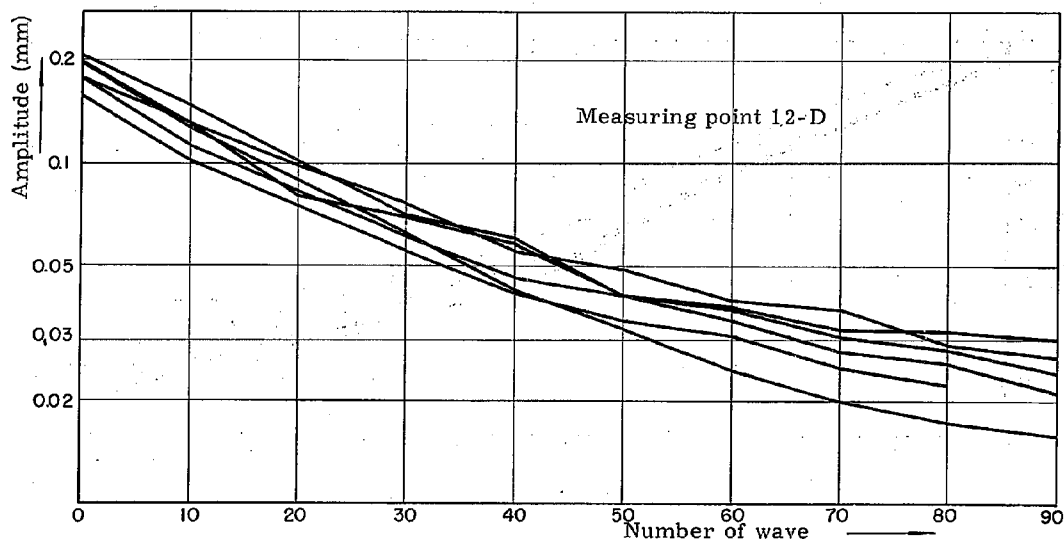
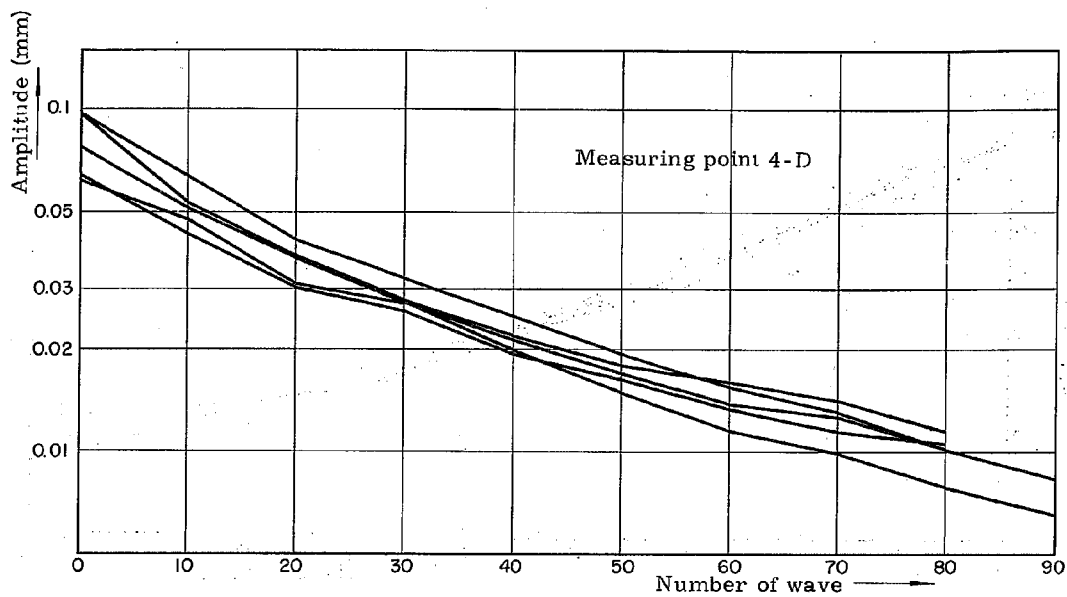
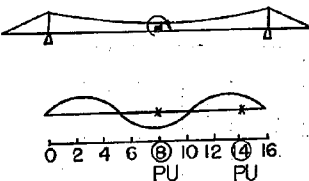


Fig. 6-5

Relation between
number of wave
and amplitude

Measuring point	8-D, 14-D	
Mode of oscillation	Torsional with two nodes	
Logarithmic decrement	0.017 ~ 0.029	
Resonant frequency	4.17 %s	

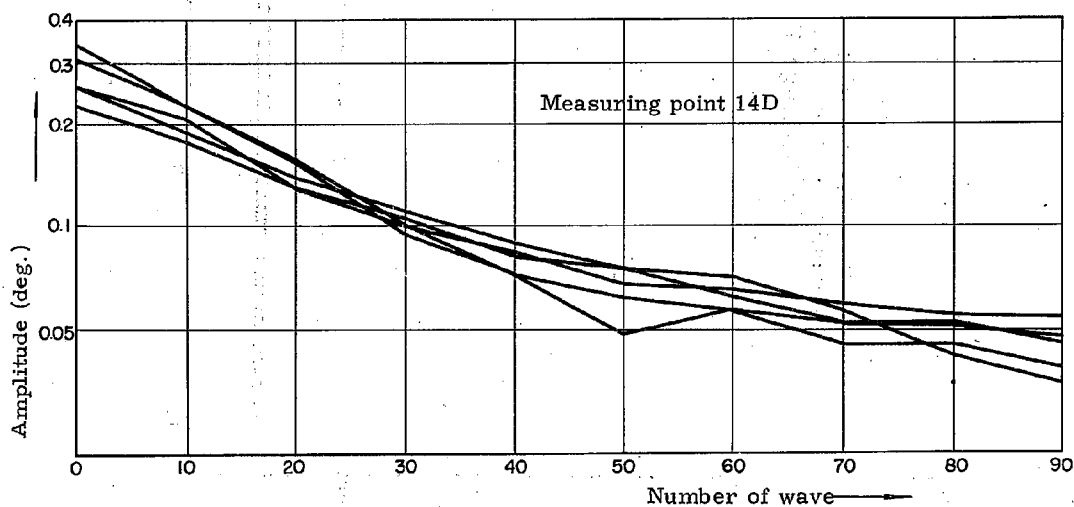
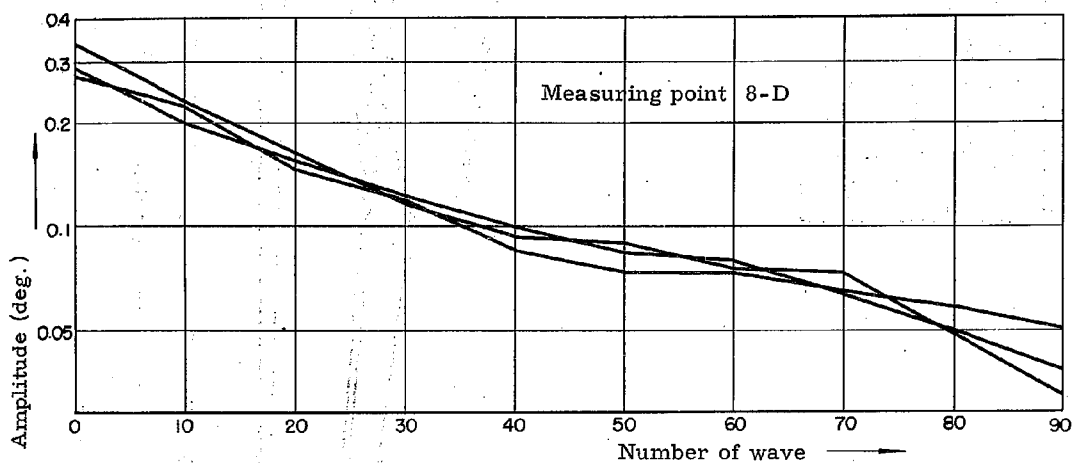


Fig. 6-6

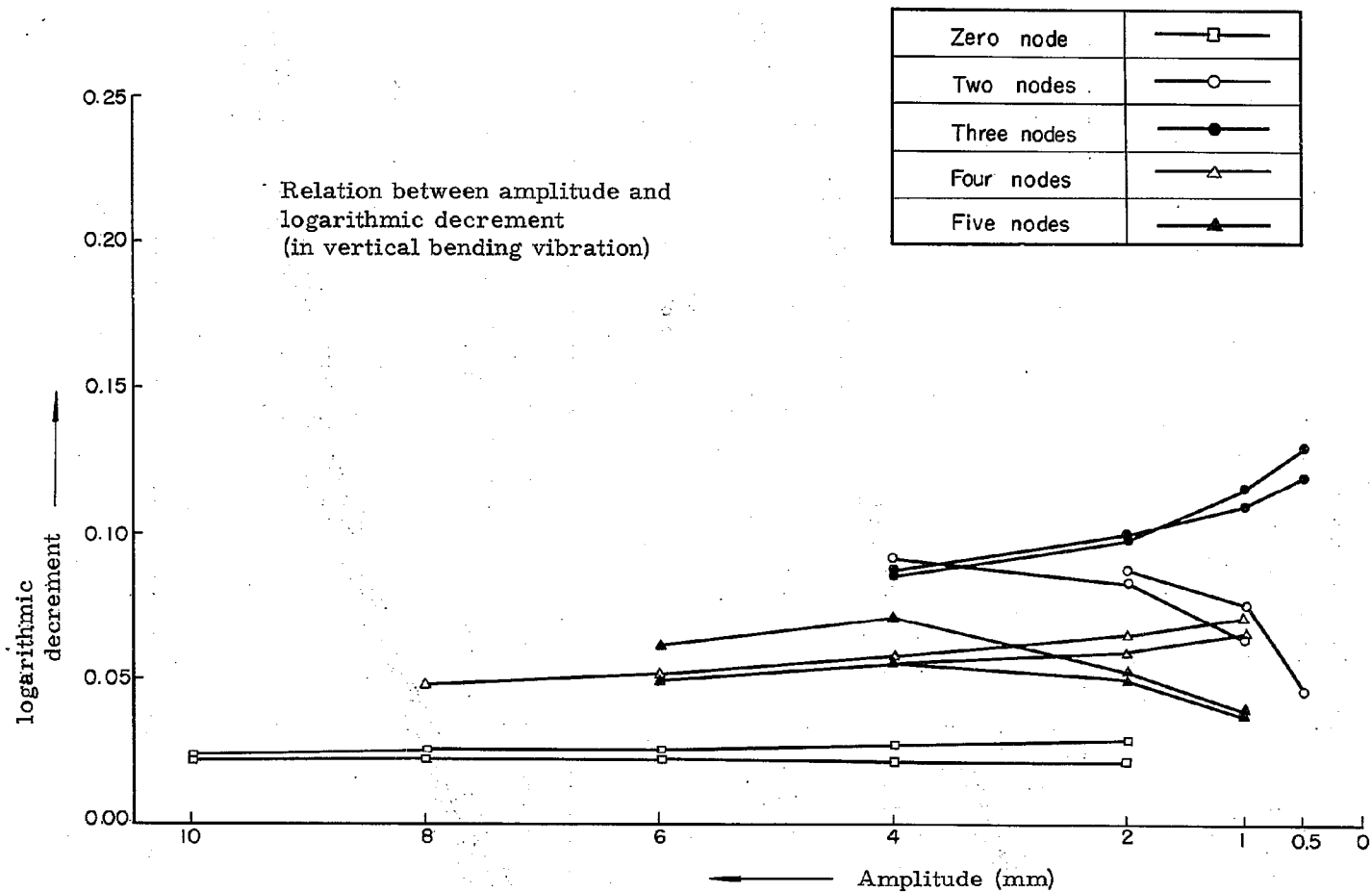


Fig. 7-1

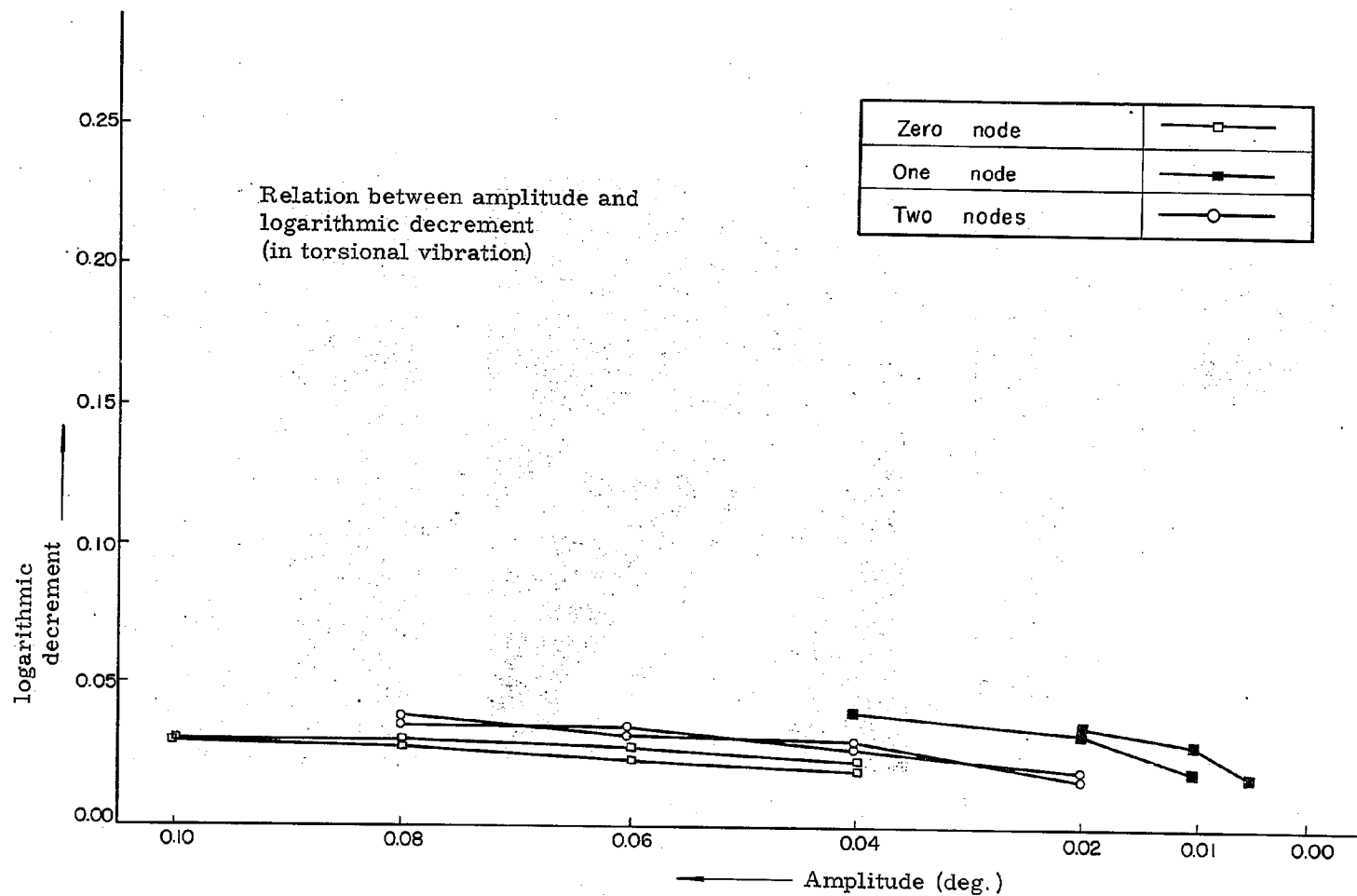


Fig. 7-2

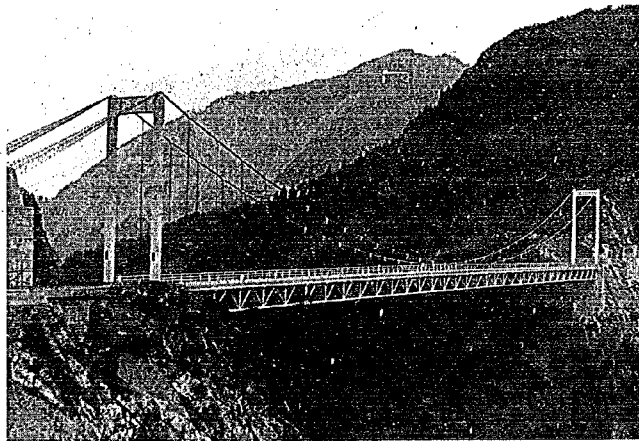


Photo - 1 Whole view of the Kompira Bridge

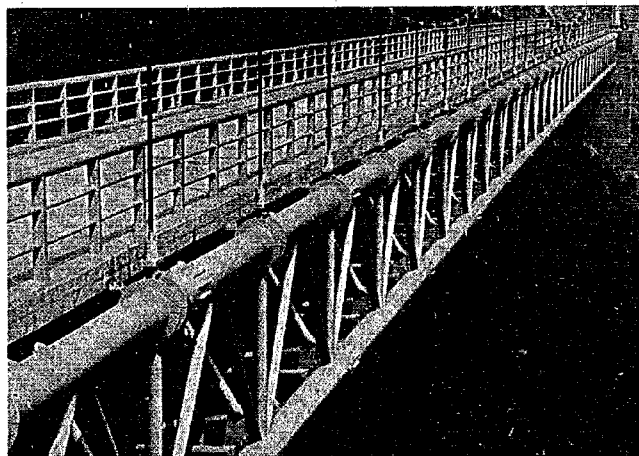


Photo - 2 Close-up view of the bridge

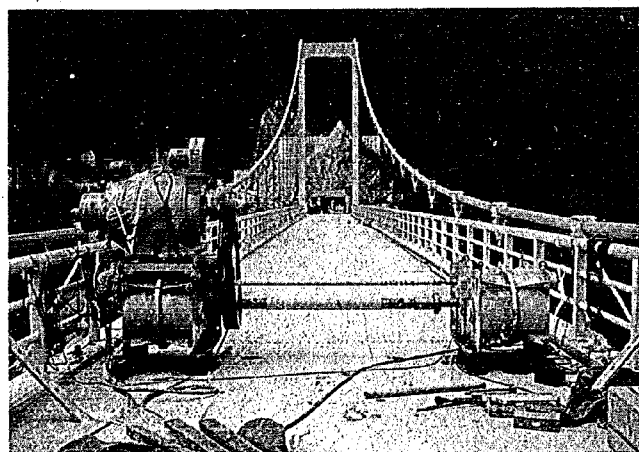


Photo - 3 Twin-exciter set up on the deck

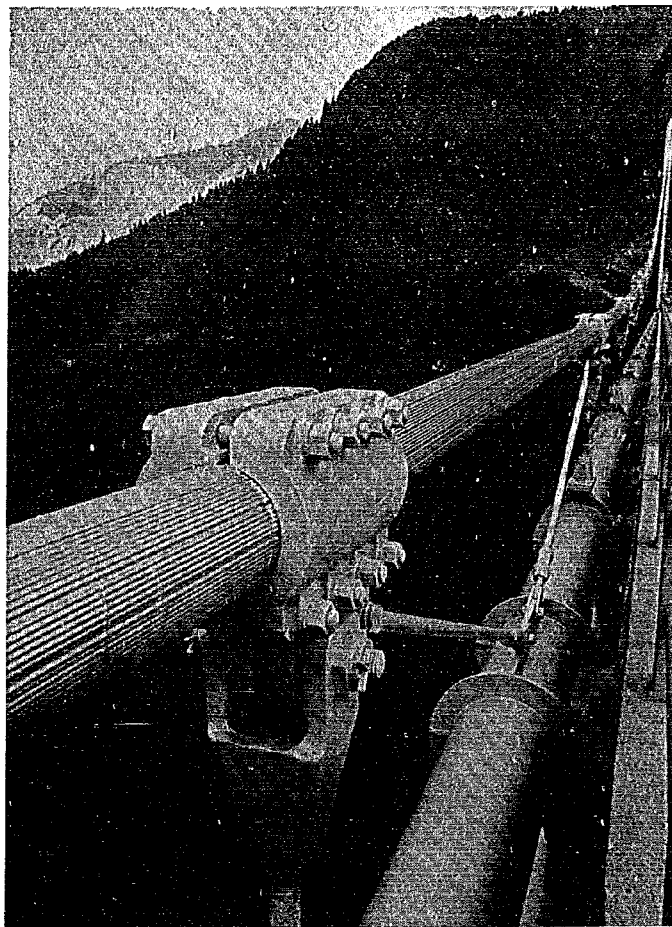


Photo - 4 Center ties

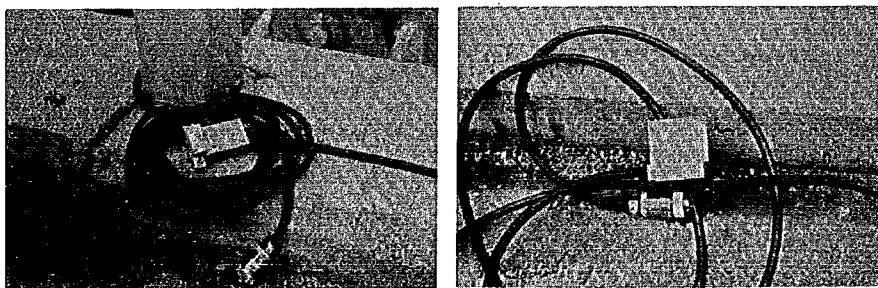


Photo - 5 Accelerometer (5g on the left, 2g right)