

Design of Concrete Dams

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PREFACE

This report is written in English for engineers come from abroad to understand general design procedures of concrete dams in Japan.

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1. INTRODUCTION

Design of structures including dams is a process of studies for ensuring their functional performance and for selecting the most economical way of completing their construction. In case of dams, these studies should be made to assure that the storage function as well as the functional and structural requirements for releasing flood waters in a necessary time can be satisfied most safely and economically under the given conditions in perfect balance with the surrounding environment. In general, the safety and economic efficiency mentioned here tend to come into conflict with each other, but the safety rather than the economic efficiency of dam construction is of great importance to society. The designer should therefore give precedence to safety over all other elements when he encounters any uncertain situation of condition in the said process of studies, although it is theoretically possible for him to work out an optimum design satisfying both requirements harmoniously.

For the safe design of a dam, it is necessary to estimate the working load to be imposed on the dam body accurately. The working load comprises those working constantly such as water pressure and dam's own weight and those expected to be imposed in the future such as seismic load. The designer is required to estimate the load to be imposed in the future accurately on the basis of the dam site conditions.

Another important thing to be done in the process of design is to make clear the resistant characteristics of the structure on which the said load is to be imposed. In the design of a concrete dam, it is necessary to know the resistant characteristics of concrete (or design the mix proportion of concrete to give the required resistant characteristics to it) and to know the resistant characteristics of the bedrock upon which the dam is to be built because the ground is the final supporting point of all structures including dams. In the design of a fill dam, it is necessary to know the strength and deformation characteristics of the dam body materials and the foundation as well as their resistance to seepage flow. Thus, the design of a dam safe from the danger of rupture becomes possible only when all necessary data on the loads and the resistance characteristics of dam body materialy and foundation are made available. The design is worked out employing the concept of safety factor which will be explained later in detail.

We now proceed to the economic aspect of design. Under a given condition,

there can be an infinite number of designs in which the required degree of safety is ensured, and the most economical one of them is called "optimum design." Designers often consider that the dam volume including the appurtenant structures such as stilling basin is the most important for determining the optimum design, but this is not necessarily correct. Let us assume that there are a plural number of alternative dam sites. Since the greater part of the dam construction cost consists of the quantity of the dam body materials, the approximate total cost can be obtained by multiplying the unit cost per m by the dam volume. If the unit cost per m is the same at all sites, then it is correct to select the optimum design on the basis of the volume of the dam body materials. However, the unit cost required for piling up 1 m of the dam body materials consists of a diversity of expenses. In the case of concrete dams, for instance, it comprises the expenses for aggregate gathering, crushing and transportation, the expenses for concrete manufacturing, transportation and placing, and even the expense for constructing the aggregate material transportation road, so that all these component expenses must be carefully examined and estimated in order to complete the optimum design.

In actuality, however, there have been few cases where the dam site was selected on the bases of such detailed cost estimation and in most cases, it is selected mainly on the bases of the total dam volume and the possibility of spillway construction and foundation treatment without making an economic comparison. Nevertheless, the designer should make every effort to make his design economically optimum in the selection of dam type, dam site and abutment locations even if the information available on hand is limited.

2. DAM TYPE AND FACTORS IN TYPE SELECTION

a. general

Dams can be classified in different ways according to the purpose of classification. In general, they are classified by the purpose of use and the kind of material. When based on the purpose of use, they are classified as follows.

- a. Storage dam
- b. Intake dam
- c. Sabō dam

The storage dam is further classified into "single-purpose dams" constructed for a single purpose such as irrigation and power generation and "multipurpose dams" constructed for two or more purposes.

When classified by the kind of material, dams are broadly divided into a concrete dam and a fill dam. The concrete dam is further classified into the following four types according to the structure.

- a. Gravity dam
- b. Hollow gravity dam
- c. Arch dam
- d. Buttress dam

The fill dam is also classified as follows by the structure.

- e. Zoned type dam
- f. Homogeneous type dam
- g. Facing type dam

When people speak of the type of a dam, they usually mean one of the type classified above (a. g.), so that any reference to dam type in the following pages will be based on the above classification.

Both the safety and the economic efficiency are largely influenced by which of the above-listed types is selected for construction at the given site. A brief explanation is given below on the structural characteristics of each type of dam because the selection of the optimum dam type calls for a clear understanding of such characteristics.

Both gravity dam and hollow dam have a simple structure in which the external load is resisted with the own weight of dam body and the water pressure and other loads are transmitted to the bedrock, and they are usually designed by two-dimensional analysis. The hollow gravity dam has a hollow inside the dam body to alleviate the relatively large uplift working on

the dam base and to make the dam volume smaller than that of the gravity dam. The buttress dam has a plate structure that resists the water pressure, and its buttress transmits the working load to the bedrock.

The arch dam has a structure in which the working load is transmitted to the bedrock on either bank by its arch action, but a part of the load is transmitted to the base by the cantilever action. Since the direct stress is the main stress component generated by the arch action, this structure is advantageous to concrete which has a large compressive strength and can make the dam volume far smaller than that required for the gravity dam which generates mainly bending stress. Concrete dams including those with a large height have been constructed in many places in Japan so long as the dam site topography and geology are satisfactory because of their stable structure.

The zoned type dam has a structure in which the impervious zone formed with earth material is covered by filter material on both sides to prevent the piping of the earth material, with permeable rock material filled on the outside of the filter material to make the dam body stand erect.

The homogeneous type dam is constructed with fine-grained earth material in the greater part of its dam body. The fine-grained earth material means the material having a 50 grain diameter of less than 5 mm or a permeability coefficient of less than 5×10^{-4} cm/sec.

The facing type dam has an impervious wall of asphalt concrete or reinforced concrete or other material on the upstream side of the permeable zone.

One of the characteristics of fill dams is that almost all kind of bedrock is available because it has a large area of contact with the dam body and consequently the stress working on it is made small. The homogeneous type dam has a low strength of dam body material, and its deformation after completion is large, while the facing type dam has its small thickness of the impervious walls. These two types have not been selected so often in Japan for high dams, the largest height being about 30 m for the former type and about 70 m for the latter.

Selection of the optimum dam type should essentially be based on the study of economic aspects. However, considering the fact that the survey method varies according to the dam type, it is extremely difficult to collect and consolidate, at the initial stage of survey, all accurate data necessary for making a comparative study of alternative types. In addition, attention

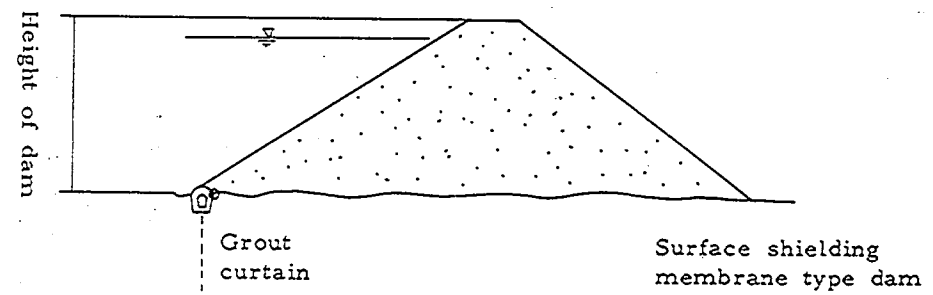
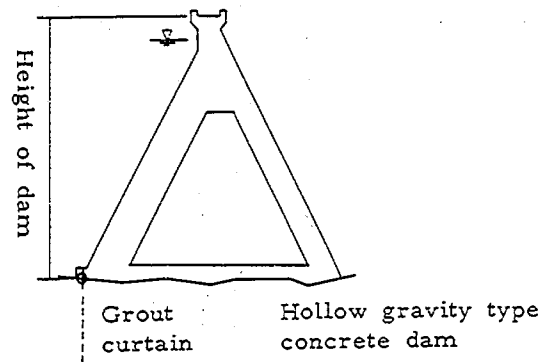
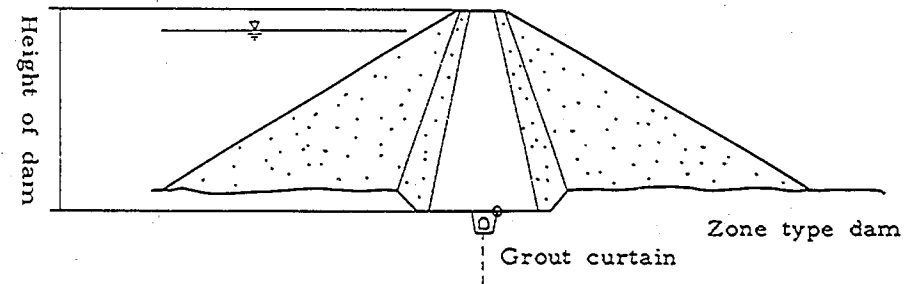
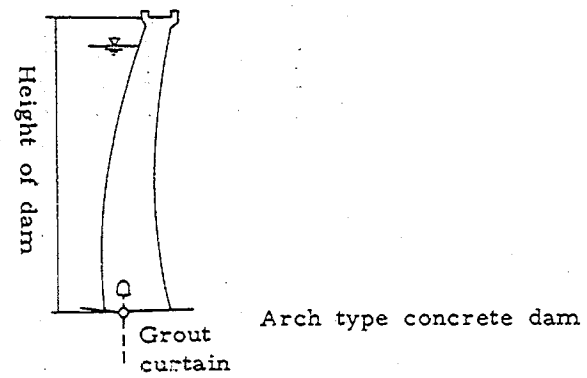
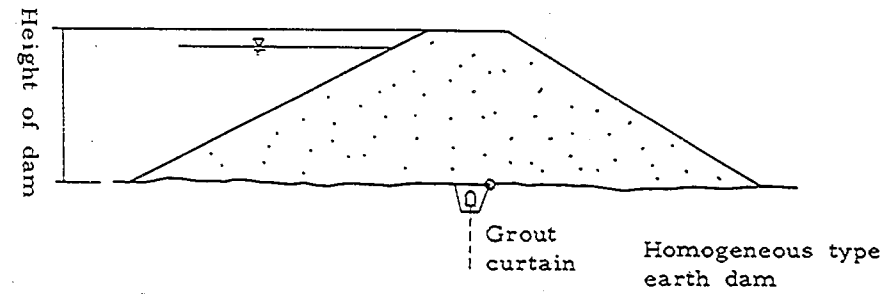
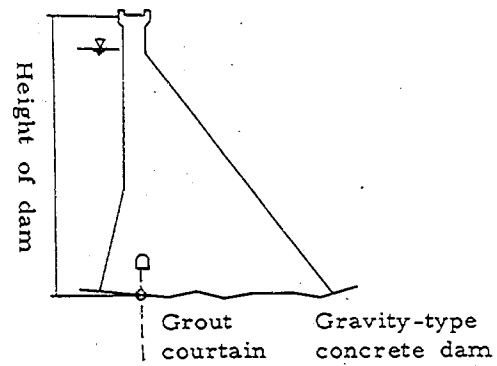


Fig. 1 Types of Dams

must be paid to factors like environmental preservation that cannot be readily cope with or included in the construction cost. Furthermore, to ensure the smooth progress of the dam construction work, the dam type should be determined at least tentatively at the initial stage of the feasibility study. In many cases, therefore, the dam type selection is based not on a rigid economic comparison but on the qualitative judgement. For this reason, a brief explanation is given below on the factors to be considered in selecting the dam type from the qualitative aspect.

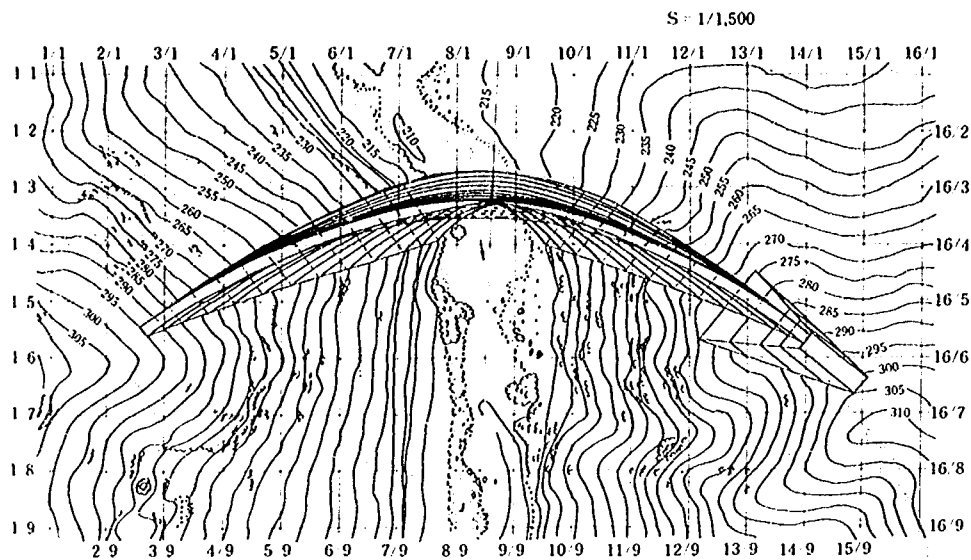
The direct factors bearing upon the dam type selection include the dam site natural conditions such as topography, geology, natural environment, climate and hydrology and the local conditions such as the availability of embankment materials and transport means. The indirect factors to be considered include the purpose and size of the dam, construction period, and manpower availability. Some of these factors carrying a relatively heavy weight are explained below in relation to different dam types.

b. Topography and geology

The dam site topography has a direct bearing upon the dam volume, so that it is the most important factor as the economic efficiency in the dam type selection. Special attention must be paid to the valley width, valley slope, and chain of mountains stretching in the river flow direction (expressed by contour lines). The greater the valley width, the more advantageous becomes the fill dam construction. This is because the cost of embankment materials required for the fill dam construction is $1/5 \sim 1/8$ of that needed for the concrete dam construction, and this gap widens with the increase in the total dam volume.

Even if the valley width is large, the gravity dam is often selected so long as the geological condition permits. This is because the fill dam is economically less advantageous than the gravity dam in certain cases for the purpose of flood control as will be described later. If the valley is steep and has a small width, it is disadvantageous to the fill dam construction. This is because such a topography places a limit on the operation of large construction machinery and reduces the stability of the impervious core. If the valley is steep, the arch dam construction is sometimes advantageous so long as the geological condition is favorable.

The chain of mountains stretching in the river flow direction is important to the selection of abutment locations. In general, contour lines



(a) Topography suitable for an arch type concrete dam

Fig. 2 Types of dams and topography

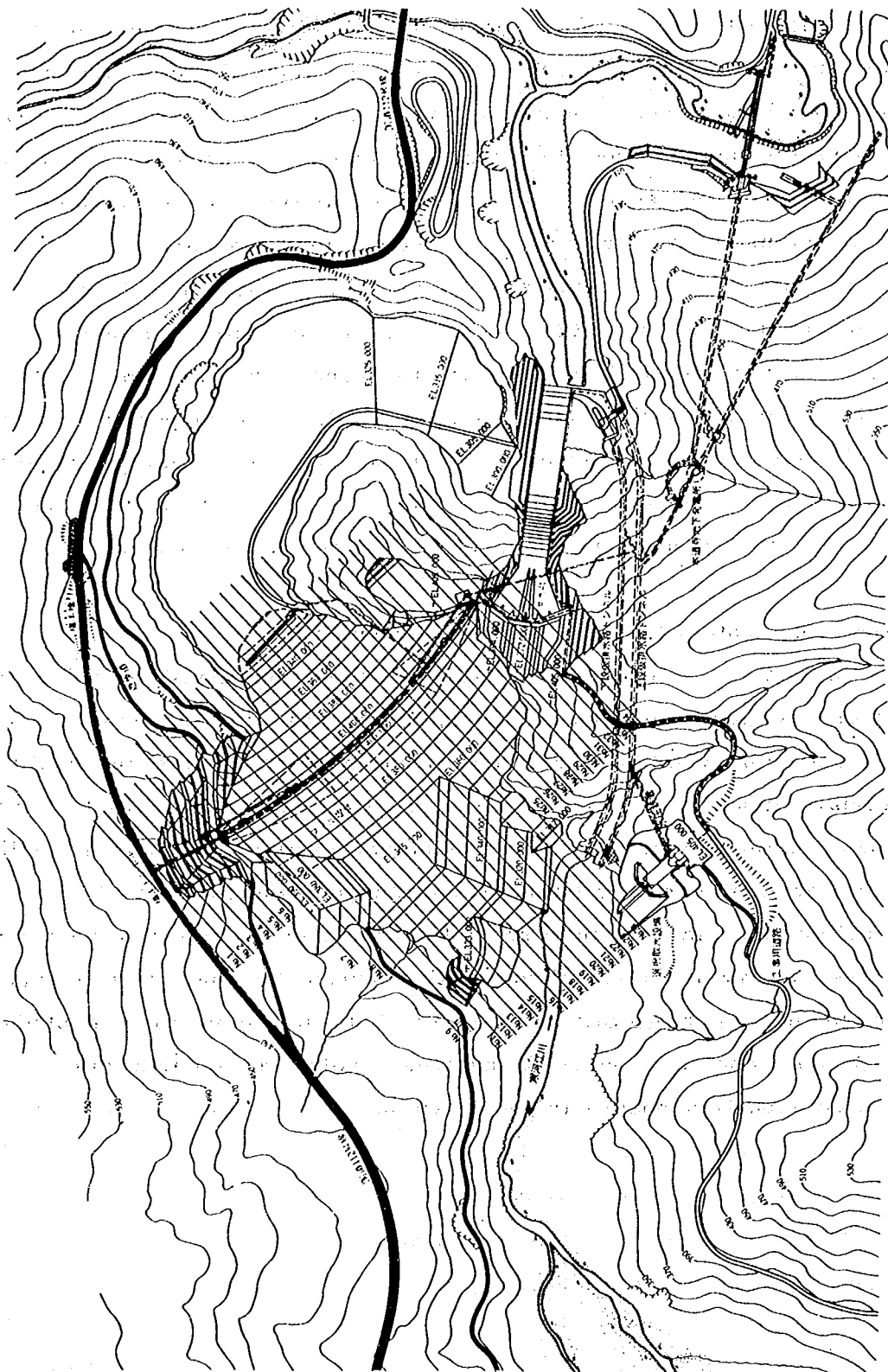
running straight in parallel with each other are desirable, but a protruding topography suited for spillway installation is sometimes found advantageous to the fill dam construction. For the arch dam construction, a topography becoming narrower toward lower reaches is desirable rather than a straight valley configuration,

We now proceed to the geological condition. The hard and sound bedrock is a favorable condition for all types, but the necessary conditions of foundation rock differ from type to type. The arch dam calls for a hard and sound bedrock at all elevations in addition to the above-mentioned topographic conditions by reason of its structural characteristics. The gravity dam varies the required conditions of bedrock according to the dam height. As the required bedrock characteristics lessens according to the decrease in the dam height, the gravity dam can be the second most adaptable type to the geological condition after the fill type.

The fill dam is adapted to all geological conditions. In certain cases, the fill dam can be built on a sandy gravel foundation or even an earth foundation. However, the zoned type dam should preferably be built on a firm bedrock if it is large in height, and its impervious core zone should be built on such sound bedrock enough to support a gravity dam.

c. Hydrology and flood discharge

In the case of fill dams, the outlet structures cannot be installed inside



Sagae Dam

Fig. 2 (b) Topography suitable for a fill type dam

or on the dam body, so that they must be built adjacent to or apart from the dam. The construction cost of the outlet structure varies largely according to the dam site topography and geology, but it is more directly influenced by the flood discharge and consequently by the required size of the spillway. Table 1 shows the design flood discharge and catchment area for fill dams and concrete dams. As seen in the table, the fill type is not selected so many on the mainstream of rivers with a large flood discharge.

Overflow during the fill dam construction causes the same degree of influence and damage to the downstream areas as one after completion. Accordingly, the diversion channel needed throughout the fill dam construction period is built for a probable flood discharge with a return period of about 20 years which is relatively large (a return period of one year is usually adopted for concrete dams). Thus, it is necessary to note that the construction of the diversion channel incurs a considerably high cost.

d. Construction materials and natural environment

Selection of the fill type presupposes that the voluminous embankement materials are available and can be obtained with ease in the vicinity of the dam site. Accordingly, it is necessary to confirm the volume and quality of such materials at an early stage of survey. It is also necessary to carry out suitable countermeasures to assure that borrowpits and quarries will maintain the original harmony with the surrounding environment after the material collection. In certain cases, it is required to select borrow-pits and quarries in careful consideration of the surrounding natural scenery. Thus, environmental preservation must always be borne in mind in planning the fill type dam construction as it involves a voluminous earthwork and requires a construction road with large width.

Table 1 Types of Dams and Design Flood Discharge

Name of dam	Name of river system	Catchment area (Km ²)	Design flood discharge (m ³ /sec)	Flood probability	Type of dam
Daisetsu	The Ishikari	291, 6	1, 000	1/100	Fill-type dam
Tokachi	The Tokachi	592, 0	1, 800	1/100	Fill-type dam
Isarigawa	The Ishikari	113, 3	600	1/100	Fill-type dam
Pirika	The Goshiribetsu	115, 0	1, 000	1/100	Fill-type dam
Shichigashuku	The Abukuma	236, 6	1, 750	1/150	Fill-type dam
Minase	The Omono	172, 0	1, 300	1/100	Fill-type dam
Shirakawa	The Mogami	205, 0	1, 400	1/100	Fill-type dam
Sagae	The Mogami	231, 0	2, 000	1/100	Fill-type dam
Sagurigawa	The Shinano	76, 2	1, 100	1/100	Fill-type dam
Tedorigawa	The Tedor	247, 9	2, 400	1/100	Fill-type dam
Kuzuryu	The Kuzuryu	Direct 184, 5 Indirect 117, 0	1, 500	1/80	Fill-type dam
Ryūmon	The Kikuchi	Dam 26, 5 The Kikuchi 91, 6 The Chikugo 154, 0	540	1/100	Fill-type dam
Fukuchi	The Fukuchi	32, 0	1, 000	1/50	Fill-type dam
Naramata	The Tone	Direct 60, 1 Indirect 35, 3	370	1/100	Fill-type dam
Urayama	The Arakawa	51, 6	1, 000	1/100	Fill-type dam
Misogawa	The Kiso	55, 1	650	1/100	Fill-type dam
Agigawa	The Kiso	81, 8	850	1/100	Fill-type dam
Tokuyama	The Kiso	254, 5	1, 920	1/100	Fill-type dam
Terauchi	The Chikugo	51, 0	300	1/150	Fill-type dam
Yuda	The Kitakami	583, 0	2, 200	1/57	Concrete dam
Sonohara	The Tone	Direct 493, 9 Indirect 113, 7	2, 350	1/100	Concrete dam
Ookawa	The Agano	825, 6	2, 600	1/100	Concrete dam
Nagashima	The Ooi	534, 3	6, 600	1/100	Concrete dam
Yahagi	The Yahagi	504, 5	2, 300	1/80	Concrete dam
Maruyama	The Kiso	2, 049, 0	6, 600	1/100	Concrete dam
Yokoyama	The Kiso	471, 0	2, 800	1/80	Concrete dam
Ohno	The Yura	354, 0	2, 400	1/60	Concrete dam
Oodo	The Niyodo	688, 9	6, 000	1/60	Concrete dam
Matsubara	The Chikugo	491, 0	2, 770	1/100	Concrete dam
Kawabegawa	The Kuma	470, 0	3, 520	1/80	Concrete dam
Tsuruta	The Sendai	805, 0	4, 600	1/100	Concrete dam
Takayama	The Yodo	615, 0	3, 400	1/80	Concrete dam
Ikeda	The Yoshino	1, 904, 0	11, 300	1/80	Concrete dam
Sameura	The Yoshino	Direct 417, 0 Indirect 45, 0	4, 700	1/80	Concrete dam

Source : "Multi-purpose Dams in Japan"

3. LOADS

All loads expected to work on the dam while it is functioning must be taken into consideration as design external forces according to the need. Tables 2 and 3 show the loads required to be estimated and incorporated in the design under the "Ordinance Concerning the Structures of River Facilities. The necessary loads are different from each others according to reservoir conditions and dam type. A brief explanation is given below on some of the listed loads which need to be given special attention.

Table 2 Loads Stipulated by the Dam Structure Regulations

(Loads acting on dam body)

Regulation Article 6 Listed below are the loads which will act on dam body and foundation ground, and they are classified on the bases of the types of dams and the water levels of reservoirs.

Types of dams Water level of reservoir	Gravity-type concrete dam	Arch-type concrete dam	Fill-type dam
1 When the water level in the direct upstream of non-overflow sec- tion of a dam is below the normal water level or the surcharge water level	W, P, Pe, I, Pd, U	W, P, Pe, I, Pd, U, T	W, P, I, Pp
2 When the water level in the direct upstream of non-overflow sec- tion of a dam is equal to the design flood water level	S, P, Pe, U	W, P, Pe, U, T	W, P, Pp

Remarks :

The symbols, W, P, Pe, I, Pd, U, Pp, and T, listed above represent the respective loads which are defined below.

W ; the dead weight of dam body

P ; the force exerted by the hydrostatic pressure of stored water

P_e ;	the force caused by the silt in the reservoir
I ;	the force of inertia of the dam body at the time of an earthquake
P_d ;	the force exerted by the dynamic water pressure of stored water at the time of an earthquake
U ;	the uplift pressure caused by stored water
P_p ;	porosity (water pressure caused by the seeping water inside a dam body and foundation ground)
T ;	the force exerted by temperature change inside a dam body

Table 3 Combination of Loads

Types of dams State of reservoir	Gravity-type concrete dam	Arch-type concrete dam	Fill-type dam
Normal water level and surcharge water level	Dead weight Hydrostatic pressure Dynamic water pressure at the time of an earthquake Ice pressure Silt pressure Force of inertia at the time of an earthquake Uplift pressure	Dead weight Hydrostatic pressure Dynamic water pressure at the time of an earthquake Ice pressure Silt pressure Force of inertia at the time of an earthquake Uplift pressure Temperature load	Dead weight Hydrostatic pressure Force of inertia at the time of an earthquake Porosity
Design flood water level	Dead weight Hydrostatic pressure Silt pressure Uplift pressure	Dead weight Hydrostatic pressure Silt pressure Uplift pressure Temperature load	Dead weight Hydrostatic pressure Porosity

Types of dams State of reservoir	Gravity-type concrete dam	Arch-type concrete dam	Fill-type dam
Lowest water level	Dead weight Force of inertia at the time of an earthquake Hydrostatic pressure Dynamic water pressure at the time of an earthquake Uplift pressure	Dead weight Force of inertia at the time of an earthquake Hydrostatic pressure Dynamic water pressure at the time of an earthquake Uplift pressure Temperature load	
Sudden lowering of water level			Dead weight Hydrostatic pressure Force of inertia at the time of an earthquake Porosity
Intermediate water level			Dead weight Hydrostatic pressure Force of inertia at the time of an earthquake Porosity
When emptied	Dead weight Force of inertia at the time of an earthquake	Dead weight Force of inertia at the time of an earthquake	
Immediately after completion			Dead weight Force of inertia at the time of an earthquake Porosity

* Listed below are the states of a reservoir, in which the seismic coefficient to be used for seismic force can be reduced to one

half (1/2) :

- 1 surcharge water level,
- 2 at the time of a sudden lowering of water level in a dam whose water level is not lowered as usual,
- 3 when the dam is emptied, and
- 4 immediately after completion of the dam.

* Intermediate water level means a level between the lowest and the Normal water level which is got by trial and error.

In the case of concrete dams, consideration must be given to the own weight of dam body, hydrostatic pressure, silt pressure, uplift, inertia force during earthquake, hydrodynamic pressure during earthquake, and temperature load (for arch dams only).

The silt pressure in the vertical direction is the gravity of mud in water, and the horizontal silt pressure is expressed by the following equation.

$$P_e = C_e W_i d \quad \dots\dots\dots (1)$$

where, P_e : Horizontal silt pressure (t/m^2)

C_e : Coefficient of silt pressure

W_i ; Unit weight of silt in water (t/m^3)

d ; Thickness of silt (m)

It is difficult to obtain the accurate values of C_e and W_i at the stage of design, so that $C_e = 0.6$ and $W_i = 1.0 t/m^3$ are generally adopted in the design of concrete dams.

The uplift works on the entire area of the section for reviewing the stability of concrete dams, and is considered to be a force imposed perpendicularly on that section. Observation of its distribution has been made quite frequently, the representative existing data is shown in Fig.3.

In the actual design, however, the uplift having the linear distribution shown in Table 4 is considered according to the presence or absence of the drain hole. As for the uplift inside the dam body, the distribution in the case where no drain hole is provided is applied. The short-time influences of waves or changes in the reservoir level need not be considered for uplift. The uplift statistics shows no differences whether the drain hole is provided or not. This may seem somewhat strange, but it is to be noted that only small dams built on hard bedrocks have no drain system and all dams constructed by the Ministry of Construction are usually provided with a drain system.

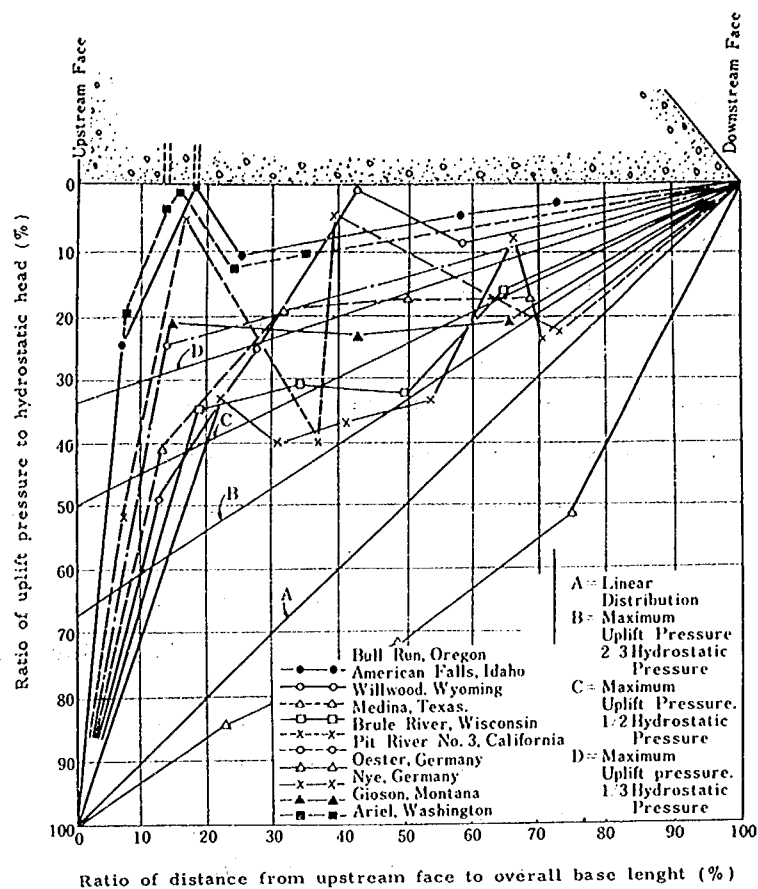


Fig. 3 Measurement example of uplift pressure

Table 4 Values of uplift pressure acting upon gravity-type concrete dam

Presence or absence of drain holes	Uplift pressure		
	Upstream end	Position of drain hole	Downstream end
When drain holes are provided	Water pressure on the side of upstream	The value obtained by adding more than one-fifth of the difference between upstream water pressure and downstream water pressure to the water pressure on the side of the downstream	Water pressure on the side of downstream
When drain holes are not provided	The value obtained by adding more than one-third of the difference between upstream and downstream water pressures to the water pressure on the side of downstream		Water pressure on the side of downstream

The inertia force during earthquake, which works horizontally on the dam body, is expressed by the following equation.

$$I = WK_d \quad \dots\dots\dots (2)$$

where, I : Inertia force of dam body during earthquake

W : Own weight of dam body

K_d : Design seismic coefficient

The most important seismic coefficient is determined with the values shown in Table 5 according to the foundation condition and dam type, as criteria for each zone shown in Fig. 4 according to the past seismic action,

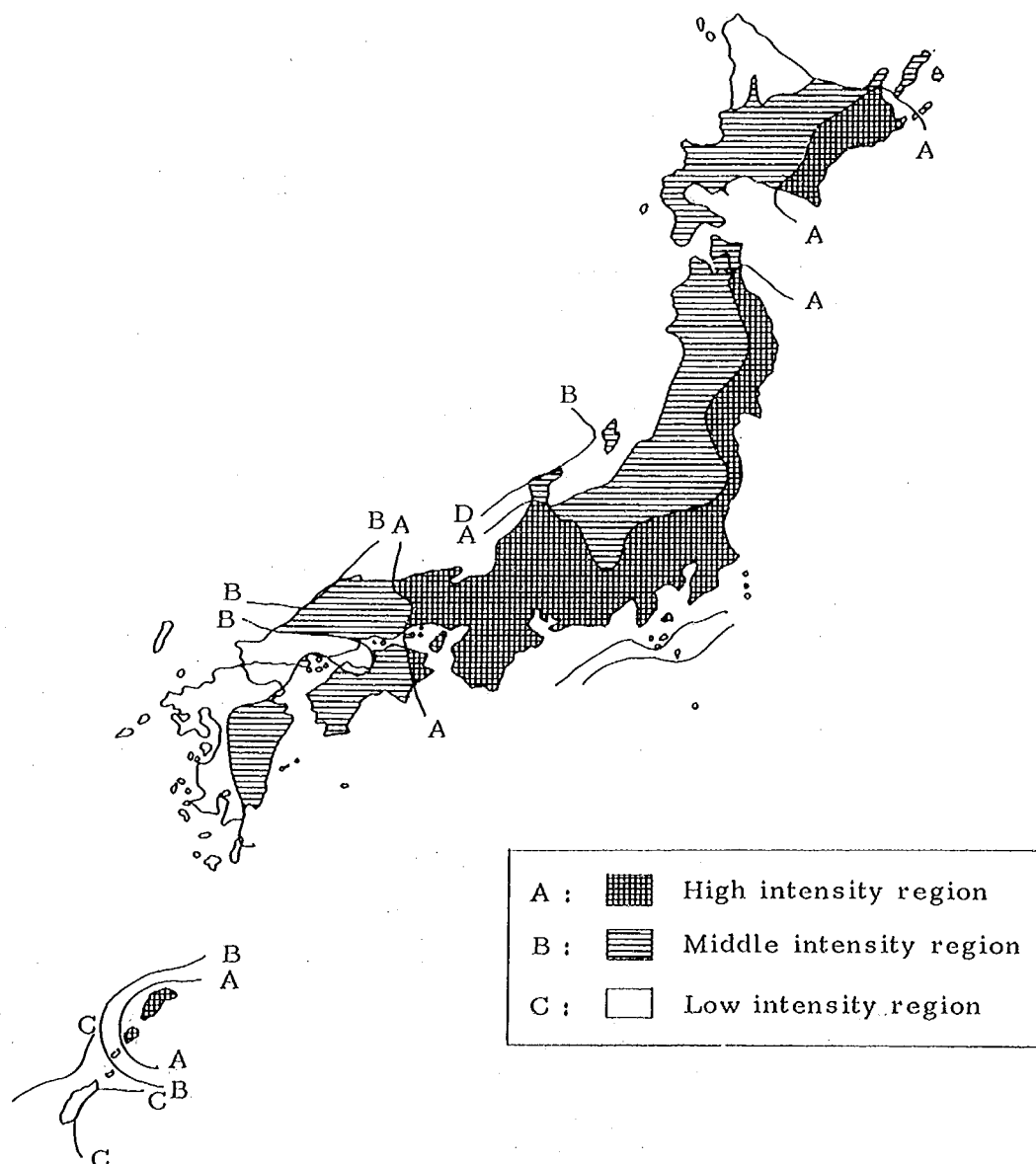


Fig. 4 Regional distribution on the bases of seismic action

Table 5 Design seismic intensity for dams

	Conditions of dam foundation	Gravity-type concrete dam and hollow gravity-type concrete dam	Arch-type concrete dam	Rock fill dam	Homogeneous type earth dam
Region A	Ordinary rock foundation	0,12 ~ 0,15	0,24 ~ 0,30	0,15	0,15 ~ 0,18
	Heavily weathered or creviced rock foundation, or geology after the new tertiary period and unconsolidated rock foundation	0,15	-	0,15 ~ 0,18	0,20 ~ 0,22
	Sandy gravel foundation Soil foundation	-	-	0,20	0,25
Region B	Ordinary rock foundation	0,12	0,24	0,12 ~ 0,15	0,15
	Heavily weathered or creviced rock foundation, or geology after the new tertiary period and unconsolidated rock foundation	0,15	-	0,15	0,18
	Sandy gravel foundation Soil foundation	-	-	0,18	0,22
Region C	Ordinary rock foundation	0,10 ~ 0,12	0,20 ~ 0,24	0,10 ~ 0,12	0,12
	Heavily weathered or creviced rock foundation, or geology after the new tertiary period and unconsolidated rock foundation	0,12	-	0,12	0,15
	Sandy gravel foundation Soil foundation	-	-	0,15	0,18

Source : "Technological Standards for River and Erosion Control Works (draft)" (incomplete)

The hydrodynamic pressure during earthquake is obtained by the application of Westergaard's formula or Zanger's empirical formula. Westergaard's formula is used when the upstream face is nearly vertical, whereas Zanger's formula is used when it has a mild gradient. Westergaard's formula, shown below, is also used when the upper half or more of the upstream face is nearly vertical.

$$P_d = 0.875 W_0 K_d \sqrt{H h} \quad \dots\dots\dots (3)$$

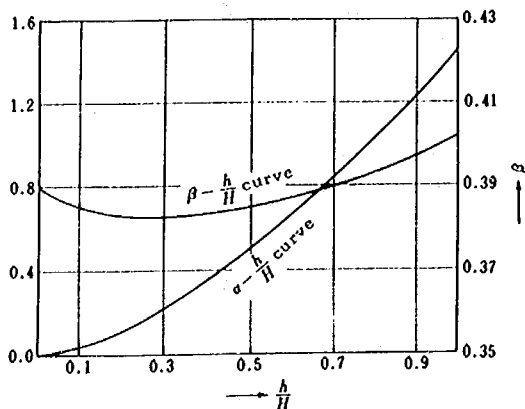
where, P : Hydrodynamic pressure (t/m²)
W : Unit weight of water (t/m³)
k : Design seismic coefficient
H : Water depth from the reservoir water surface to the foundation (m)
h : Water depth from the reservoir water surface to the optional working point of hydrodynamic pressure

Zanger's formula is as shown below.

$$P_d = C W K H \quad \dots\dots\dots (4)$$

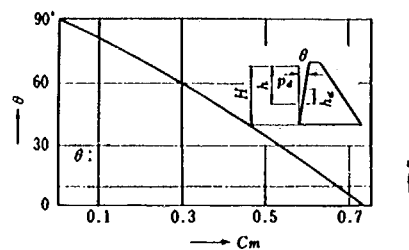
$$\text{where } C = \frac{C_m}{2} \left[\frac{h}{H} \left(2 - \frac{h}{H} \right) + \sqrt{\frac{h}{H} \left(2 - \frac{h}{H} \right)} \right]$$

C_m indicates the value of C when P_d reaches its peak. As for the total hydrodynamic pressure (ΣP_d) and the height from the foundation of its working point (h_d), ΣP_d = 0.583 W₀ K H² and h_d = 0.4 H according to Westergaard's formula and ΣP_d = α $\frac{C_m}{2}$ · W₀ K H² sec θ and h_d = β h according to Zanger's formula. α and β are obtained from Fig. 5.1 and C_m from Fig. 5.2.



(1) and α and β curve

Angle of slope to the vertical axis of dam upstream face



(2) θ - curve

Fig. 5 Dynamic water pressure

The temperature load need not be considered in the design of gravity dams but it must be taken into account for arch dams. It is obtained by estimating the change in the internal temperature expected to take place after grouting the contraction joints. This is because the temperature drop leads concrete to the tensile stress side. In general, the temperature rise is conservative because it is on the compression side, but it tends to increase the thrust and this needs to be examined whenever necessary.

4. DESIGN OF GRAVITY DAMS

4.1 Introduction

The design of gravity dams can be classified into the most suitable site selection work, the basic design for determining the dam body gradient on the upstream and downstream sides, the detailed design of miscellaneous appurtenant structures and the design of outlet structures and stilling basin. This chapter deals mainly with the basic design of a dam body with some complementary explanation of other necessary items.

The site selection is very important to the optimum design of gravity dams. The optimum design means, as explained in Chapter 1, a design that makes the gravity dam construction most economical, so that the site selection must be worked out not only to minimize the concrete volume but also on the basis of a comprehensive judgement of the layout of temporary facilities, arrangement of stilling basin, foundation treatment and other factors. However, the process of studies for selecting a damsite is difficult to explain in a systematic and plain manner because the damsite conditions vary from place to place. The following are the general cautions to be paid during determining location of a dam.

It is advisable to divide all elements affecting the work into two groups for a comparative study, that is, those which are difficult to solve or incur a large amount of cost for solution and those which can be solved with some endeavors and expenditure. Table 6 shows the elements to be considered in selecting a damsite. The designer should determine which of the two groups each element belongs to, and work out a rational design after careful consideration of the advantages and disadvantages resulting from the solution or exclusion of each element.

Table 6 Considerations when determining a damsite

Topography	Valley width, raised ridge top, landslide topography, long slope caused by excavation
Geology	Fault, pervious zone, shape and form of hard rock line (plane shape and form, downstream down, downstream up, etc.)
Spillway, Scour protection stilling basin (pool)	Positioning, alignment of scour protection stilling basin
Construction equipment	Positioning of concrete placing equipment (for cable crane, valley width and the foundation of the fixed and moving parts)

4.2 Study of Bedrock

Before proceeding to the dam design, we need to study the bedrock that supports the dam. In the design of dams, the bedrock should be considered in terms of its strength, deformation and cutoff performance. When viewed from the hardness of rock that bears upon its strength, we find that the bedrock presents varying phases ranging from the compact Palaeozoic sandstone to the unsolidified Neogene sedimentary rock.

If a fragment is sampled from this mass of bedrock and subjected to the same strength and deformation tests as conducted on concrete, it will be found to be superior to concrete in almost all cases. The bedrock can be considered as a mass of uninterrupted chain of rock fragments, but this is not a thorough understanding because it has the following two characteristics not found in concrete.

- 1 It contains discontinuous planes such as cracks and joints.
- 2 It is not homogeneous like concrete, and often involves faults, fractured zones and alteration zones which are extremely weak in strength and deformation.

For this reason, the bedrock is far inferior to rock fragments in strength, deformation and cutoff performance. As a supporting base of the dam, the bedrock should be considered as a discontinuous body.

Although the bedrock has discontinuous planes and heterogeneous portions, it is sometimes found to be superior to concrete in strength and deformation. If a dam is to be built on such a hard bedrock, its design is very easy as it needs to be worked out only for the dam body.

In general, however, the bedrock is inferior to concrete in strength, deformation and cutoff performance, and this is particularly so in Japan where all well-conditioned damsites have already developed and it is now found inevitable to built dams in geologically poor-conditioned places. In cases like this, it is necessary to examine, in addition to working out the design of the dam body, whether or not the bedrock itself will break due to the forces transmitted from the dam body and if it will undergo an extreme deformation and break the dam body. For this reason, it is often required to design the dam body not only to resist the water pressure but also to fill the conditions for assuring the safety of the bedrock. Thus, the design of dams is largely influenced by the characteristics of the bedrock on which they are to be built.

The bedrock strength and deformation bear closely upon the safety of dams as explained above. How, then, are these two Characteristics estimated? Generally, the bedrock strength is expressed by its shearing strength and its deformation by its elastic modulus (or deformation modulus). The bedrock strength is expressed by its shearing strength because the forces transmitted from the dam body is resisted mainly by the shearing resistance and its rupture is a shearing rupture.

The first problem in determining the bedrock strength and deformation is how to evaluate, in a systematic manner, the bedrock characteristics that vary from place to place. The bedrock has an area of thousands of square meters only in the base to contact directly with the dam, and a volume of several million cubic meters only within the reach of stress. Direct evaluation of whole bedrock characteristics is thus impossible, and this has given rise to the concept of bedrock classification. The method of bedrock evaluation and classification generally employed in Japan is explained below.

At the stage when the geological structure has been grasped and a geological map has been prepared with reasonable accuracy, the bedrock is broadly grouped into 4~6 classes according to the elements considered to govern the bedrock strength and deformation. It is difficult to select such governing elements adequately so that in actuality, the rock hardness, frequency of cracks, and condition of cracks are usually used as such elements. Table 7 shows the bedrock classification standard prepared at the construction site of Kawaji Dam (arch dam) under the supervision of Okamoto.

The bedrock evaluation work starts with the preparation of this classification standard at the dam site. Then, the bedrock is classified at intervals of 1 m on the basis of this standard by geological observation of exploring tunnels and boring cores. Upon completion of this work, the bedrock is classified along a line (tunnels and bore holes constitute a line in the extensive area of dam site). On the basis of this line, the geologists estimate the invisible inside characteristics of bedrock making use of their expert knowledge and experience as well as available geological maps. Needless to say, the distribution and continuity of faults and the direction of bedding serve as important data for estimating the bedrock class at this time. The bedrock in the entire dam site area is thus classified and the bedrock classification map is prepared.

At the next stage, the in-situ test is conducted to determine the strength and deformation for each bedrock class. The test is conducted for 3 - 5

Table 7 Classification criteria for bedrock (Kawaji Dam)

Character of rock	Symbols	Foundation	Remarks
Hardness of rock	A	New and hard	Sparks occur when struck with a hammer.
	B	Partly soft because of partial weathering	Cracks are caused when struck a heavy blow with a hammer.
	C	Completely weathered and softened	Hammer's pick can dig.
Gap of crack	I	30 cm or more	
	II	10 to 30 cm	
	III	10 cm or less	
State of crack	a	Adhered closedly	
	b	Opening	
	c	Filled with clay	

Overall judgement		Reclassified combinations						
A		A1a						
B		A1b	A1c	A11a	B1a			
C	H	A11b	A111a	B1b	B11a			
	M	A11c	B1c	B11b	B111a	C1a		
	L	A111b	A111c	B11c	C1b	C1c	C11a	
D		B111b	B111c	C11b	C11c	C111a	C111b	C111c

samples for both strength and deformation, and the average is adopted for each bedrock class. If the strength and deformation of the same bedrock class show a large dispersion, it indicates that the governing elements selected for bedrock classification are inadequate, so that re-examination of the governing elements becomes necessary. It is known, however, that the bedrock classification has been conducted successfully in most cases using the aforementioned elements.

We now proceed to the method of in-situ shearing test and deformation test.

Fig.6 shows the loading plate used in the shearing test in which the vertical stress working on the bedrock is varied by changing the vertical load to obtain the corresponding breaking strength τ .

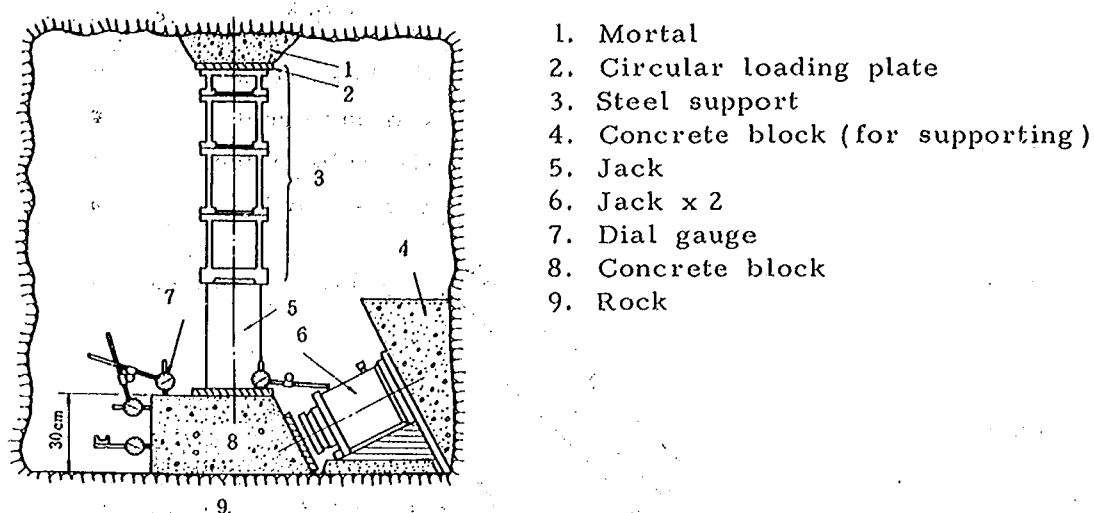


Fig. 6 Shearing testing apparatus

The shearing strength at this time, τ is expressed as follows by Coulomb.

$$\tau = \tau_0 + f \sigma \dots\dots\dots (5)$$

where, τ_0 ; Cohesion (or initial shearing strength)

f ; Coefficient of internal friction

Fig.7 shows the results of the shearing test conducted for each bedrock class at the Ohishi Dam site. The results shown were successful.

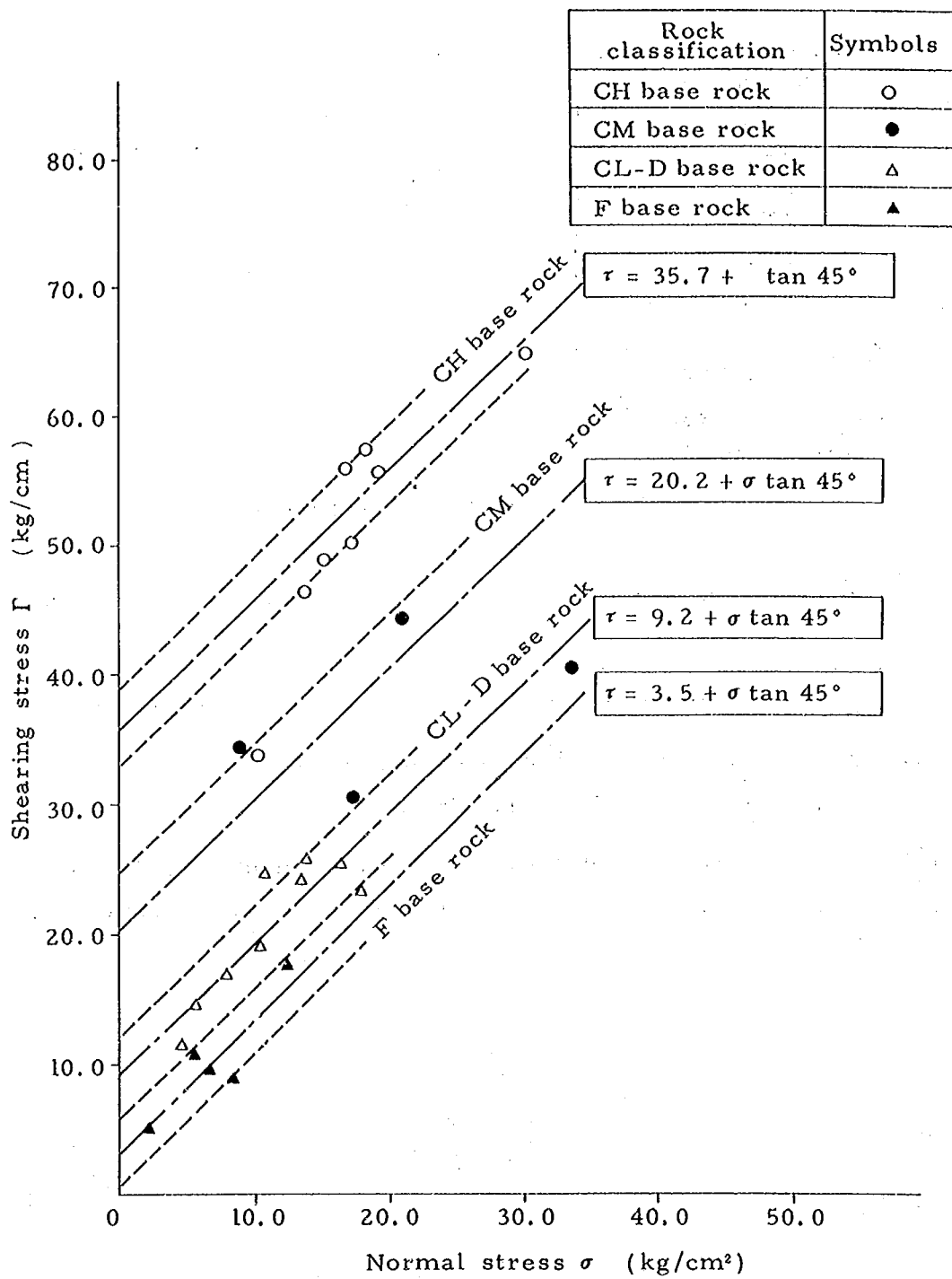


Fig. 7 Results of shearing test at Ohishi Dam

Fig.8 shows the deformation test equipment. In this test, the load is imposed on the bedrock using a rigid loading plate such as the uniform displacement occurs and the elastic modulus E is obtained from the following equation using the relationship between the load and the displacement as solution of Boussinesq's semi-infinite body.

No.	
1	Bearing section
2	Bearing plate
3	Spherical seat
4	Bearing column
5	Joint
6	Hydraulic jack
7	Load pedestal
8	Test foundation
9	Dial gauge
10	Base rock
11	Standard beam
12	Standard beam regulating abutment
13	Pressure-tight rubber hose
14	Pressure gauge
15	Oil pump

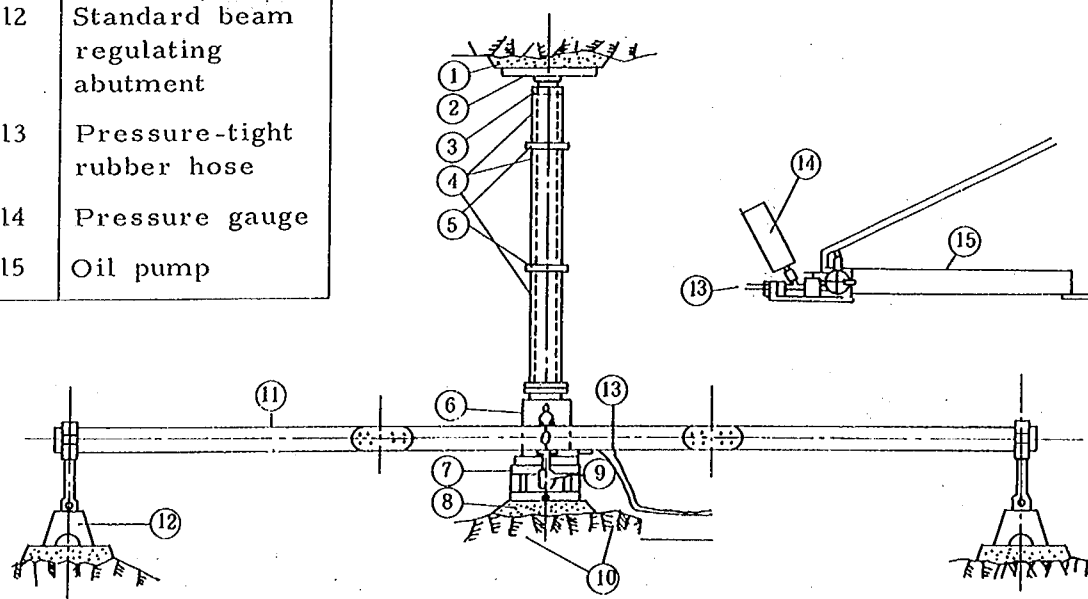


Fig. 8 Deformation testing apparatus

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obtained by the above process are indispensable for the analysis of bedrock stability and internal stress. In the design of dams of great importance or those to be built in geologically unfavorable places, tests should be conducted to obtain these design values.

4.3 Requirements for stability of Gravity Dams

In the "Design Criteria for Dams" established by the Japanese Committee of Large Dams, the following requirements are stipulated for ensuring the stability of gravity dams.

- (1) On the upstream side, no tensile stress should be generated within the vertical zone perpendicular to the dam upstream slope.
- (2) Safety from shearing or sliding.
- (3) Compressive stress and tensile stress should be smaller than the maximum allowable value.

The first requirement listed above (no vertical tensile stress on the upstream face) is enforced to design the dam shape so that the resultant of all working load will be imposed on the core of the estimated section. It is known well as the Theory of Middle Third, and is a little more severe than the stability condition for preventing fall down by assuring that the resultant will come within the estimated section. This condition is imposed because if tensile stress is generated on the upstream face, the bedrock is liable to become loose, causing the reservoir water to infiltrate into it and raise the uplift. This causes changes in the initial conditions considered at the stage of design and is therefore dangerous.

The second requirement is expressed by Henny's formula which indicates that the safety factor for shear sliding should be 4 or larger.

Henny's formula is as follows,

$$n = \frac{fV + \tau_0 l}{H} \dots\dots\dots (7)$$

where, n ; Safety factor for shear sliding

f ; Coefficient of internal friction

V ; Vertical force (including uplift) working on unit width of of shearing surface (t/m)

τ_0 ; Shearing strength of bedrock, concrete, or their contact surface, whichever is the smallest (t/m)

l ; Length considered for shearing resistance (m)

H : Shearing force per unit width (including that during earthquake) (t/m)

The third requirement (compressive and tensile stress should not exceed the limiting value) is out of the question in the design of dam section unless the dam height exceeds 150 m.

Thus, in the design of gravity dams, it is required to fill only the first two conditions which are

- a. No tensile stress is generated on the upstream face
- b. Required safety factor for shear sliding is provided

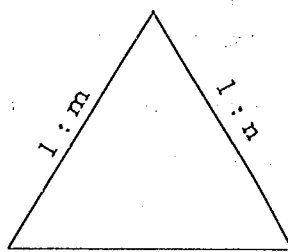
Recent years have seen increasing difficulty in constructing gravity dams in geologically unfavorable places, and this means that it has become more and more difficult to fill the latter condition,

4.4 Standard Design Method of A Gravity Dam

a. general

In the standard design of gravity dams, it is common to employ the method of fundamental triangle having a unit width (m) in the direction of dam axis. This is a two-dimensional method, and the three-dimensional method is very rare in Japan.

To design this triangular section means to determine the combination of its upstream slope (m) and downstream slope (n) to make them most economical (smallest) according to the design requirements mentioned in the preceding section.



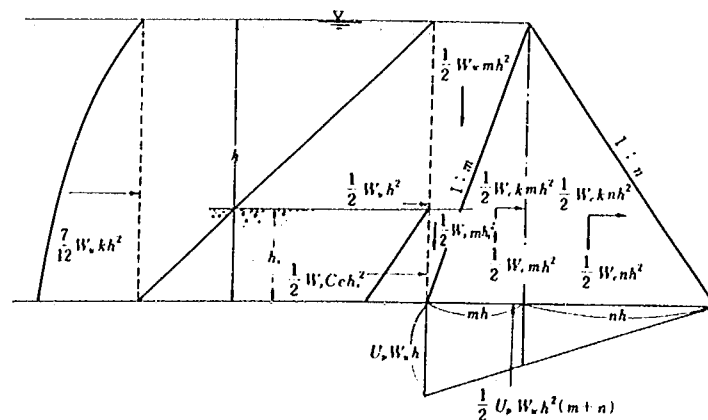
Fundamental Triangle

On the basis of this triangle, corrections are effected to the additional weight to be imposed by roads, handrail, gates, etc. and to the hollow portions such as conduit pipes, galleries, etc. and the safety of the final dam section is checked to complete the design.

In the following explanation, therefore, the process of studies to determine the upstream and downstream slopes of the fundamental triangle is

dealt with at first, with an example of construction design shown to help understand the actual process of design. The design discussed in this section is called "standard design" because the explanation is given on the case where the upper structure is designed with the bedrock condition expressed simply by the average shearing strength. The design of the upper structure to be built on a unfavourable bedrock will be explained in Section 4.6.

Loads working on the gravity dam are the own weight of dam body, hydrostatic pressure, uplift pressure, silt pressure, seismic force and ice pressure, of which the ice pressure is disregarded here as it need not be considered in most cases. The dam should be designed so as to be able to fill the most exacting condition shown in Table 3. The condition considered here is the case where seismic force and hydrodynamic pressure work fully and simultaneously, but as described earlier, other cases of load combination must be checked in the actual design.. It is to be noted that if the normal water level is very low, the severest load condition is not necessarily this case, but the surcharge water level plus 50% of seismic force occasionally become the severest load condition to be considered in the design of the fundamental triangle. Each external force imposed as load and its moment is shown below.



External Force and its Moment

Explanation of symbols

- h ; Dam height, reservoir water level
- hs ; Silt level
- m, n ; Upstream and downstream slopes
- Wc ; Unit weight of concrete

W_w ; Unit weight of water
 W_s ; Unit weight of silt in water
 k ; Dam body seismic coefficient
 U_p ; Coefficient of uplift pressure
 C_e ; Coefficient of silt pressure

Hydrodynamic pressure is expressed by the following equation.

$$P_o = \frac{7}{8} W_w k \sqrt{Hh} \quad (\text{Westergraad's formula of approximation})$$

Table 8 External Force and Moment Caused by External Force

	Load	Symbol	External force	Distance from upstream face to working point	Moment cause by external force
Horizontal force	Water pressure	H_w	$\frac{1}{2} W_w h^2$	$\frac{1}{3} h$	$\frac{1}{6} W_w h^3$
	Silt pressure	H_s	$\frac{1}{2} W_s C_e h^2$	$\frac{1}{3} h_s$	$\frac{1}{6} W_s C_e h_s^3$
		H_c	$\frac{1}{2} (m+n) W_c kh^2$	$\frac{1}{3} h$	$\frac{1}{6} (m+n) W_c kh^3$
	Hydrodynamic pressure	H_{kw}	$\frac{7}{12} W_w kh^2$	$\frac{2}{5} h$	$\frac{7}{30} W_w kh^3$
	$\Sigma H_i = \frac{1}{2} W_w h^2 + \frac{1}{2} W_s C_e h_s^2 + \frac{1}{2} (m+n) W_c kh^2 + \frac{7}{12} W_w kh^2$ $\Sigma M_i = \frac{1}{6} W_w h^3 + \frac{1}{6} W_s C_e h_s^3 + \frac{1}{6} (m+n) W_c kh^3 + \frac{7}{30} W_w kh^3$				
Vertical force	Weight of water imposed on upstream face	V_w	$\frac{1}{2} m W_w h^2$	$\frac{1}{3} mh$	$\frac{1}{6} m^2 W_w h^3$
	Silt weight	V_s	$\frac{1}{2} m W_s h_s^2$	$\frac{1}{3} mh_s$	$\frac{1}{6} m^2 W_s h_s^3$
	Own weight of dam body	V_c	$\frac{1}{2} (m+n) W_c h^2$	$\frac{2m+n}{3} h$	$\frac{1}{6} (m+n)(2m+n) W_c h^3$
	Uplift pressure	V_u	$-\frac{1}{2} (m+n) U_p W_w h^2$	$\frac{1}{3} (m+n) h$	$-\frac{1}{6} (m+n)^2 U_p W_w h^3$
	$\Sigma V_i = \frac{1}{2} m W_w h^2 + \frac{1}{2} m W_s h_s^2 + \frac{1}{2} (m+n) W_c h^2 - \frac{1}{2} (m+n) U_p W_w h^2$ $\Sigma M_i = \frac{1}{6} m^2 W_w h^3 + \frac{1}{6} m^2 W_s h_s^3 + \frac{1}{6} (m+n)(2m+n) W_c h^3 - \frac{1}{6} (m+n)^2 U_p W_w h^3$				

The process of determining the upstream and downstream slopes of the fundamental triangle is explained according to each design condition.

- b. Requirement for focussing resultant of external forces in middle third of horizontal section

The requirement must be filled in order not to generate vertical tensile stress on the upstream side, but the dam is also required to be resistant to seismic force working in the upstream direction when the reservoir is empty. It is to be noted that concrete dams are generally not very resistant to external forces working in the reverse (upstream) direction.

- 1) In case the dam is subjected to a seismic force working in the upstream direction during its construction or immediately after its completion.

$$x_0 \leq \frac{2}{3}(m+n)h$$

$$x_0 = \frac{\text{Moment working on downstream side when reservoir is empty}}{V}$$

Since the external forces to be considered here are the inertia force during earthquake and the own weight of dam body, we may rewrite the above equation as

$$x_0 = \frac{\frac{1}{6}(m+n)W_c kh^3 + \frac{1}{6}(m+n)(m+2n)W_c h^2}{\frac{1}{2}(m+n)W_c h^2}$$

$$= \frac{1}{3}(k+m+2n)h$$

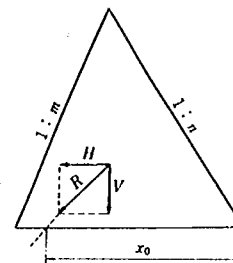
Therefore, it follows that

$$2(m+n) - (k+m+2n) \geq 0$$

$$m \geq k \dots\dots\dots (8)$$

As seen above, the upstream slope, m, can be taken at a value equivalent to the seismic coefficient, k. In the Design Standard, the seismic coefficient is required to be 50% of the normal value for the case when the reservoir is empty.

In actuality, generation of vertical stress or a minor degree of bedrock loosening on the downstream side is permissible because it does incur an increase of uplift as it would on the upstream side. Under the latter condition, the most economical section can be obtained by making the upstream face perpendicular or slightly inclined (about 0.05). For this reason and for the ease of construction work, the upstream face is often designed to be perpendicular,



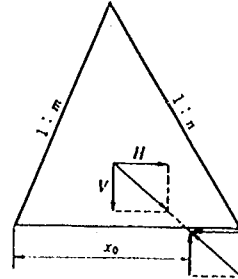
- 2) Requirement for not causing tensile stress on the upstream side when hydrodynamic pressure and seismic force work at the firm power water level.

The stability condition in this case is also expressed by the following equation.

$$x_0 \leq \frac{2}{3}(m+n)h$$

$$x_0 = \frac{\text{Sum total of moments due to horizontal and vertical forces}}{V}$$

Substitution of the external forces and moments shown in Table 8 into the above equation and multiplication of both members by $\frac{6}{h^3}$ leads to the equation,



$$(m+n)^2 (W_c - W_u U_p) - (m+n) \left[\left\{ W_c - 2W_u - 2W_s \left(\frac{h_1}{h} \right)^2 \right\} m + W_c k \right] - \left[\left\{ W_s \left(\frac{h_1}{h} \right)^3 + W_u \right\} m^2 + W_u + W_s C_c \left(\frac{h_1}{h} \right)^3 + \frac{7}{5} W_u k \right] \geq 0 \quad \dots\dots\dots (9)$$

By rewriting the above equation as

$$A(m+n)^2 - B(m+n) - C \geq 0$$

we obtain

$$m+n \geq \frac{B + \sqrt{B^2 + 4AC}}{2A} \quad \dots\dots\dots (10)$$

where, $A = W_c - W_u U_p$

$$B = \left\{ W_c - 2W_u - 2W_s \left(\frac{h_1}{h} \right)^2 \right\} m + W_c k$$

$$C = \left\{ W_s \left(\frac{h_1}{h} \right)^3 + W_u \right\} m^2 + W_u + W_s C_c \left(\frac{h_1}{h} \right)^3 + \frac{7}{5} W_u k$$

Removing the silt pressure and the hydrodynamic pressure for simplicity produces the following equation.

$$m+n \geq \frac{1}{2(W_c - W_u U_p)} \left[(W_c - 2W_u)m + W_c k + \left\{ (W_c^2 + 4W_u^2 - 4U_p W_u^2)m^2 + 2W_c(W_c - 2W_u)km + 4W_u(W_c - U_p W_u) + W_c^2 k^2 \right\} \right] \quad \dots\dots\dots (11)$$

To obtain the most economical section means to seek the least possible value of $m+n$. In equation (10), the values of all factors excluding m are known for the dam to be designed. This means that $m+n$ can be obtained by determining its only variable, m .

Accordingly, it is necessary to obtain the value of m which makes the value of $m+n$ (Eq. 10 shown above) the smallest.

Since $2A$ is a constant, it follows that

$$f(m) = B + \sqrt{B^2 + 4AC}$$

$$= \alpha_1 m + \alpha_2 + \sqrt{(\alpha_1 m + \alpha_2)^2 + 4A(\alpha_3 m^2 + \alpha_4)} \dots\dots\dots (12)$$

where, $\alpha_1 = W_c - 2W_u - 2W_s \left(\frac{h_t}{h}\right)^2$

$$\alpha_2 = W_c k$$

$$\alpha_3 = W_s \left(\frac{h_t}{h}\right)^3 + W_u$$

$$\alpha_4 = W_u + W_s C_c \left(\frac{h_t}{h}\right)^3 + \frac{7}{5} W_u k$$

$$A = W_c - W_u U_p$$

$$\frac{df}{dm} = \alpha_1 + \frac{\alpha_1(\alpha_1 m + \alpha_2) + 4A\alpha_3 m}{\sqrt{(\alpha_1 m + \alpha_2)^2 + 4A(\alpha_3 m^2 + \alpha_4)}} \dots\dots\dots (13)$$

α_1 , α_2 , α_3 and A are self-explanatory plus from their respective expressions.

When $\alpha_1 \geq 0$, $\frac{df}{dm}$ assumes a plus value in the region where $m \geq 0$, so that f makes a monotonous increase. This means that if m is to be taken at a plus value ($m < 0$ is not adopted in consideration of an earthquake in the upstream direction when the reservoir is empty), the most economical section is the one having a perpendicular upstream face ($m = 0$).

when $\alpha_1 < 0$ and for example in the case where $m = 0$, $\frac{df}{dm}$ becomes

$$\alpha_1 + \frac{\alpha_1 \alpha_2}{\sqrt{\alpha_2^2 + 4A\alpha_4}}, \text{ so that } \frac{df}{dm} < 0.$$

If m is taken at a sufficiently large value, $\frac{df}{dm} > 0$, so that f assumes the minimum value in the region where $m \geq 0$. This m , expressed by the following equation, is the upstream slope that gives the most economical section.

$$m = \frac{-\alpha_1 \alpha_2 \alpha_3 + \sqrt{(\alpha_1 \alpha_2 \alpha_3)^2 + (\alpha_1^2 \alpha_3 + 4A\alpha_3^2) \alpha_2^2 \alpha_4}}{\alpha_1^2 \alpha_3 + 4A\alpha_3^2} \dots\dots\dots (14)$$

In certain cases, the most economical section is given by m which is not zero (does not make the upstream face perpendicular). By some further consideration of this fact, it can be seen that

when $\alpha_1 < 0$, $\left(\frac{h_t}{h}\right)^2 > 0.15$ and $\frac{h_t}{h} > 0.387$, provided that

$$W_c - 2W_u - 2W_s \left(\frac{h_t}{h}\right)^2 < 0, \quad W_c = 2.3t/m^3, \quad W_u = 1.0t/m^3, \quad W_s = 1.0t/m^3$$

In other words, the situation mentioned above could occur in case the silt depth is relatively large in comparison with the reservoir water

depth,

When $h_s/h = 0,50$ for example, the upstream slope that produces the most economical section is $m = 0,07$. Even in such cases, however, it is quite possible that the section is designed with $m = 0$ for the ease of construction work because the difference between the two values is very small. Fig. 10 shows the upstream slope that produces the most economical section and the sum total of m and n ,

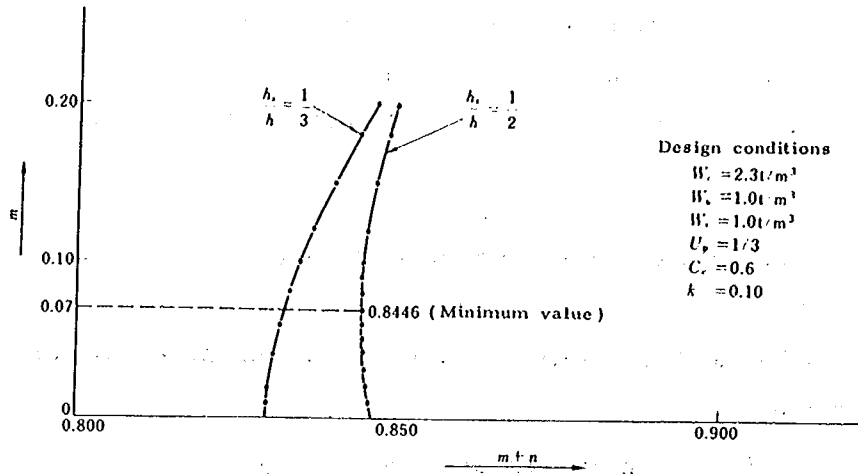


Fig. 10 Upstream slope (m) producing most economical section and sum total upstream and downstream slopes ($m+n$)

The above study was made for the base which is the largest section, and we now proceed to upper sections. Of all load terms in Eq. 9, only the silt pressure and the hydrodynamic pressure change on other sections than the largest section. The silt pressure becomes zero above the silt surface, and the hydrodynamic pressure also changes as its load intensity is expressed as

$$P_0 = \frac{7}{8} W_s k \sqrt{Hh}$$

Eq. (11) produced by removing the silt pressure and the hydrodynamic pressure from Eq. (9) includes no h term. This means that when these two loads are removed, the fundamental triangular section filling the condition of Middle Third is determined not by the dam height but by the upstream and downstream slopes. Strictly speaking, the two loads cannot be disregarded as explained earlier, so that at the lateral cantilevers this design becomes somewhat conservative. But since the difference is relatively small, it is often done to employ the upstream and downstream slopes of the largest section for all cantilevers.

c, Requirement for providing sufficient safety factor for sliding ($F_s \geq 4$)

The section of gravity dams recently been determined by this condition in many cases due to the increasing dam height and weak bedrocks. In this case, Henny's formula shown below is applied as explained in Section 4.3.

$$F_s = \frac{\tau_0 l + fV}{H} \geq 4$$

where, τ_0, f ; Initial shearing strength and coefficient of internal friction of dam body material or bedrock

l ; Length of shearing surface

H, V ; Total horizontal and vertical force working on sliding surface

The above equation can be rewritten as follows,

$$4H \leq \tau_0 l + fV$$

Substitution of H and V shown in Table 8 for consolidation of $(m+n)$ leads to the equation

$$\begin{aligned} (m+n) \left(\frac{2\tau_0}{h} + W_c f - W_w U_p f - 4W_c k \right) + mf \left\{ W_u + W_s \left(\frac{h_s}{h} \right)^2 \right\} \\ - \left\{ 4W_w + 4W_s C_r \left(\frac{h_s}{h} \right)^2 + \frac{14}{3} W_w k \right\} \geq 0 \\ m+n \geq \frac{4W_w + 4W_s C_r \left(\frac{h_s}{h} \right)^2 + \frac{14}{3} W_w k - mf \left\{ W_u + W_s \left(\frac{h_s}{h} \right)^2 \right\}}{\frac{2\tau_0}{h} + W_c f - W_w U_p f - 4W_c k} \dots\dots\dots (16) \end{aligned}$$

Removing the silt pressure and the hydrodynamic pressure for simplicity produces the following equation

$$m+n \geq \frac{4W_w - mfW_u}{\frac{2\tau_0}{h} + W_c f - W_w U_p f - 4W_c k} \dots\dots\dots (17)$$

Eq. (16) produced for consolidation of $(m+n)$ is a simple equation of m , so that $(m+n)$ can be obtained by determining m as seen in the equation of Middle Third. The coefficient working on m is $-f \left\{ W_u + W_s \left(\frac{h_s}{h} \right)^2 \right\}$ which assumes a minus value. This means that it is advantageous to determine m at a large value, because the milder the upstream slope, the greater and more conservative become the upstream side water volume and silt weight.

$\left(\frac{h_s}{h} \right)$ included in the numerator is not determined by the dam height because it is only the ratio of silt depth to dam height. On the other hand, $\frac{2}{h}$ occupies a large portion of the denominator and decreases with the increase of dam height (h). In other words, if τ_0 is constant, $(m+n)$ must be increased according to increasing dam height because of this condition. $(m+n)$ determined by the condition of Middle Third is hardly influenced by

dam height, which means that this condition becomes more and more influenced by dam height, which means that this condition becomes more and more influential with the increase of dam height. Conversely speaking, if an economical section is to be obtained even if the dam height to increased, the bedrock strength must be correspondingly large. Fig.11 shows the shearing strength required according to the dam height to provide a safety factor of 4 for sliding for the dam section determined by the condition of Middle Third. As seen in the figure, the bedrock is required to have a shearing strength of more than 250 t/m² for construction of a dam having a height of 100 m. However, it is very rare to expect such a great strength from the bedrock, so that the actual design section is so determined as will fill the conditions of Middle Third and of sliding at the same time, it will be most economical. These relationships are shown in Fig.12(a) in which $\frac{\tau_0}{h}$ is taken as parameter. For example, if dam height = 100 m, bedrock shearing strength = 200 t/m², and $f = 1,0$, $\tau_0/h = 2$, so that the most economical section satisfying both conditions is the one having $m = 0,4$ and $m+n = 0,9$ which can be obtained from the intersection of the two conditions and consequently, $n = 0,5$.

But in case of bad and weak bedrock, it is not proper to base the design only on the fundamental triangular section. As explained later, various research activities are conducted for design in such cases.

It is to be noted that the fundamental triangular section can be designed to determine the upstream and downstream slopes filling the both conditions only when it shows little difference from the section determined by the condition of Middle Third. It is also to be noted that Fig.12(b) shows that if $\tau_0 = 250$ t/m², the two conditions do not meet within the region where $m > 0$ and therefore, the section determined by the condition of Middle Third automatically fills the requirement for safety factor for shear sliding.

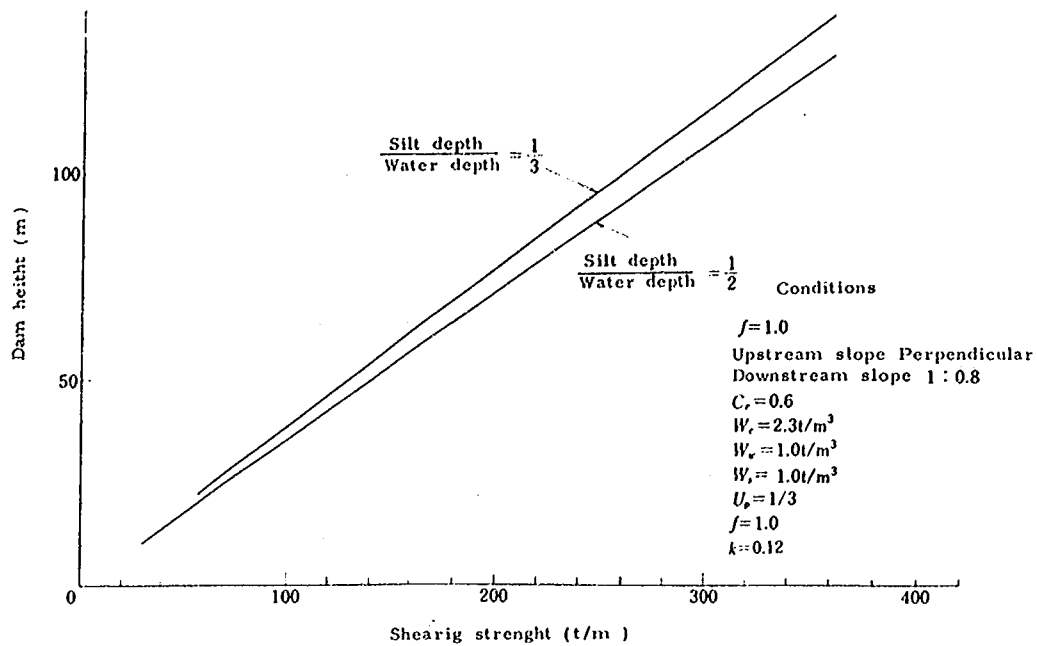


Fig. 11 Dam height and required shearing strength of bedrock
(Design dam section determined by the condition of Middle Third)

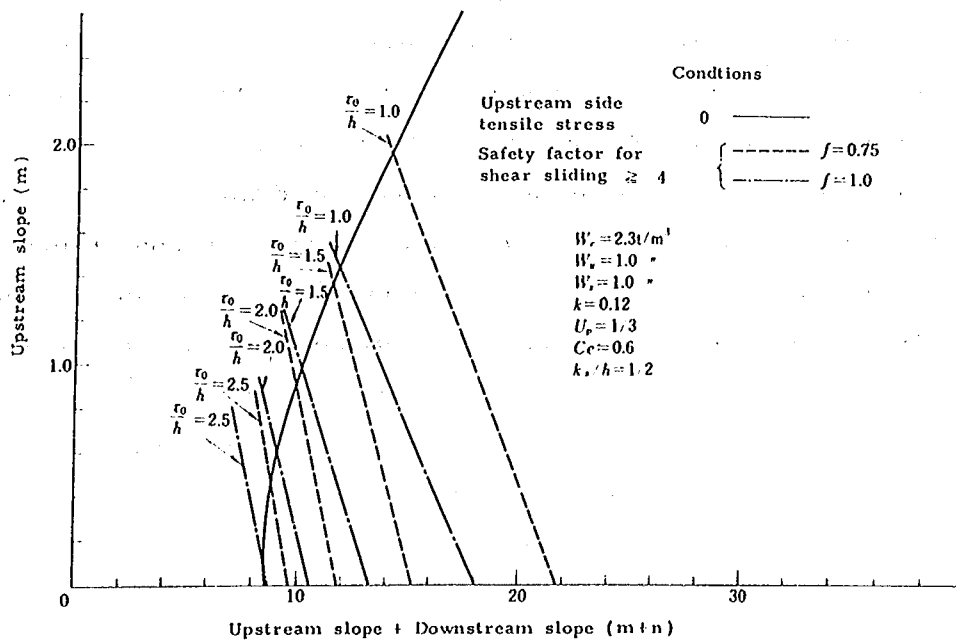


Fig. 12 (a) Upstream & downstream slopes satisfying design conditions (I)

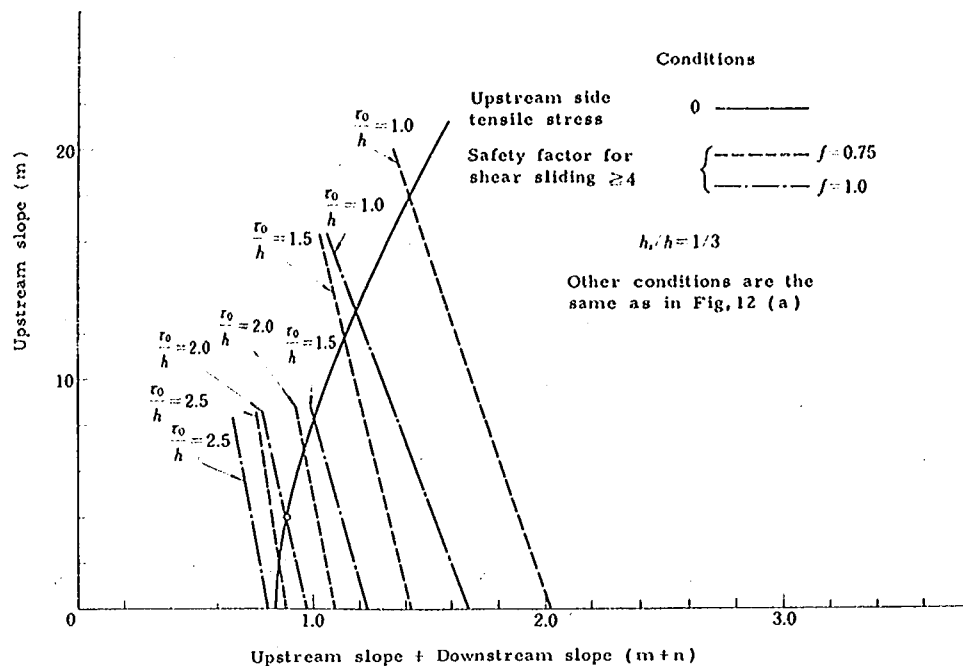


Fig. 12 (b) Upstream & downstream slopes satisfying design conditions (II)

d, Requirement for holding stress below allowable limit

Stress generated in gravity dams is very small and does not present any problems except when constructing large dams exceeding 150 m in height. In the design of such high dams, the internal stress must be obtained not by the simple method of extreme fiber stress but by the analysis based on the theory of elasticity. Levy, Mononobe and Brahts worked out a theoretical solution effective for relatively simple boundary conditions, but the calculation methods they employed are all complicated or time-consuming. On the other hand, the finite element method has come to be applied very frequently with the recent development of electronic computers. And this method produces a solution accurate enough for the practical purpose with relative ease. Fig.6 shows a sample of calculations using F.E.M.

The simple method of obtaining extreme fibre stress is explained below, which is considered as the standard calculation method to discuss the stress problem.

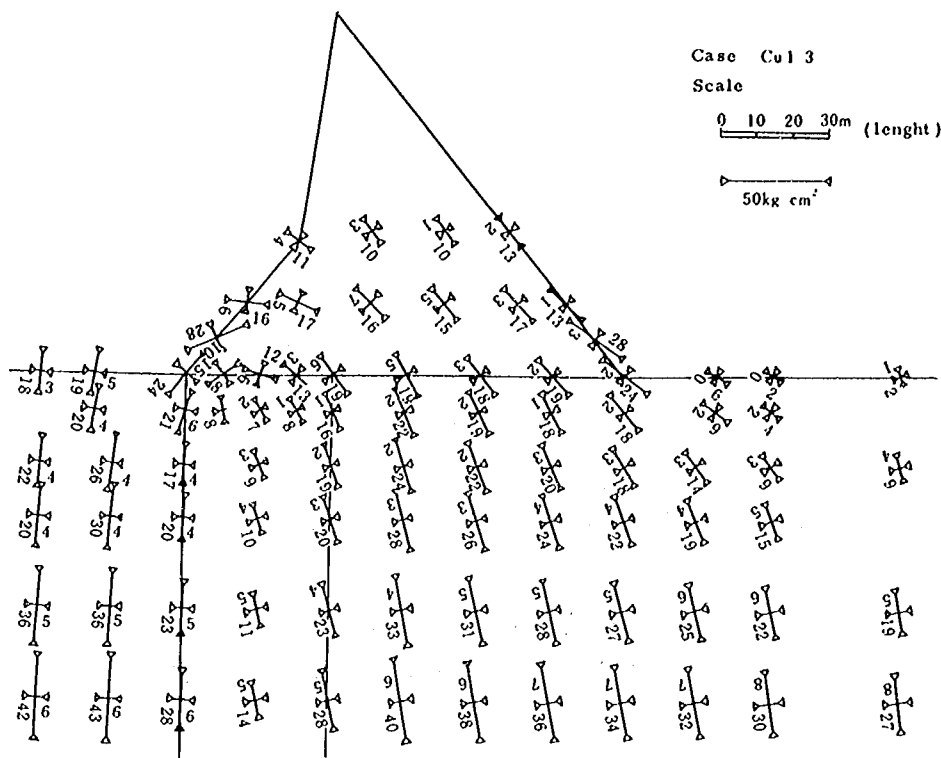


Fig. 13 Stress analysis of dam body and foundation by finite element method

The vertical stress and the bending stress of the surface considered are expressed by the following equation.

$$\text{Intensity of vertical stress} = \frac{V}{A}$$

$$\text{Intensity of bending stress} = \frac{M}{I} y = \frac{V_e}{I} y$$

where, A : Cross-sectional area of base ($A = B$

when unit width is considered)

M : Moment working on the base neutral axis

I : Moment of inertia of area relative to the base neutral axis ($I = \frac{B^3}{12}$)

e : Distance from the base center to the working point of resultant ($e = x_0 - \frac{B}{2}$)

B : Base length

y : Distance from the base center to the point of bending moment to be obtained

The downstream face stress intensity (σ_d) and the upstream face stress intensity (σ_u) are expressed by the following equations.

$$\sigma_d = \frac{V}{B} + \frac{Vc}{I}y = \frac{V}{B} + \frac{\frac{Vc}{B^3} \cdot \frac{B}{2}}{\frac{12}{B}} = \frac{V}{B} \left(1 + \frac{6c}{B} \right)$$

$$\sigma_u = \frac{V}{B} + \frac{Vc}{I}y = \frac{V}{B} + \frac{\frac{Vc}{B^3} \left(-\frac{B}{2} \right)}{\frac{12}{B}} = \frac{V}{B} \left(1 - \frac{6c}{B} \right)$$

The above equations can be rewritten as follows.

$$\sigma_d = \frac{V}{B} \left(1 + \frac{6c}{B} \right) \quad \dots\dots\dots (18)$$

$$\sigma_u = \frac{V}{B} \left(1 - \frac{6c}{B} \right)$$

The maximum vertical compressive stress is not the largest stress generated in structures. The largest stress is generated at the downstream dam edge, and it is expressed by the following equation.

$$\sigma_1 = \sigma_d (1 + n^2) \quad \dots\dots\dots (19)$$

To ensure stability against compressive load, therefore, it is necessary to hold the above-mentioned stress below the allowable maximum compressive stress of dam body of foundation.

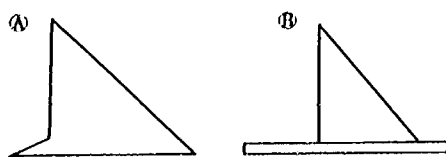
4.5 Point Safety Factor and Adoption of Fillet

a. general

As mentioned in the section 4.3, when the height of the dam exceeds 100 meters, when the shearing strength of the bedrock is relatively small, or when there exist weak layers with small shearing strength near the base on which a gravity dam is built, the condition that the shearing friction safety factor $F_s \geq 4$, which is the second designing condition, becomes dominant in designing a gravity dam.

This requirement must also be fulfilled in the horizontal section of a gravity dam, but generally the shearing strength of concrete is big and it is easy to place concrete having required shearing strength depending on the height of the dam. In this respect, the concrete dam body itself is not a matter of big concern.

Therefore, the regulation of the shearing friction safety factor bears importance in the contact face between the rock bed and concrete or within the bedrock. As for the contact face between the bedrock and concrete, the satisfying conditions can be the forms (A) or (B). In other words,



$$n = \frac{\tau_0 l + fV}{H} \text{ leads to } l = \frac{nH - fV}{\tau_0}$$

so if l is allowed to be sufficiently large, it is possible to satisfy $n \geq 4$.

The designing method like this seems to be handled too conveniently, therefore, it seems necessary to re-examine the matter based on the original meaning of Henny's formula.

Henny proposed the formula previously mentioned based on Coulomb's internal friction theory and the results of the shearing rupture caused by compression of a concrete cylindrical test piece. According to this formula, the shearing friction safety factor of a structure is determined only by the external force applied to it and the strength of the material used, and the partial breaking on the shearing face is not significant.

This idea has been applied to the ultimate design, and the reason for the ignorance of partial value derives from the hypothesis that if the value has partially reached the allowable critical stress of the material, the plastic deformation of the material takes place developing the redistribution of stress, and breaking does not occur until the whole shearing face reaches the ultimate state of equal stress.

This hypothesis can be applied to a substance with complete plasticity which does not receive a repetitive load. However, despite the fact that the rock bed which provides the base for a dam possesses plastic deformation features to a certain degree, the plastic deformation as large as to allow complete unification of stress does not occur. This is apparent in the In-situ shearing tests on rock beds currently conducted. In other words:

- 1) In the shearing test of rock bed, the deformation curve of stress which occurs in the block possesses two bending points before reaching the breaking point, and deformation begins at 30% of breaking load, developing into a eminent deformation at 60% of breaking load. This indicates that breaking is taking place partially within the rock bed.
- 2) When displacement becomes extreme, strain energy stored within the rock bed reduces and displacement is caused mainly by slidings between rock beds.

3) According to the experiments conducted at the Kurobe No. 4 Dam, when a repetitive load was applied which was larger than the second bending point of displacement curve of stress (60% of breaking load), deformation gradually developed and reached rupture in the end.

In the dam, a repetitive load is always applied due to the variations or reservoir water level or the reasons earlier mentioned, so it is difficult to apply the hypothesis of the ultimate design to the rock bed. Instead, it is necessary to investigate whether partial breaking causes total breaking or not. In other words, between the case in which a portion of small breaking safety factor exists partially and that the whole section possesses an almost uniform safety factor, the former has greater possibility of breaking by a small load than the latter despite the fact that the safety factors obtained from Henny's formula are the same between them. In fact, the real safety factor is smaller in the former. Regarding how to get the partial breaking safety factor, Mr. Ishii and others have proposed the following.

b. Definition of spot (point) safety factor :

The breaking condition formula of the rock bed is practically expressed by Coulomb's formula.

$$\tau_u = \tau_0 + f\sigma \quad f = \tan \phi \dots\dots\dots (20)$$

Provided ;

- τ_0 ; Initial shearing strength
- f ; Internal friction coefficient
- ϕ ; Internal frictional angle
- τ_u ; Shearing strength
- σ ; Direct stress applied to the breaking surface

So the following value is defined to be the partial shearing friction safety factor n' (this is called point safety factor),

$$n' = \frac{\tau_0 + f\sigma_n}{\tau_n}$$

Provided,

- σ_n : Vertical stress applied on the assumption that a slide takes place at a certain point
- τ_n : Shearing stress which works in the direction in which a slide is supposed to take place at a certain point,

With the application of this formula, we can calculate the distribution of

point safety factor along the presumed sliding face with the help of stress value and strength of the material.

Next, let's consider the meaning of point safety factor. Intuitively it is possible to think that breaking of the point begins when the point safety factor becomes 1. There are two possible cases for such a state. One is when the material strength reduces and other is when the load increases. Let's look at the former case. The strength of the rock bed is expressed by τ_0 and f as indicated in the formula (20). So the following three cases can be considered possible causes for the reduction of strength leading into breaking.

- ① τ_0 alone lowers to $1/n$
- ② Both τ_0 and f lower to $1/n$
- ③ τ_0 lowers to $1/n$ and f to m/n

In the case of ① $1 = \frac{\tau_0/n + f\sigma}{\tau}$

So it becomes $1 = \frac{\tau_0 + fn\sigma}{n\tau}$

The safety factor in this case is equal to that obtained when both σ and τ are multiplied by n , and it coincides with the safety factor at the time of breaking when the load is multiplied by n . So the safety factor in this case is designated as load safety factor (N_L).

$$N_L = \frac{\tau_0}{\tau - f\sigma} \dots\dots\dots (22)$$

In the case of ②,

$$1 = \frac{\tau_0/n + f\sigma/n}{\tau}$$

so it becomes

$$1 = \frac{\tau_0 + f\sigma}{n\tau}$$

The safety factor in this case coincides with the safety factor at the time of breaking when τ alone is multiplied by n . The safety factor in this case is designated as strength safety factor (N_S).

N_S is expressed as :

$$N_S = \frac{\tau_0 + f\sigma}{\tau}$$

In the case of ③,

$$1 = \frac{\tau_0/n + m/n f\sigma}{\tau}$$

so we can obtain

$$1 = \frac{\tau_0 + m\sigma}{n\tau}$$

The safety factor in this case coincides with the safety factor at the time of breaking when τ is multiplied by n and σ by m . The combined safety factor in this case (N_m) is as follows :

$$N_m = \frac{\tau_0 + n\sigma}{\tau}$$

Next, we consider the safety factor from the viewpoint of engineering significance. If we compare these safety factors numerically, they stand commonly as $n > 1$, $m > 1$, $n > m$. So from the formula, the following relations apparently exist :

$$N_L > N_m > N_s$$

and here the load safety factor shows the largest value and the strength safety one the smallest. The degree or load uncertainty is small as stated earlier and it is hardly possible that the total load becomes n times. This writer believes that, considering the ambiguity of strength estimation of the rock bed or the deterioration of the rock bed over long periods of time, we should provide some allowance to the strength. In this respect, strength safety factor should be adopted as the safety factor.

The assumption that τ_0 and f lower to $1/n$ seems to be a little too severe viewed from the results of the shearing tests conducted at each site, and in principle, the combined safety factor (N_m) best represents the reality. However, as it requires a complicated process to find out the difference between n and m at each site and moreover the formula itself will lose its generality, the difference between N_s and N_m had better be considered allowance. In Henny's formula, the word safety factor has been used vaguely, but it is apparent that it has meant strength safety factor.

In investigating the point safety factor of the contact face between the rock bed and concrete or within the rock bed, we need to know the stress condition of the point in question. In the past, the method to obtain the stress value relied on photoelasticity experiment a model testing, but today, it is easily obtained by the finite element method. Therefore, although the idea to examine the partial safety factor was proposed a long time ago, there have been few cases in which examinations were carried out from the viewpoint of the point safety factor because the process to get stress was complicated and required much labor and cost. However, during the past

ten years, it has become the common method of examination.

Generally, the stress at a certain point is expressed by the principal stress. Suppose the first principal stress is σ_1 , the third stress is σ_3 , and line element lies urthen the area containing σ_1 forming angle θ from σ_1 as indicated in the Figure 13.

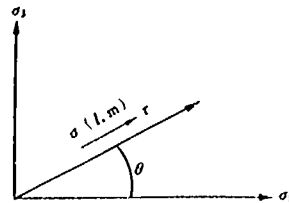


Fig. 13

We now try to obtain the direct stress and shearing stress which work on this line element. Suppose AB is Δl in the infinitesimal triangle shown in the right, we get the following :

$$BC = \Delta l \cos \theta \quad AC = \Delta l \sin \theta$$

The balance of power in the horizontal direction is :

$$\sigma_1 \Delta l \sin \theta = \sigma_n \Delta l \cdot \sin \theta + \tau \Delta l \cdot \cos \theta \quad (a)$$

The balance of power in the vertical direction is as follows :

$$\sigma_3 \Delta l \cdot \cos \theta = \sigma_n \Delta l \cdot \cos \theta - \tau \Delta l \cdot \sin \theta \quad (b)$$

From (a) $\times \sin \theta$ + (b) $\times \cos \theta$ we get :

$$\sigma_n = \sigma_1 \sin^2 \theta + \sigma_3 \cos^2 \theta = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3) \cos 2\theta \quad (25)$$

From (a) $\times \cos \theta$ - (b) $\times \sin \theta$ we get :

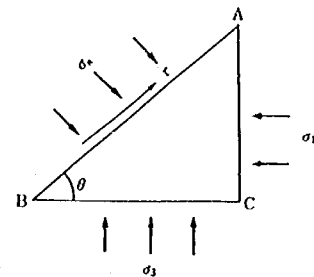
$$\tau = \sigma_1 \sin \theta \cdot \cos \theta - \sigma_3 \sin \theta \cdot \cos \theta = \frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\theta \quad (26)$$

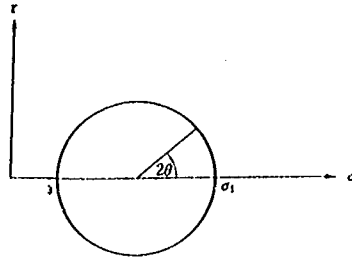
In addition, from the formulas (25) and (26), we get :

$$\left\{ \sigma_n - \frac{1}{2}(\sigma_1 + \sigma_3) \right\}^2 + \tau^2 = \left\{ \frac{1}{2}(\sigma_1 - \sigma_3) \cos^2 \theta \right\}^2 + \left\{ \frac{1}{2}(\sigma_1 - \sigma_3) \sin^2 \theta \right\}^2 \\ = \left\{ \frac{1}{2}(\sigma_1 - \sigma_3) \right\}^2 \quad (27)$$

So σ_n and τ make a circular locus around a co-ordinate $(0, \frac{1}{2}(\sigma_1 + \sigma_3))$. This is what is called Mohr's circle. It is convenient to memorize the coordinate transformation of stress by means of the shape of Mohr's circle.

In other words, the stress, σ_n and τ of the surface inclined by θ from σ_1 is expressed by the coordinate value of a certain point within the circle which has an angle of 2θ from σ_1 of Mohr's circle.





When principal stress is given to a certain point, the sliding point safety factor of that point differs depending on the direction of slide but there must be a certain direction which provides the smallest value, Let's examine the matter,

The strength safety factor has been shown in the expression (23), That is ;

$$N_s = \frac{\tau_0 + f\sigma}{\tau}$$

If you substitute the above expression with those of (25) and (26), you get the following ;

$$N_s = \frac{\tau_0 + f\left[\frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta\right]}{\frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta} \dots\dots\dots (28')$$

If you solve θ with the smallest value of N_s by the condition $\frac{\partial N_s}{\partial \theta} = 0$, you get ;

$$\theta = + \left\{ \frac{\pi}{4} - \frac{1}{2} \tan^{-1} \left(\frac{f}{N_s} \right) \right\}$$

$$N_{s, \min} = \frac{f(\tau_0 + f\sigma_1)(\tau_0 + f\sigma_3)}{\frac{1}{2}(\sigma_1 - \sigma_3)}$$

In this way, when a state of stress is given to a certain point, it is possible to discover the direction which has the greatest potentiality for sliding as well as obtain the sliding point safety factor of that direction. The smallest value of safety factor is designated as minimum point safety factor.

When a weak strength layer like a fault exists within the bedrock, you first obtain vertical stress σ and shearing stress τ which work on that plane, and then you can get the point safety factor along the weak plane by the following expression ;

$$N_t = \frac{\tau + f\sigma}{\tau}$$

Provided ;

τ ; Shearing strength of the weak layer

f_f ; Internal frictional angle of the weak layer

However, if the inside of the weak layer is relatively large, the direction with the greatest potentiality for sliding exists within the weak layer, therefore, it is not sufficient enough just to take the direction along the weak layer for the direction of sliding. We should check the direction with the minimum point safety factor we have mentioned earlier and its value,

Let us now introduce a case of investigation we conducted at a model dam on the distribution of the point safety factor we have previously defined,

According to the design standards, it is regulated, as we have already mentioned, that the safety factor expressed by Henny's formula must satisfy 4, and the result obtained from it can be considered the safety factor of the whole sliding plane, We now describe the actual distribution of the point safety factor of the dam designed under this condition, The stress along the contact face and distribution of the point safety factor of the dam designed under the following conditions (Fig.14(a)) will be like that shown in the Figure 14(b),

Height of dam	: 100 m	Earthquake coefficient	: 0.05
Downstream slope	: 1 : 0.8	Concrete unit weight	: 2.3 t/m ³
Upstream slope	: 1 : 0.05	Uplift pressure coefficient	: 0.4
Hydrostatic pressure action depth	: 100 m	Silt	: None

Here, we adopted $\tau_0 = 220 \text{ t/m}^2$ and $f = 0.65$ within the rock bed.

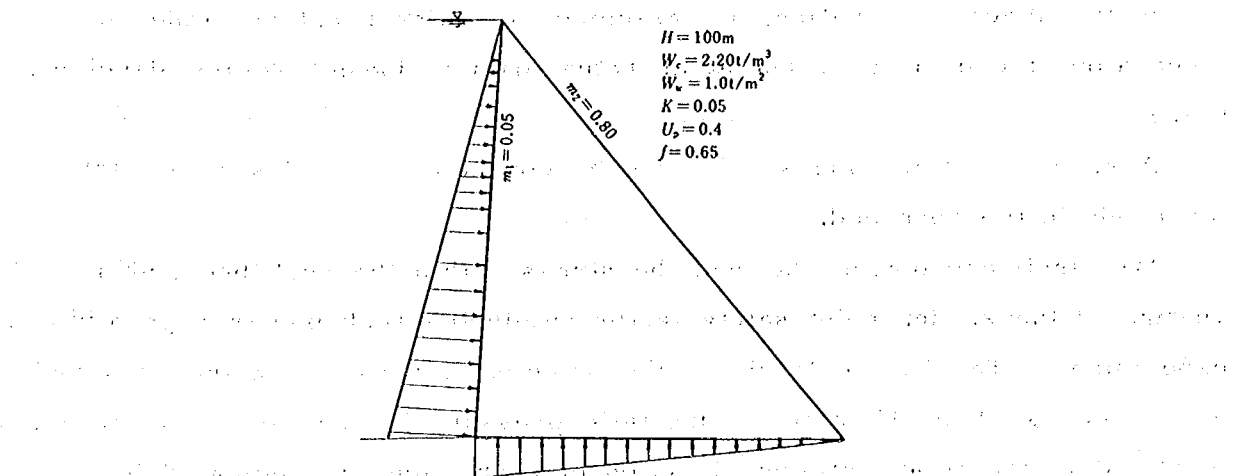


Fig. 14 (a) External load condition

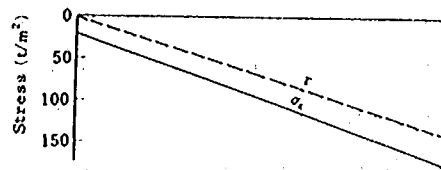


Fig. 14 (b) Stress distributions of a dam base

We obtained the results as described in the Figure 15 regarding the distribution of point safety factor along the dam base.

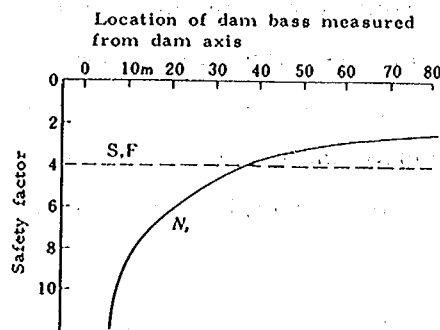


Fig. 15 Distribution of shearing friction safety factor F_s and that of point safety factor N_s along the base of a gravity dam

Here, Henny's formula is Safety factors $F_s = 4$ and it is indicated by a dotted line. The minimum point safety factor is 2.4, clearly indicating at most part of the downstream of the dam point safety factor is lower than Henny's 4.

In the above examination, we assumed that slide breaking plane developed along the dam base, but at an actual dam it doesn't always develop there.

A similar examination should also be conducted on slides which take place within the rock bed.

Mr. Ishii and others obtained the stress within the rock bed and investigated the sliding point safety factor within the rock bed through a model experiment. The Figure 16 shows the principal stress within the rock bed which occurs when 100 meter hydrostatic pressure is applied. The minimum point safety factor and sliding direction of each point are indicated in the Figure 17.

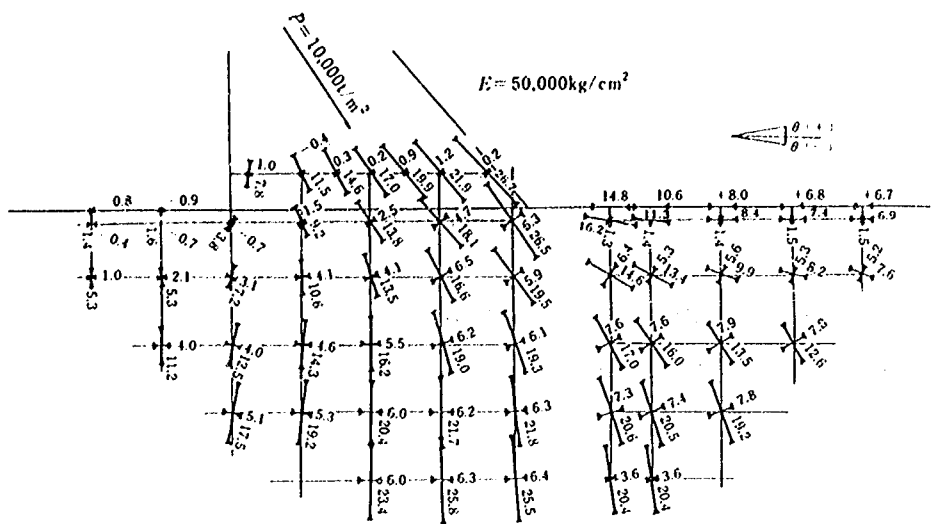


Fig. 16 Stress distribution inside the rock bed of a gravity dam

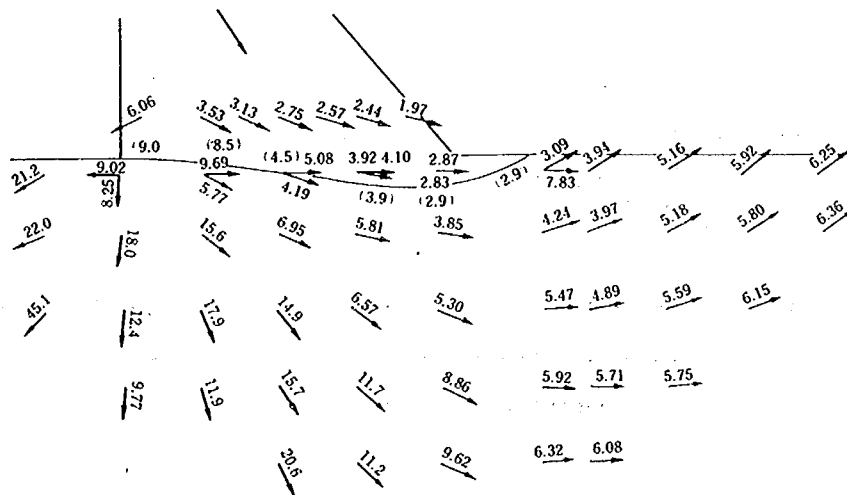


Fig. 17 Distribution chart of sliding plane and point safety factors inside the rock bed of a gravity dam

The thick solid line shows the shape of the sliding line when the safety factor becomes minimum. The table 9 describes the comparison between the point safety factor along this sliding line and that of the base.

Table 9

Safety factor along the dam base obtained by Henny's formula	5.7
Average point safety factor along the dam base	6.1
Point safety factor along the sliding plane	2.9 ~ 9.0
Average safety factor along the sliding plane	5.3

From these results, we can get the following conclusion.

- 1) The sliding plane locates in the lower rock bed at the downstream part of a dam base and it extends a little farther downstream from the downstream end of the dam. Also, the average point safety factor is a little lower in the sliding plane than in the dam base. The shape of this sliding plane resembles that of the breaking plane we commonly observe in in-situ shearing tests of rock bed.

However, from macroscopic point of view, we can say that the sliding plane is located very close to the dam base and that there isn't much difference in safety factor between the dam base and the sliding plane. So we can safely say that the conventional method to apply Henny's formula to dam base is appropriate.

- 2) The base of a gravity dam at an upstream section shows a considerably high safety factor but it goes down at downstream sections. In fact, it is about half of the value obtained from Henny's formula near the downstream end.

As we have observed, the shearing friction safety factor of a gravity dam secures more than 4.0 as a whole but it shows around 2.5 as the minimum point safety factor at the downstream end of a dam. Also, it is quite possible that the point safety factor further drops at the downstream section when the breaking plane locates through the base rock.

c. Engineering judgement of point safety factor :

We have observed that even if the shearing friction safety factor has attained 4 according to Henny's formula, the point safety factor stays at 2.0 ~ 2.5 near the downstream end of the contact face. Furthermore, it even becomes lower than 1 in extreme cases when a portion with low shearing

strength like a fault exists in a layer relatively close to the surface within the rock bed. Then, how can we judge dam's safety using the point safety factor from the engineering point of view? If we offer our conclusion first, we consider 2 as the yardstick.

This derives from the following reasons.

- 1) Shearing strength is commonly described by ultimate strength. In this case, rock bed possesses yield point (singular point or inflection point) as seen in an experimental case of the Kurobe No.4 Dam, and it broke when applied stronger repetitive load than the yield point. This singular point appears at about 60% level of the ultimate load. In that case, the point safety factor reaches the singular point when it is $1/0.6 = 1.7$.
- 2) Due to ambiguity of strength estimation and stress analysis, some allowance should be given.

At the Nagawado arch dam of Tokyo Electric Power Company, they use a similar strength point safety factor defined as "loose coefficient" when they check stability of the bed rock. At this dam, the loose coefficient has to be larger than 1, and they bring out affirmative judgement to the check if the loose coefficient is not smaller than 2 as a whole. Their investigation is based on two dimensional plane slice. In the case of an arch dam, quite different from a gravity dam, there exists initial stress within the actual base rock bed (Bady Force), and three dimensional dispersion of stress is conspicuous. Therefore, they seem to have taken such a judgement, thinking that it is too safe to use two dimensional dam thrust for a calculation.

d. Adoption of fillet :

When base rock bed is solid enough and can be settled by the Middle Third conditions, the point safety factor retains about 2.5 at the minimum and such local safety alone is good enough to guarantee sufficient safety of the dam. Now, when the section can be determined by the shearing friction safety factor, such designs as (A) and (B) of page 42 can be possible if we stick only to the average safety factor based on Henny's formula which we have mentioned earlier. We call the added part of the upstream section such as the design (A) "fillet". Mr. Kobayashi and others have examined these designs paying attention to the distribution of the point safety factor. Let's talk about the results of their examination.

First they investigated how the distribution of this point safety factor varied when downstream slope was changed while bottom length was kept

constant, using a model gravity dam with $\tau_0 = 150 \text{ t/m}^2$, $f = 1.0$, $h = 100 \text{ m}$.

Generally the vertical stress σ_d and shearing stress τ_d at the downstream end of a dam becomes as follows when no external force works on downstream face :

$$\frac{\sigma_{sd}}{\tau_d} = \frac{1}{m^2} \dots\dots\dots (31)$$

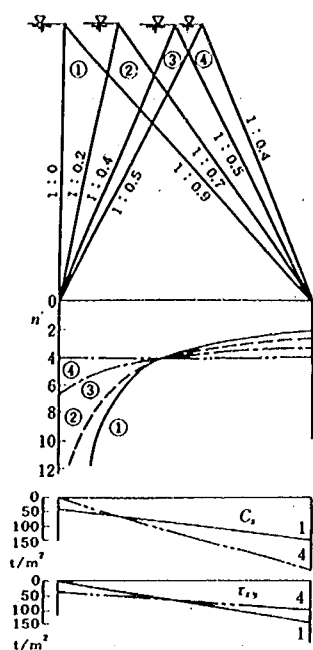
If we substitute strength point safety factor with the above, we get the following :

$$N_t = \frac{\tau_0 + f\sigma_m}{\tau} = \frac{\tau_0 + f\sigma_{sd}}{\tau_d} = \frac{\tau_0}{\tau_d} + f \cdot \frac{\sigma_{sd}}{\tau_d} \dots\dots\dots (32)$$

So, point safety factor at downstream becomes larger as downstream slope is steeper. In other words, when downstream slope is steep, the direction of principal stress at downstream end becomes almost vertical against the dam base making it difficult to cause sliding.

The Figure 18 shows the results of investigation. When upstream slope is gentle and downstream slope is steep, the distribution of point safety factors at the dam base becomes uniform. So varying dam slope provides an effective means for unification of point safety factors.

We have already mentioned that, when upstream slope is gentle and downstream slope is steep while the dam base is kept constant, shearing friction safety factors of Henny's formula become advantageous because we gain water weight on upstream surface. We also understand that it is more advantageous when upstream slope is almost vertical observed from the conditions of Middle Third. From this fact, we had an idea to keep the downstream slope at the value determined by the Middle Third conditions and fix a fillet at upstream to compensate the shortage of shearing friction safety factor, We thought of extreme designs like (A) and (B) when we ignored the distribution of stress and point safety factors.



	1	2	3	4
Upstream slope	0, 0	0, 2	0, 4	0, 5
Downstream slope	0, 9	0, 7	0, 5	0, 4
τ_0	t/m ² 150	t/m ² 150	t/m ² 150	t/m ² 150
f	1, 00	1, 00	1, 00	1, 00
k	0, 05	0, 05	0, 05	0, 05
U	0, 3	0, 3	0, 3	0, 3
W_c	2, 3	2, 3	2, 3	2, 3
W_w	1, 0	1, 0	1, 0	1, 0
Dynamic water pressure and Silt pressure	W_w 15%	15%	15%	15%
σ_{xd} Upstream end	t/m ² 41	35	18	5
σ_{xd} Downstream end	t/m ² 155	186	229	254
τ_{xy} Upstream end	t/m ² 0	9	25	38
τ_{xy} Downstream end	t/m ² 139	130	114	102
n^i Upstream end	∞	20, 8	6, 72	4, 13
n^i Downstream end	2, 37	2, 58	3, 31	3, 98
n Average	3, 56	3, 75	3, 93	4, 02

Dam's height 100 m

Fig, 18 Stress and point safety factor at dam bottom (gravity dam)

Then, they carried out a model experiment regarding slope of fillet. The experiments were conducted to obtain stress distribution based on the two dimensional photo-elasticity theory in three cases : a gravity dam, height 100 m, downstream slope 1 : 0.8, upstream slope 1 : 0.4 as indicated in the Figure 19 ; the same dam, fixing a fillet which has the slope of 1 : 1.0 against the vertical part at upstream ; and fixing a fillet which has the slope of 1 : 2.0 against the vertical part.

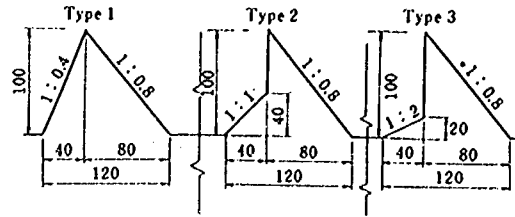


Fig. 19 Gravity dams' sections

They put into consideration only dead weight and water pressure load as the load for experiments. For dead weight, they loaded by mercury based on Biot theory into a model made of epoxy resin mixed with plastic material with high photo-elastic sensitivity, and for water pressure load, they loaded it with a lever onto the model made of epoxy board.

The distribution chart of vertical stress and shearing stress obtained through the experiments are shown in the Figure 20.

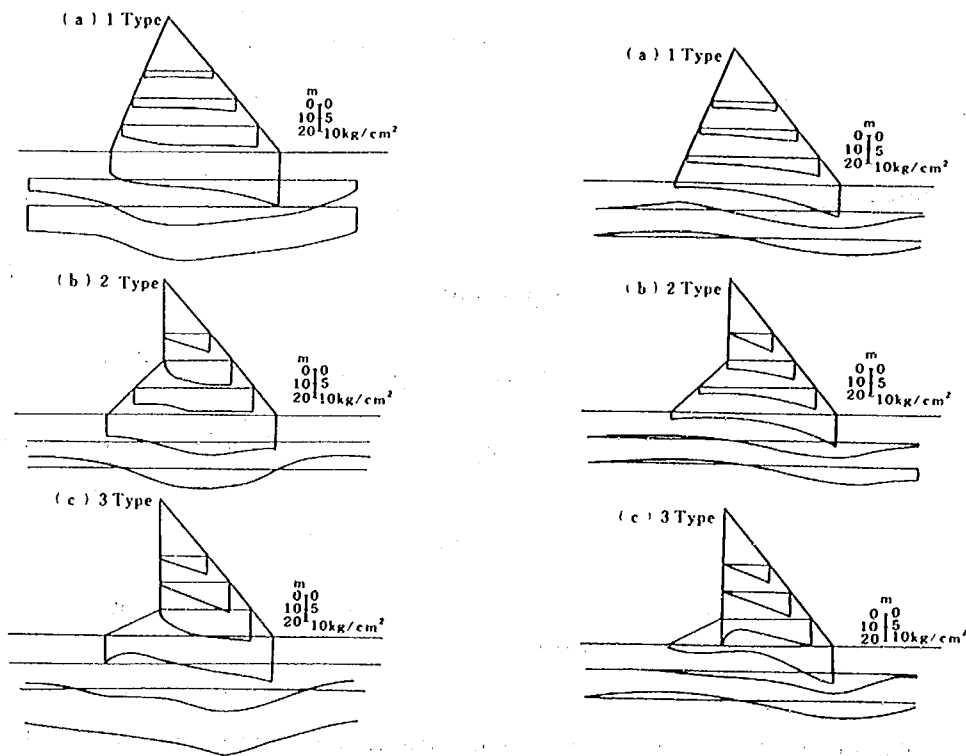


Fig. 20 (a) Distribution chart of vertical stress

Fig. 20 (b) Distribution of shearing stress

We show in the Figure 21 the distribution of point safety factors at dam base obtained through calculation based on these figures under the condition of $\tau_0 = 100t/m^2$, $f = 1.0$, and uplift pressure coefficient $= 0.3$.

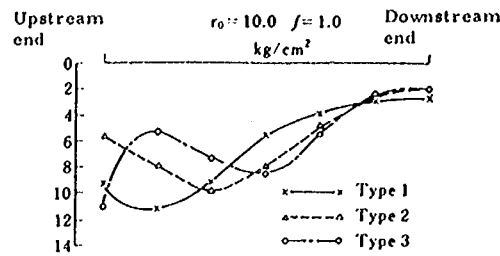


Fig. 21 Distribution chart of point safety factors

From these figures, we can conclude as follows.

- 1) When there is no fillet, shearing stress which occurs at upstream part is small and point safety factor is large, but when a fillet is fixed, the point safety factor at that point lowers.
- 2) In case of a fillet with the slope of 1 : 2.0, point safety factor becomes large at upstream end, and it is feared that the effect is not sufficient enough in the thin part of the fillet with gentle slope.
- 3) Point safety factor at downstream end is almost constant in any case being about half of the safety factor obtained by Henny's formula. It seems that point safety factor at downstream end is controlled mainly by downstream slope.
- 4) Stress concentration occurs in the place where a fillet is installed and its influence is relatively limited when its slope is 1 : 1.0 but it extends to a wide area when its slope is made into 1 : 2.0.

These experiments were conducted on the assumption that the module of elasticity of the dam body and rock bed were equal. So it's still too early to talk about its generality from these results alone. However, we can at least say that, when a large fillet is installed as in the experiment, there won't occur excessive stress concentration at the place of its installment if its slope is made as gentle as 1 : 1.0, and the distribution of point safety factors become uniform at the dam base. It is advantageous in these respects.

The point safety factor at downstream end is not influenced by the fillet. So it's better to apply a method to make downstream slope steeper.

We felt intuitively that the (A) and (B) plans were extremely convenient, and now we understand that they are inappropriate from the theoretical point of view as well. It is generally desirable to make the slope of fillet about 0.8 - 1.0.

4.6 Example Design of A Gravity Dam when the Base is in Unfavourable Condition

We explained in the previous chapter about the general approach to the case when the design of gravity dam is determined by the condition of shearing friction safety factor of the base $F_s \geq 4$. In short, it is necessary, in such a case, to examine the matter paying attention to point safety factor in addition to average shearing friction safety factor. When safety factor obtained by Henny's formula and point safety factor do not satisfy the standards, we can generally raise such safety factors by enlarging the section so that the contact face with the base rock bed becomes large. However, it will increase construction cost depriving the advantages of gravity dams. So fill-type dam may substitute it. A gravity dam is constructed when it is impossible to make a fill-type dam or when it is more economical to build a gravity dam. In this case, to enlarge the gravity section is not the only way to secure safety. For example, it is often taken to replace the weak part of base rock with concrete. There are various engineering methods for improving rock bed, but this subject belongs to the category of "Foundation treatment". So we explain, based on actual cases, about what we investigated and what methods we took when we encountered with a case in which the base rock bed contained defects.

Oodo dam is a gravity dam with a height of 100 m. Speaking of its geographical features, clay slate and sandstone are distributed in the upstream part of the dam, and schalstein and chart surrounded by schalstein in layer form or in lenticular form are distributed at the dam base or further downstream. The clay slate and sandstone were in good quality but the schalstein were softened by the effect of serpentining.

The strength and deformation properties corresponding to the grades of rock quality to be induced from in-situ rock tests. Incidentally, as serpentining is considered highly influential upon strength, we put it into consideration when we made rock classifications.

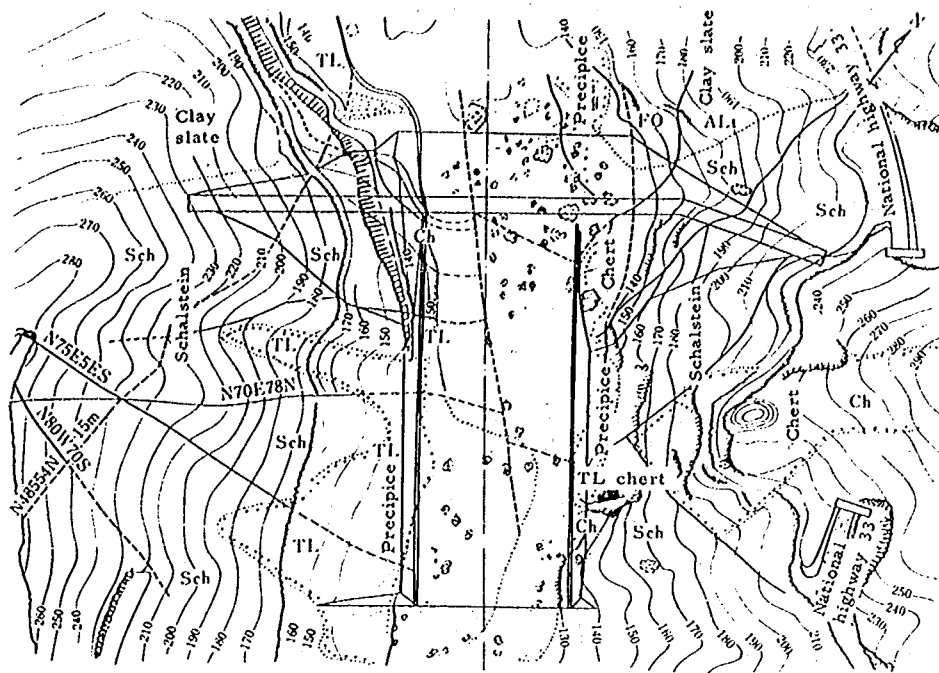


Fig. 22 Geographical plan of Oodo dam

Table 10 Shearing strength of rock bed

Rock grade	Shearing strength	Static modulus of elasticity
CH	200t/m ²	50,000kg/cm ²
CM ~ CH	150t/m ²	20,000 30,000kg/cm ²
CM	80t/m ²	12,000kg/cm ²

Table 11 Contrast between rock grades and the degree of serpentine

Rock grades	Degree of serpentine			
	CL	CM	CM ~ CH	Over CH
Serpentine is progressing conspicuously	97%	81%	40%	12%
Serpentine is progressing to a certain degree	3%	19%	60%	88%
Almost no signs of serpentine	0	0	0	
Extension of exploring pit (m)	118	167	33.5	101.5

If these values are design shearing strength, the dam section of H=100m dam is apparently determined by the conditions of sliding, and the dam section is shown in the Figure 23. The strength of rock bed is not uniform against the section and we examined the shearing friction safety factor in the following way.

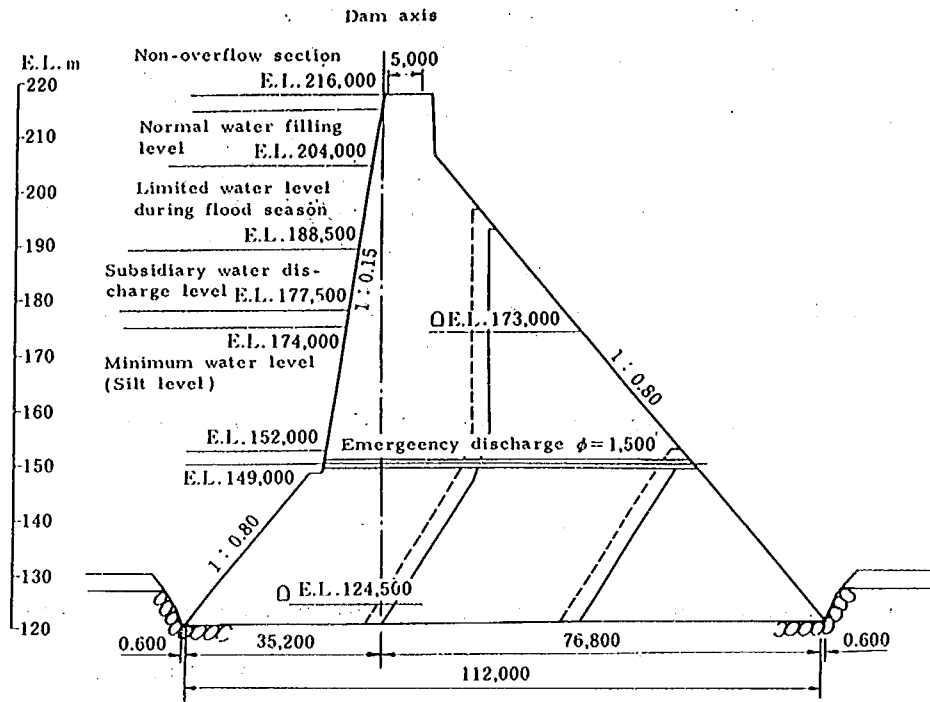


Fig. 23 Standard section of non-overflow section of A dam

- 1) Obtain the average strength of the base rock bed for τ_0 and f and apply Henny's formula to it.
- 2) Divide sliding plane into various zones with different strength and calculate by the next expression.

$$N = \frac{\sum_{i=1}^m (\tau_{0i} + f_i \cdot \sigma_i) l_i}{H} \dots \dots \dots (33)$$

Provided :

- m : Number of divided zones
- τ_{0i} : Shearing strength of rock bed at each zone
- f_i : Friction coefficient of rock bed at each zone
- σ_i : Average normal stress of rock bed at each zone
- l_i : Resistant length at each zone

The dam section was determined by these methods, but it was quite doubtful whether the application of the average safety factor (Henny's safety

factor) is sufficient enough in dealing with the rock bed made of combinations of parts different in strength and deformation. So we examined the actual situation of force transmission and the distribution of point safety factors.

We made comparison, for the sake of judging safety, of the case when weak layers with weak strength and large deformation existed in the upstream and downstream sides. We will show the results of only the cases where the weak layer occupies 1/2 of the whole base rock bed. The following is how we conducted our examinations.

Regarding the dam body and rock bed as elastic bodies, we conducted stress analysis by means of finite element method. We checked the following facts for this stress value.

- 1) Whether a large tensile stress has developed in both dam body and rock bed.
- 2) Whether compressive stress has developed more than allowable stress.
- 3) The distribution of point safety factors.

The results of stress analysis can be described in principal stress as in the Figure 24. Also, the distribution of point safety factors along the dam base is indicated in the Figure 24. The actual analysis was carried out based on estimated geographical features, and also we examined various cases other than that. The main points of our conclusion are as follows.

- 1) When a part of the rock bed is soft, the force of the dam body is transmitted to the hard and rigid rock bed in concentrated form and the stress of soft part of the rock tends to be small.
- 2) The downstream end is the spot where the maximum force is transmitted, and if a border exists between weak layer and hard rock in this spot, point safety factor partially lowers due to stress concentration.
- 3) There is almost no problem with regard to the weak layer which is located in the middle of the dam base or in the upstream section.
- 4) If a weak layer exists in the downstream section of the dam and the upstream section of the dam is composed of hard rocks, tensile stress develops in the horizontal direction near the upstream end of the dam.

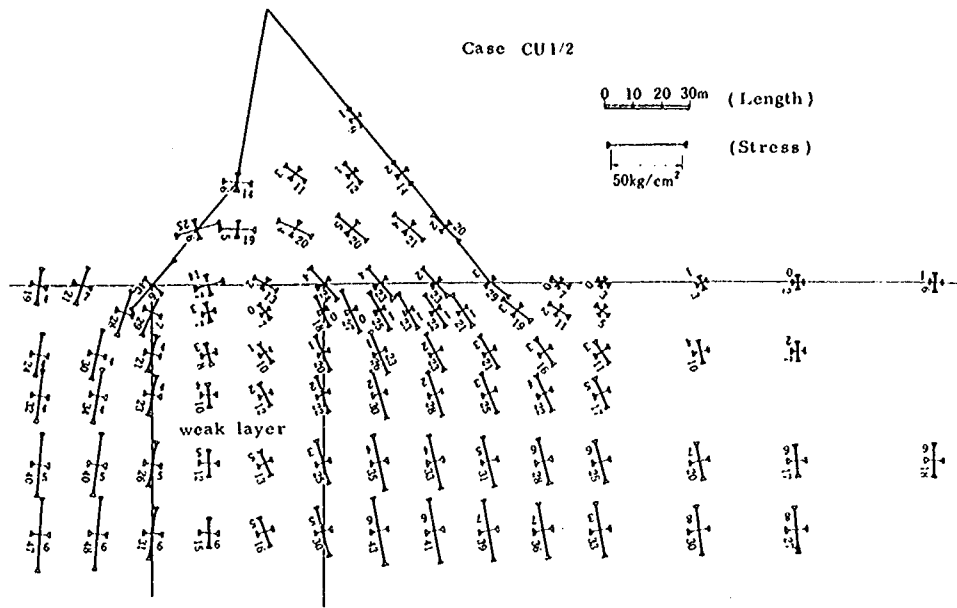


Fig. 24 (a) A case when weak layer occupies 1/2 of the dam base in the upstream section

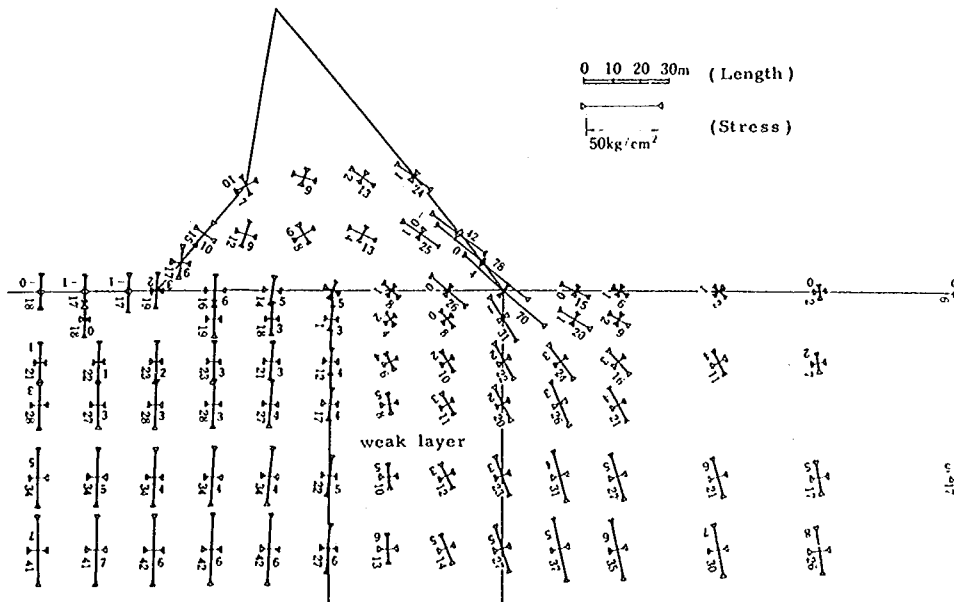


Fig. 24 (b) A case when weak layer occupies 1/2 of the dam base in the downstream section

At the Oodo dam, conclusions will be drawn whether special treatment needs to be taken as soon as geographical features are clarified by excavating. In any case, a problem will arise only when a weak layer is discovered at the bottom end of the dam. In that case, a part of the weak

layer at the bottom end of the dam can be replaced by concrete to secure sufficient engineering stability.

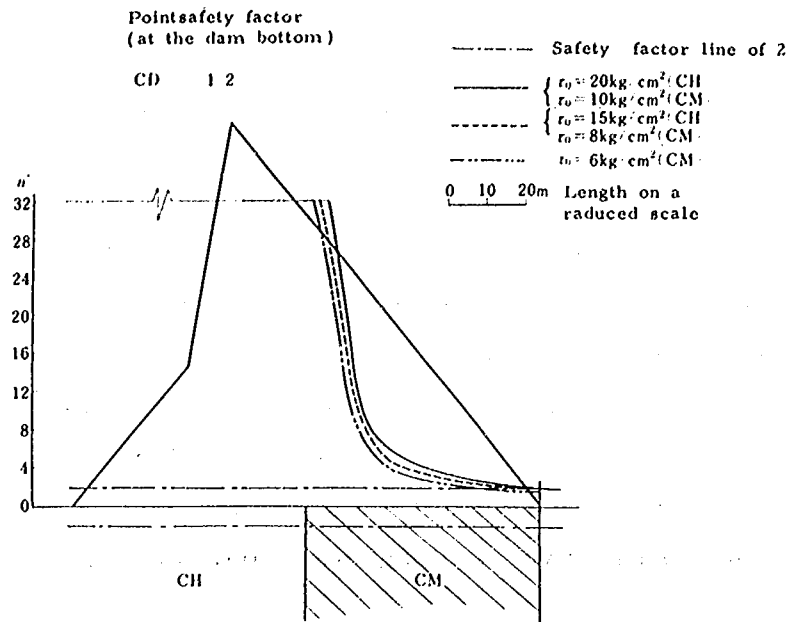


Fig. 25 (a) Distribution of point safety factor

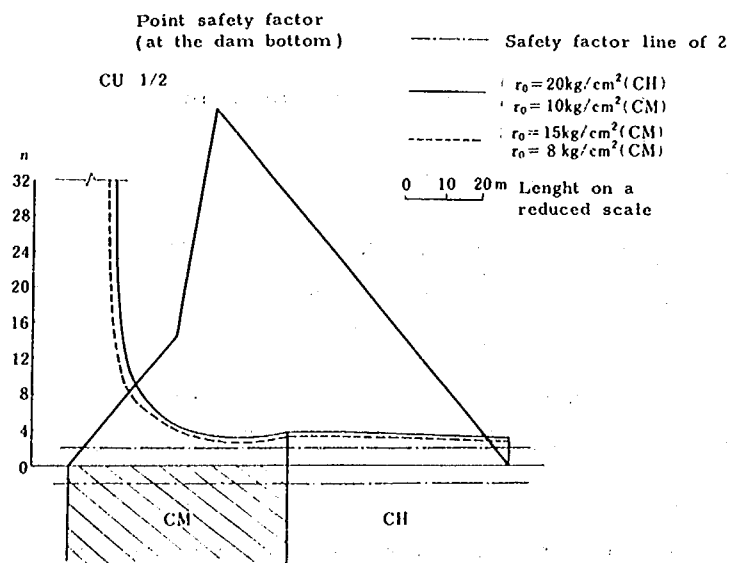


Fig. 25 (b) Distribution of point safety factor

4.7 Detailed Design of Concrete Dams

4.7.1 Design of joints

a. general

The concrete placed dissipates heat of hydration as it cures. Unless the heat of hydration is removed in a positive way, the concrete temperature may rise as high as 50°C. Although the dam concrete settles at an final and stable temperature in a long time, large transient temperature differences in it can exist after the concrete has attained sufficient rigidity, causing tensile stress in the concrete to crack. In order to absorb the contraction of concrete for the purpose of preventing the random cracking in the concrete, the concrete placing work is carried out in blocks into which the dam is divided. The contact surfaces between the blocks along the dam axis are called the transverse joints. Where the dam structure has some thickness along the river running direction, joints should also be provided in the planes normal to the river running direction, and are called the longitudinal joints.

Designed to prevent the cracking in the concrete, these joints must be treated to ensure a sufficient watertightness when they are cured open at an ultimate temperature.

b. Joint spacing

The factors that determine the spacing of transverse joints include the following.

- 1) Foundation conditions
- 2) Dam appurtenances (spillway, overflow gate, etc.)
- 3) Contracting characteristics of concrete
- 4) Capacity of conveying concrete

As regards the foundation conditions, it should be borne in mind that, if a joint falls on a weak zone or an irregular part of the foundation, its opening can propagate into the bedrock. Thus, full engineering considerations should be done to remove irregular foundations and not to arrange the joints directly at weak layers.

The dam appurtenances can govern the location of joints. It should be borne in mind that no joints be provided in the piers or near the outlet conduits.

The most important factor is the contracting characteristics of concrete; if the joint spacing is too wide, cracks will develop in the blocks, and if it

is too narrow, it will hamper grouting. Empirical data show that the optimum joint spacing is about 15 m and that a joint opening of about 2mm will be sufficient for grouting.

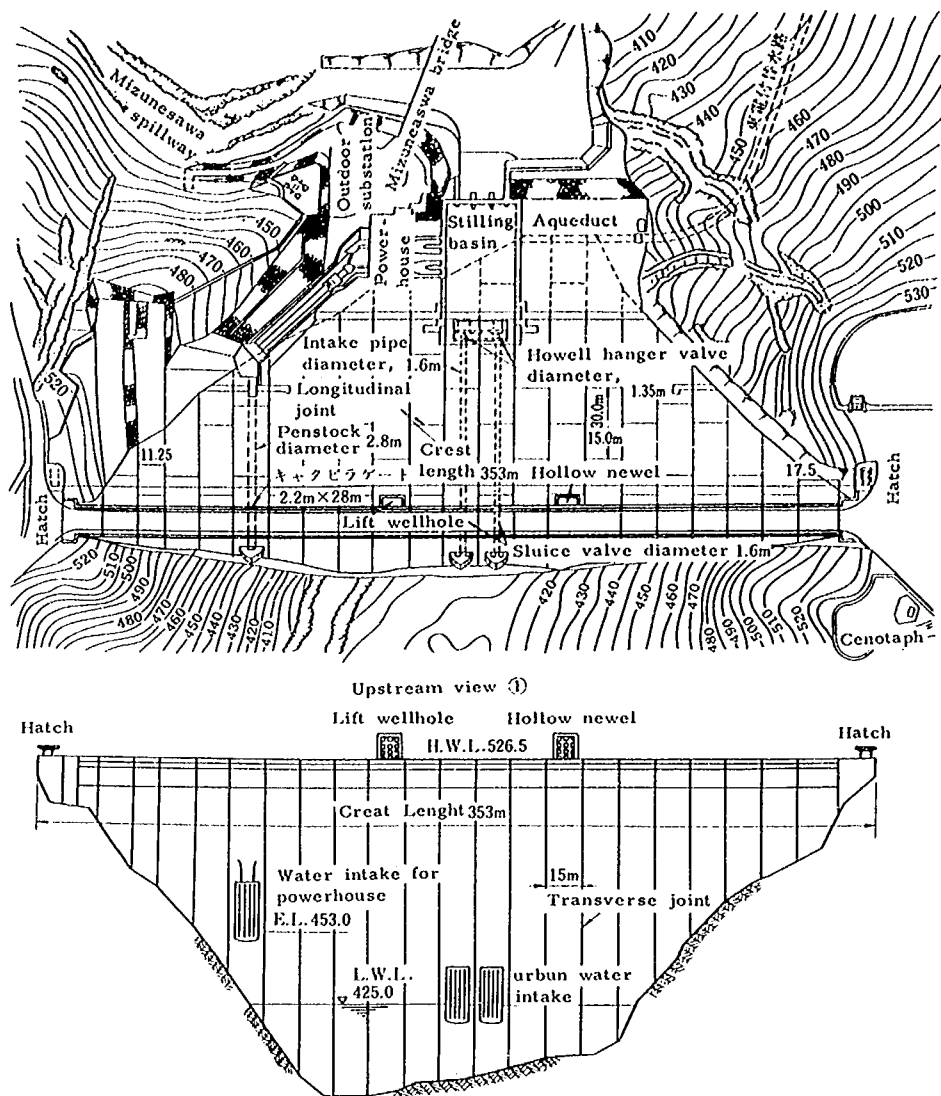


Fig. 26 Layout of joints (example)

Where the block size does not correspond to the batcher plant, the concrete placing period will be prolonged to disadvantage, as intermittent concrete placing leads to the development of cold joints (non-artificial joints).

If a given dam is high or the bedrock is poor in quality, the thickness of the dam must naturally be increased, which in turn requires longitudinal joints. The factors that determine the spacing of longitudinal joints are essentially the same as with the transverse joints. While in almost all

cases, the spacing of the transverse joints is around 15 m, the longitudinal joints are spaced 15 m to 50 m, This difference is due to the following reasons,

- 1) As the gravity dam follows a two-dimensional design, the separation in the dam body due to transverse joints is no matter theoretically, But the longitudinal joints develop discrete parts in the cross section of the dam body, and it should be avoided as far as we can.
- 2) Cracks parallel to the dam axis are far less significant from the viewpoint of watertightness than those normal to the dam axis, and are safer as they tend to close automatically under water pressure,
- 3) Well-designed cooling practice can increase the spacing of longitudinal joints,
- 4) Prior construction cases show that the opening of the longitudinal joint is smaller compared with that of the transverse joint, and it is necessary to widen the spacing of longitudinal joints for successfully grouting,
- 5) Intersection between longitudinal joints and outlet pipes, etc, is not desirable, and the number of longitudinal joints should be minimized,

There is an actual case where a dam with a maximum width of 70 m was constructed by placing concrete in layers without providing longitudinal joints, but it has not been reported for random cracks to generate, So at present, it is generally practised to provide longitudinal joints at an interval of about 40 m,

c. Precautions on arrangement of joints

The transverse joint is provided to rise from the bedrock perpendicularly up to the crest to have a single plane parallel to the river flow direction, On the other hand, the arrangement of longitudinal joints need some consideration,

Usually, the longitudinal joints are arranged to stagger with the transverse joints, (See Fig, 27)

Where the upstream face of the dam is provided with a slope or is filleted, no longitudinal joint usually is provided in the upstream face. This is because water seepage is likely to make a discrete section in the dam at the longitudinal joint. Even if it is required to provide a longitudinal joint at such a place, it should be terminated at a position about 5 m below the upstream face. Where the longitudinal joint is exposed to the downstream face, it is usually turned up just short of the downstream face so that it

Table 12 Maximum block sizes for the existing dams

Name of dam	Type	Height (m)	Dam volume (m ³)	Block (m)	Lift height (m)	Artificial cooling method
Okutadami	Gravity	157.0	1,670,000	18x37.5	2.0	Pipe
Sakuma	Gravity	155.5	1,222,000	15x45	1.5	Pipe, Pre
Ogochi	Gravity	149	1,676,000	15x30	1.5	Pipe, Pre
Tagokura	Gravity	145.0	1,960,000	18x37.5	1.5	Pipe, Pre
Kusaki	Gravity	140.0	1,300,000	15x35	2.0	Pipe
Shimokubo	Gravity	129	1,193,000	15x25	1.5	Pipe
Ikari	Gravity	112	480,000	15x25	1.5	Pipe
Kazaya	Gravity	100.0	566,000	17x51	2.0	Pipe, Pre
Kuromatagawa No. 1	Gravity	91.0	289,000	15x72	1.5	Pipe, Pre
Ishidegawa	Gravity	87	423,000	15x30	1.5	Pipe
Sonohara	Gravity	76.5	207,000	15x70	1.5	Pipe
Nukabira	Gravity	76.0	470,000	15x70	1.5	Pipe
Kurobe No. 4	Arch	186.0	1,590,000	18x55	3.0	Pipe
Nagawado	Arch	155.0	700,000	16x35	2.0	Pipe
Yahagi	Arch	100	256,000	15x28	2.0	Pipe
Shorenji	Arch	82	120,000	15x24	2.0	Pipe
Takayama	Arch-gravity	67.0	185,000	15x36	1.5	Pipe

* Pipe : Pipe cooling
Pre : Precooling

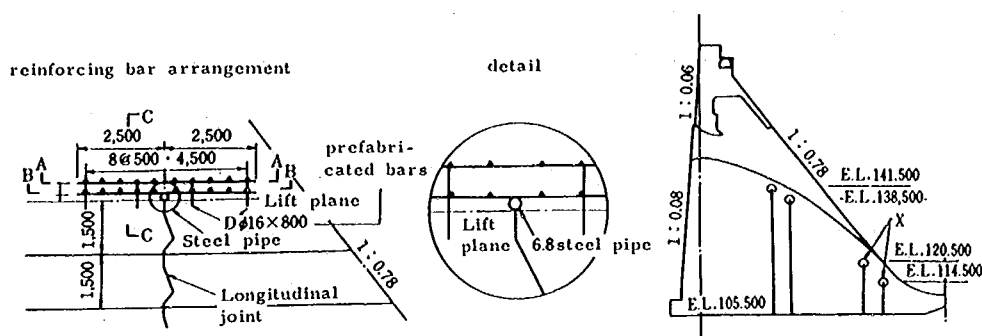


Fig. 27 Termination treatment of a longitudinal joint (example)

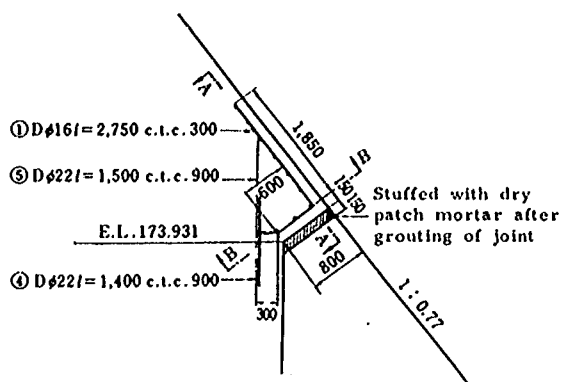


Fig. 28 Detailed view of reinforced termination treatment for longitudinal joint ($S = 1:30$)

will be normal to the downstream face and to eliminate corners.

It is a general practice to provide keys for the transmission of shearing force.

The following are the concepts for the joint grouting.

According to the current two-dimensional design of gravity dam, it is not always necessary to integrate the transverse joints in the direction of dam axis by grouting. However, the longitudinal joint needs to be integrated by grouting for the purpose of communicating the stress in the downstream direction.

There are many existing dams which used grouting for transverse joints. Currently, this practice was omitted. Theoretically, there is no need of providing keys for the transverse joints. In Japan, however, there are no cases which omitted the keys for transverse joints. The joint grouting and keys for the transverse joints may well be understood as a safety allowance. As already discussed, the joint grouting lets the dam behave three-dimensionally, and something of three-dimensional checkups will be necessary. If the keys alone are provided, the author thinks that there will be necessary. If the keys alone are provided, the author thinks that there will be no necessity to check up the dam three-dimensionally.

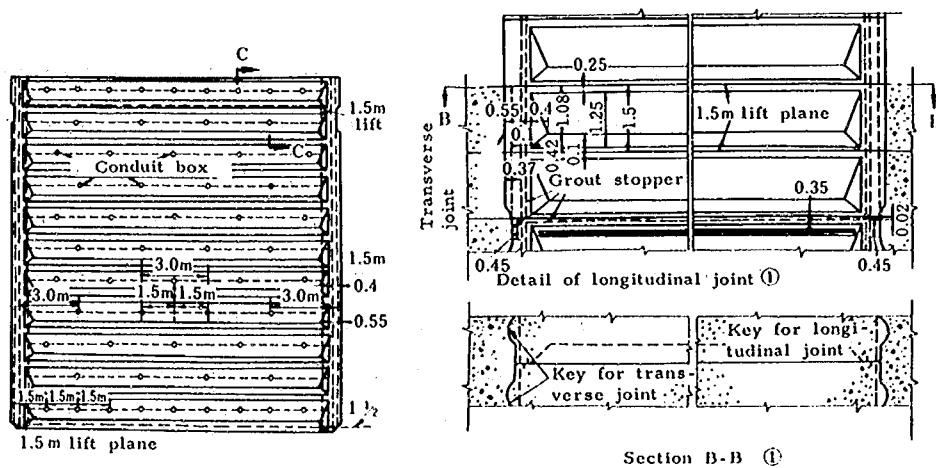
d. Design of keys

Usually, the joints are provided with keys for the purpose of ensuring reliable transmission of forces. For the transverse joints, perpendicular keys are provided as illustrated in Fig. 29. From the viewpoint of two-dimensional design, these keys are allowances. But, these keys are ex-

Figure 10 consists of two diagrams. The top diagram, labeled 'Section A-A (d)', shows a cross-section of a concrete structure with a grout stopper and a key for longitudinal joint. The bottom diagram, labeled 'Detailed view of transverse joint (d)', shows a longitudinal section of a transverse joint between adjacent blocks, including labels for 'Upstream face', 'Downstream face', 'Grout stopper', 'Key for longitudinal joint', and 'Longitudinal joint between adjacent blocks'.

Keys and joint grouting offer a sense of security.

As shown in the following, the longitudinal joints usually are provided with horizontal keys.



It is necessary to transmit the perpendicular shearing force by way of this key. As the grouted material of joints shows a less resistance of shearing force, the horizontal key surface should be arranged normal to the primary principal stress for the purpose of protecting the joint from the shearing stress. In this case, the direction of the primary principal stress

varies from place to place in the dam ; if we strictly satisfy this, we have to make the horizontal key, different in shape and size at each place. But this is troublesome work for forming, it is generally practised to use the shape near the toe where the perpendicular shearing force is maximized as representative of all the sections.

4.7.2 Design of a gallery

a. general

Galleries are installed inside the dam body for various purposes. They are installed in the dam axis direction and the direction normal to it. Any hollow parts, like galleries, in the dam body are unfavourable to the dam, and the galleries should be installed with all possible care. The reasons why the gallery is necessary in the dam body are as follows.

- 1) Working space for installing the drains necessary for reducing the uplift.
- 2) Working space for grouting the foundation.
- 3) Working space for cooling and joint grouting.
- 4) Space for monitoring the development of cracks after construction.
- 5) Space for operating the outlet works.

Every gallery is used for multi-purposes. The following gives a briefing on the engineering considerations for the gallery with respect to these purposes.

The gallery designed for foundation grouting is installed parallel to the dam axis and near the dam surface, and is used also for draining work. Needless to say, the gallery is as spacious as capable of accommodating a drilling machine. According to the design standards, this gallery is required to be located twice its height or more apart from the upstream face of the dam. According to U.S.B.R., it is ruled that the gallery floor should be set 1.5 m or more apart from the bedrock and that the gallery should be separated from the upstream face of the dam by more than 6 m or at least 10% of the water depth at the level of gallery. The grouting to be carried out from this gallery is curtain grouting.

The gallery explained above is also used for draining work. Where the reduction of uplift is particularly important or where it is required to check the actual distribution of uplift with the design uplift, a crossing gallery to the dam axis (that is, parallel to the riverstream) is sometimes installed for draining purposes. In this case, also, there should be a space enough to accommodate a drilling machine.

The gate control room and its access gallery are determined by the size of related facilities, and usually need a considerably large space. Careful reinforcement is also required to protect the space from stress concentration. Where it is impossible to expose the pipings for cooling and joint grouting to the dam surfaces, these pipings are often run through this gallery. It should be borne in mind that the pipings for joint grouting are required to be set at a grouting lift of about 15 m. The gallery for monitoring the dam body (inspection gallery) is used to check for cracks and for the safety control and maintenance of the entire dam. The galleries already explained serve as an inspection gallery.

b. Reinforcement of a gallery

The surroundings of the gallery are subject to stress concentration, and require iron reinforcement, accordingly. In order to minimize the stress concentration, the gallery should preferably have a circular cross section. Not only for the sake of working space economy, but for the simplicity of construction work, the gallery cross section usually is made rectangular.

There are various methods for determining stress concentration. They include model test, photoelastic test, finite element method, and a theory concerning the stress concentration around a hole in a two-dimensional plane, etc. No theoretical solution has yet been obtained about the rectangular cross section. For a large hollow space, like gate control room, the two-dimensional assumptions are no longer practical. To handle this kind of problems, the photoelastic test and finite element method are generally accepted as an effective one. Compared with the photoelastic test, the finite element method is handier in calculating the stress. But the finite element method has a drawback in that. It is not so good to calculate concentrated stress by its nature. For the purpose of improving the accuracy of stress concentration analysis, it is required to fine down the meshes for analysis. For the arch dam, the gallery should be reinforced as required by the analysis of stress concentration around it.

In the case of gravity dams, and particularly when an intended gallery is not so large, the arrangement of reinforcement bars may be determined by following the examples of the existing dams of comparable sizes. There is a widely accepted theoretical method in determining the stress concentration around a circular cross section such as a pipe's. It assumes a case where stress is acting uniformly on an infinite plane having a hole.

Let us assume an infinite plane and a compressive stress σ acting uniformly on its two opposed sides. A theoretical solution for the stress around a circular hole in the plane is given by the following formula.

Referring to Fig. 31, assume the stresses σ_x and σ_y on the x- and y-axis which act in the directions normal to the x- and y-axis respectively. Then, these stresses are given as follows.

$$\begin{aligned}\sigma_y &= \frac{1}{2} \sigma \left(2 + \frac{x^2}{r^2} + 3 \frac{r^4}{x^4} \right) \\ \sigma_x &= \frac{1}{2} \sigma \left(\frac{x^2}{y^2} - 3 \frac{r^4}{y^4} \right)\end{aligned}\dots\dots\dots(35)$$

($x=r, y=0$),
($x=0, y=r$)

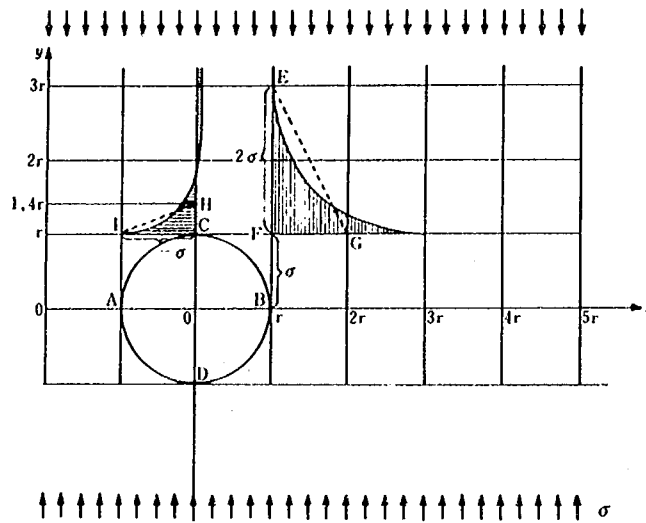


Fig. 31 Stress concentration around a hole

At the crown of the hole ($x=r, y=r$), the stresses σ_y and σ_x are calculated as follows.

$$\begin{aligned}\sigma_y &= 3\sigma \\ \sigma_x &= -\sigma\end{aligned}\dots\dots\dots(36)$$

The minus sign in σ_x denotes a tensile stress ; At the crown of the hole tensile stress works. The stress concentration diminishes quickly as it goes far away from the circular hole. As illustrated in Fig. 31, the excess stresses can practically be approximated by the triangles CHI and EFG. Taking F_t and F_c as the sums of excessive stresses in the y- and x-axis direction respectively, the following are determined.

$$\begin{aligned}F_t &= -0.2r \sigma \\ F_c &= r \sigma\end{aligned}\dots\dots\dots(37)$$

Where stresses act in both x- and y-axis directions the solution can be obtained by superimposing the solution for the x-axis and that for the y-axis

upon each other.

In practice, it is very difficult to apply the above theoretical method to the solution of stress concentration in the dam. But if a geometry is so simple as a circular conduit pipe in an arch dam, the stress concentration around the pipe can be determined by taking the principal stresses in the upstream and downstream faces determined in the dam design as axial stresses in the plane illustrated in Fig. 31.

In actuality, however, cases are few which permit approximation by two-dimensional plate stresses, and if it is desired to provide an unprecedentedly large hollow, necessary analysis has to be done.

4.7.3 Design of Seals

As already discussed, some opening develops between the blocks. In order to prevent leakage from the reservoir through this clearance, it is necessary to provide watertightness at a joint near the upstream face of the dam. We usually use a device consisting of various materials, and is called a seal.

Usually, the seal is buried while placing concrete. The seal must satisfactorily follow the opening of the joint without breakdown when the dam concrete has attained an ultimate steady-state temperature and it must also have sufficient durability.

The materials for seals consist of copper, stainless steel, synthetic materials like polyvinyl chloride and rubber. The widely used metal seals are Z type and M type illustrated in Fig. 32. Each of these types is designed to be able to follow the opening of the joint. Among those two types, M type is more capable of following opening.

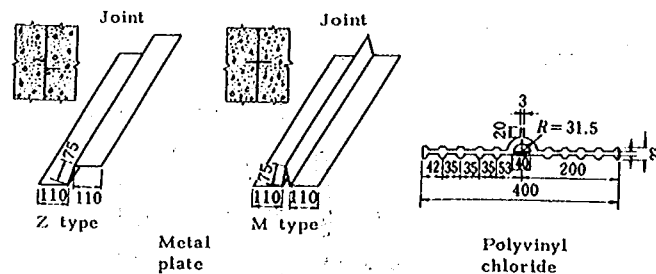


Fig. 32 Shapes of seals.

Where polyvinyl chloride is employed, it is usually designed as illustrated in Fig. 32.

When the transverse joint is to be grouted, another seals, grout stoppers

are necessary. In this case, polyvinyl chloride has been used increasingly in recent years. Polyvinyl chloride is less costly and easier to handle compared with metals. It can be rolled up for ease of transportation, and the number of tying can be minimized. But polyvinyl chloride is a little harder to tie than metals.

In case grouting is not applied to the transverse joints, small holes of seals directly lead to the seepage of water toward the downstream side. Therefore, seals of the transverse joints must be made with utmost care. The mistake often made is that the welding operator makes small holes and ignores them. In case grouting is not carried out for the transverse joints, the horizontal construction joints may become a weak point when the concrete at the upstream corner is defective in quality because of insufficiency of vibrating work and green cut, it may cause water leakage. The concrete placing and green cut at the upstream corner should be conducted with circumspection.

In some cases, joint drains are accommodated for the purpose of protecting leakage through weak horizontal construction joints. The gathered leakage water is usually guided into the gallery.

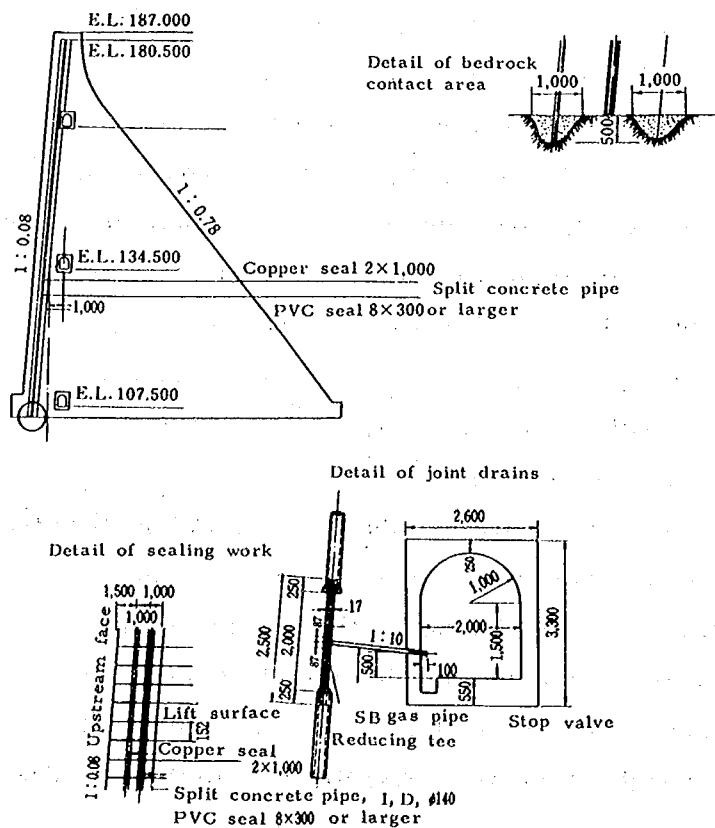


Fig. 33 An example of joint drains and double seals

4.7.4 Design of drain holes

The water passing through the grout curtain or leaking around the grout curtain works to lift the dam body (uplift). For the purpose of reducing the uplift and blocking up the leaks in the dam foundation, drain holes are provided right behind the grout curtain. The provision of drain holes is imperative as the uplift has a direct bearing on the stability of dam body. As for the arch dam, the influence of uplift is less than on the gravity dam. But there are many arch dams having drains. This is because the drains can not only reduce the uplift but also offer valuable information, such as water leakage rate and water color, necessary for the safety control and maintenance of the dam body.

Generally speaking, stable dams will have the water leakage reduced or stabilized at a constant level in a few years after start of water storage. If the leakage water gets coloured, it is suspected that something is wrong with the foundation (e.g., piping inside the foundation).

Another problem is how many drain holes be installed and what way they should be arranged for the purpose of fulfilling the design requirements for the gravity dam.

There is no established theory for handling this problem. But it is easily understood that essentially the quality of bedrock and the effect of grout curtain will be a paramount factor governing the design of drain holes. But it is almost impossible to discuss quantitatively how to reflect these conditions peculiar to each site in the layout of drain holes. For this reason, there is no way at present but to take recourse to the experience and knowledge of expert engineers who can design the drain hole system based on the layout and size of drain holes in the existing dams and the measured uplift and by taking into account the foundation conditions at a given site. Not that there is no standard layout of a kind at all. Here we will discuss the drain hole layout.

Usually, the number of drain hole rows to be installed is one. Where the foundation is poor, two rows or more may be installed. As regards the size of drain hole, it should be noted that if the diameter of the drain hole is too small, it will be encrusted with metallic salts freed from water and blocked up draining action ineffective. In this sense, the drain hole size should be large enough. Usually, the drain hole is sized to 46 mm to 76 mm and 66 mm in average diameter.

Another problem is how deep to dig drain holes into the foundation.

This is also concerned basically with the foundation conditions. According to the design criteria of U.S.B.R. 5), the depth of the drain hole is so as to be 20% to 40% in terms of water pressure (water depth). In Japan, nowadays the depth of a drain hole is about 5 m, it is shallower than the end of consolidation grouting holes. The prime concern is what spacing should be provided between drain holes. It is reported that the draining effect is improved none the better for the reduction of the spacing to as small a value as less than 1.5 m. Usually, therefore, the spacing is set at 5 to 7.5 m.

The drilling of the drain holes is usually carried out from the grouting gallery.

The time when to drill the drain holes is just after consolidation and curtain grouting. The concrete is blaced with guide pipes beforehand, and the drain holes are drilled with a drilling machine from inside the gallery. The drilled holes are each provided with a pipe and tipped with a cock and a pressure gauge for the purpose of checking the quantity of the leakage per drain hole and total leakage volume from all the drain holes. An example of layout is shown in detail in Fig. 34.

4.8 Design of Hollow Gravity Dams

4.8.1 Advantages of hollow gravity dams

The hollow gravity dam is provided with an upstream cutoff wall called the diamond head, and its inside is hollow. This structure has the following advantages,

- 1) Since the upstream slope of the dam is moderate (1 : 0.5 or so), the stability of dam body against slide and breakdown can be increased because of adding the weight of water and silt deposit on the upstream face.
- 2) Since the bedrock is exposed in the bottom of the cavity, the uplift on the dam body can be reduced,
- 3) By virtue of the advantages (1) and (2) above, the concrete volume for the dam body can be cut down without sacrificing the stability of the dam, (about 70% of the concrete consumption compared with a gravity dam)

This advantage is particularly significant where a high dam constructed in a wide valley,

- 4) The hollow gravity dam has a large area for dissipation of hydration

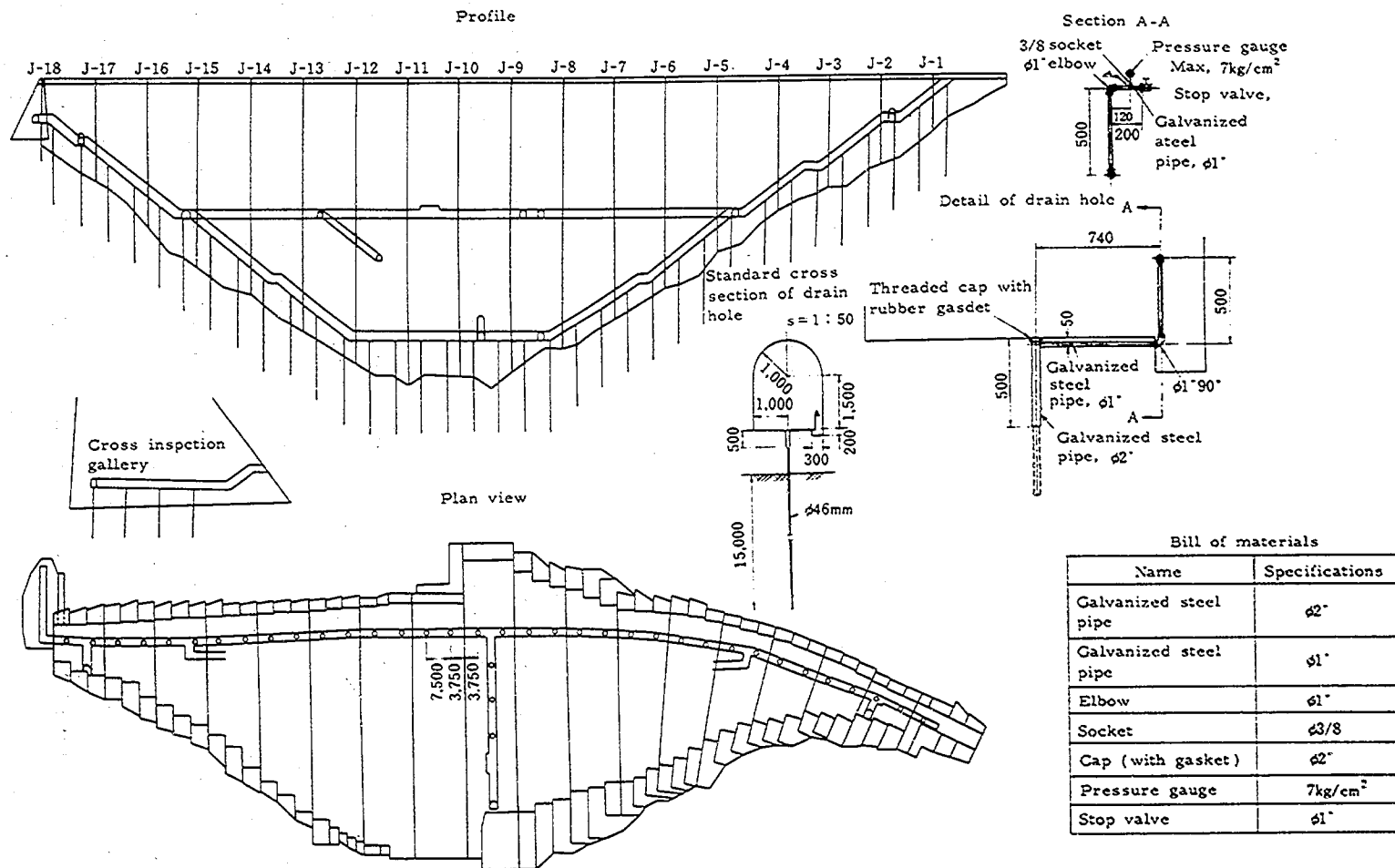


Fig. 34 An example of drain system

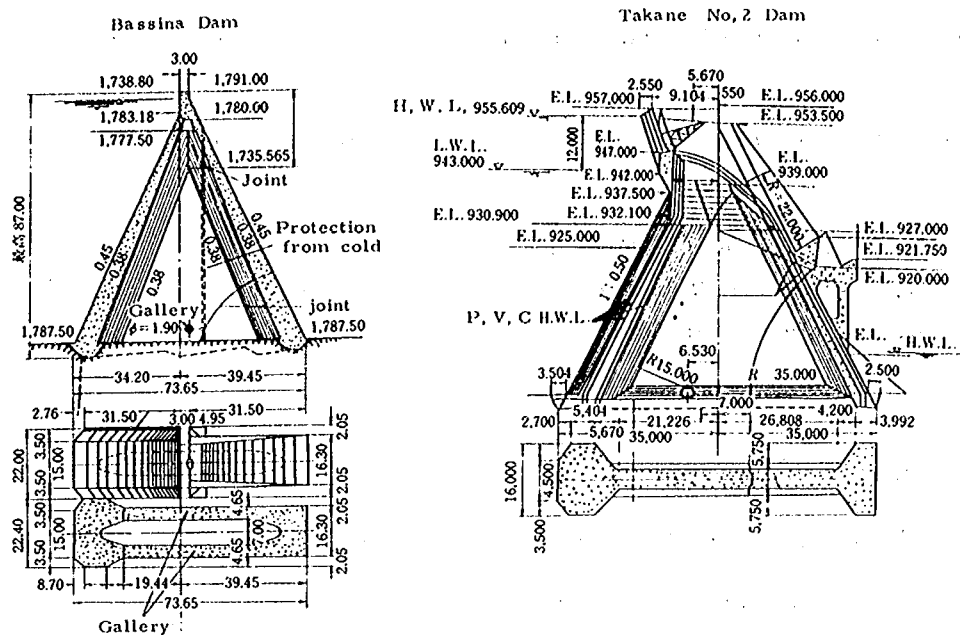


Fig. 35 An example of a hollow gravity dam

heat from the concrete. In addition, the concrete thickness is no more than several meters, and the concrete temperature is lowered to an ultimate steady-state value during the winter of the year of concrete placement without special cooling facilities which are indispensable for the solid gravity dam. If it is possible to avoid excessive temperature rise or steep temperature gradient during the period of construction, the costs for artificial cooling work can be saved, and at the same time the lift can be increased.

- 5) In the hollow gravity dam, each block is provided with a cavity. Even at a such site where much time and labor is necessary for the improvement of poor foundation, the drilling work and grouting work can be carried out from the cavity, not disturbing concrete placing work.
- 6) The cavity is useful in laying out penstock, outlet pipe and other various facilities, and also serves the inspection and maintenance after completion of the dam.
- 7) The placing volume of concrete can be almost equalized as the castable area changes little with level of construction lifts, and the scale and capacity of temporary facilities can be reduced, accordingly.
- 8) Since the distribution of point safety factors can be equalized, the safety factor for shear sliding can be increased substantially.

The drawbacks the hollow gravity dam has are as follows.

- 1) The engineering work is complicated, and the cost for forms becomes high.
- 2) The hollow gravity dam needs wide foundation, so excavation work is considerably large.
- 3) The flood control during construction work is difficult compared with the solid gravity dam.
- 4) Much time and labor is required for the internal cleaning of dam and removal of forms.
- 5) The hollow gravity dam lacks rigidity against seismic forces in the dam axis direction.

The merits and demerits of the hollow gravity dam have been explained. Of the disadvantages, items (1), (2) and (4) are too significant to overlook in these days when labor cost sometimes exceeds materials cost. Even with a substantial reduction on concrete volume, the total construction cost remains almost on a level with the solid gravity dam's. For this reason, few hollow gravity dams have been constructed in recent years.

Table 13 Major hollow gravity dams in Japan

Name of dam	Dam height (m)	Dam volume (10 ³ m ³)	Year of completion
Ikawa	103	460	1957
Ohmorigawa	72	135	1959
Hatanagi No. 1	125	587	1962
Yokoyama	81	292	1964
Takane No. 2	85	159	1969
Uchinokura	69	168	1973

The latest hollow gravity dam constructed in Japan is Uchinokura Dam which was completed in 1973.

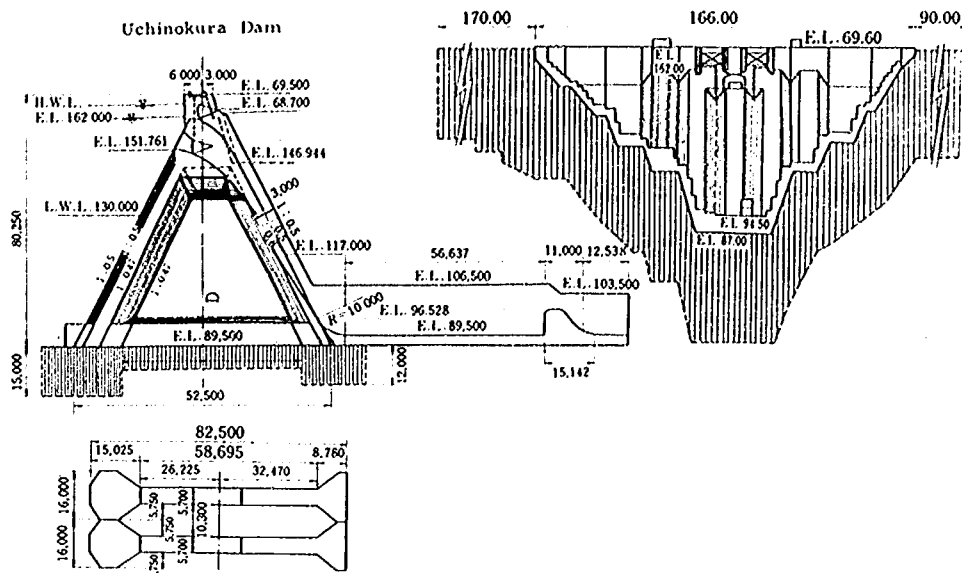
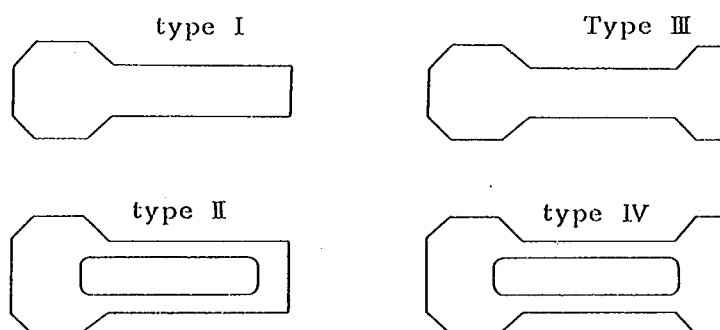


Fig. 36 An example of a hollow gravity dam

4.8.2 Horizontal section of hollow gravity dam

The hollow gravity dam is classified into the following four types according to the horizontal cross section of unit blocks used.



Types I and II have one web, while types III and IV have two. Types II and IV form a continuous wall on the downstream side, while types I and III make the downstream wall discontinuous.

The features of these types are enumerated below.

- 1) Comparison between closed downstream end types (II, IV) open downstream end types (I, III)

Unlike the open end types, the closed end types do not respond so quickly to the temperature change in the cavity. The upstream face of the diamond head is always submerged into water, and is little subjected to temperature change throughout the year. On the other hand, the

diamond head of the open end type experiences a temperature change of around 20°C cyclically between winter and summer. This repetitive temperature stress could sometimes result in generating fracture of concrete.

- 2) Comparison between two-web types (III, IV) and one-web types (I, II)
When a seismic force is given in the direction normal to the web, types III and IV are considered higher in stability than types I and II so far as a unit block is concerned. But there is no noticeable difference between them when the stability of the dam as a whole is evaluated.

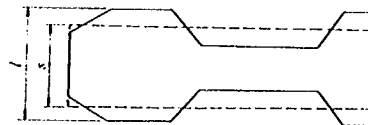
4.8.3 Design of a hollow gravity dam

The design conditions and loading of the hollow gravity dam are just the same as those of the solid gravity dam. The downstream slope is also designed based on the basic triangular cross section according to the same procedure as used in the designing of the gravity dam. What makes difference, however, is that the cavity must be taken into account as discussed below.

First, it is necessary to determine which of the horizontal cross sections should be employed.

When a horizontal cross section is selected, it is substituted by a gravity dam having an equivalent cross-sectional area as illustrated as follows ;

$$\frac{S}{l} = s : \text{width of the cross section of an equivalent gravity dam}$$



By adjusting the loads and others with factor "s", it is possible to carry out two-dimensional design for unity width quite the same manner as with the gravity dam. Adjustment must be done like below.

$$\text{Unit weight of concrete} \quad W_c' = W_c \times s \quad (39)$$

In the paragraphs relating to the gravity dam, we have already studied that the cross section of the dam is governed by the conditions relating to shear sliding rather than by the middle third conditions when the dam height becomes large.

In those cases, a study should be made to place concrete over the entire bottom area of the foundation in order to increase shearing resistance. It is often effective in spite that uplift becomes larger.

As regards the web, care should be taken not to change its thickness

abruptly within the horizontal cross section. This is because to prevent stress concentration at the connection between the diamond head and web. The thickness of the web has much to do with aseismic stability in the dam axis direction, and should not be made so small. As F.E.M. has made marked progress in recent years, the bar arrangement, etc. should be designed according to the analysis of stress distribution in the diamond head.

In the hollow gravity dam, the transverse joints are provided between the adjoining heads, and the keys are usually omitted. The longitudinal joints are shaped circular as shown Fig. 35.

5. DESIGN OF ARCH DAMS

5.1 Introduction

The arch dam takes up water pressure and communicates it to the bedrock on both banks by its arching function. It is an excellent structure that can use concrete strength to allowable limit. In an arch dam the stress developed in the concrete is large and the force the bedrock on either bank must take up is extremely large. Thus, the arch dam is required to be designed to fulfil the following requirements.

- 1) The stress developed in the arch dam body must not exceed the allowable stress intensity of concrete.
- 2) The foundation must be safe against shear breakdown.

The magnitude of stress in the arch dam will be well understood if compared with the stress intensity of a gravity dam of a comparable size. The gravity dam takes a maximum stress at the foot; when the dam height is about 100 m, it will be about 30 kg/cm^2 . On the other hand, the arch dam usually shows a design stress of about 80 kg/cm^2 . Besides, this high stress acts over a wide range of the dam body, and is not confined locally.

In 1959, an unprecedented flood attacked Riviera, France, claiming the lives of 500 people all of a sudden. The cause of the disaster was the breakdown of Malpasset Dam, an arch dam in the upper reach of Riviera.

The stability and safety of the arch dam is solely dependent on its foundations; every arch dam engineer has dedicated his efforts to evaluate the strength and deformation of the foundation bedrock as accurate as possible, corroborate its stability and to ensure the stability of an arch dam to be built on it.

Here we shall learn the design theory of arch dam, design procedures and stress analysis method briefly, and then safety engineering considerations for the foundation bedrock and a foundation stability analysis method.

5.2 Theory of Arch

As will be explained later, the arch dam is considered a combination of an arch element and a cantilever element. Basically, however, it exists for its arching function. Here we shall see fundamental formula for determining the arch elements.

Let us assume that the thrust, bending moment and shearing force at the crown of an arch are H_0 , M_0 and V_0 , respectively, that the

corresponding forces at a given cross section are H , M and V , respectively, and then that the Rotation, radial displacement and tangential displacement at the crown are $(\Delta\theta)_0$, $(\Delta\gamma)_0$ and $(\Delta S)_0$, respectively. Then, the displacement of the crown due to deformation of part ds is given by the following formula. (See Fig. 37)

$$\left. \begin{aligned} d(\Delta\theta)_0 &= \frac{M ds}{E_c I} \\ d(\Delta\gamma)_0 &= \frac{My ds}{E_c I} - \frac{H}{E_c A} \sin \phi ds + \frac{KV}{E_c A} \cos \phi ds \\ d(\Delta S)_0 &= -\frac{Mx ds}{E_c I} - \frac{H}{E_c A} \cos \phi ds - \frac{KV}{K_c A} \sin \phi ds \end{aligned} \right\} \dots\dots\dots (40)$$

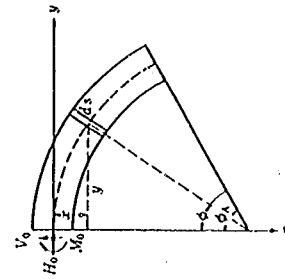


Fig. 37

Let us now assume the following,

- α ; coefficient of rotation of bedrock due to the bending moment at the abutment
- α_2 ; coefficient of radial displacement of bedrock due to bending moment at the abutment
- γ ; coefficient of radial displacement of bedrock due to the shearing force at the abutment
- β ; coefficient of tangential displacement of bedrock due to thrust at the abutment

Then, the displacement of the crown due to the deformation on the lefthand bank is given by the following formula,

$$\left. \begin{aligned} (\Delta\theta)_0 &= \int_0^{\phi_A} \frac{M ds}{E_c I} + {}_L M_A \alpha + {}_L V_A \alpha_2 \\ (\Delta\gamma)_0 &= \int_0^{\phi_A} \left\{ \frac{My}{E_c I} - \frac{H}{E_c A} \sin \phi + \frac{KV}{G_c I} \cos \phi \right\} ds \\ &\quad + ({}_L M_A \alpha + {}_L V_A \alpha_2) {}_L y_A \\ &\quad + ({}_L M_A \alpha_2 + {}_L V_A \gamma) \cos \phi_A - {}_L H_A \beta \sin \phi_A \\ (\Delta S)_0 &= \int_0^{\phi_A} \left\{ -\frac{Mx}{E_c I} - \frac{H}{E_c A} \cos \phi - \frac{KV}{G_c I} \sin \phi \right\} ds \\ &\quad - ({}_L M_A \alpha + {}_L V_A \alpha_2) {}_L x_A \\ &\quad - ({}_L M_A \alpha_2 + {}_L V_A \gamma) \sin \phi_A - {}_L H_A \beta \cos \phi_A \end{aligned} \right\} \dots\dots\dots (41)$$

Where,

$$\left. \begin{aligned} M &= M_0 + H_{0x} + V_{0y} - M_L \\ H &= H_0 \cos \phi - V_0 \sin \phi + H_L \\ V &= H_0 \sin \phi + V_0 \cos \phi - V_L \end{aligned} \right\} \dots\dots\dots (42)$$

in which M_L , H_L and V_L are the bending moment, thrust and shearing force due to the external load acting on the section from the crown and a given point,

By substituting Eq, (42) into Eq, (41), we obtain the following formula (43),

$$\left. \begin{aligned} (\Delta \theta)_0 &= {}_L A_1 M_0 + {}_L B_1 H_0 + {}_L C_1 V_0 - {}_L D_1 \\ (\Delta \gamma)_0 &= {}_L C_1 M_0 + {}_L B_2 H_0 + {}_L C_2 V_0 - {}_L D_2 \\ (\Delta S)_0 &= {}_L B_1 M_0 - {}_L B_3 H_0 - {}_L B_2 V_0 + {}_L D_3 \end{aligned} \right\} \dots\dots\dots (43)$$

Where,

$$\left. \begin{aligned} {}_L A_1 &= \int_0^{S_A} \frac{ds}{E_c I} + \alpha \\ {}_L B_1 &= \int_0^{S_A} \frac{x ds}{E_c I} + \alpha \cdot x_A \cdot \alpha_2 \cdot \sin {}_L \phi_A \\ {}_L C_1 &= \int_0^{S_A} \frac{y ds}{E_c I} + \alpha \cdot y_A + \alpha_2 \cdot \cos {}_L \phi_A \\ {}_L B_2 &= \int_0^{S_A} \frac{xy ds}{E_c I} + \frac{1}{2} (3 + 5 \nu_c) \int_0^{S_A} \frac{\sin \phi \cdot \cos \phi}{E_c A} ds \\ &\quad + \alpha \cdot x_A \cdot y_A - \beta \sin {}_L \phi_A \cdot \cos {}_L \phi_A + \gamma \sin {}_L \phi_A \cdot \cos {}_L \phi_A \\ &\quad + \alpha_2 \cdot y_A \cdot \sin {}_L \phi_A + \alpha_2 \cdot x_A \cdot \cos {}_L \phi_A \\ {}_L C_2 &= \int_0^{S_A} \frac{y^2 ds}{E_c I} + \int_0^{S_A} \frac{\sin^2 \phi + \frac{5}{2} (1 + \nu_c) \cos^2 \phi}{E_c A} ds \\ &\quad + \alpha \cdot y_A^2 + \beta \cdot \sin^2 {}_L \phi_A + \gamma \cdot \cos^2 {}_L \phi_A + 2 \alpha_2 \cdot y_A \cdot \cos {}_L \phi_A \\ {}_L B_3 &= \int_0^{S_A} \frac{x^2 ds}{E_c I} + \int_0^{S_A} \frac{\cos^2 \phi + \frac{5}{2} (1 + \nu_c) \sin^2 \phi}{E_c A} ds \\ &\quad + \alpha \cdot x_A^2 + \beta \cdot \cos^2 {}_L \phi_A + \gamma \cdot \sin^2 {}_L \phi_A + 2 \alpha_2 \cdot x_A \cdot \sin {}_L \phi_A \\ {}_L D_1 &= \int_0^{S_A} \frac{M_L}{E_c I} ds + {}_A M_L \cdot \alpha + {}_A V_L \cdot \alpha_2 \\ {}_L D_2 &= \int_0^{S_A} \frac{M_L \cdot x}{E_c I} ds + \int_0^{S_A} \frac{H_L \cdot \sin \phi + \frac{5}{2} (1 + \nu_c) V_L \cdot \cos \phi}{E_c A} ds \\ &\quad + {}_A M_L \cdot \alpha \cdot y_A + {}_A H_L \cdot \beta \cdot \sin {}_L \phi_A + {}_A V_L \cdot \gamma \cdot \cos {}_L \phi_A \\ &\quad + {}_A M_L \cdot \alpha_2 \cdot \cos {}_L \phi_A + {}_A V_L \cdot \alpha_2 \cdot y_A \\ {}_L D_3 &= \int_0^{S_A} \frac{M_L \cdot x}{E_c I} ds + \int_0^{S_A} \frac{H_L \cdot \cos \phi + \frac{5}{2} (1 + \nu_c) V_L \cdot \sin \phi}{E_c A} ds \\ &\quad + {}_A M_L \cdot \alpha \cdot x_A - {}_A H_L \cdot \beta \cdot \cos {}_L \phi_A + {}_A V_L \cdot \gamma \cdot \sin {}_L \phi_A \\ &\quad + {}_A M_L \cdot \alpha_2 \cdot \sin {}_L \phi_A + {}_A V_L \cdot \alpha_2 \cdot x_A \end{aligned} \right\} \dots\dots\dots (44)$$

In the same way, the displacement due to the deformation on the righthand bank side is given as follows,

$$\left. \begin{aligned} M &= M_0 + H_{0x} + V_{0y} - M_R \\ H &= H_0 \cos \phi - V_0 \sin \phi + H_R \\ V &= -H_0 \sin \phi - V_0 \cos \phi - V_R \end{aligned} \right\} \dots\dots\dots (45)$$

Hence,

$$\left. \begin{aligned} (\Delta \theta)_0 &= -{}_R A_1 \cdot M_0 - {}_R B_1 \cdot H_0 - {}_R C_1 \cdot V_0 + {}_R D_1 \\ (\Delta \gamma)_0 &= -{}_R C_1 \cdot M_0 - {}_R B_2 \cdot H_0 - {}_R C_2 \cdot V_0 + {}_R D_2 \\ (\Delta S)_0 &= +{}_R B_1 \cdot M_0 + {}_R B_3 \cdot H_0 + {}_R B_2 \cdot V_0 - {}_R D_3 \end{aligned} \right\} \dots\dots\dots (46)$$

Where,

$$\left. \begin{aligned} {}_R A_1 &= \int_{S_A}^0 \frac{ds}{E_c I} + \alpha \\ {}_R B_1 &= \int_{S_A}^0 \frac{x ds}{E_c I} + \alpha \cdot {}_R x_A - \alpha_2 \cdot \sin {}_R \phi_A \\ {}_R C_1 &= \int_{S_A}^0 \frac{y ds}{E_c I} + \alpha \cdot {}_R y_A - \alpha_2 \cdot \cos {}_R \phi_A \\ {}_R B_2 &= \int_{S_A}^0 \frac{x y ds}{E_c I} + \frac{1}{2} (3 + 5 \nu_c) \int_{S_A}^0 \frac{\sin \phi \cdot \cos \phi}{E_c A} ds \\ &\quad + \alpha \cdot {}_R x_A \cdot {}_R y_A - \beta \cdot \sin {}_R \phi_A \cdot \cos {}_R \phi_A + \gamma \cdot \sin {}_R \phi_A \cdot \cos {}_R \phi_A \\ &\quad - \alpha_2 \cdot {}_R y_A \cdot \sin {}_R \phi_A - \alpha_2 \cdot {}_R x_A \cdot \cos {}_R \phi_A \\ {}_R C_2 &= \int_{S_A}^0 \frac{y^2 ds}{E_c I} + \int_{S_A}^0 \frac{\sin^2 \phi + \frac{5}{2}(1 + \nu_c) \cos^2 \phi}{E_c A} ds + \alpha \cdot {}_R y_A^2 \\ &\quad + \beta \cdot \sin^2 {}_R \phi_A + \gamma \cdot \cos^2 {}_R \phi_A - 2 \alpha_2 \cdot {}_R y_A \cdot \cos {}_R \phi_A \\ {}_R B_3 &= \int_{S_A}^0 \frac{x^2 ds}{E_c I} + \int_{S_A}^0 \frac{\cos^2 \phi + \frac{5}{2}(1 + \nu_c) \sin^2 \phi}{E_c A} ds + \alpha \cdot {}_R x_A^2 \\ &\quad + \beta \cdot \cos^2 {}_R \phi_A + \gamma \cdot \sin^2 {}_R \phi_A - 2 \alpha_2 \cdot {}_R x_A \cdot \sin {}_R \phi_A \\ {}_R D_1 &= \int_{S_A}^0 \frac{M_R}{E_c I} ds + {}_A M_R \cdot \alpha + {}_A V_L \cdot \alpha_2 \\ {}_R D_2 &= \int_{S_A}^0 \frac{M_R \cdot y}{E_c I} ds + \int_{S_A}^0 \frac{H_R \cdot \sin \phi - \frac{5}{2}(1 + \nu_c) V_R \cdot \cos \phi}{E_c A} ds \\ &\quad + {}_A M_R \cdot \alpha \cdot {}_R y_A + {}_A H_R \cdot \beta \cdot \sin {}_R \phi_A - {}_A V_R \cdot \gamma \cdot \cos {}_R \phi_A \\ &\quad + {}_A M_R \cdot \alpha_2 \cdot \cos {}_R \phi_A + {}_A V_R \cdot \alpha_2 \cdot {}_R y_A \\ {}_R D_3 &= \int_{S_A}^0 \frac{M_R \cdot x}{E_c I} ds - \int_{S_A}^0 \frac{H_R \cdot \cos \phi + \frac{5}{2}(1 + \nu) V_R \cdot \sin \phi}{E_c A} ds \\ &\quad + {}_A M_R \cdot \alpha \cdot {}_R x_A - {}_A H_R \cdot \beta \cdot \cos {}_R \phi_A - {}_A V_R \cdot \gamma \cdot \sin {}_R \phi_A \\ &\quad - {}_A M_A \cdot \alpha_2 \cdot \sin {}_R \phi_A + {}_A V_A \cdot \alpha_2 \cdot {}_R x_A \end{aligned} \right\} \dots\dots\dots (47)$$

Eqs, (45) and (47) are identical, Thus we can simplify the terms as follows,

$$\left. \begin{aligned} {}_L A_1 + {}_R A_1 &= A_1 \\ {}_L B_1 + {}_R B_1 &= B_1 \\ {}_L C_1 + {}_R C_1 &= C_1 \\ {}_L B_2 + {}_R B_2 &= B_2 \\ {}_L C_2 + {}_R C_2 &= C_2 \\ {}_L B_3 + {}_R B_3 &= B_3 \\ {}_L D_1 + {}_R D_1 &= D_1 \\ {}_L D_2 + {}_R D_2 &= D_2 \\ {}_L D_3 + {}_R D_3 &= D_3 \end{aligned} \right\} \dots\dots\dots (48)$$

Thus, we finally obtain the following formula,

$$\left. \begin{aligned} A_1 M_0 + B_1 H_0 + C_1 V_0 &= D_1 \\ C_1 M_0 + B_2 H_0 + C_2 V_0 &= D_2 \\ B_1 M_0 + B_3 H_0 + B_2 V_0 &= D_3 \end{aligned} \right\} \dots\dots\dots (49)$$

By solving Eq, (49), M_0 , H_0 and V_0 can be determined, and M , H , and V at a desired point can thus be determined by making use of Eqs, (42) and (45). The displacement of the crown can be determined from Eq, (43). The displacement at an arbitrarily selected point on the lefthand bank side can be determined from Eq, (44), provided that the lower limit of the integral sign in Eq, (44) is rewritten as S , the point (x, y, ϕ) is transformed into a point according to a system of coordinates having the normal to the arch center line at the given point as the x -axis and the tangential line as the y -axis, and that M_0 , H_0 and V_0 in Eq, (43) are taken as M , H and V . The displacement at an arbitrarily selected point on the righthand bank side can also be determined by transforming Eqs, (46) and (47) just the same manner as above,

In the case of a monocentric, uniform-thickness arch, I and A are constant, $ds = r d\phi$, $x = r(1 - \cos \phi)$, and $y = r \sin \phi$. In addition, M_L , H_L , V_L , M_R , H_R , and V_R are given by simple trigonometric functions. Namely, Eqs, (44) and (47) become solvable trigonometric functions,

In the case of nonuniform-thickness arch, I and A are no longer constant, and M_L , H_L , V_L , etc, cannot be expressed by simple trigonometric functions and are hard to solve theoretically. For this reason, various approximation methods are tried,

5.3 Stress Analysis of An Arch Dam

Since the arch dam makes the maximum use of the strength the concrete can display, it is of paramount importance to precisely determine the stress in the dam body. The methods of determining the stress in the arch dam are largely classified into the following three,

- 1) A method in which an arch dam is taken as a lattice structure consisting of arch elements and cantilever elements, (Trial Load Method)
- 2) A method based on a theory of elasticity,
- 3) A method using a model test,

The Trial Load Method developed by the U.S. Bureau of Reclamation has been only one to analyze stress distribution in an arch dam for a long time.

The three-dimensional finite element method is only a method available under item (2) above. Before entering into detailed discussion of these stress analysis methods, we shall see a prevailing design procedure for arch dams as the stress analysis methods should be implemented effectively

in the process of arch dam design procedure.

The explanation here assumes that a geological map for a given dam site is already on hand.

The arch dam design procedures consist of two major processes ; one in which efforts are made to determine a safe, economical shape of the arch dam, and the other in which efforts are made to locate suitable abutment for ensuring the stability of the foundation. But, these processes are not conducted severally, but should be conducted in close coordination.

- 1) First, the central angle and thickness of arch dam and the shape of cantilever are determined by referring to the design of the existing dams of comparable sizes.
- 2) With reference to a given geological plan view, the shape of the dam is determined on the assumption of its abutment.
- 3) A stress analysis is then conducted on that dam shape to calculate the stress in the dam body and the thrust acting upon the dam foundation.
- 4) The calculated stresses are checked with the allowable stress, and the dam geometry is corrected as required.

The stability of the foundation is evaluated according to the calculated thrust, and the depth of abutment is corrected as required. Then, a second design draft is prepared.

The steps above are repeated according to the new draft until the design plans are boiled down.

Finally, the best plan which fulfills the safety and stability requirements of the dam foundation and at the same time offers the maximum economy is singled out of alternative plans. In this way, the actual designing work is carried out on a trial-and-error method. For the purpose of determining the optimum arch dam design, it is necessary to elucidate qualitative and quantitative relationships among the various factors that govern the design of the arch dam. Among those, the following factors have the most significant effect on the design and stress conditions of the arch dam.

- 1) Central angle of arch.
- 2) Thickness of an arch dam.
- 3) Shape of crown cantilever.

An example will be enough to show how complicated the design processes are : We (Dam Structure Division) had to discuss as many as 30 cases before reaching a final plan. In each case, the effects of each factor are systematically studied. What stress analysis method should we employ

in the design processes, then? We use Trial Load Method in this process, and for the final plan, it is recommended to check up the stresses according to a finite element method based on strict elasticity theory or to a model test method.

The reasons for this recommended procedure are as follows.

- 1) The cost for analysis is less.

Trial Load Method : ¥50,000 to 100,000/case

Three-dimensional finite element method : ¥5,000,000/case

Model test method : ¥30,000,000/case

- 2) Trial Load Method is the poorest of all so far as the accuracy of analysis is concerned. Trial Load Method is an approximation itself, and has some specific tendency in its results. At present, this tendency has been well understood by comparing the difference between Trial Load Method and a model test method as to the results of analyses on the designs of the existing arch dams. Accordingly, the analytical results obtained by Trial Load Method can be interpreted to compensate for the poor accuracy, allowing us a practical engineering judgement,

5.3.1 Trial Load Method

The underlying principle of this method is to consider a complex structure of arch dam as a combination of structurally analyzable systems.

This method is subclassified into two :

- 1) arch cantilever method, and
- 2) tilted arch method.

Of the two; the tilted arch method has not yet been applied to actual dam design in Japan; and is omitted here. We shall discuss the arch cantilever method alone, accordingly.

In the arch cantilever method, the arch dam is reckoned as a structure using a horizontal arch and a perpendicular cantilever beam in combination to split loads.

Now let us assume that the water pressure is "p", the load to be borne by the arch is "Pa" and the load to be borne by the cantilever beam is "Pc". Then, the following formula is established.

$$p = p_a + p_c \dots\dots\dots (50)$$

The load will be distributed on condition that the deformation of the horizontal arch due to "Pa" and that of the cantilever beam must take place

equally at any point of the dam body.

Then, there arises a question about the nature of deformation and the condition adjusting the deformation all over the dam.

It is practically impossible to make all the types of deformation go well with all the points. It is therefore necessary to prune down insignificant elements and simplify the calculation to the extent not to affect the accuracy of analysis.

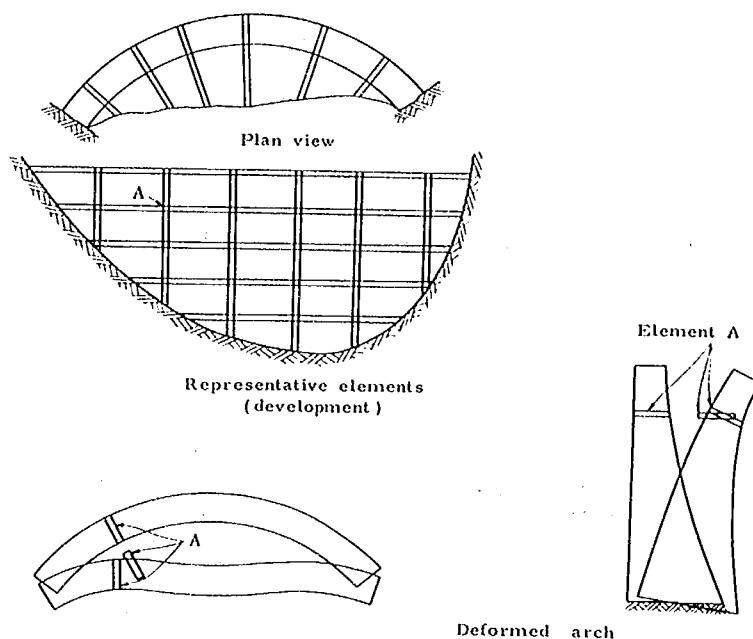


Fig. 38 Deformations of an arch dam

When an arch dam is divided into an arch and a cantilever beam, the following deformations are conceivable.

- 1) Radial deflection and rotation on the radial axis.
- 2) Tangential deflection and rotation on the tangential axis.
- 3) Perpendicular deflection and rotation on the perpendicular axis.

Of the most governing importance among them is the radial displacement. An analytical method designed to adjust the radial displacement (deflection) alone with the arch cantilever element is called the radial adjustment calculation.

Another method in which the linear displacements in the radial and tangential directions and the rotations on the tangential axis and perpendicular axis out of the six displacements are adjusted is called the complete adjustment calculation. In the complete adjustment calculation, the linear displacement in the perpendicular direction and the rotation displacement

around the radial axis are often left out of adjustment since they are negligibly small compared with other displacements.

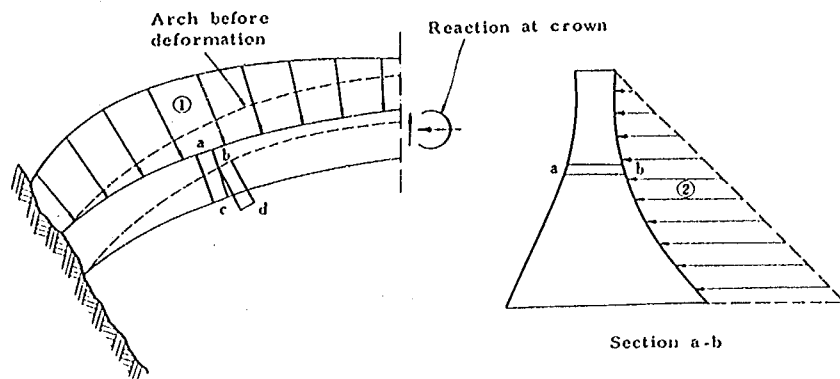


Fig. 39 Application of horizontal load

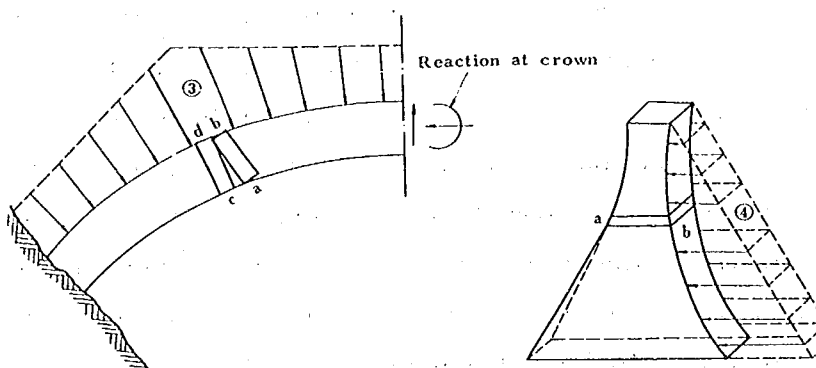


Fig. 40 Radial adjustment

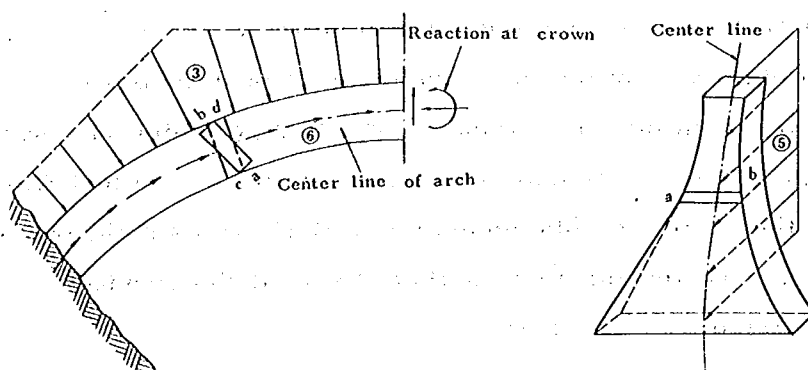


Fig. 41 Tangential adjustment

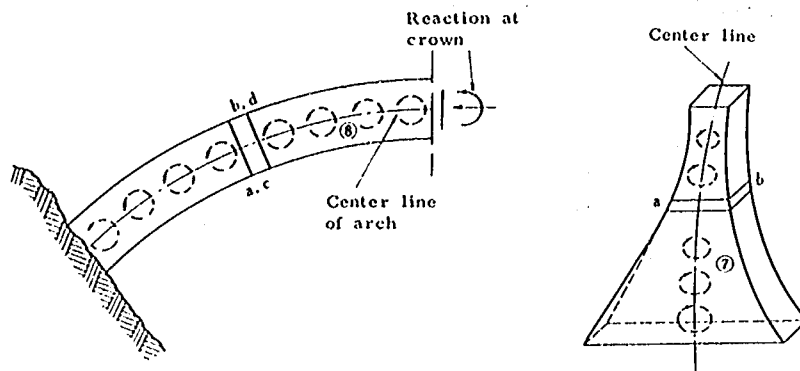


Fig. 42 Torsional adjustment

What should be borne in mind here is the fact that an assumed load is necessary for the purpose of adjusting each elementary displacement of the cantilever beam. For example, as illustrated in Fig. 41, there is necessary a load acting in the arch direction (tangential direction) for the purpose of adjusting the tangential displacement. Actually, however, this load is nonexistent, and it is required to apply on the cantilever beam a load equal in size, but opposite in direction to the aforesaid load. Two equal, but opposed loads to be applied on the two structural elements for the purpose of analytical convenience are called the self-balancing loads, and are often used in this kind of analysis. Another problem is at what point the adjustment of these displacements is to be carried out.

The larger the number of adjusting points is, the better will the accuracy of analysis become. But, this will just be realized at the cost of money and time. The recent progress in computer science, however, has made it feasible to carry out numerical computation. Prior to the advent of electronic computer, symmetrical arch dams used to be designed, only for the purpose of reducing the number of analytical points by symmetrical arrangement. In the past, so-called crown cantilever method was often employed which the radial displacements of arch and cantilever beam were adjusted only at crown on assumption that the load distribution is kept constant in the horizontal direction.

Since the load distribution in the horizontal direction does not vary largely, there is no need of locating so many adjusting points. The increase in the number of adjusting points will not do much toward improvement in the accuracy of analysis. In the stress analysis program in the possession of the Dam Structure Division which is based on load splitting method, the

points at which stress analysis is carried out by adjusting the displacements are as illustrated in Fig. 43.

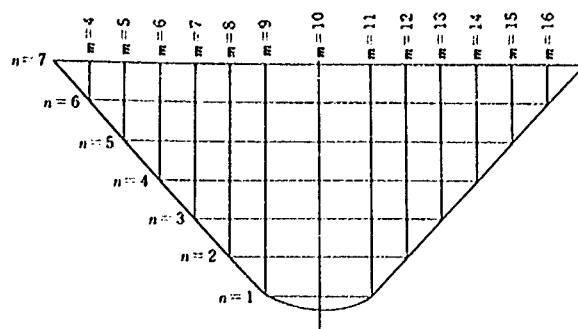


Fig. 43 Elements of an arch dam

As illustrated, the number of arch elements is 6, while the cantilever elements are arranged on the maximum cross section of the arch dam and at the abutments of each arch element.

For the adjustment of the displacement of cantilevers, there are two methods-Trial Load Method and the method to solve the equation simultaneously. Trial Load Method⁸⁾ was developed by the U.S. Bureau of Reclamation, and has been used for a long time until computers become in use for stress analysis. It is in the assumption of the deformation of foundation bedrock and assignment of unit load that the trial load method is characterized. To adjust the deflection at each intersection is nothing but to solve simultaneously equations with the loads to be borne by one element at each intersection being taken as an unknown quantity. In actuality, however, the solution of a system of simultaneous equations in many unknowns is not so easy a task. With reference to Fig. 43, for example, if a given arch dam is a symmetrical one and if it is only necessary to carry out radial adjustments, we must to solve a system of simultaneous equations in 28 unknowns. Prior to the development of electronic computer, this was almost impossible. In Trial Load Method, therefore, the deformation of each element is calculated on the assumption of some load splitting, and this load splitting is adjusted depending on the differences in deformation between arch element and cantilever element. Then, a second calculation of displacements is carried out. The steps are repeated until the load is split so that the displacement of the arch elements balance those of the cantilever elements. Thus, the designer requires some experience in assigning the load distribution. If he acquires experience and skill, he will be able to shake down

the load distribution in a minimum number of repetitions. The most important thing in this method is the flair to perceive the effect of a change in the load distribution on the degree of change in displacement and the ability to figure out the displacement according to a given load distribution. The Trial Load Method is also featured by many charts and diagrams available for the convenience of calculations. Nevertheless, it is not rarely that the design takes several years so far as it is based on Trial Load Method. The number of cases calculated by Trial Load Method so far was comparatively less, also. Today, we can enjoy the benefits of computer, and can easily solve simultaneous equations. It is quite natural that analysis method has changed from Trial Load Method to simultaneous equation method.

Now let us see the simultaneous equation method for stress analysis, together with the assumption used in the load splitting method which has been completed according to Trial Load Method.

For the sake of simplicity, let us now consider a symmetrical arch dam.

See the illustration below. Let us assume that the radial deflection of the cantilever at point (m, n) is $cr(\Delta\gamma)_{m,n}$, that the load shared by the cantilever at point (m, n) is $crP_{m,n}$, and that the load shared by the arch at point (m, n) is $arP_{m,n}$.

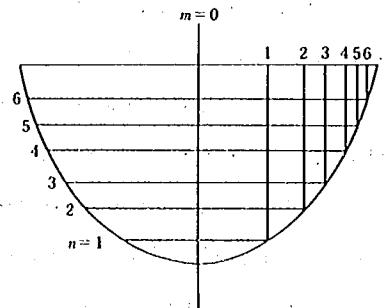
Then, the basic definition for the load splitting method is given by the following formula,

$$\left. \begin{aligned} cr(\Delta\gamma)_{m,n} &= ar(\Delta\gamma)_{m,n} \\ arP_{m,n} &= P_{m,n} - crP_{m,n} \end{aligned} \right\} \dots\dots\dots (51)$$

On the other hand, $cr(\Delta\gamma)_{m,n}$ is rewritten as follows,

$$\begin{aligned} cr(\Delta\gamma)_{m,n} &= CI(\Delta\gamma)_{m,n} + \sum_{t=1}^6 crP_{m,t} \cdot cr(\Delta\gamma)_{m,t}^{n-1} \\ &\quad + \cos(\phi_{m,m} - \phi_{m,n}) \sum_{k=0}^m (P_{k,m} - crP_{k,m}) ar(\Delta\gamma)_{k,m}^{n-1} \\ &\quad + \sin(\phi_{m,m} - \phi_{m,n}) \sum_{k=0}^m (P_{k,m} - crP_{k,m}) ar(\Delta\gamma)_{k,m}^{n-1} \dots\dots\dots (52) \end{aligned}$$

Here we shall discuss the terms in the expression (52). The term, $CI(\Delta\gamma)_{m,n}$, denotes the initial deflection at point (m, n) . The initial deflection means a deflection due to a load other than the water pressure, say, seismic inertial force applied in advance on the cantilever. It does not mean that such non-hydrostatic force is applied only on the cantilever, but that it is imposed on the cantilever for the purpose of convenience. The non-hydrostatic force on the cantilever is offset by an increase in the hydrostatic load to be borne by the arch element. Namely, this means nothing less than preparing a system of equations using all the known loads, including



hydrostatic ones, and unknown quantities.

As regards the dead load, whether the cantilever elements bear up it totally or the cantilever elements and arch elements go shares with each other is determined depending on the timing of actual joint grouting. Thus, the dead load should be processed as the situation may demand. The program currently in use by the Dam Structure Division is designed to permit entry of this condition.

$$\sum_{l=m}^6 cr P_{m,l} cr(\Delta \gamma)_{m,m}^{m,l}$$

Where, $cr(\Delta \gamma)_{m,m}^{m,l}$ denotes the radial deflection at point (m,n) of the cantilever element (m) due to a unity load at point (m, l). The term, $cr P_{m,l}$ is the load to be shared by the cantilever (m) at point (m, l).

$\sum_{l=m}^6$, means the sum of radial displacements at point (m,n) of the cantilever (m) due to the loads assigned to the cantilever elements from the base (point (m,m) here, i. e., abutment) to the level 6. The reason why the integration is limited to the level 6 is that the section 6-7 usually is provided with a crest notch; namely, that all the loads in this section are to be shouldered by the cantilevers as no arching action is expected. Accordingly, the displacement due to the loads in the section 6-7 is included in the initial deflection.

$$\cos(\phi_{m,m} - \phi_{m,n}) \sum_{k=0}^6 (P_{k,m} - cr P_{k,m}) ar(\Delta \gamma)_{m,m}^{k,m} :$$

The term, $ar(\Delta \gamma)_{m,m}^{k,m}$, denotes the radial displacement at point (m,m) of the arch element (which is at the base elevation of the cantilever (m)), of the level (m) due to the unity load at point (k,m). The term, $(P_{k,m} - cr P_{k,m})$, denotes the load shared by the arch element. the term, $\sum_{k=0}^m$, stands for the total of the radial displacements at point (m,m) due to the loads shouldered by the arch elements lying from the crown to the abutment.

The term, $\cos(\phi_{m,m} - \phi_{m,n})$, denotes the adjustment of the radial direction at point (m,m) to that at point (m,n). To put it simply, the above is interpreted that the reference point (i. e., base) of the cantilever (m) is already displaced in the radial direction owing to the deformation of the arch element at the same elevation. This comes from the assumption that the cantilever is mounted on a horizontal arch.

$$\sin(\phi_{m,m} - \phi_{m,n}) \sum_{k=0}^m (P_{k,m} - cr P_{k,m}) ar(\Delta s)_{m,m}^{k,m} :$$

This is merely a transformation of the aforesaid radial displacement

into the tangential displacement. Unless the base of the cantilever (m) agrees with the radial direction at point (m, n), the tangential displacement should naturally have a radial component at point (m, n).

We have just discussed the radial displacement of the cantilever element. Now let us examine the radial displacement of the arch element.

The radial displacement of the arch element $ar(\Delta\gamma)_{m,n}$, is expressed as follows.

$$ar(\Delta\gamma)_{m,n} = a_1(\Delta\gamma)_{m,n} + \sum_{k=0}^n (P_{k,n} - crP_{k,n}) ar(\Delta\gamma)_{m,n}^k + K_n \sum_{i=1}^6 crP_{n,i} \left[crV_{n,i}^n ar(\Delta\gamma)_{m,n}^i + crH_{n,i}^n as(\Delta\gamma)_{m,n}^i \right] \dots\dots\dots (53)$$

Where, the term, $a_1(\Delta\gamma)_{m,n}$, signifies the initial deflection of the arch element just as in the case of the cantilever element. The term, $\sum_{k=0}^n (P_{k,n} - crP_{k,n}) ar(\Delta\gamma)_{m,n}^k$, denotes the total of the radial displacements of the arch element at point (m, n) due to the load shared by the arch element at the level n. The last term shows the radial displacement at point (m, n) of the arch element at the level n due to concentrated load, such as shearing force, communicated from the cantilever element on the abutment of same arch element. With all explained above, we have exhausted the displacements of the arch element.

Here, let us assume :

$$cr(\Delta\gamma)_{m,n} = ar(\Delta\gamma)_{m,n}$$

The unknown quantity is the load to be shared by the cantilever element, i. e., $crP_{m,n}$, which exists only at the intersection of a cantilever element and an arch element. In the example taken up here, there are 28 intersections. Since the equations can be established as many as the intersections, the number of equations becomes equal to the number of unknowns, making it possible for us to determine $crP_{m,n}$.

If $crP_{m,n}$ is determined, we can also determine the load to be borne by the arch element by making use of the formula, $arP_{m,n} = P_{m,n} - crP_{m,n}$.

What should be borne in mind here is that we compensate for the load taken in the discussion of the initial deflection.

For example, if seismic inertial force is applied on the cantilever element to determine the initial deflection, the true load to be borne by the cantilever element ($crP'_{m,n}$) is given by the following.

$$crP'_{m,n} = crP_{m,n} + c/P_{m,n} \text{ (where, } c/P_{m,n} \text{ : seismic inertial force)}$$

The same applies to the determination of the true load to be borne by the arch element,

As the load distribution has been determined, we can determine the stress and displacement of a single cantilever and a single arch by applying the cantilever and arch theories.

In the complete adjustment calculation, the load splitting is carried out just the same way as above for the tangential displacement and rotation about the perpendicular axis, and is reflected in the final determination of displacement and stress. In actuality, however, the loads to cause tangential displacement and rotation do not exist as external loads, and therefore the self-balancing loads of equal size and opposite signs are determined,

The factors that are expected to have considerable effects on the accuracy of the load splitting method derived from Trial Load Method are as follows,

- 1) As regards the variable-radius arch, the radial direction of the crest does not fall in with that of the bottom, and the cantilever assumes a twisted form. But this twist effect is not taken into account,
- 2) The deformation of the foundation bedrock is predicated upon Vogt's formula concerning the mean deformation of a semi-infinite rectangular foundation under a load. In actuality, the foundation is not a flat one, but takes a complicated three-dimensional pattern. Nevertheless, the following assumptions are used in the formula,
 - (1) The foundation is developed as a flat plane,
 - (2) The axis is straight,
 - (3) Use of equivalent developed planes of rectangular form, each having an equal area and an equal aspect ratio,
 - (4) The same load always develop the same amount of deformation,

Of these assumption, the most problematic is 3) ; namely, the fact that the deformation constant is determined as a function of (b/a) which is the ratio of the length (b) of a rectangular foundation plane to its width (a) ,

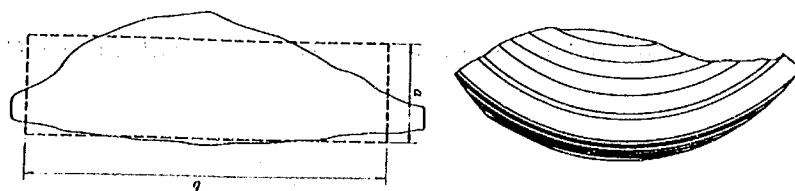


Fig. 44 Equivalent development drawing

This assumption is at variance with reality, because the width (a) is small near the riverbed, but is larger the nearer it goes to the upper

abutment compared with actual ones,

In other words, the assumption has a tendency to undervalue the displacement of the bedrock at the under-riverbed arch abutment and overvalue the displacement of the bedrock at the upper abutment.

Accordingly, it is apparent that restraint of the cantilever near the riverbed happens to be evaluated larger than is actually, while that at an elevated place is estimated less. As explained later, the load splitting method produces a result as if the upstream face of the cantilever base near the crown would develop an excessive tensile stress. According to the model test, however, this tensile stress is subtracted by 20 to 30 kg/cm². The difference will probably be due to the assumption 3). As already, discussed, the load splitting method simplifies the stress analysis, and is put to use from the beginning of designing. It is recommended to carry out the radial adjustment calculation at first and then complete adjustment calculation when the design plan is nearly final.

The cost for computation per case is about ¥50,000 for radial adjustment and about ¥100,000 for complete adjustment if a variable-radius nonuniform-thickness arch is handled.

As example of stress analysis according to the load splitting method is shown in Figs. 45 through 48.

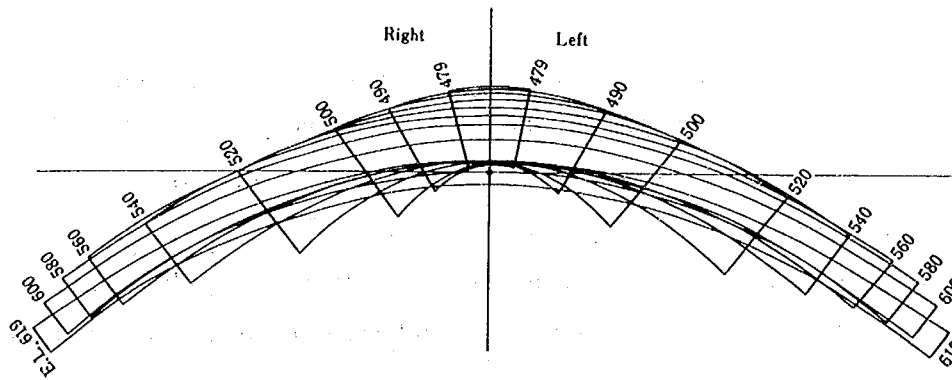


Fig. 45 Plan view of arch (Kawaji Dam) (Final plan : Case 28)

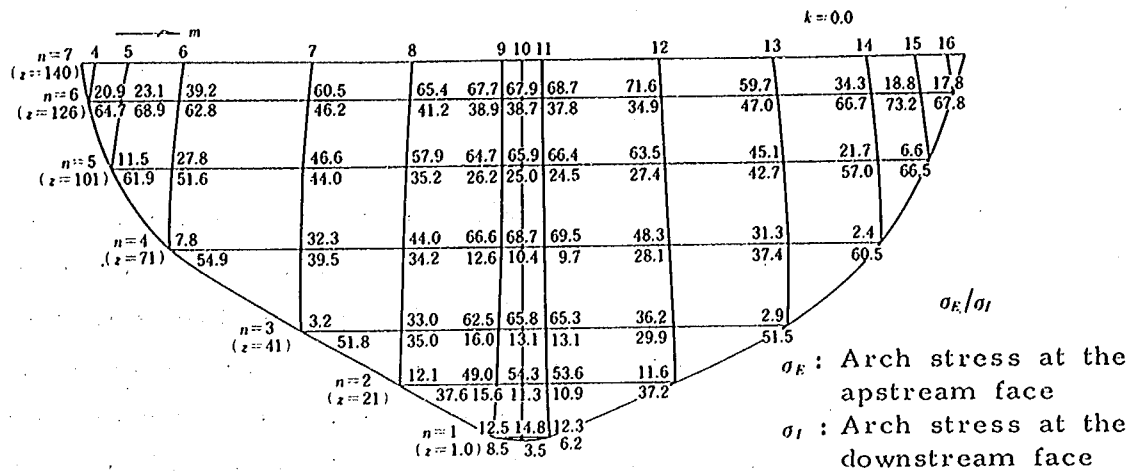


Fig. 46 Stress intensity of arch (normal load condition)

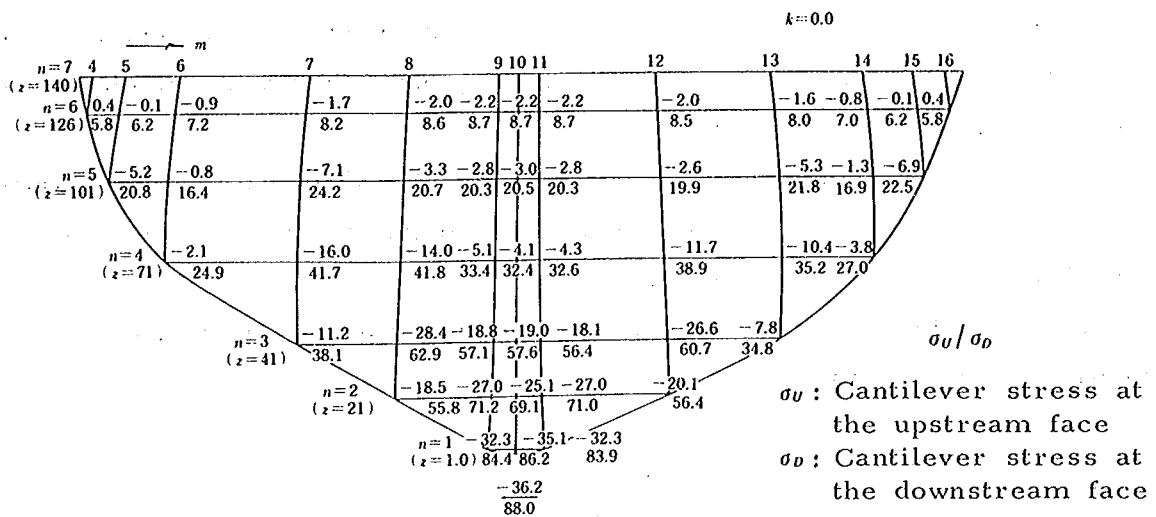


Fig. 47 Stress intensity of cantilever (normal load condition)

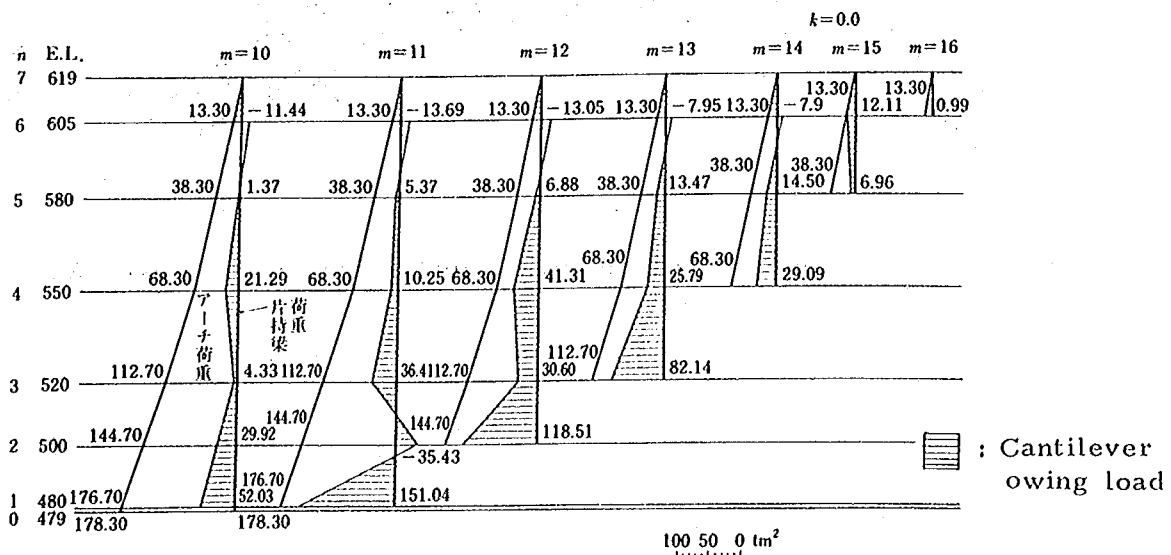


Fig. 48 (a) Load splitting diagram (normal load condition)

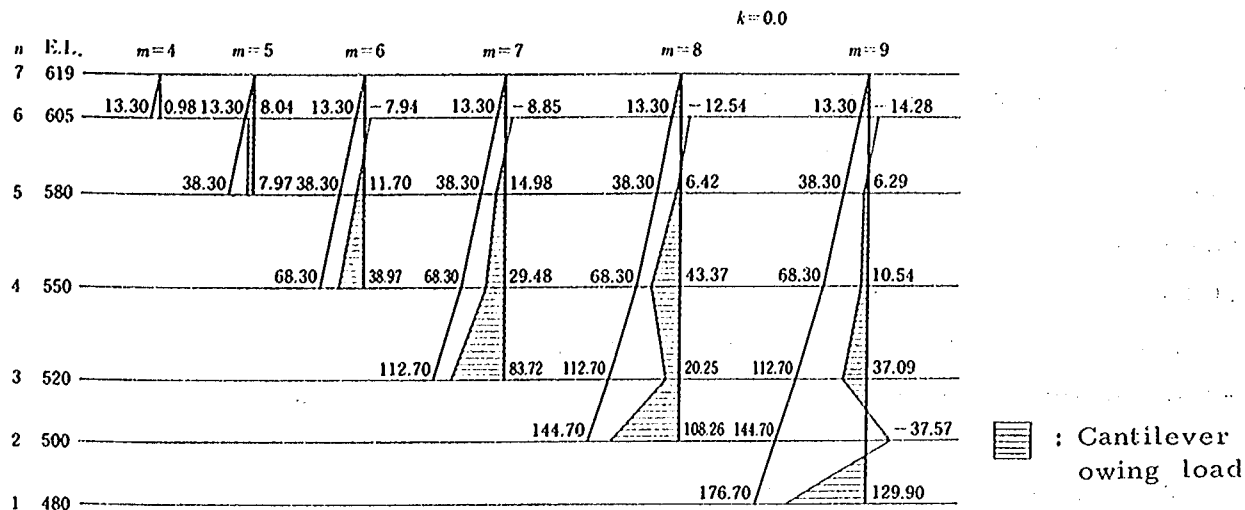


Fig. 48 (b) Load splitting diagram (normal load condition)

5.3.2 Stress Analysis by Model Tests

a. general

Generally, in the experiments using a model, the following three conditions must be satisfied by the actual object and the model.

- 1) The relationships between the stress and strain of the materials of the two are similar.
- 2) The model is geometrically similar to the actual object.
- 3) The two have the same breaking mechanism.

The first condition means the requirement that the relationships between the stress and strain of the two are similar.

If the experiments are limited to the elastic range, however, only that the model is made of elastic material and that the model has an equal Poisson's ratio to the actual object are required.

The second condition is self-evident.

The third condition is required only when carrying out the breaking test, being not required like the first condition so long as the experiments are made within the elastic range.

In addition to these conditions, to increase the accuracy of the measurement, the model must be deformed to a large extent since the stress is calculated by measuring the strain. For this purpose, it is possible to make a model of a large scale or to apply a heavy load, but the most practical method is to make a model of small Young's modulus.

The followings have been used as comparatively satisfying materials of these conditions so far.

- 1) Pumise concrete
- 2) Resin
- 3) Mixture of gypsum and diatom earth

A serious defect of the pumise is that it is difficult to reduce its Young's modulus, resulting in larger models. The resin, in general, has larger Poisson's ratio than concrete, and considerably large plastic deformation, but has an advantage that it can provide models with small Young's modulus.

The most popular material of these is the mixture of gypsum and diatom earth. In the model experiments conducted by Public Works Research Institute on the arch drum, only this material was used.

The mixture has not only sufficient elasticity, but also a Poisson's ratio almost the same as that of concrete. In addition, its Young's modulus can be varied easily from 5,000 kg/cm² to 100,000 kg/cm². Defects of this material are that the experiment must be conducted under completely dry condition, if possible, because its Young's modulus varies extremely depending on the humidity, and it cannot be used in the break test as it is considered to have a different breaking mechanism from that of concrete.

b. Loading device

The model is made using the mixture of gypsum and diatom earth with a reduced ratio from 1/100 to 1/200. Following two methods are available for loading the model.

- 1) Using a liquid like mercury with a large specific gravity.
- 2) Using a hydraulic jack

In the method of using the liquid (1), mercury having the largest specific gravity among the liquid is used. The mercury is applied to the upstream of the model by using gloves. The loading device by this method is comparatively simple. Since the mercury method has a small load ratio, the error in the measured strain is large.

In the jack method, 20 to 30 units of jack are installed on the frame mounted in the upstream of the model to load the model. This method has a larger load ratio to produce greater strain. Defects of this method are that the load cannot be distributed uniformly in the upstream, but concentrated to certain portions, resulting in nonuniform distribution of the stress around the jack and the difficulty in making the direction of the resultant force, obtained when replacing the distributed force with the concentrated force, agree with the actual direction.

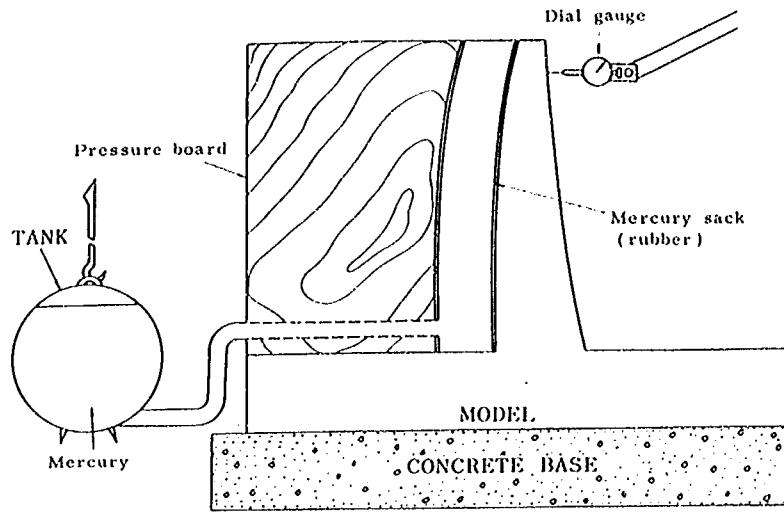


Fig. 49 Mercury type loading device

c. Measurement

The experiment is conducted mainly by measuring the strain and deflection within the elastic range to convert them into the bending moment and stress.

The strain is measured using an electric resistance strain meter shown in Fig. 51,

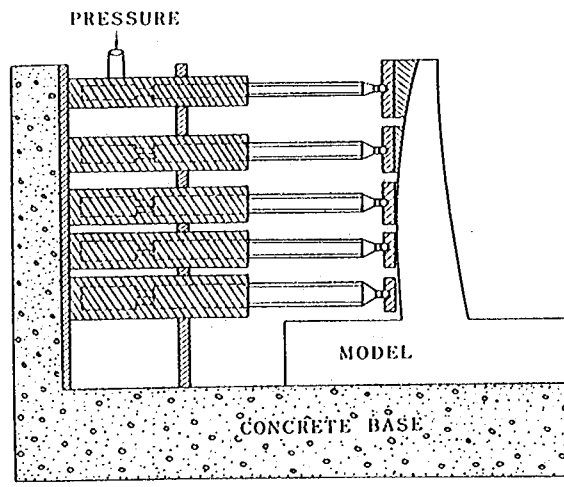


Fig. 50 Oil jack type loading device

The measurement principle is that when the R_s is adjusted in such a way that the current does not flow through G , the resistance R_i of the strain meter is obtained by

$$R_1 = \frac{R_2}{R_4} \times R_3$$

where, R_1 is an active gauge attached to the object, R_2 is a dummy gauge to prevent the effect of the temperature variation, R_2 is the same kind of strain gauge as R_1 and is attached to the same kind of object as R_1 is attached in a state free from any external force, Since two models are made in most cases, R_1 and R_2 are attached to each of them,

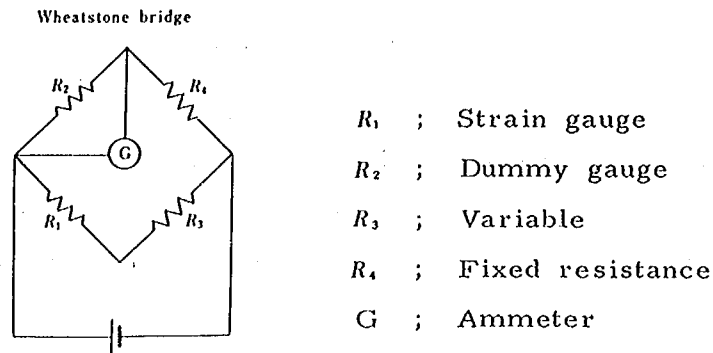


Fig. 51 Principle of strain measurement

d, Stress

The stress is calculated from the arranged value of the strain measured, According to the Hooke's law, the following equations are obtained from the strains in three directions,

$\epsilon_h, \epsilon_v, \epsilon_d$: Strains obtained by the model experiment in horizontal, vertical, and slant direction respectively.

n, E_m, μ, P : The reduced scale, Young's modulus, Poissons's ratio, force applied to the dam surface of the model.

$\sigma_h, \sigma_v, \tau_{hv}$: Horizontal stress, vertical stress, shearing stress respectively of the model.

$\sigma_{hp}, \sigma_{vp}, \tau_{hvp}$: Horizontal stress, vertical stress, shearing stress, respectively of the actual object.

Stresses of a point due to a vertical strain and horizontal strain are,

$$\left. \begin{aligned} \sigma_v &= \frac{E_m}{1-\mu^2} (\epsilon_v + \mu \epsilon_h) + \frac{\mu}{1-\mu} P \\ \sigma_h &= \frac{E_m}{1-\mu^2} (\epsilon_h + \mu \epsilon_v) + \frac{\mu}{1-\mu} P \\ \tau_{hv} &= \frac{E_m}{2(1+\mu)} (2\epsilon_d - (\epsilon_h + \epsilon_v)) \end{aligned} \right\} \dots\dots\dots (54)$$

$$\left. \begin{aligned} \epsilon_v &= \frac{1}{E_m} \{ \sigma_v - \mu (\sigma_h + P) \} \\ \epsilon_h &= \frac{1}{E_m} \{ \sigma_h - \mu (\sigma_v + P) \} \end{aligned} \right\} \dots\dots\dots (55)$$

$$\epsilon_v + \mu \epsilon_h = \frac{1}{E_m} \{ \sigma_v (1 - \mu)^2 - \mu (1 - \mu) P \}$$

The stress of the actual object is

$$\left. \begin{aligned} \sigma_{HP} &= \frac{n}{G} \cdot \frac{E_m}{1 - \mu^2} (\epsilon_h + \mu \epsilon_v) + \frac{n}{G} \cdot P \cdot \frac{\mu}{1 - \mu} \\ \sigma_{VP} &= \frac{n}{G} \cdot \frac{E_m}{1 - \mu^2} (\epsilon_v + \mu \epsilon_h) + \frac{n}{G} \cdot P \cdot \frac{\mu}{1 - \mu} \\ \tau_{HVP} &= \frac{n}{G} \cdot \frac{E_m}{2(1 + \mu)} \{ 2\epsilon_D - (\epsilon_h + \epsilon_v) \} \end{aligned} \right\} \dots\dots\dots (56)$$

where, G : Load strength

n : Reduced scale of the model

e. Evaluation of the model test in the stress analysis

Although the model experiment is one of the effective methods of the stress analysis, it is not suitable to use this method from the first stage of designing because it requires great amount of labor and expense. Being relatively inexpensive as an alternative method in those days when the load splitting method was made by manual calculation, the model experiment has often been employed. At present, however, despite several hundred times of labor and expense required as compared with the load splitting method, it is not so good way from a view of accuracy.

Public Works Research Institute designs the arch dam by the load splitting method and conducts the stress checking only on the final design by the model test. At present the Institute intends to abolish this method with the appearance of the finite element method. A discussion on the precision of the model experiment method is given below.

Main causes for the error are considered as follows :

- 1) The material of the model is not in accordance with the specification.
- 2) Error due to the poor workmanship
- 3) Error due to the loading device
- 4) Error due to the strain meter

Error (2) is comparatively small and negligible. Although the errors (3), (4) are comparatively large, the error due to the improper loading except a concentrated loading, output loss of the jack, intrinsic error to the strain meter, can be eliminated to a considerable extent by a careful experiment or increased number of measurements. The total of these errors are estimated to be about 5 kg/cm², converted into the stress of the actual object.

The greatest error is considered to be produced when the material (1) does not have the target uniformity and Young's modulus. As mentioned

before, gypsum varies its property greatly depending on the minute change in its water content and ambient humidity. At present the model is made by molding and adhered to the abutment using an adhesive. Since the mold is thin and wide, the air bubble inside the mold does not escape sufficiently when charging the gypsum. Considering from the fact that a test piece to determine the characteristic of the model is examined after sufficiently exhausting the air at completely dried condition in the oven, the model is thought to be insufficiently dried and to have a considerably small Young's modulus. Unless a part of the model is examined from time to time, the characteristics of the model are far lower than the expected value as compared with those of the test value because the model is examined under the adverse conditions. Although the difference between the characteristics of the test piece and the model have not examined yet, it is estimated to be more than 20% because Young's modulus varies by 20% between the test pieces.

As mentioned above, although the model experiment is an excellent stress analysis method in the point that it employs less assumption than calculation method, it should be noted that the error will be greater unless the model materials, etc. are examined sufficiently.

5.3.3 Comparison between the Stress Analysis Results by Load Splitting Method and by Model Experiment

The load splitting method has widely been practiced because electronic computers are available in its calculation, although its precision of analysis is rather low. If the errors of the load splitting method are grasped both qualitatively and quantitatively, the errors will not matter much in the actual practice of the method.

Dam Structure Division of Public Works Research Institute has conducted the model experiments on more than ten arch type dams of their design to check the stress of the final designs and has compared the model experiment results with the analysis values by load splitting method to clarify the relationship between the two.

These are summerized as follows :

- 1) Horizontal stress in the upstream has a little difference between the experiment result and arranged calculation value in radial adjustment.
- 2) Horizontal stress at the low elevation of crown in the downstream obtained by the experiment shifts to the tension side by 10 to 30 kg as

compared with the arranged calculation value in radial adjustment.

- 3) Vertical stress in the upstream at the side cantilever base obtained by the experiment shifts to the compression side by 20 to 30 kg/cm² as compared with the arranged calculation value in radial adjustment.
- 4) Vertical stress at the upper part of the downstream at the side cantilever base obtained by the experiment shifts to the tension side by 10 to 20 kg as compared with the arranged calculation value in radial adjustment.
- 5) In parabolic arch dam, the principal stress parallel to the rock at the bottom of the downstream obtained by the experiment shifts to the tension side by 5 to 10 kg as compared with the arranged calculation value in radial adjustment.

For the dam with deep and U-Shaped valley like Kawamata dam, the experiment results almost agree to the calculation value, while for the dam with wide and V-Shaped valley the arch bending moment of the arranged calculation value in radial adjustment at the lower elevation has a tendency to be larger than the experiment value.

This difference is considered to be caused by the lack of torsion adjustment during the calculation process. It is presumed that because the effect of the torsion around the vertical axis is small at the side cantilever due to the steep slope of the abutment in U-shaped dam, the difference between the experiment value and the arranged calculation value in radial adjustment is comparatively small, while in V-shaped dam abutment slope is easy with greater torsion effect, resulting in larger difference between the two.

Generally speaking, the load splitting method is an excellent analysis, the cause for the above errors are considered as follows :

- 1) Equivalent development method is employed for calculating the foundation deformation.
- 2) The effect of the twisted cantilever is not taken into consideration.
- 3) The adjustment of displacement in vertical direction of the cantilever is omitted.

Among these, the error by the assumption of (1) is presumed to be the largest. In the load splitting method, it is almost impossible to eliminate the tension stress on the cantilever upstream base during the designing process, while in experiment this stress value shifts to the compression side greatly. The cause for this is presumed that the deformation at the

crown base is evaluated too small in the assumption in the calculation of foundation deformation, causing a large tension stress by too strong restraint as a cantilever.

Fig. 52 ~ Fig. 59 show the deflections and stresses of Kawamata dam as a typical U-shaped dam, and Shōrenji dam as a typical V-shaped dam.

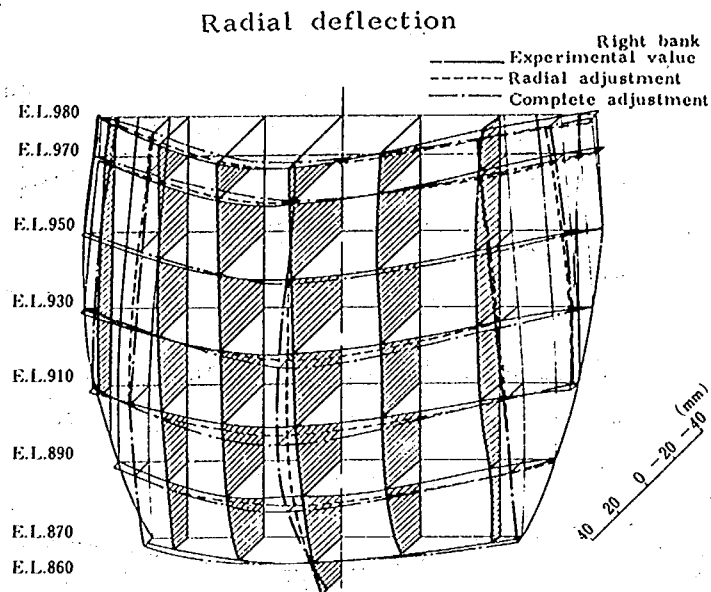


Fig. 52 Comparison between load splitting method and model experiment method (at Kawamata Dam)

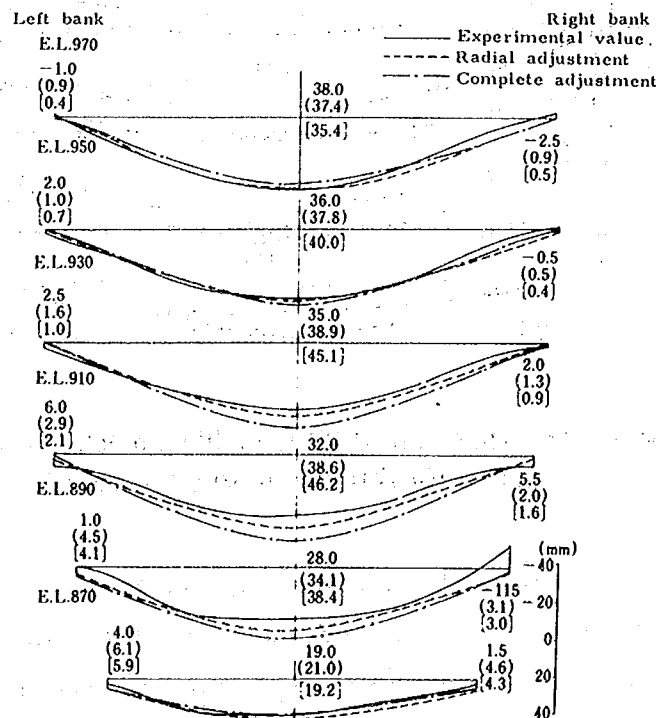


Fig. 53 Radial deflection (at Kawamata Dam)

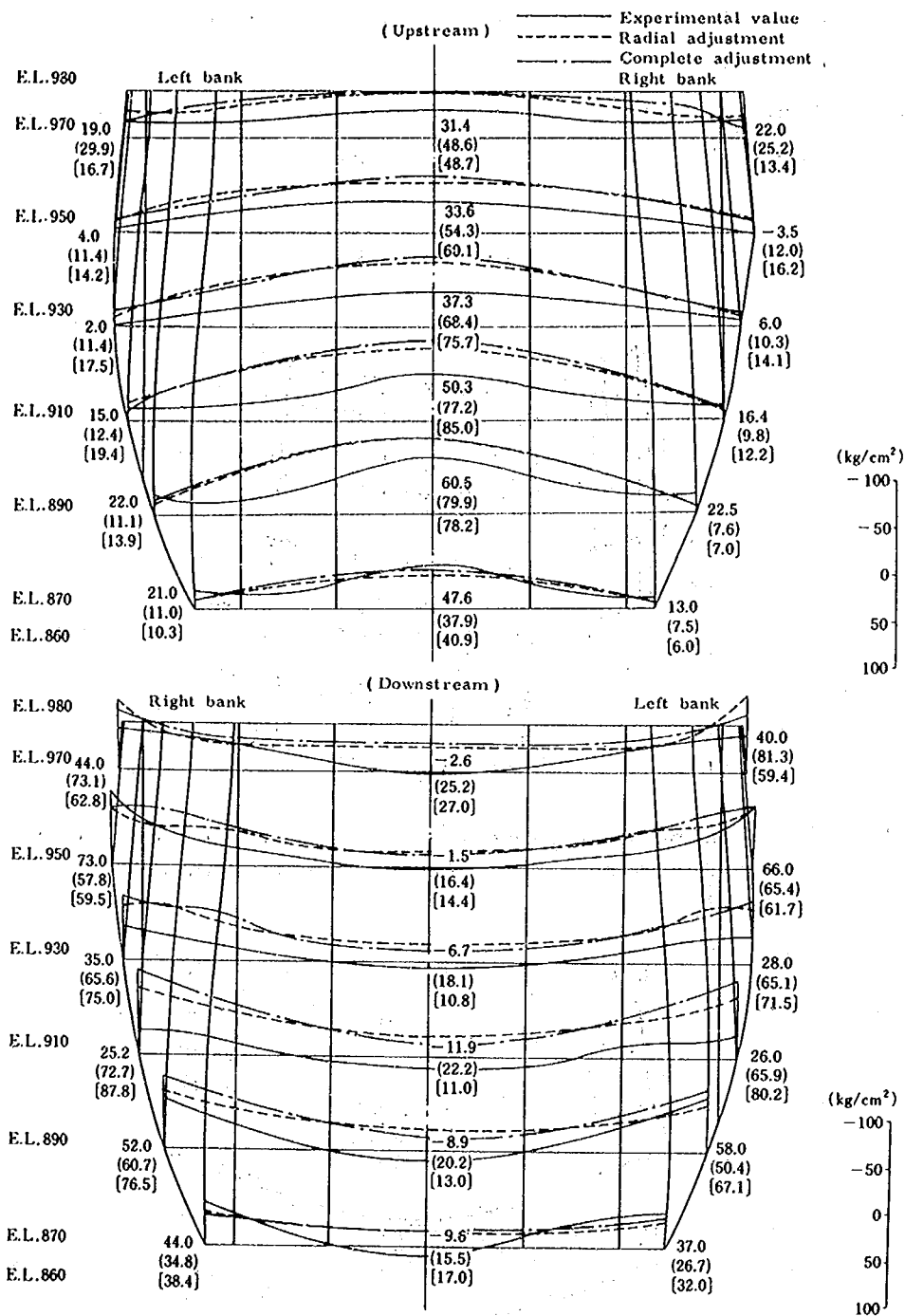


Fig. 54 Horizontal stress distribution diagram
(at Kawamata Dam)

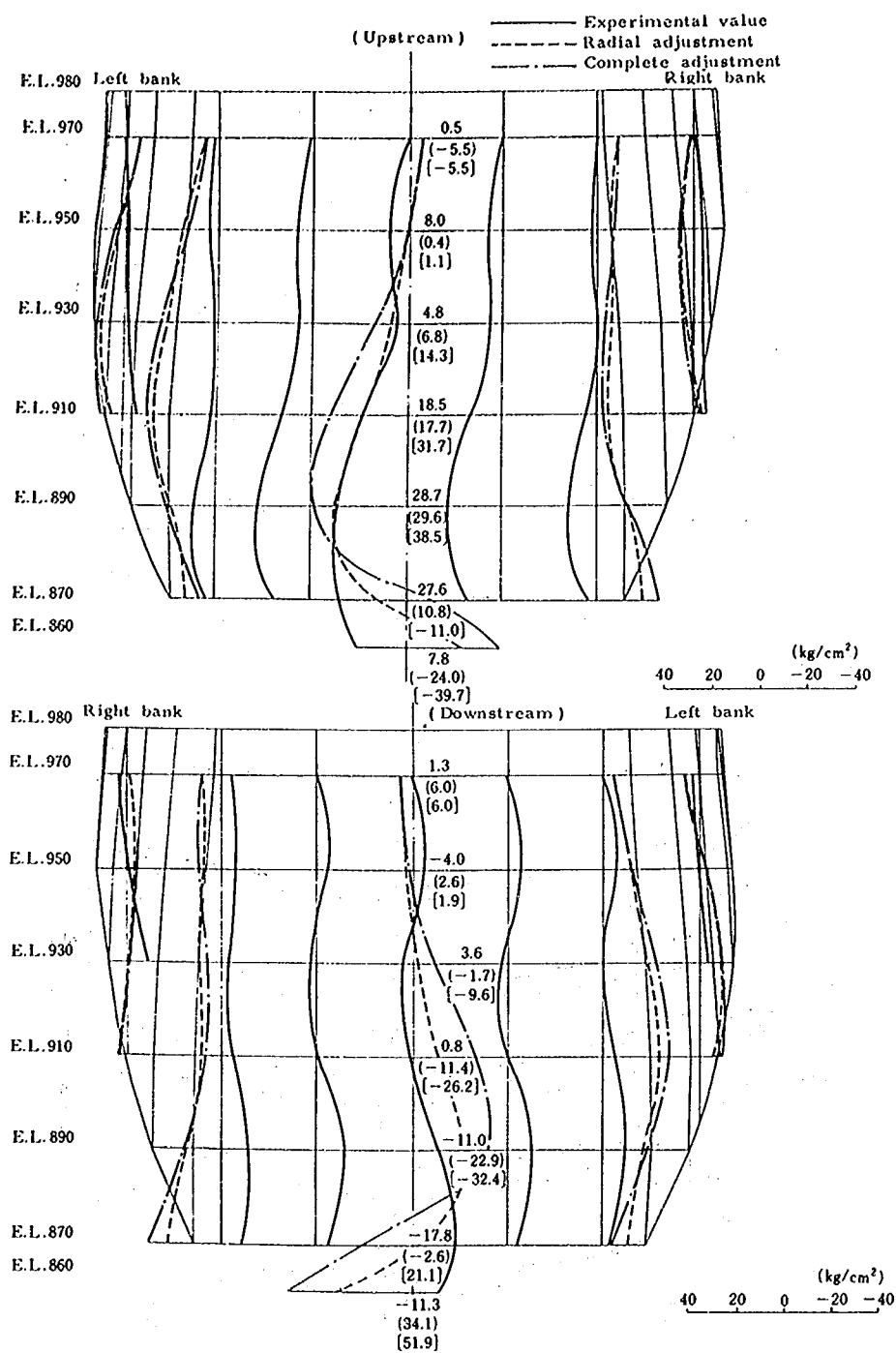


Fig. 55 Vertical stress distribution diagram
(at Kawamata Dam)

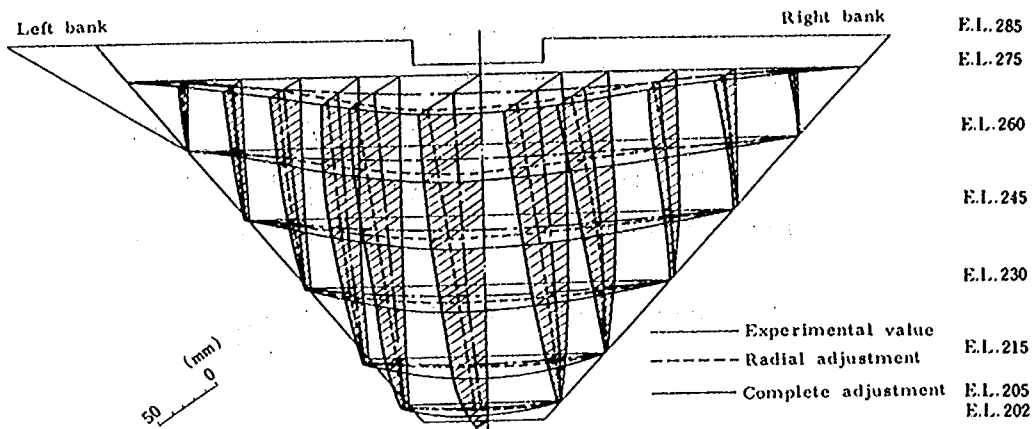


Fig. 56 Radial deflection of the Shorenji Dam

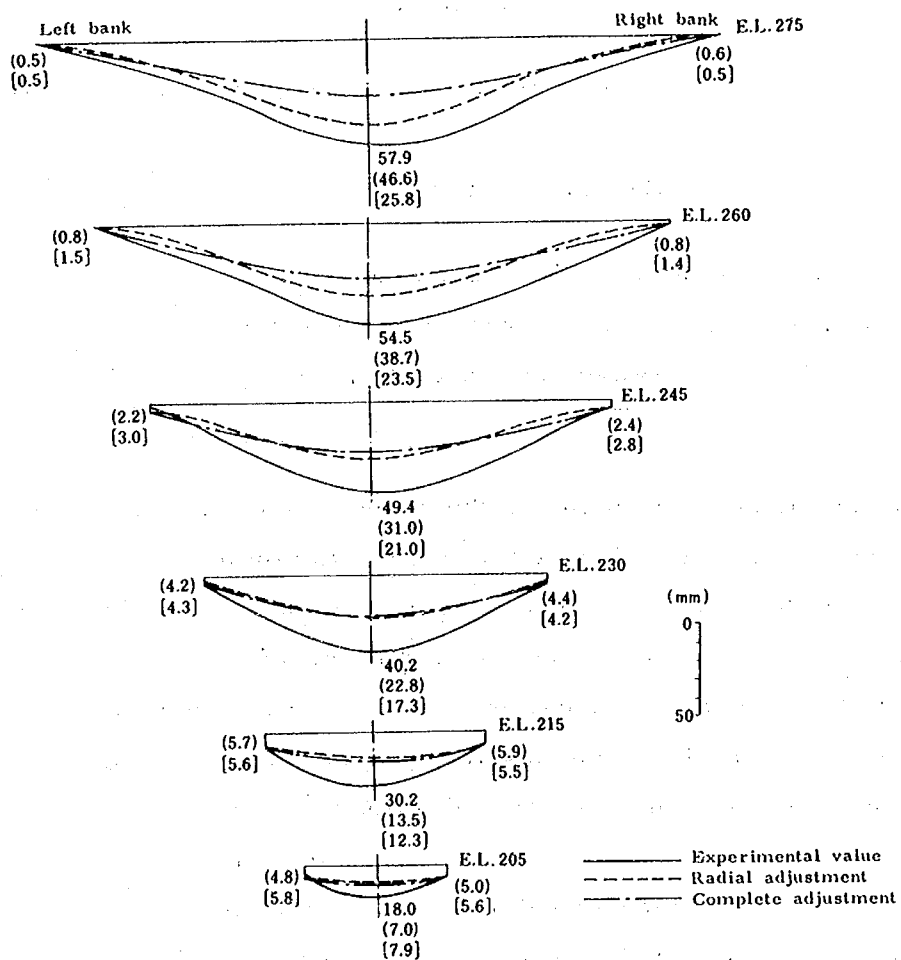


Fig. 57 Radial deflection (at Shorenji Dam)

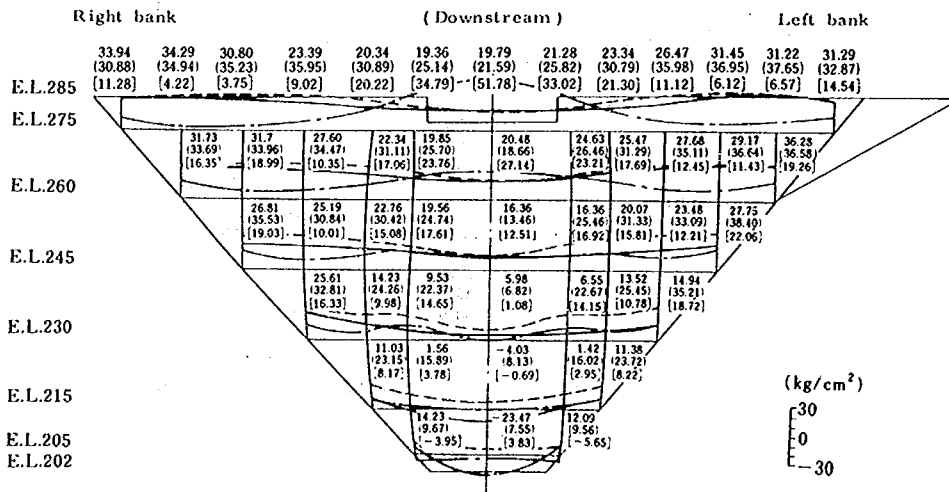
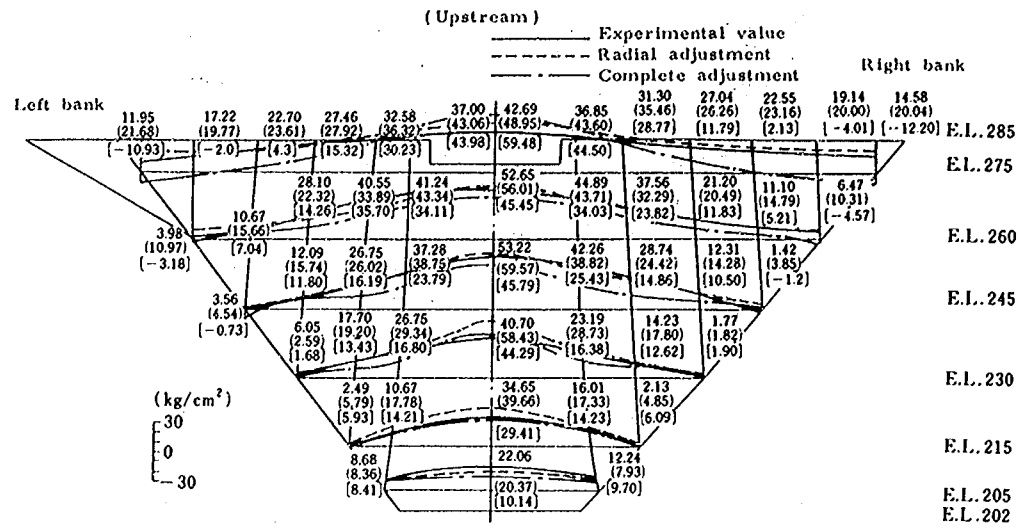


Fig. 58 Horizontal stress distribution diagram
(at Shorenji Dam)

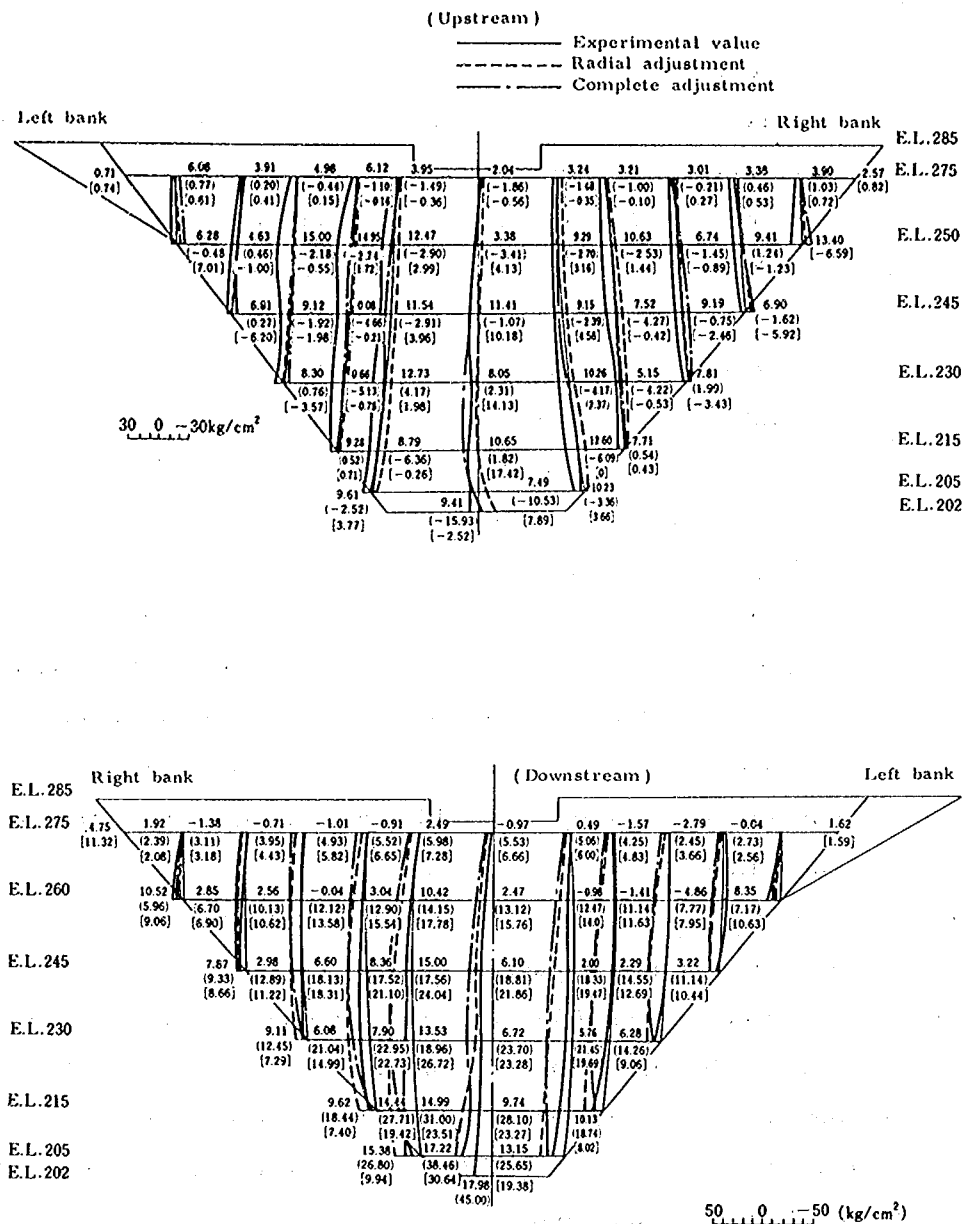


Fig. 59 Vertical stress distribution
(at Shorenji Dam)

5.4 Stability Analyses for Foundation of Arch Dams

a. general

The safety of the foundation rock is very important for arch dams. The fact that some failures of arch dams in the past have been caused by the defective foundation evidently proves this. In the gravity dam or rock fill dam, the partial defect of the foundation rock not necessarily leads to the destruction of the whole dam. The destruction of the arch dam instantly follows the break of its foundation. This is a reason why careful examinations are required on the stability of the arch dam foundation.

As mentioned before, at the designing of the arch dam two works are done simultaneously, one is to let the working stress of the dam body allowable stress range, and the other is that the required foundation safety is secured by adjusting the location of abutments to the suitable position. It is not easy, however, to satisfy the two at the same time. Stress analysis is made for the original design and we use the thrust obtained by stress analysis to check the stability of the foundation. The results of check very rarely have a suitable safety factor, in most cases being too small or too sufficient. To save the labor for such designing, Shibata has introduced the idea of "the required hard rock line", which is outlined below.

The shear friction safety factor by Henny's equation is employed for the calculation of the arch dam foundation stability. First we assume a sliding line S_i in a horizontal section (See Fig. 60), based on the assumption that the sliding occurs at the upstream end of the dam. Resultant force R , including uplift and inertia force acts on the sliding line. The required length l_i which satisfies the required shear friction factor can be obtained as follows from the Henny's equation.

$$l_i = \frac{h \cdot H_i - f_i \cdot V_i}{\tau_i} \dots\dots\dots (57)$$

where, l_i : The required length along the assumed sliding line S_i

τ_i : Shearing strength along the assumed sliding line S_i

f_i : Shearing friction factor along the assumed sliding line S_i

H_i : Component of R parallel to the assumed sliding line S_i

V_i : Component of R vertical to the assumed sliding line S_i

Results obtained from the calculations on different sliding lines are shown in Fig. 60. A curve enveloping these points is given. This curve is called "required shoulder".

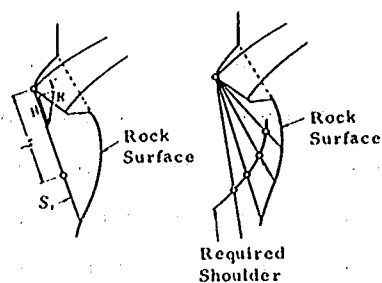


Fig. 60 Required shoulder

In case the hard rock surface crosses this required shoulder line, the safety factor is less than the required value. In case the space between the two is too wide, the safety factor is too large, showing the too deep abutment in the rock base, which means uneconomical design works. Thus, if the dam is designed by taking into account the idea of "required shoulder", not small amount of labor and time can be saved. When a satisfactory original design is made, the required shoulder line is prepared at each elevation. It is easy to find the most reasonable and economical location of the dam by studying the required shoulder line overlapped with the geological map. The inter-relationship between the required shoulder line and actual hard rock line tells us suitable location of abutment or the quantitative value of the dam span to be corrected. By repeating the above step on the corrected original design, the final design can be completed in a short time. For the dams designed by the Institute, location of abutment has been determined in this way.

Since the safety factor " n " is taken as 4, $n=4$ is secured at every elevation in two-dimensional condition. In case weak layers like fault exist in the foundation rock, the idea of the required shoulder line cannot solve the problem. In case a number of such faults exist in the foundation, although it is correct to proceed designing by taking into account the safety from its first stage, the designing work becomes very complicated. It is not a good way to design a dam including its stability from its first stage. Therefore, in general, location of abutment is designed by the idea or required shoulder line by considering the effects of the faults. For this purpose, the safety factor is determined with allowance (for example $n=4.5$), or the required shoulder line is calculated with some allowance in the designing strength of the foundation rock. Thus, allowances are provided when checking the stability of each weak layer.

The foundation rock designed in this way can be understood to have 4 or more shearing slide safety factor for even three-dimensional sliding macroscopically, having a suitable safety factor.

The stability of each weak layer should be studied respectively. In this case the study has been made mainly by Henny's equation through two-dimensional slice, too. Studies on the shearing friction safety are made in general, but in this case analysis way and technological judgement on safety factor is difficult. The problems about this are as follows :

- 1) Handling of the analysis method by Henny's equation
- 2) Technological judgement on safety factor by two dimensional analysis
- 3) Technological judgement on point safety factor

The problem (1) means the following. As shown in Fig. 61 in case faults exist at the inner part of the abutment of the arch dam, a sliding break along the fault is expected. The way of handling the Henny's equation for getting the safety factor is very uncertain.

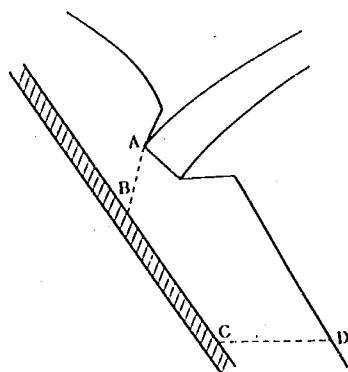


Fig. 61 Study on the slide along the fault

$$n = \frac{\tau l + fV}{H}$$

Henny's equation is shown by the above, but it is not clear where the force H , V should be applied. It is evident that resultant force is supported by all resistance along $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$. But we can not find the forces working to each length,

The author have studied various problems on the handling of Henny's equation when we studied the stability of the faults existing in the foundation rock of Kawaji dam. In effect, all the stress in the foundation rock (fault) has been calculated by the finite element method and the various sliding surfaces are assumed. The safety factor has been studied on summed-up

stress along the sliding surface,

$$n = \frac{\sum \tau_{oi} l_i + \sum f_i \sigma_i}{\sum \tau_i} \dots\dots\dots (58)$$

where, τ_{oi} ; Initial shearing strength of a minute part along the assumed sliding line

f_i ; Shearing friction factor of a minute part along the assumed sliding line

l_i ; Length of a minute part along the assumed sliding line

σ_i ; Vertical stress on a minute part along the assumed sliding line

τ_i ; Shearing stress on a minute part along the assumed sliding line

If the minimum safety factor is obtained in this way, the limit safety factor can be obtained by taking the ratio of the total applied force to the resistance force, In the analysis in which the calculation of up to the internal stress is performed, the simplicity of Henny's equation is lost,

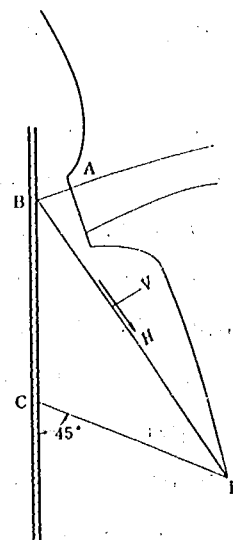
The calculated internal stress has shown that the vertical stress along the part of the assumed sliding line, $\overline{AB}$ which were considered to be created in parallel to the upstream face acts mainly as tension, resulting in the absence of strength,

A rough study by Henny's equation of the fault location shown in Fig, 64. can be made as follows ;

- 1) Approximation of the break line
 $\overline{AB}$ should be vertical to the abutment,
 $\overline{CD}$ should be 45° to the fault surface,
 (* The position of C should be determined to be minimum by trial and error,)

2) Calculation method

Thrust is divided into two directions ;
 a force parallel (H) to and vertical (V)
 to the average direction of the assumed sliding line $\overline{BD}$,



$$n = \frac{\tau_f \overline{BC} + \tau_s \overline{CD} + f_l V}{H} \dots\dots\dots (59)$$

* For safety, inner friction factor of the fault is taken,

where, H ; Destructive force

V ; Friction resistance force

τ_f ; Initial shearing strength of the fault

r_k ; Initial shearing strength of the foundation rock

f_f ; Inner friction factor of the fault

By changing the thrust to the average direction of the slide, the partial supporting resistance of CD can be added considerably, An example of the test results of the study on Kawaji dam is shown in Table 14,

Left bank fault F-2 (E, L, 520 m)

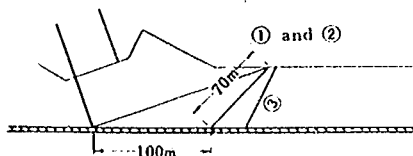


Table 14 Safety factor comparison by analysis method

Calculation method	Minimum safety factor by trial
1 Thrust is applied to fault surface	2,1
2 Thrust is applied to average slide direction	3,5
3 Calculation by two-dimensional F, E, M,	3,2

b, Technological judgement of the safety factor by two-dimensional analysis

Technological meaning of the safety factor obtained by the above -mentioned Henny's equation has been considered, It is intuitively understood and evident, considering from the examples of destruction that the breaking style of the arch dam foundation rock is third-dimensional, Considering from such a fact, it is obvious that safety factor at a certain elevation does not have a direct connection to the safety factor at the actual whole break, In this point, the designing of the arch dam differs from that of the gravity dam basically, to which two-dimensional designing can be applied, Therefore, in case the parts with low safety factor are limited to a specific elevation in second-dimensional analyses, the dam cannot be said to be dangerous only for this reason, In other words, it is not recommendable to make the abutment with sufficient depth just to satisfy the safety factor 4 at a certain elevation, resulting in the excessive excaration for other elevations, This does not contradict the idea that in the required shoulder line the foundation should be determined in such a way that the Henny's safety factor according to two dimensional analysis satisfies 4, That the safety factor is 4 at every

elevation means that the whole safety factor is a little more than 4, which is not excessive.

It is impossible to discuss the safety of the base just for a reason that at a certain elevation layer exists in low safety factor against sliding, This should be discussed according to the results of three-dimensional analysis, and this is similar to the actual breaking style of the base.

One example of those three dimensional analyses is the rocky-pillar method which was used for checking if the rocky-pillar separated by the faults at Kawamata dam could be stable. This calculation method is as follows ;

The pillar can be considered as a cantilever with an elastic floor (fault) in the back. Therefore, to know the external force which the pillar must support, the reaction force from the back should be obtained. The reaction force from the back is calculated on the assumption that the deflection of the cantilever beam of the pillar is equal to the deflection of the fault material in the back side and in the back hill. Approximation of the foundation deformation is based on the trial calculation analysis method by the U, S, Bureau of Reclamation,

$$\Delta_1 = P_m \left(\frac{T''}{E'} + \beta' \right) \dots\dots\dots (60)$$

where, Δ_1 ; Deformation of the back hill and back fault material

P_m ; Reaction force from the back

T'' ; Thickness of the back fault material

E' ; Young's modulus of the back fault material

β' ; Base constant

Deflection of the pillar as a cantilever at the elevation of m,

$$\Delta_2 = \sum \delta_{n,m} (P_n - p_n l_n) \dots\dots\dots (61)$$

In the equation, $\delta_{n,m}$ is the deflection of the cantilever due to the triangular unit load,

$$\delta_{n,m} = \sum \left(M_A \alpha + V_A \alpha_2 + \sum \frac{M_A}{E R I} \Delta Z \right) \cdot \Delta Z + \left(V_A \gamma + M_A \alpha_2 + \sum \frac{V_A \cdot K}{A G} \cdot \Delta Z \right) \dots\dots\dots (62)$$

where, P_n ; Vertical component to the back hill having an external force at elevation n

p_n ; Reaction strength by P_i at elevation n

l_n ; Back width of the pillar at elevation n

α, α_2, γ ; Foundation constant
 G ; Shearing modulus
 E_R ; Young's modulus of the foundation
 A ; Pillar section area
 K ; Shape constant

The reaction force of the back hill, p_m , can be obtained as the solution of the following simultaneous equation,

$$p_m \left(\frac{T''}{E} + \beta' \right) = \sum_n \delta_{n,m} (P_n - p_n \cdot l_n) \dots\dots\dots (63)$$

Thus, a force is obtained by deducting the reaction force from the external force, The force must be supported by the shearing resistance at the back and bottom of the rocky pillar, The limit safety factor is shown by the following equation like Henny's equation,

$$S_F = \frac{1}{H} (\tau_R \cdot A_R + f \Sigma W + \tau_F \cdot \Sigma A_F) \dots\dots\dots (64)$$

where, H ; The force supported by the pillar
 ΣW ; Weight of the pillar
 τ_R ; Shearing resistance strength of the foundation rock at the bottom of the pillar
 A_R ; Bottom area of the pillar
 τ_F ; Shearing resistance strength of the back fault material
 ΣA_F ; Area of the back fault
 f ; Internal friction factor of the foundation rock

The stability of the foundation rock at Kawamata dam has been evaluated by performing the calculations mentioned above for each elevation and for other assumable pillars,

The author has studied the following method to check the stability of the pillar separated by the faults which are nearly vertical and running in both upstream and downstream direction, Despite complicated and laborious work required before calculating the supporting strength of the back, the afore-mentioned rocky-pillar calculation requires the Young's modulus of the fault and lacks in accuracy in assuming the deformation of the foundation, In addition this equation does not hold unless the downstream is open surface, Then, the simpler, following method has been selected,

- 1) The assumed pillar break has a shape made by enveloping the lines which minimize the safety factor in two-dimensional analysis by the Henny's equation dimensional slice,

- 2) Deformation of the pillar is not taken into consideration.
(In this meaning, this is called "rigid body sliding method".)
- 3) The force vertical to the fault is supported by the back hill of the fault.
Therefore, the force parallel to the fault becomes the force to shear the pillar, resulting in the slide along the fault surface.
- 4) Resistance forces are the shearing strength of the bottom base, shearing strength of the fault, shearing friction resistance due to the weight of the hill, and the shearing friction resistance due to the force vertical to the fault.

In this case, the problems are the assumptions in (2) and (3). Considering from the nature of the problem, slide along the fault surface does not matter, but the problem lies in the fact that the force vertical to the fault is regarded as the shearing friction resistance. Taking the fault material as gelatine like, the fault can not withstand the force applied to it. Being small and solid in this case, (30 cm to 100 cm), (Young's modulus; 5,000 to 8,000 kg/cm²), the fault has been regarded as being capable of transmitting the force to the back hill as in the case of no fault. For this reason, the calculation has been simplified. The equation is as follows :

$$S_F = \frac{f_R(W-U) + \tau_R A_B + f_f(V-U\gamma) + \tau_F A_F + \tau_R A_R}{H} \dots\dots\dots (65)$$

where, H ; Component of the thrust parallel to the fault face

V ; Component of the thrust vertical to the fault face

U ; Uplift acting on the bottom of the pillar

$U\gamma$; Up acting on the fault face

f_R, f_f ; Internal friction factor of the rock and fault

τ_R, τ_f ; Initial shearing strength of rock and fault

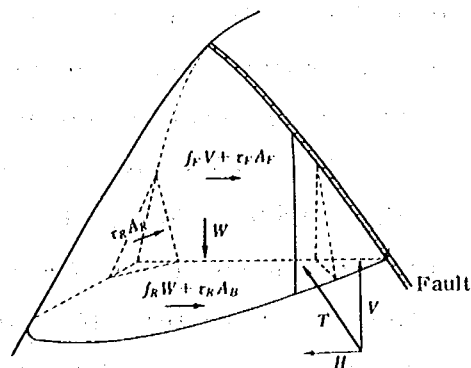
W ; Weight of the pillar

A_R ; Area of the shearing section of the side rock

(CD of the two-dimensional slice)

A_B ; Area of the bottom

A_F ; Area of the back fault



F-7 fault at the left bank of Kawaji dam under study has shown the safety factor of 1 by two-dimensional analysis, using Henney's equation, while 20 by this rigid body sliding method. This is an example in which a great difference has been obtained between three-dimensional stability analysis and two dimensional one. In Kawaji dam, those which can not ensure the safety factor 4 by two-dimensional analysis are analyzed by the above-mentioned rigid body sliding method to guarantee the safety of the base. Considering the calculation error of the rigid body sliding method, the author placed the standard of technological safety judgement at 6. In Nagawado dam, the safety of the foundation is also studied by the rigid body sliding method.

In the above-mentioned methods, whether it is two-dimensional or three-dimensional, the basic idea is based on the ratio between the working force and resisting force. This is to judge the technological stability by the limit safety factor, and as mentioned in the item of the gravity dam this is insufficient, requiring the judgement on the safety factor against stress break at a local part.

Since it is very difficult to obtain the three-dimensional stress-distribution in the foundation of the arch dam, the stress by two-dimensional stress is obtained first and checking is made on the point safety factor explained in the chapter of the gravity dam. Since the arch dam is a three-dimensional structure, the judgement should be made after obtaining third-dimensional stress conditions. In this point the arch dam differs from nearly two-dimensional gravity dam. The stress at Managawa dam was analyzed in the rock including the fault using three-dimensional finite element method. The result shows that the stress dissipates greatly from just up the dam crest and from the dam base into the foundation, resulting in far faster dissipation of the stress as compared with two-dimensional stress conditions.

In case the weight of the rock is taken into consideration, at the point 20 m inside the abutment, the majority of the stress is caused by the weight of the rock, while the effect by the thrust is very slight.

Thus, in the case of the arch dam, judgement by the point safety through two-dimensional stress condition has a possibility to evaluate the condition too dangerous. This may be due to the fact that two-dimensional stress condition is not maintained at the third-dimensional condition and in the two-dimensional condition the initial stress is not taken into consideration. Therefore, three-dimensional stress must be calculated to obtain the accurate local safety factor (point safety factor) in the rock of the arch dam.

Only the theory of elasticity is available for this calculation. Can the properties of the rock foundation with great non-uniformity be analyzed? Many difficult problems lie before us. It is needless to say, of course, judgement by two-dimensional analysis is effective from technological point of view.

5.5 Arch Dam in A Parabola Shape

As stated in the previous section of the design location of abutment of arch dams, the consideration to stabilize the foundation of arch dams is specially necessary beyond other type dams. When the rock foundations on the downstream have weak strata or when the abutments of the dam are put on the thin roofs, location of abutment becomes inevitably deep. The depth of the rooting is directly related to the quantity of both concrete and excavation and greatly affects economy. Therefore, it is quite natural that every possible effort is made to have the rooting as shallow as possible. For this purpose, there are roughly two counter-measures as below :

- 1) The rock foundation on the downstream should be improved and the strength of the foundation be increased so that the dam has the shallow rooting.
- 2) By the design of the dam body, the thrust to the base rock foundation should be put to effective operation, to have the shallow rooting.

Regarding (1), it is a measure often adopted, by replacing the nearby abutments or the strata on the downstream with concrete, and also by conveying the force to the deep part of a mountain with the stress conveying wall (concrete wall). These methods, however, frequently damage the foundation of a dam (expressed in the term of looseness) during the course of construction works and since the degree of damages cannot be judged quantitatively. As such, the measure (1) which keeps the stabilization of the foundation by improving the rock-foundation should be adopted only when the deep rooting does not produce a good effect or where remarkable economical effects can be expected by partial improvements.

If the measure (2) is possible, the safety of the foundation is increased smoothly. It would be the best measure. In this sense, it is an arch dam in a parabola shape that have come to appear on the stage. As stated in the section of the required shoulder line, the direction of the thrust greatly affects the stabilization of the foundation. As showing the drawing on the right the required shoulder line varies in its length according to the direction

of the thrust, in case of i) and ii). Namely, the thickness required on the downstream is different. If AB should be the strata, in case of ii), the safety factor of sliding in more, while that of i) is less. In general, as factors which form the thrust (T), there are the thrust (H) by arch action and shearing force (V). The degree toward the downstream of T from abutment is affected by $\frac{V}{H}$. The greater the shearing force becomes, the more the degree toward the downstream of T.



Another factor is the central angle of arch that regulates the direction of abutments of a dam. As a matter of fact, as the degree which affects the direction of the thrust, the central angle is overwhelmingly great.

In designing Kawamata dam, considering that to have the thrust toward the mountain side increases the safety of the poor foundation rocks where many strata and predominant seams existed. Komai and Shibata looked into the relationship between the central angle of arch and the degree of the thrust toward the mountain side by using symmetrical, circular arch of equal thickness and equal semi diameter. The angle to be formed by the line against the base surface and the Resultant force (T) of the arch thrust (H) and the shearing force (V) can be figured out by the following formula, when the central angle of arch is ϕ_A ;

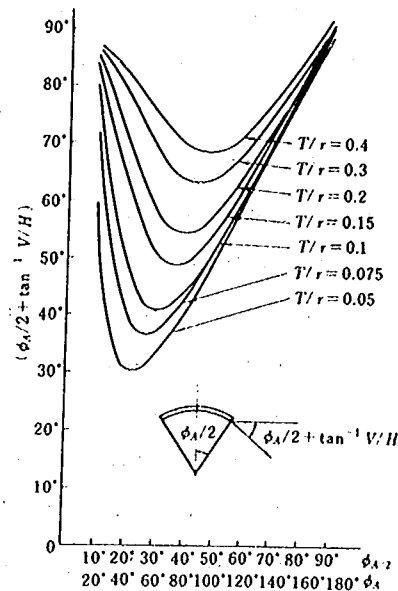
$$\phi = \frac{\phi_A}{2} + \tan^{-1} \frac{V}{H} \quad (66)$$

The Fig, 62 shows this relationship, from which it is understandable that when the ratio $\frac{T}{r}$, the thickness of a dam, T and arch semi diameter, r is 0,1, $\phi_A \doteq 60^\circ$, when $\frac{T}{r}$ is 0,05, $\phi_A = 45^\circ$ and thus the thrust faces toward the mountain side in a greatest extent.

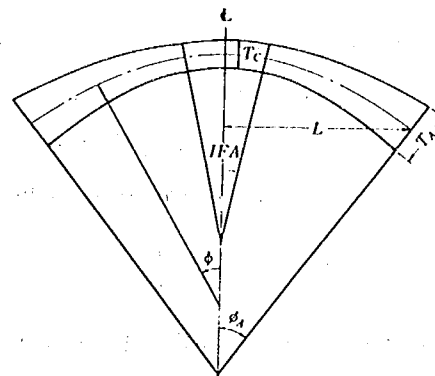
From this drawing, it can generally be said that there is a tendency that the smaller the central angle is and the greater the arch semi diameter, the thrust faces toward the mountain side. However, when the central angle is made smaller and the curve ratio semi diameter greater, it will form an arch shape very close to the straight line, so that in the vicinity of arch crown where the bending moment is great, there arises a great tensile stress. When Kawamata dam was designed, it was not possible to solve this problem fundamentally. The central angle of the arch dam at that time was about 110° , however, the safety of the base rock foundation was successfully increased, by adopting arch central angle less than 90° , having

the thrust face toward the mountain side.

Even at the time of designing Kawamata dam, it was intuitively possible to anticipate that the excessive tensile stress arisen in the vicinity of arch-crown by having the small arch central angle and great curve ratio semi-diameter can be avoided by having the curve ratio semi-diameter in only the vicinity of arch-crown small, and the great curve ratio. But, at that time, the application analysis was carried out manually and it was practically difficult to carry out the application analysis of the arch dam of a complicated shape.



Fig, 62 Retation between direction of thrust and center angle



Fig, 63 Arch for radius of variation

- 1) $\lambda = 0$, $\phi_A = \phi_A$ or $\lambda = 0$, $\phi_A = 0$,

In case $T_c = T_A$, equal thick arch with a core

- 2) In case $\lambda = 0$, $T_c \neq T_A$, un-equal thick arch with a core

- 3) In case $\lambda = -3$, arch in which arch centerline is parabola

- 4) In case $\lambda > 0$, variable semi-diameter arch in which the curve ratio semi-diameter is continuously small from crown to abutment

- 5) In case $\lambda < 0$, it is possible to handle the variable semi-diameter arch

in which the curve ratio semi-diameter is great continuously from crown to abutment

Then, the geometrical characteristics of the parabola shall be taken into consideration. The parabola expressed in

$$x = \frac{1}{4p}y^2 \quad \text{shall be considered,}$$

the focus of the parabola be $C_0 (p, 0)$
any one point on the parabola be $p(x, y)$,
The cross point of the line against P
point and x axis be C ,

$$\angle PCO = \phi \quad \angle PC_0O = \theta$$

Passing P , drawing parallel PE of x axis,
then from the nature of parabola

$$\angle EPC = \angle CPC_0$$

$$\theta = 2\phi$$

$$PC_0 = C_0C$$

$$PC_0 = 2p - PC_0 \cos \theta = 2p - PC_0 \cos 2\phi$$

$$\therefore PC_0 = \frac{2p}{1 + \cos 2\phi} = p \sec^2 \phi$$

$$\therefore x = PC_0 - p = p \sec^2 \phi - p = p \tan^2 \phi$$

$$y = PC_0 \sin 2\phi = 2p \tan \phi$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\phi}}{\frac{dx}{d\phi}} = \frac{\sec^2 \phi}{\tan \phi \sec^2 \phi} = \cot \phi$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\phi} \left(\frac{dy}{dx} \right) \frac{d\phi}{dx} = -\operatorname{cosec}^2 \phi \times \frac{1}{2p \tan \phi \sec^2 \phi} = -\frac{1}{2p} \cot^3 \phi$$

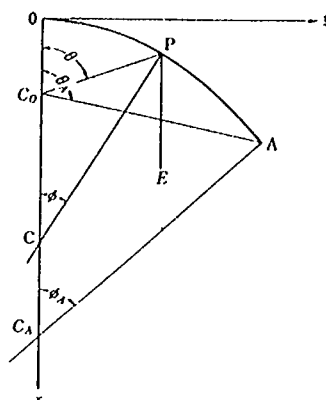
Therefore, the curve ratio semi-diameter r ,

$$r = \frac{\left(1 + \left(\frac{dy}{dx} \right)^2 \right)^{\frac{3}{2}}}{\left(\frac{d^2y}{dx^2} \right)} = 2p \sec^3 \phi = 2p \cos^{-3} \phi \quad \dots \dots \dots (67)$$

Namely, the curve ratio semi-diameter is variable in proportion to $\cos^{-3} \phi$.

When the parabola shape is adopted for the shape of arch dam, it is possible to reduce the tensile stress in the vicinity of arch crown, since it is possible to make the curve ratio large value in the vicinity of arch crown despite of the small central angle of arch dam, which could not be achieved by the circular arch. In other words, it is possible to have the thrust face toward the mountain side without making stress distribution of an arch dam itself worse, and as a result, the safety of the foundation is increased and thus the economical design of an arch dam becomes possible.

Shibata and Ogura have made a comparison of a circular arch (central



Fig, 64 The nature of the parabola

angle 100°) and a parabolic arch (center angle 75°). Against the comparative design proposal where stress distribution of an arch itself for both was kept equal, the total volume for a parabolic arch was $240,000 \text{ m}^3$ while that for a circular arch $252,000 \text{ m}^3$, thus the volume of a parabolic arch was less by 5% than that of circular arch. The direction of the thrust for both is shown in Fig. 66. In case of a parabolic arch, the thrust is almost at right angles against a group of the faults for example, QL~SL and as far as the safety of the foundation is concerned, there is a great difference between the two.

In the Institute, utilizing the merits of this parabolic arch, the design of arch dams by this shape has been carried out since Yahagi dam. Such dams as Susobana, Shōrenji, Hōheikyo, Managawa, Kawaji and Kawabegawa have adopted parabola shape, the central angles of which are $70 \sim 75^\circ$.

In adopting the parabola shape for Yahagi dam for the first time, as aforesaid, it was confirmed to be a superior method of design, from the standpoint of static condition, however, as far the characteristics of the dam during an earthquake nothing has been clear. Therefore, the vibration model test was done by a method in Production Technique Laboratory of Tokyo University, and a comparison has been carried out between a circular arch, whose maximum central angle is 100° , and a parabolic arch. From the results of the experimental figures, the following table has been searched for, calculating self vibration periodic time, damping constant, top part acceleration and stress.

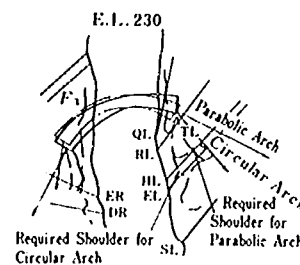
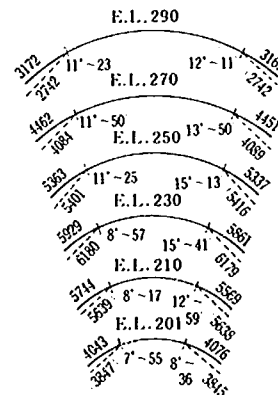


Fig. 65 The horizontal section of Yahagi dam (comparison with circular arch)



Dotted line and () shows a circular arch
Solid line shows a parabolic arch

Fig. 66 Comparison of the thrust action direction of parabolic arch and circular arch of Yahagi dam

Table 15 Comparison of the vibration characteristics of the circular arch and the parabolic arch of Yahagi dam

The condition of the water reservoir	Kinds of arch	Self vibration periodic time(sec)	Damping constant	Top part acceleration	Top part arch stress	Bottom part cantilever stress
At the time of its being empty	Parabolic arch	0.268	0.034	0.34 g	8.6	5.4
	Circular arch	0.231	0.037	0.31 g	6.6	2.4
At the time of its being full	Parabolic arch	0.388	0.120	0.53 g	22.0	22.5
	Circular arch	0.275	0.083	0.84 g	11.0	9.8

From this table, the self vibration periodic time, compared with test results during reservoir empty does not make differences however, the self vibration periodic time of the parabolic arch during reservoir full is about 0.4 second and it is rather long. It does not matter in the earthquake-resistant design.

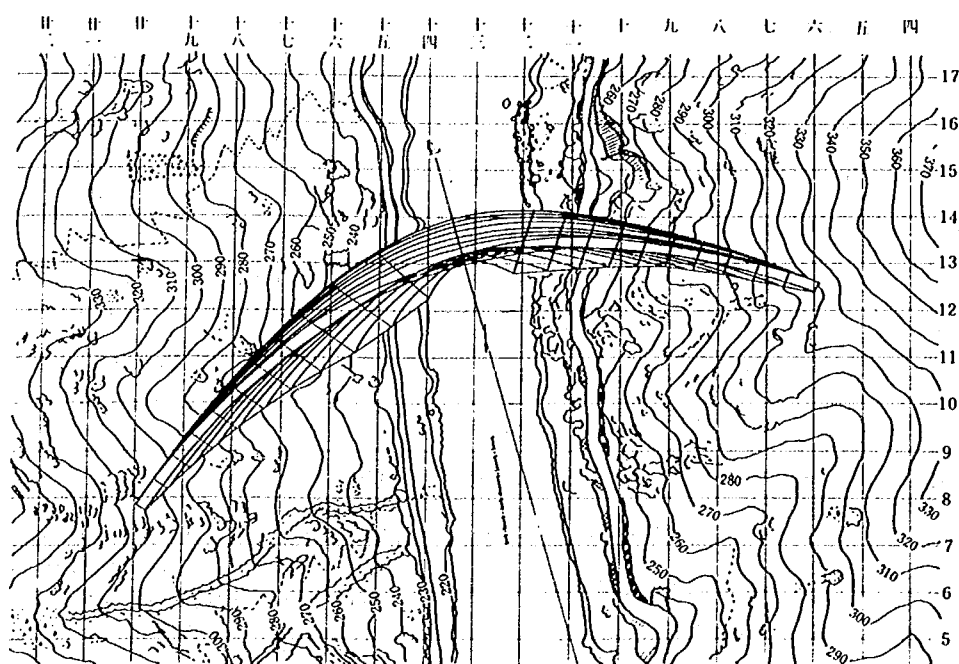


Fig. 67 Plan of Yahagi dam

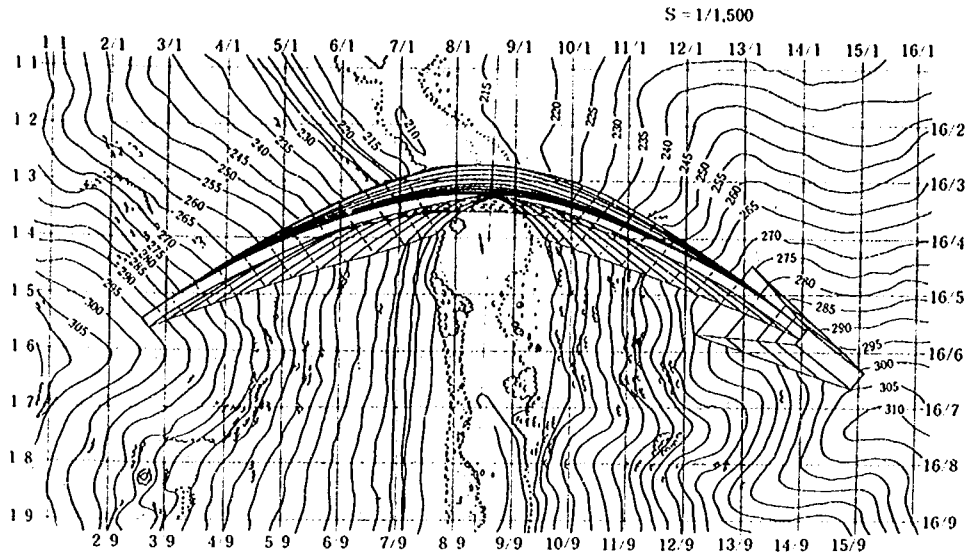


Fig. 68 Plan of Shōrenji dam

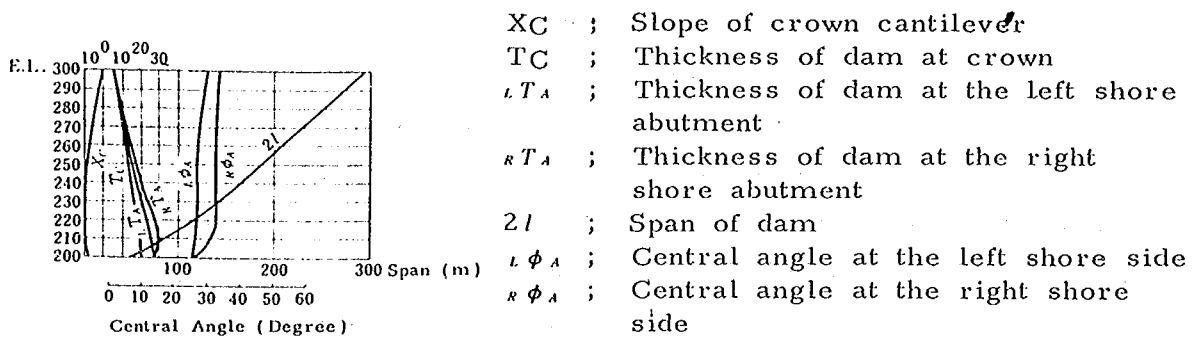


Fig. 69 Basic dimension of Yahagi dam

The maximum stress shown on the table has not arisen where the static stress becomes maximum further the stress during an earthquake changes plus and minus, which will not be dangerous. Such being the case, the result that the parabolic arch, compared with the circular arch is inferior, from the standpoint of anti-earthquake has not been obtained, so that it is dynamically the stable shape.

In case of Yahagi dam, the on-site test, installing the vibrator to the

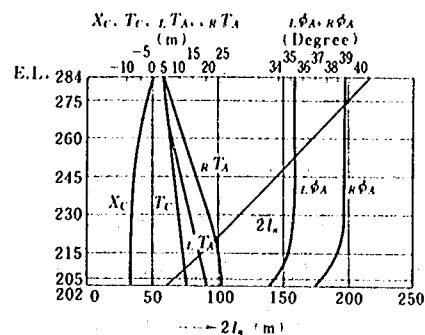


Fig. 70 Basic dimension of Shōrenji dam

prototype dam, has been carried out and the test results as to the proper periodic time and the vibration mode conform well to those of the model test.

5.6 The Detailed Design of Arch Dams

5.6.1 Cautions in the Design Coming Out from Differences between An Arch Dam and A Gravity Dam

Generally, the detailed design of the arch dam is carried out, based on that of the gravity dam, however, there are special cautions pertaining to the arch dam, which are as follows :

The arch dam, compared with the gravity dam has thin thickness, therefore, the large stress acts. Different from the gravity dam, being a three-dimensional structure, it should be a continuous structure toward the direction of the dam axis not only upwards and downwards.

When the stress is great, it requires inevitably the abundant mixture of concrete. As a result, the temperature rises abruptly right after concreting, so that there is a great possibility that the concrete might have cracks. In case of the arch dam, compared with the gravity dam, cracks produce much more damages, therefore, we should avoid them, considering to prevent seepage because of its thin section. Also, in case of the arch dam, in order to check the temperature rise right after concreting, pipe cooling is definitely necessary. Joint grouting is also indispensably necessary since the dam body should be continuous toward the dam axis. In order to satisfy this condition, it is required to reduce the temperature as far as the final stable temperature, and in this sense, it is impossible to do without pipe cleaning.

In order to avoid cracks toward the stream direction, the arch dam is also provided with transverse joints, of which spacing is the side joint 10 ~

20 meters, considering the degree of cooling and the construction capacity. In case of the arch dam, longitudinal joints are not set, because of the thin section. In the vicinity of the bottom part dam abutment, there are cases where the dam thickness is about 30 ~ 40 meters, however, if the longitudinal joints are set, the spread fluck might arise, to unify the dam, it is better not to have the longitudinal joints.

From the two reasons that the dam is thin, and it is a three-dimensional structure, specially the joint grouting should be ensured to be carried out. For this purpose, from the standpoint of the construction works, designers

should make full arrangements to reduce the temperature of the dam as far as the final stable one. Further more, the horizontal construction joints should be well green-cut and the seals should be installed carefully.

Rising from the high stress of an arch dam, the degree of the stress concentration around the openings of the main conduit pipe and galleries is remarkably great, compared with the gravity dam. In order to meet this problem, the stress calculation is done like in case of the gravity dam, but because the quantity of bars is so great that concreting on-site becomes extremely difficult. In order to ease this problem, such considerations are necessary as to reduce the number of steel bar by having larger diameter or to omit wrapping or hooking parts by using deacon steel bars.

It has been already stated that in case of the arch dam, cracks should be severely prevented, because of the thin section and so there is an example that steel bars were arranged on the surface of the arch dam, to prevent the surface cracks. But this quantity of steel bar is enormous and becomes a hindrance during the construction works, the author considers it is better to avoid it by having the secure management and construction of concrete and its ample care after concreting.

5.6.2 The Design of Supplementary Structures of Abutment

The sound rock foundation is necessary throughout the entire base of the dam, but sometimes it lacks soundness at a part. On this account, in designing the arch dam, supplementary structures are often set for abutment in order to reduce the stress, or to design the dam body economically. And, these are often adopted to reinforce the geographical defects at the dam site. There are such supplementary structures for abutment as fillet, thrust block, gravity abutment, saddle, plug and wing dam. These are explained in order as follows :

a. Fillet

Generally, as in the vicinity of abutment, the dam body has bending moment, multiplied with arch thrust the stress degree becomes greater. In order to reduce the stress degree of the dam body it has an arch of un-equal thickness and to ease the stress to the foundation. The fillet is the dam body is increased partially toward the up and down stream. We can use it advantageously to correspond to the nature of the foundation, it has nothing to do with the basic shape of the whole body of arch.

b, Thrust block

When at the upper part the valley is open or the foundation rock is weathered to a deep part, if we design the dam according to the topography or the sound rock line, the shape becomes remarkably uneven and un-symmetrical. In this case, designing an arch dam with smooth shape is very difficult and the stress concentration might arise at the uneven parts. In order to prevent this, the thrust block is set, just nearby the dam body, between the dam and the foundation, to convey the thrust from the dam to the foundation.

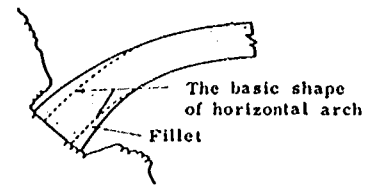


Fig. 71 Fillet

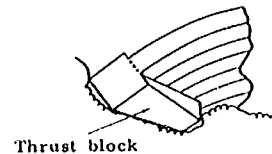


Fig. 72 Thrust block

By providing the thrust block, the shape of the dam itself can be decided, regardless of the topography or the geological partial defects.

In case of Shōrenji dam, at the upper part of the left bank, the sound rock is not available so that the thrust block is set at this part and almost symmetrical arch dam could be designed, regardless of the geological defects.

In order to design the thrust block, like in case of the gravity dam, consideration should be given to the stabilization of failure and sliding, assuming the thrust and the hydraulic pressure from the upstream as external loads. When the slope of the thrust block base is mild and the foundation of the thrust block base is weak in strength, there are some cases where the safety against sliding is very low. Necessary caution should be taken on this point.

c. Plug

As shown in Fig. 73, when the bottom part of the stream has the deep cut, into which concrete shall be filled. This concrete is called plug and it is separated from the dam itself. When there is such a geographical defect, without any plug, then the height of the dam becomes high and the dam volume and the excavation are great. Recently, even without

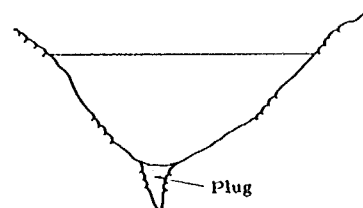


Fig. 73 Plug

such a partial cut showing on the previous drawing when the lower part of a valley is narrow, compared with the upper part, in order to have, to a certain extent, smooth shape of an arch dam, the excavation at the lower part becomes excessive and there is the problem that the long and vast slope might arise due to the results of excavation at this part. On this account, with the same idea applied to plug, such a design is often made as to fill the part of the river bed with concrete, on which the dam commences to stand. In such a case, it would be more appropriate to call footing rather than plug, and the design of the upper part of the dam is economically and easily constructed, but it is very difficult to make stress analysis to it. In other words, as to the applicable analysis of the upper part, the dam, which stands on the footing, is analyzed as an arch dam, however, since the lower footing is not an unbounded foundation, the analysis is not adequate, when calculated by the current load splitting method.

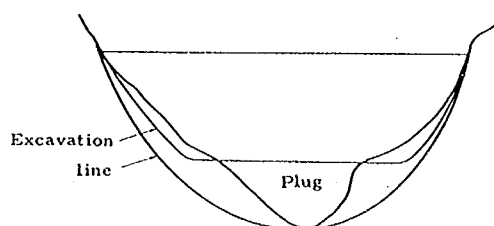


Fig. 74 Plug

In usual dealing with this case the dam of the upper part is assumed as a monolithic arch dam, on the other hand, arch dam, the lower footing is designed as a gravity dam, against hydrolic pressure and conveyed from the upper arch. Since the footing and the arch dam are, in nature, an unified structure, such a convenient analysis is greatly lack in strictness. This point can be considered by the model test, but a model in large scale is required from the standpoint of measuring technique. The most effective means to solve this problem would be a three-dimensional finite element method.

When the footing is small, no matter occurs if we use the aforementioned load splitting method in stress analysis.

Thus, this is considered to be an effective structure. As the design examples of having the footing, there are those of Yagisawa, Hōheikyo, Kawaji and Kawabegawa dam. As for Kawabegawa arch dam, the footing part was so huge as to occupy 30% of the total height and in order to check the stress condition of the dam including the footing, the model experiment was conducted.

d. Gravity abutment and wing dam

When near the top of a dam, there are defects of topography or geography, it cannot be or it is unfavourable to exert the thrust to the foundation. In such cases the artificial abutment (a kind of gravity dam) should be provided, supporting the thrust at the part, conveying it to the lower part of the foundation. This is the gravity abutment. As shown in Fig. 75, the wing dam is also required at the same time. As the examples of the gravity abutment, there are those of Korobe NO. 4 dam and Yagisawa dam.

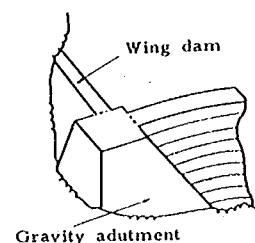


Fig. 75 Gravity abutment and wing dam

e. Saddle

As showing in Fig. 76, the saddle is used to ease the stress to the surface of the base foundation, set between the body and the foundation. By adjusting the thickness of the saddle part properly, it can cope with the uncertainty of the excavation line, and make the shape of the dam itself smooth. During excavation work, the alterations of the foundation line are disclosed, then only the revision of the saddle part is enough. It is unnecessary to revise the design of the dam.

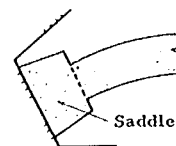


Fig. 76 Saddle

As the examples of this design, there is that of Kawamata dam, which is a symmetrical arch dam with a saddle. The intersection between the saddle and the body is a joint structure. (This is called a perimeter joint.)

REFERENCES

- 1) Japanese National Committee on Large Dams, "Design Criteria for Dams "
- 2) Shikoku District Construction Bureau, Ministry of Construction, "Final Report of Ishidegawa Dam", 1973 (in Japanese)
- 3) Ishii, Iida, Kishimoto, "Rational Design Method of Dam Foundation", Public Works Research Institute, Technical Memorandum 337, 1968 (in Japanese)
- 4) Ishii, Kobayashi, "Shearing Friction of A Gravity Dam " Public Works Research Institute, Technical Report 131-3, 1967 (in Japanese)
- 5) U. S. B. R. "Teatise on Dams Chap. 9. Gravity Dams "
- 6) Nojima, Iida and others, "Stability Analysis on Ohdo Dam " Public Works Research Institute, Technical Memodandom 717, 1972 (in Japanese)
- 7) Japanese National Committee on Large Dams, "Dams in Japan " 1973 (in Japanese)
- 8) U. S. B. R. "Treatise on Dams Chap. 10. Arch Dams "
- 9) Iida, Shibata and Others, "Relations between the Results by Trial Load Method and by Model Test on Arch Dam's Stress Analysis " Public Works Research Institute, Technical Memorandum 609, 1969 (in Japanese)
- 10) Okamoto, Shibata, "A Study of Flattening An Arch Dam " Japanese National Committee on Large Dams, Large Dams No. 57. (in Japanese)
- 11) Japanese Society of Civil Engineering, "Final Report of Kawamata Arch Dam ", 1965 (in Japanese)
- 12) Iida, "Stress Analysis of Arch Dams ", Public Works Research Institute, Technical Memorandum 1. 1965 (in Japanese)
- 13) Shibata, Ogura, "Structural Design of Yahagi Dam ", Public Works Research Institute, Technical Memorandum 1966 (in Japanese)

Design of Concrete Dams

Technical Memorandum No. 1627

March, 1981

MITSUAKI MIZUNO

Key Words: Gravity Dam, Hollow Gravity Dam,
Arch Dam, Design of Concrete Dam

Summary

This presents the standard design method of Gravity Dam and Hollow Gravity Dam. Arch theory and Trial Load Method are also explained about Arch Dam for beginners.



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