

SHEAR STRENGTH AND DYNAMIC DEFORMATION PROPERTIES OF FILL MATERIALS

February, 1988

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SHEAR STRENGTH AND DYNAMIC
DEFORMATION PROPERTIES
OF
FILL MATERIALS

by

Norihisa Matsumoto

Nario Yasuda

Synopsis :

The aim of this booklet is to present the fundamental concept of soil mechanics applicable to fill dam engineering.

The booklet consists of two parts. Part I deals with the shear strength of fill materials which is one of the most important design values in the stability analysis of fill dams. The shear strength characteristics are explained by the effective stress basis. Dependency of friction angle on confining stresses is explained by using large-scale triaxial compression test data.

Part II describes the fundamentals of soil dynamics, which are the most basic principles in earthquake resistant design of fill dams. Both part I and II include several exercises for use in lecture at Public Works Research Institute, Ministry of Construction. The authors also expect that this booklet will serve as quick reference material for practicing engineers.

key words : shear strength, triaxial test, rockfill, soil dynamics,
shear modulus, damping ratio, wave propagation

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Part I SHEAR STRENGTH OF FILL MATERIALS

1. Introduction

1.1 Classification of Fill Materials

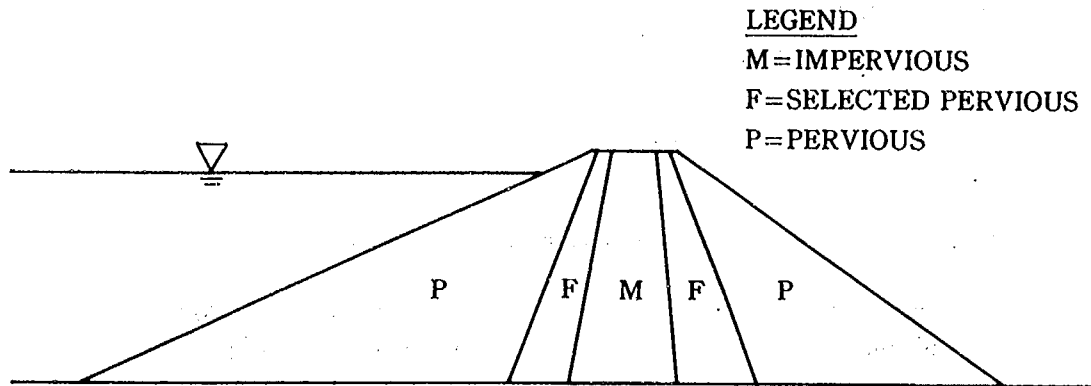


Fig. 1.1 Section of Fill Dam

1.1.1 Classification According to Particle Size

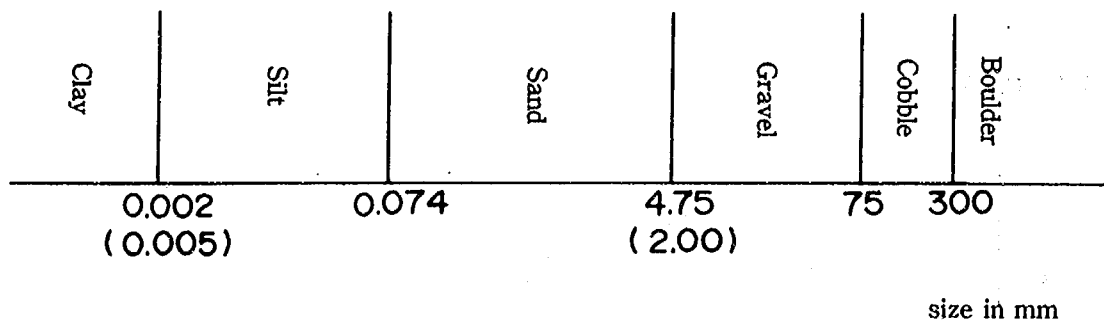


Fig. 1.2 Size

see "Unified Soil Classification"

1.1.2 Classification According to Permeability

- a. Impervious Materials
- b. Semi-pervious Materials
- c. Pervious Materials

1.1.3 Classification According to Function

- a. Core
- b. Filter
- c. Transition
- d. Shell

1.1.4 Classification According to Shear Strength Characteristics

- a. Clay or Impervious Materials
- b. Sand or Semi-pervious and Pervious Materials

1.2 Stresses within a Soil Mass

1.2.1 Stress for a Granular System

$$\sigma = \frac{\sum N}{\ell \times \ell} \quad \tau_x = \frac{\sum T_x}{\ell \times \ell} \quad \tau_y = \frac{\sum T_y}{\ell \times \ell} \dots\dots\dots (1.1)$$

Table 1.1 Typical Values for Average Contact Forces within Granular Soils (Marsal, 1963)

Particle	Diameter in mm	Average Contact Force(kgf) for Macroscopic Stress(kgf/cm ²)		
		0.07	0.7	7
Gravel	60	1.36	13.6	136
	2.0	0.00136	0.0136	0.136
Sand	0.06	$1.36 \times 10^{1/4}$	1.36×10^{-5}	1.36×10^{-4}
Silt	0.002	1.36×10^{-6}	1.36×10^{-8}	1.36×10^{-7}

1.2.2 Mohr Circle

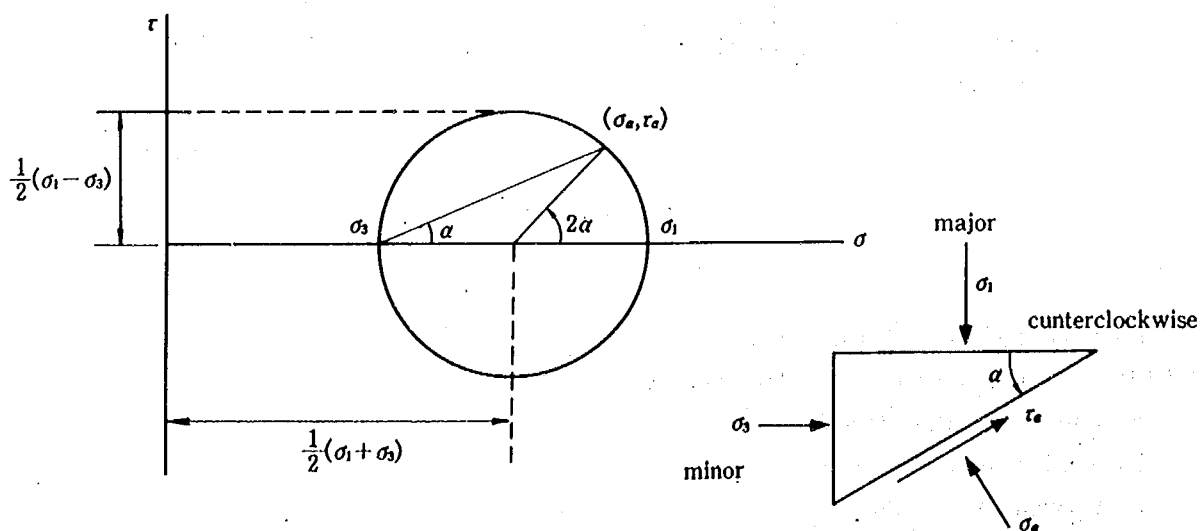


Fig. 1.3a

In Fig. 1.3a

$$\left. \begin{aligned} \sigma_a &= \sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha \\ &= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha \dots\dots\dots \\ \tau_a &= (\sigma_1 - \sigma_3) \sin \alpha \cos \alpha \\ &= \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha \dots\dots\dots \end{aligned} \right\} (1.2)$$

In Fig. 1.3b

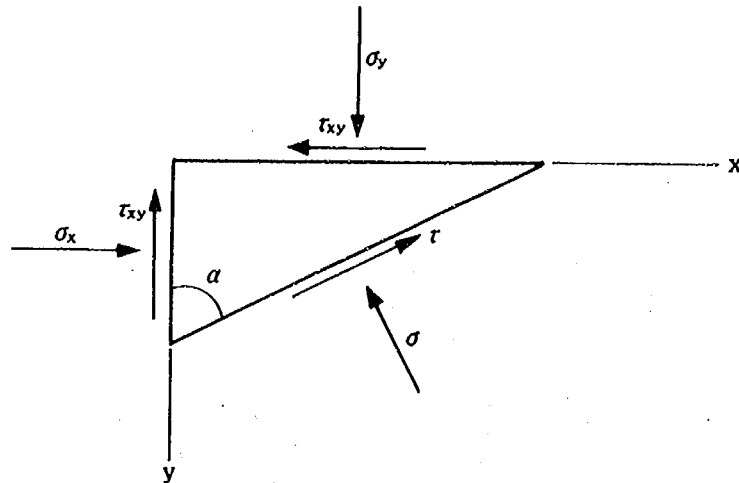


Fig. 1.3b

$$\left. \begin{aligned} \sigma &= \sigma_x \cos^2 \alpha + \sigma_y \sin^2 \alpha - 2 \tau_{xy} \sin \alpha \cos \alpha \\ \tau &= -\tau_{xy}(\cos^2 \alpha - \sin^2 \alpha) + (\sigma_y - \sigma_x) \sin \alpha \cos \alpha \end{aligned} \right\} (1.3)$$

Example 1.1

Given. Fig. 1.4

Find. Stresses on plane A-A

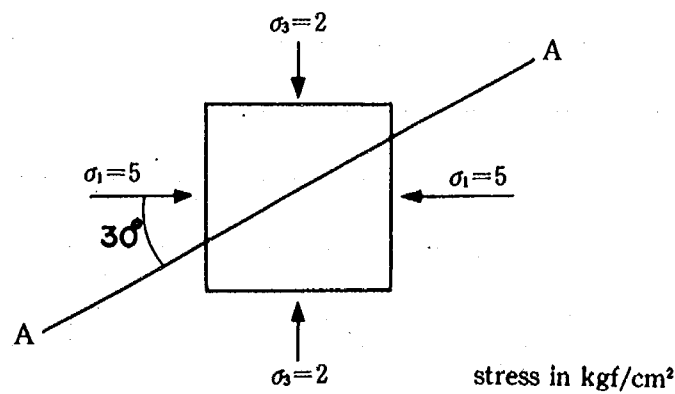


Fig. 1.4

Answer.

(1) by using Eqs. (1.2) and (1.3)

$$\sigma_1 = 5, \quad \sigma_3 = 2 \quad \alpha = 120^\circ$$

$$\sigma_a = \frac{5+2}{2} + \frac{5-2}{2} \cos 240^\circ = 3.5 - 0.75 = 2.75 \text{ kgf/cm}^2$$

$$\tau_a = \frac{5-2}{2} \sin 240^\circ = -1.30 \text{ kgf/cm}^2$$

(2) by using Fig. 1.5

$$\sigma_a = 2.75 \text{ kgf/cm}^2$$

$$\tau_a = -1.3 \text{ kgf/cm}^2$$

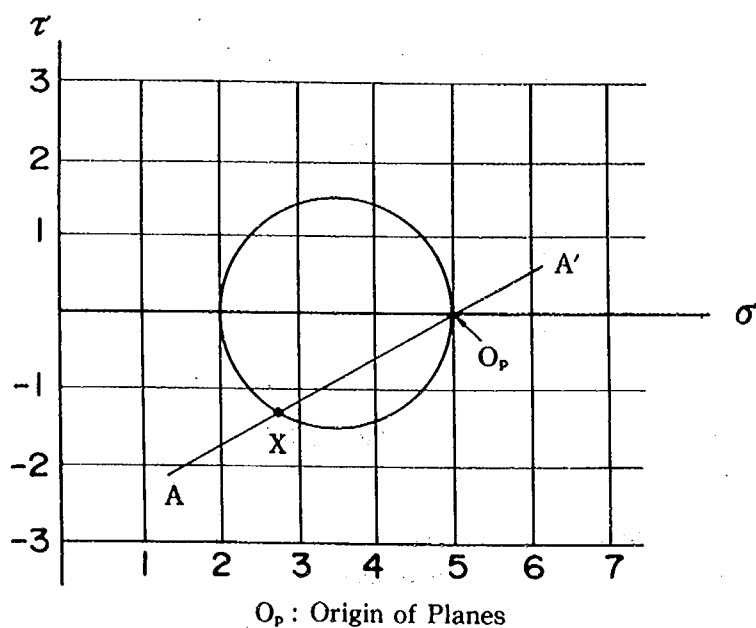


Fig. 1.5 Mohr Circle for Example 1.1

1.2.3 p-q Diagrams and Stress Paths

$$p = \frac{\sigma_1 + \sigma_3}{2} \dots\dots\dots (1.4)$$

$$q = \frac{\sigma_1 - \sigma_3}{2} \dots\dots\dots (1.5)$$

$$p = \frac{\sigma_v + \sigma_h}{2} \dots\dots\dots (1.6)$$

$$q = \frac{\sigma_v - \sigma_h}{2} \dots\dots\dots (1.7)$$

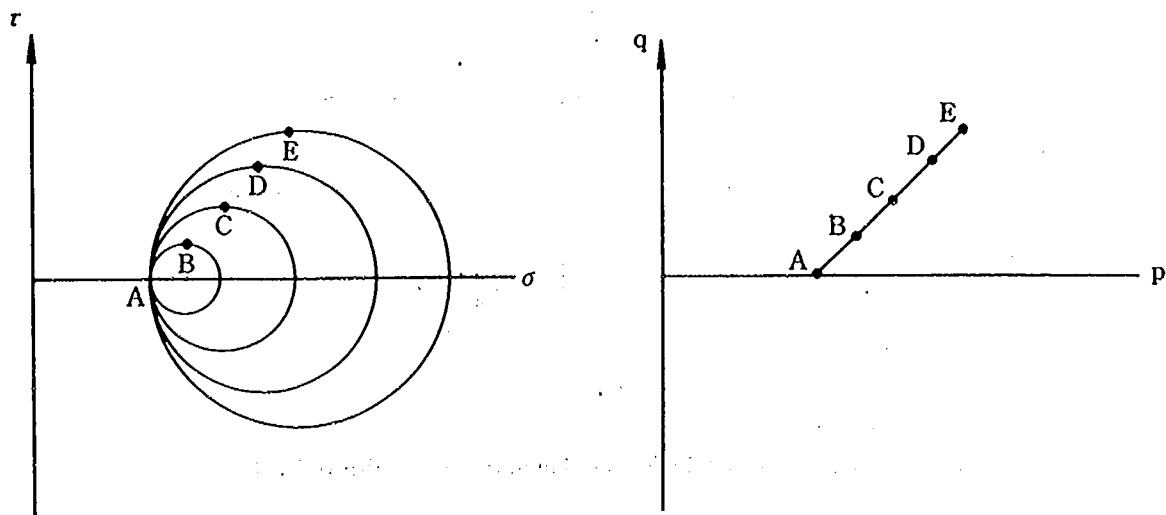


Fig. 1.6 Mohr Circle and Stress Diagram
(σ_1 increase with σ_3 constant)

1.2.4 Geostatic Stresses

Ground surface : horizontal

Nature of soil : little variation in the horizontal directions

$$\sigma_v = \int \gamma dz \dots\dots\dots (1.8)$$

$$K = \frac{\sigma_h}{\sigma_v} \dots\dots\dots (1.9)$$

K : coefficient of lateral stress or lateral stress ratio

K_o : coefficient of lateral stress at rest or lateral stress ratio at rest ($\epsilon_h = 0$)

1.2.5 Stresses due to Applied Loading

Assumption : Elasticity

a. Boussinesq's Solution (Fig.1.7)

$$\sigma_z = \frac{3 Pz^3}{2 \pi R^5} \dots\dots\dots (1.10)$$

$$\sigma_r = \frac{P}{2 \pi R^2} \left\{ \frac{3 z r^2}{R^3} - \frac{(1-2\nu)R}{R+z} \right\} \dots\dots\dots (1.11)$$

$$\tau_{rz} = \frac{3 P r z^2}{2 \pi R^5} \dots\dots\dots (1.12)$$

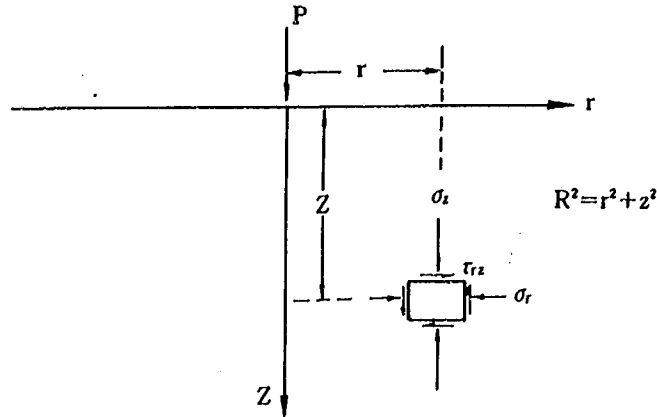


Fig. 1.7 Concentrated Force on Boundary of Semi-infinite Body

b. Circular Uniform Load (Fig. 1.8)

$$\begin{aligned}\sigma_z &= ap_0 \int (1+z\xi)e^{-\xi z} J_1(a\xi) J_0(\xi r) d\xi \\ \tau_{rz} &= ap_0 \int z\xi e^{-\xi z} J_1(a\xi) J_1(\xi r) d\xi\end{aligned}\quad \dots\dots\dots (1.13)$$

especially for $r = 0$

$$(\sigma_z)_{r=0} = p_0 \left\{ 1 - \frac{1}{\left(1 + \frac{a^2}{z^2}\right)^{\frac{3}{2}}} \right\} \quad \dots\dots\dots (1.14)$$

$$(\sigma_r)_{r=0} = p_0 \left\{ \frac{1+2\nu}{2} - \frac{1+\nu}{\left(1 + \frac{a^2}{z^2}\right)^{\frac{1}{2}}} + \frac{1}{\left(1 + \frac{a^2}{z^2}\right)^{\frac{3}{2}}} \right\} \quad \dots\dots\dots (1.15)$$

$$(u_z)_{r=0} = -\frac{2ap_0(1-\nu)^2}{E} \left(\sqrt{1 + \frac{z^2}{a^2}} - z \right) \cdot \left[1 + \frac{\frac{z}{a}}{2(1-\nu)\sqrt{1 + \frac{z^2}{a^2}}} \right] \quad \dots\dots\dots (1.16)$$

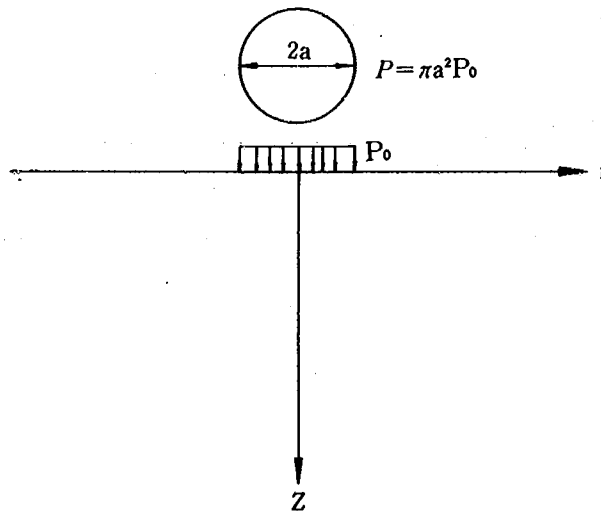


Fig. 1.8 Uniform Load on a Circular Area

1.2.6 Effective Stress

a. Definition

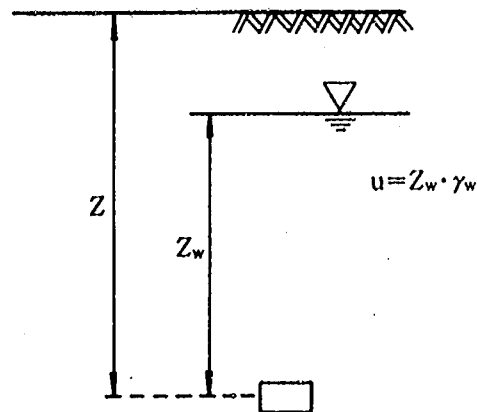


Fig. 1.9 Stresses in Subsoil

$$\bar{\sigma}_v = \sigma_v - u \quad \dots\dots\dots (1.17)$$

$$\bar{\sigma}_h = \sigma_h - u \quad \dots\dots\dots (1.18)$$

$$K = \frac{\bar{\sigma}_h}{\bar{\sigma}_v} \quad \dots\dots\dots (1.19)$$

$$\bar{p} = p - u \quad \dots\dots\dots (1.20)$$

$$q = \frac{\sigma_v - \sigma_h}{2} = \frac{\bar{\sigma}_v - \bar{\sigma}_h}{2} \quad \dots\dots\dots (1.21)$$

Example 1.2 (Fig. 1.10)

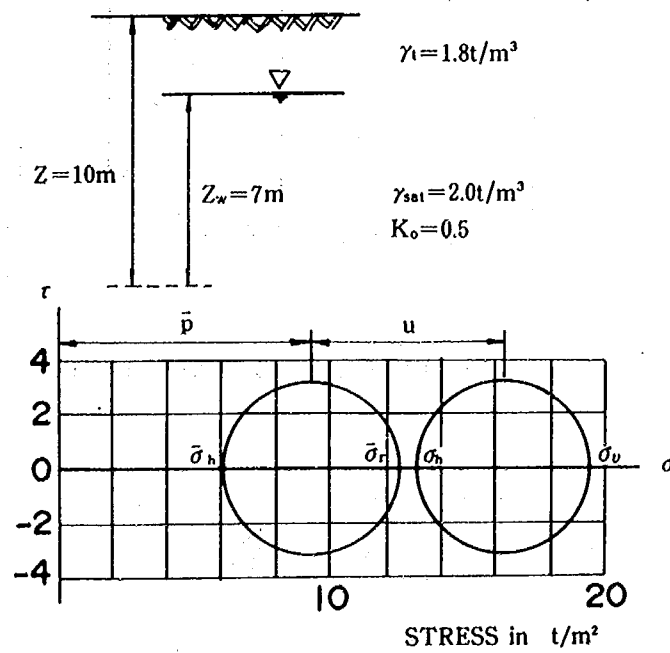


Fig. 1.10

b. Capillarity in Soil

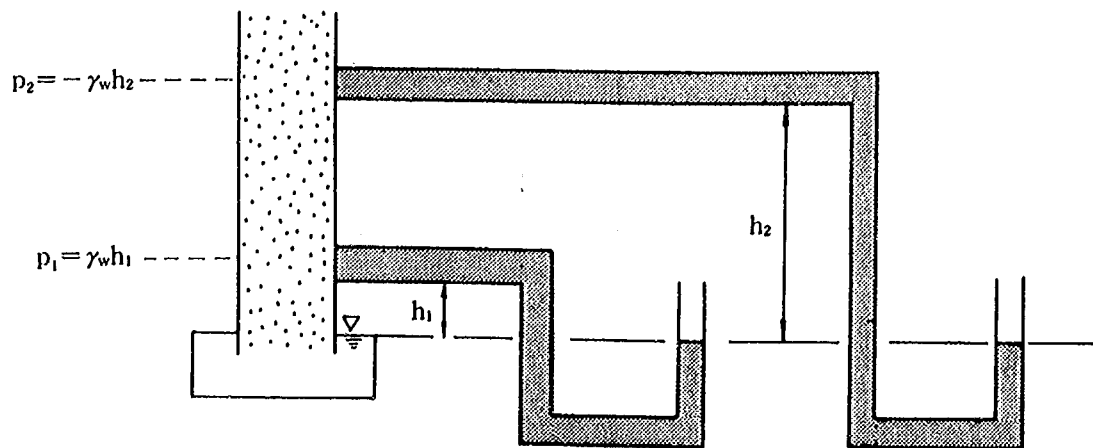


Fig. 1.11 Pressure in Capillary Water

Example 1.3 (Fig. 1.12)

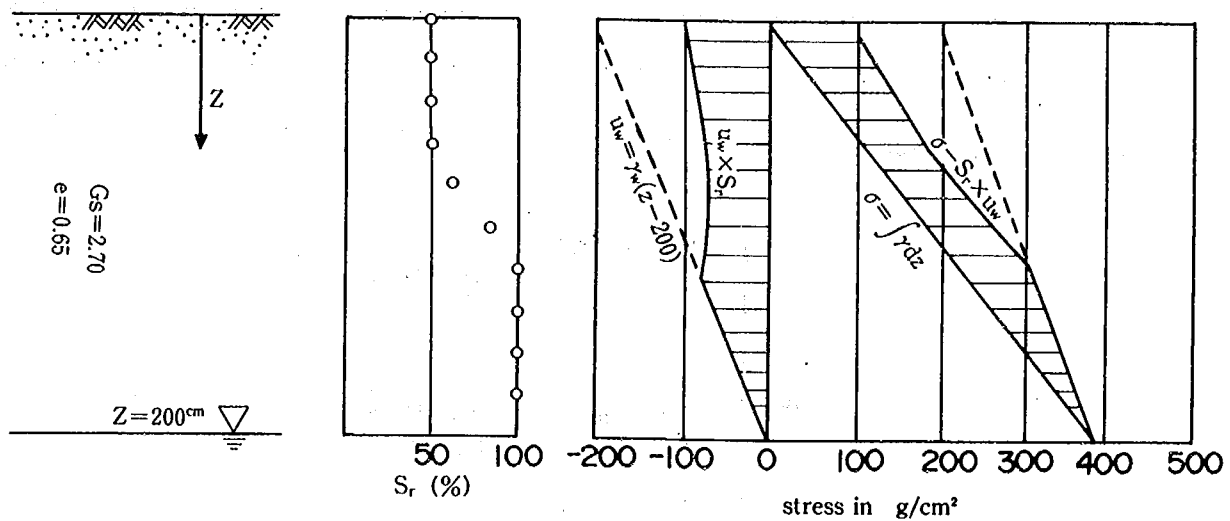


Fig. 1.12 Pressure and Stress in Subsoil

$$\begin{aligned} \sigma_z &= \gamma_t \cdot z & \bar{\sigma}_z &= \sigma_z - u_w \\ u_w &= -\gamma_w \cdot z & \text{or} & \quad u_w = -\frac{S_r}{100} \gamma_w \cdot z \end{aligned}$$

1.3 Tests to Measure Stress-Strain Properties

a. Isotropic Compression

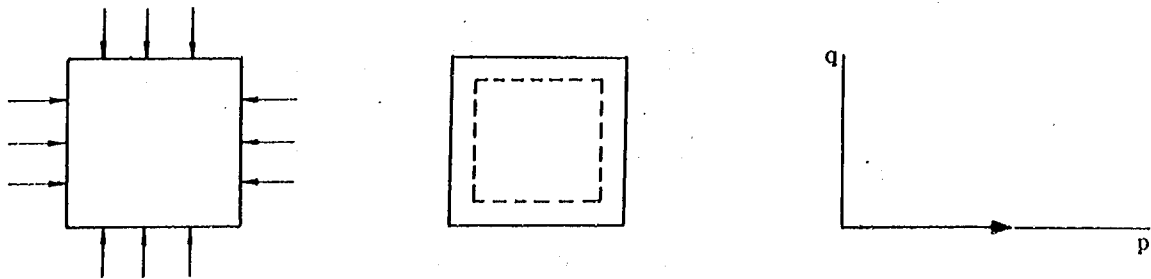


Fig. 1.13 Isotropic Compression

b. Confined Compression (Oedometer Test)

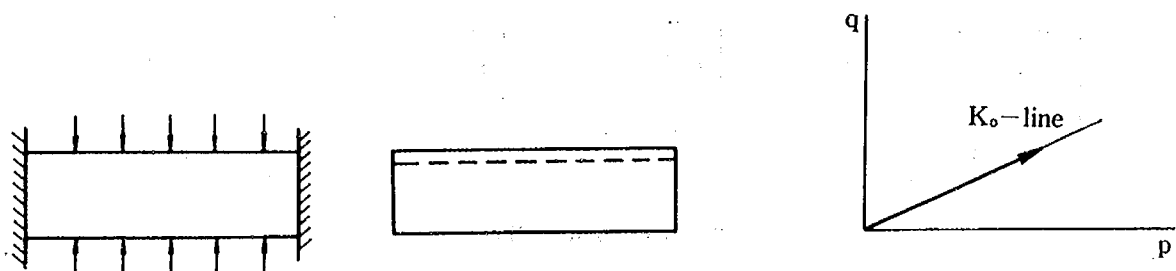


Fig. 1.14 Confined Compression

c. Triaxial Compression

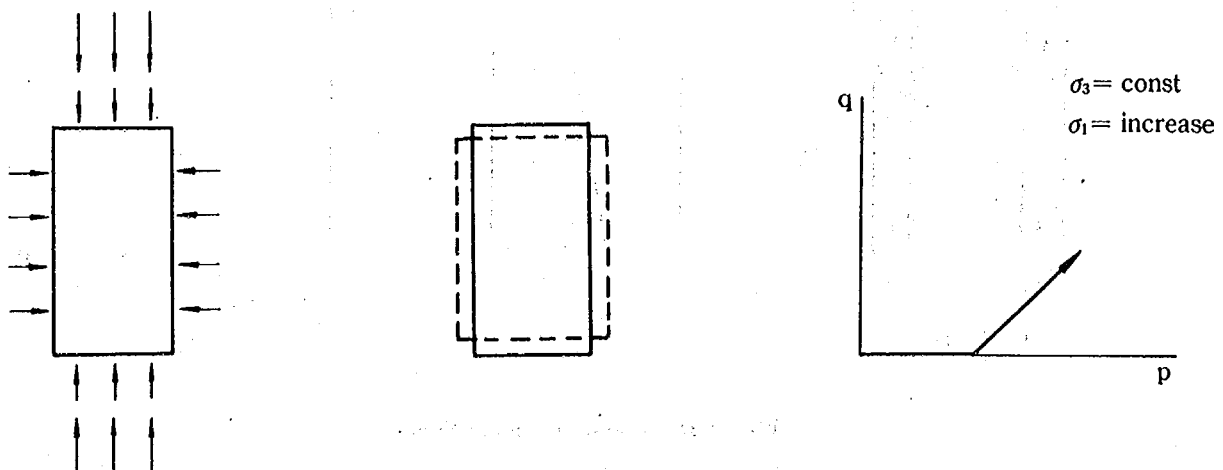


Fig. 1.15 Triaxial Compression

d. Direct Shear

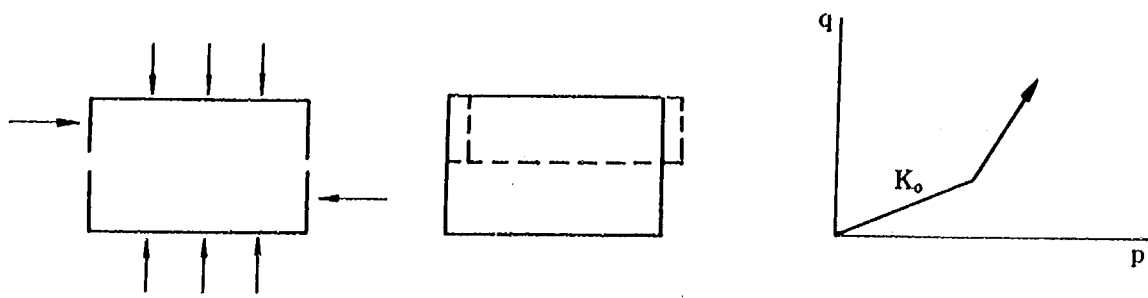


Fig. 1.16 Direct Shear

e. Simple Shear

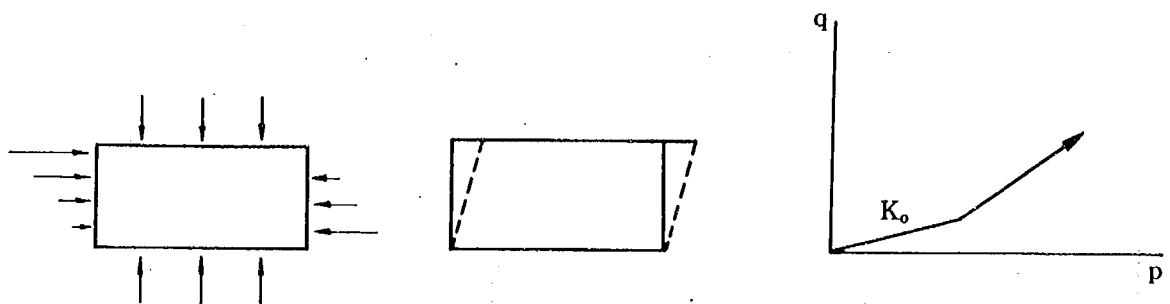


Fig. 1.17 Simple Shear

f. Torsional Simple Shear

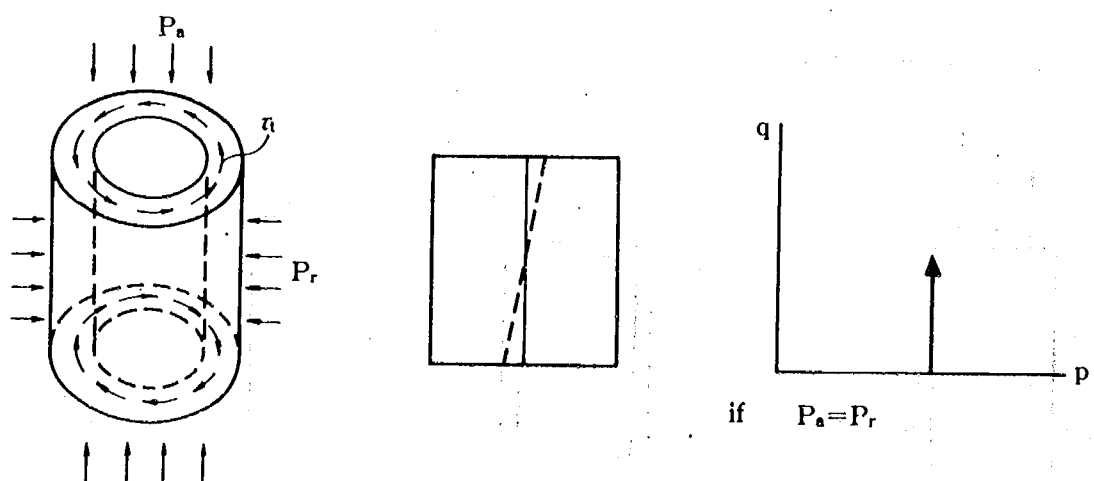


Fig. 1.18 Torsional Simple Shear

Exercises

E1.1 Given. Fig. 1.19

Find. Stresses on plane A-A

- (1) by using Mohr circle
- (2) by using Eqs. (1.2) and (1.3)

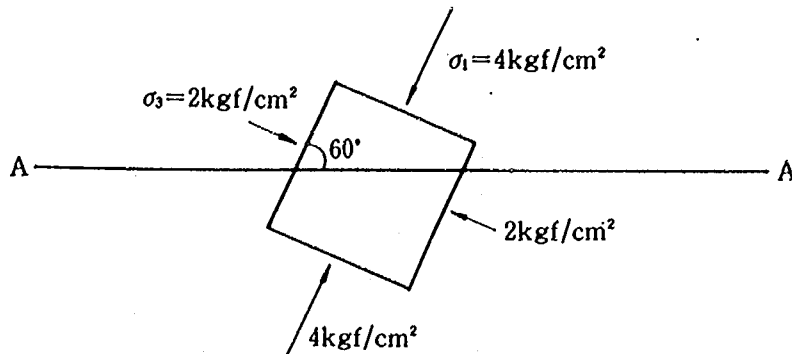


Fig. 1.19

E1.2 Given. Fig. 1.20

Find. Magnitude and direction of the principal stresses.

- (1) by using Mohr circle
- (2) by using Equation

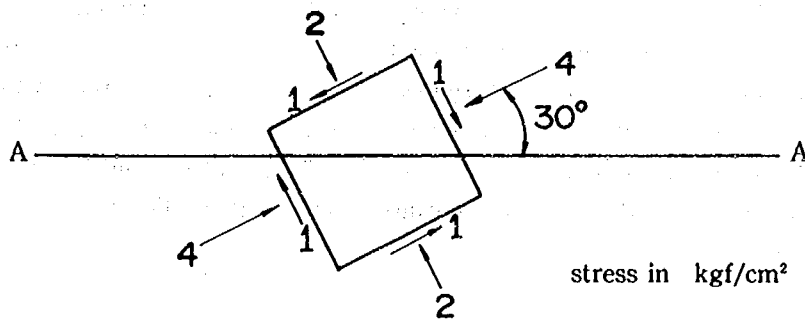


Fig. 1.20

E 1.3 Given. Ground with horizontal surface is subjected to uniform loading on a circular area as shown in Fig. 1.21.

Find. Stress paths for points A to D.

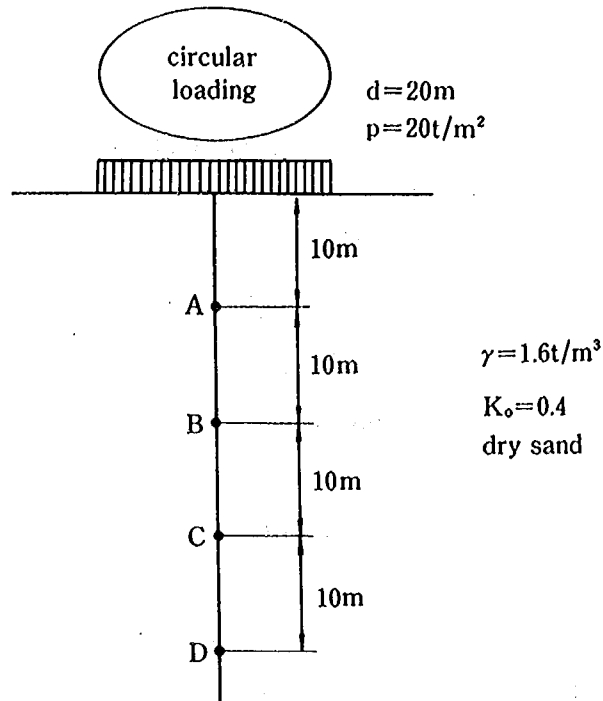


Fig. 1.21

E 1.4 Given. In triaxial tests, initial conditions and loading conditions are given as follows.

- (1) At first $\sigma_r = \sigma_a = 2 \text{ kgf/cm}^2$. σ_a remains constant while σ_r increases to 11.66 kgf/cm^2 .
- (2) At first $\sigma_r = \sigma_a = 2 \text{ kgf/cm}^2$. σ_r remains constant while σ_a increases to 12.89 kgf/cm^2 .
- (3) At first $\sigma_r = \sigma_a = 2 \text{ kgf/cm}^2$. σ_a remains constant while σ_r decreases to 0.31 kgf/cm^2 .
- (4) At first $\sigma_r = \sigma_a = 2 \text{ kgf/cm}^2$. σ_r remains constant while σ_a decreases to 0.34 kgf/cm^2 .
- (5) At first $\sigma_r = \sigma_a = 2 \text{ kgf/cm}^2$. σ_a increases to 3.41 kgf/cm^2 while maintaining $\sigma_r = 4.0 - \sigma_a$.

Find. Stress paths for these loading conditions on a p - q diagram.

2. Shear Strength of Sand and Rockfill

2.1 Shear Strength of Dry Sand and Rockfill

Example 2.1

Given. Following data are obtained from standard triaxial test, i.e. keeping σ_r constant while increasing σ_a . The parent rock of the materials is Andesite. Specimens are in dry condition.

Confining Pressure(kgf/cm ²)	Peak Axial Stress(kgf/cm ²)	Peak Value of q	Value of p at Peak
2.0	12.22	5.11	7.11
4.0	21.22	8.56	12.56
6.0	30.22	12.11	18.11
8.0	37.7	14.85	22.85

Find.

- (1) Angle of internal friction ϕ_0 at each confining pressure.
- (2) Angle of theoretical failure plane.

Solution.

Angle of internal friction ϕ_0 and angle of failure plane are given by relations shown in Fig. 2.1.

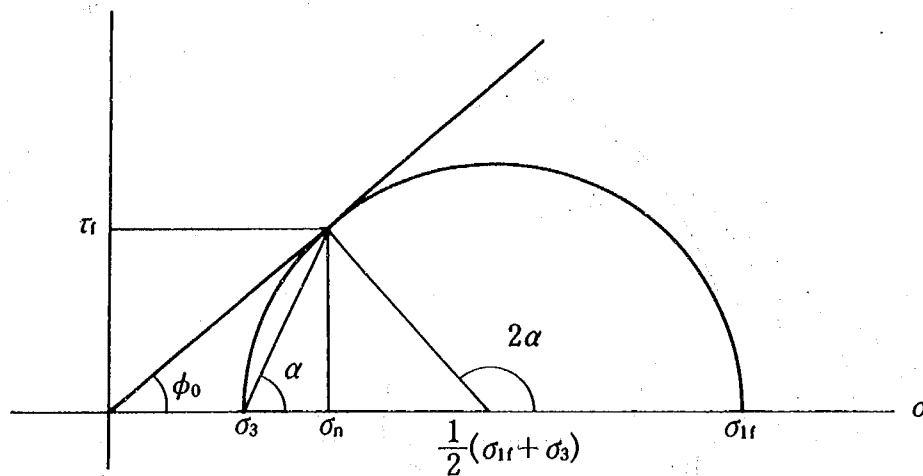


Fig. 2.1 Relations between ϕ_0 and principal stresses at failure

$$\sin \phi_0 = \frac{\frac{1}{2}(\sigma_{1f} - \sigma_{3f})}{\frac{1}{2}(\sigma_{1f} + \sigma_{3f})} = \frac{q}{p} = \frac{\left(\frac{\sigma_1}{\sigma_3}\right)_f - 1}{\left(\frac{\sigma_1}{\sigma_3}\right)_f + 1} \quad \dots\dots\dots(2.1)$$

$$\alpha_f = 45^\circ + \frac{\phi}{2} \quad \dots\dots\dots(2.2)$$

$$\tan^2 \alpha_t = \frac{\sigma_{1t}}{\sigma_{3t}} \dots\dots\dots (2.3)$$

$$N_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \left(\frac{\sigma_1}{\sigma_3} \right)_t \dots\dots\dots (2.4)$$

N_p : flow factor

σ_3	$\frac{\sigma_{1t}}{\sigma_{3t}}$	$\frac{q}{p}$	ϕ	α_t
2.0	6.11	0.719	45.9°	68.0°
4.0	5.28	0.682	43.0°	66.5°
6.0	5.04	0.669	42.0°	66.0°
8.0	4.71	0.650	40.5°	65.3°

Mohr envelope is shown in Fig. 2.2 and P – q diagram in Fig. 2.3.

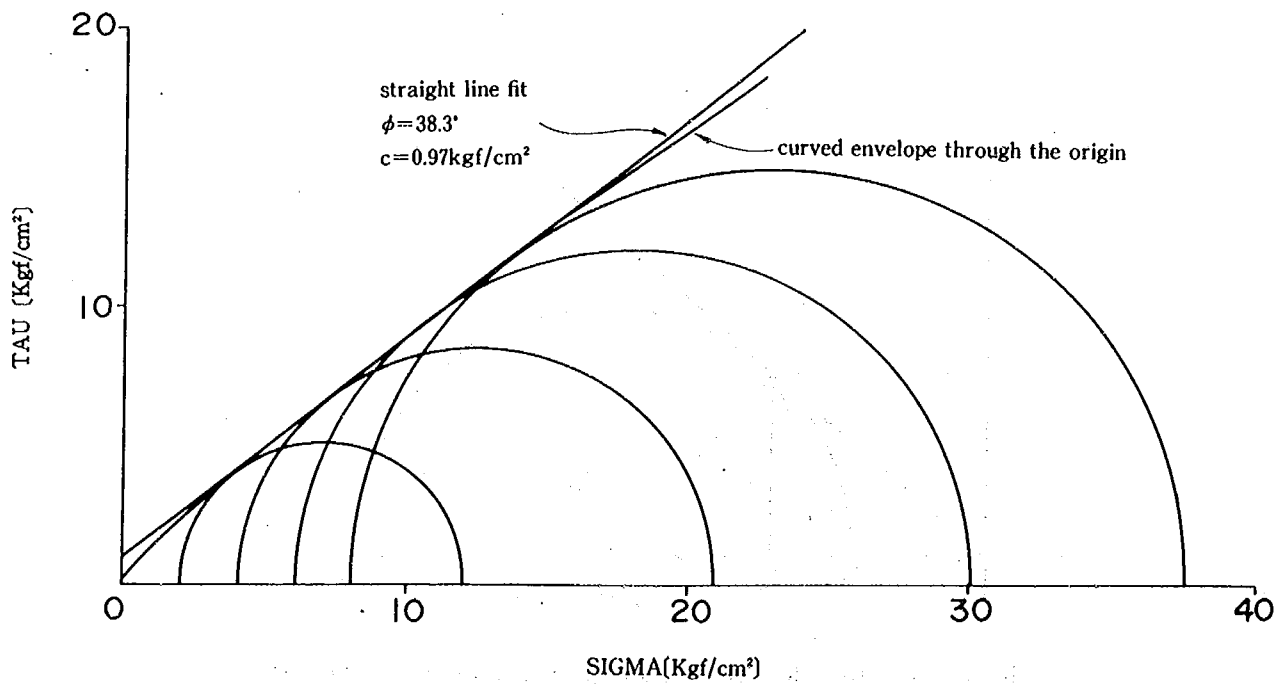


Fig. 2.2 Mohr envelope for Example 2.1

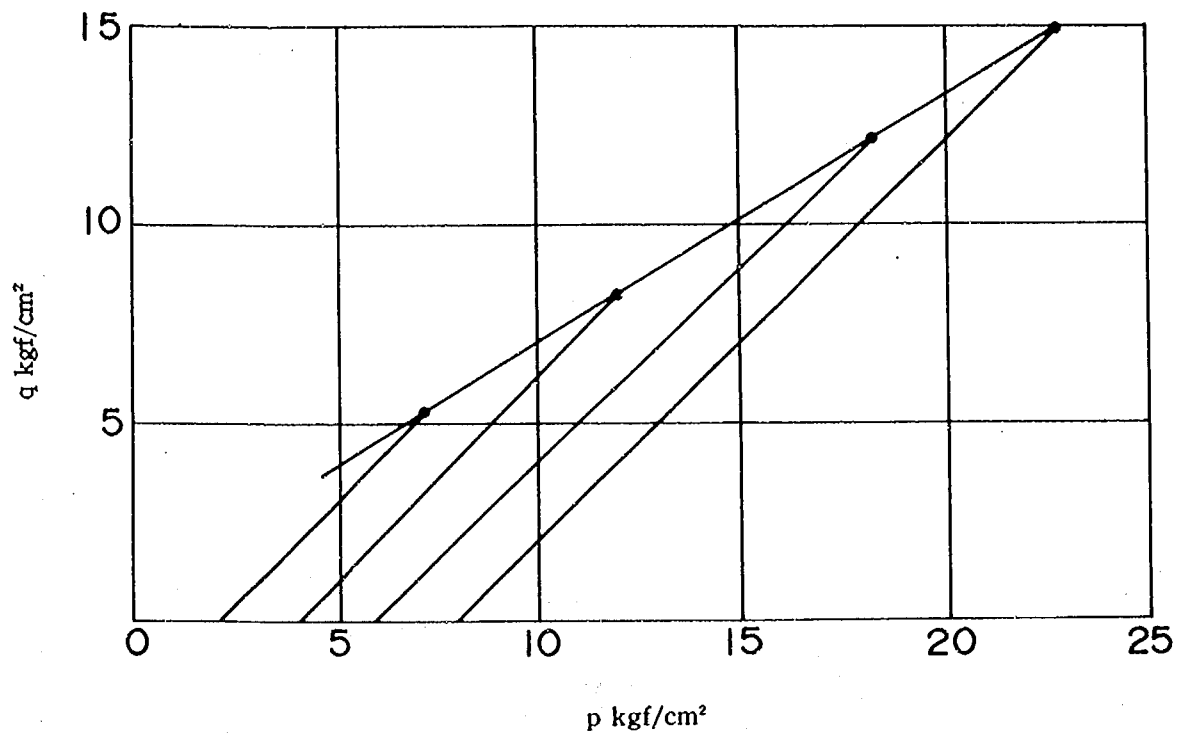


Fig. 2.3 p-q Diagram for Example 2.1

2.1.1 Effect of Confining Stress

- a. Mohr envelope. (Otto Mohr, 1882)

$$\tau_f = f(\sigma_n) \dots\dots\dots (2.5)$$

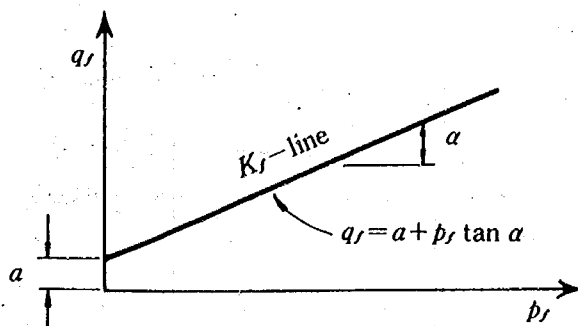
- b. Mohr-Coulomb failure law.

$$\tau_f = c + \sigma_n \tan \phi \dots\dots\dots (2.6)$$

- c. In case of straight line through the origin.

$$\tau_f = \sigma_n \tan \phi \dots\dots\dots (2.7)$$

- d. Finding c and ϕ from p-q diagram



$$\sin \phi = \tan \alpha$$

$$c = \frac{a}{\cos \phi}$$

2.1.2 Shear Strength of Rockfill

Example 2.2

Given. Test results of standard triaxial compression test are shown in Fig. 2.4.

Find. Mohr envelope, and relations of $\phi_0 - \sigma_3$ and $\sigma_n - \tau_t$.

Solution. Figs. 2.5, 2.6, and 2.7.

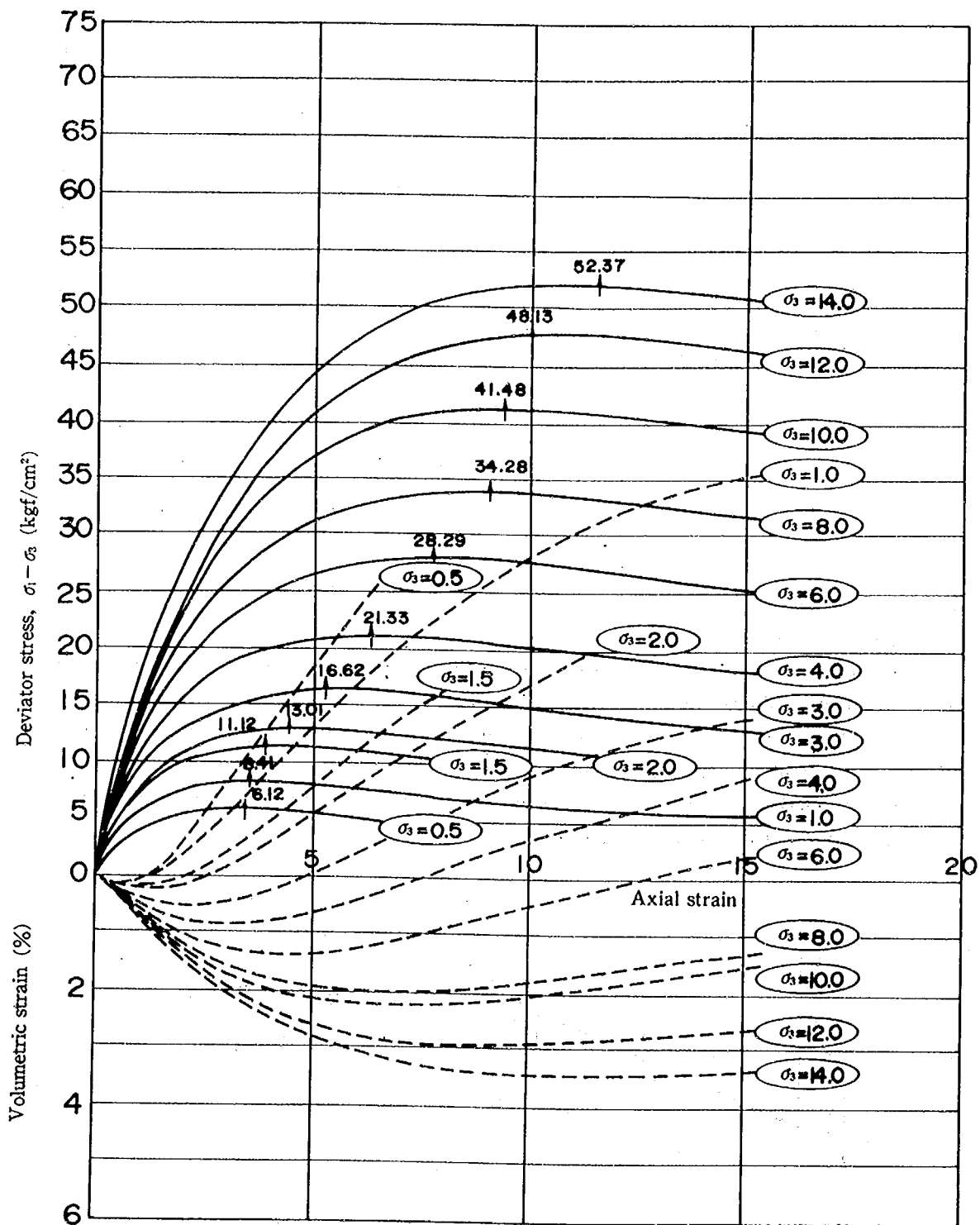


Fig. 2.4 Triaxial tests for various confining pressures

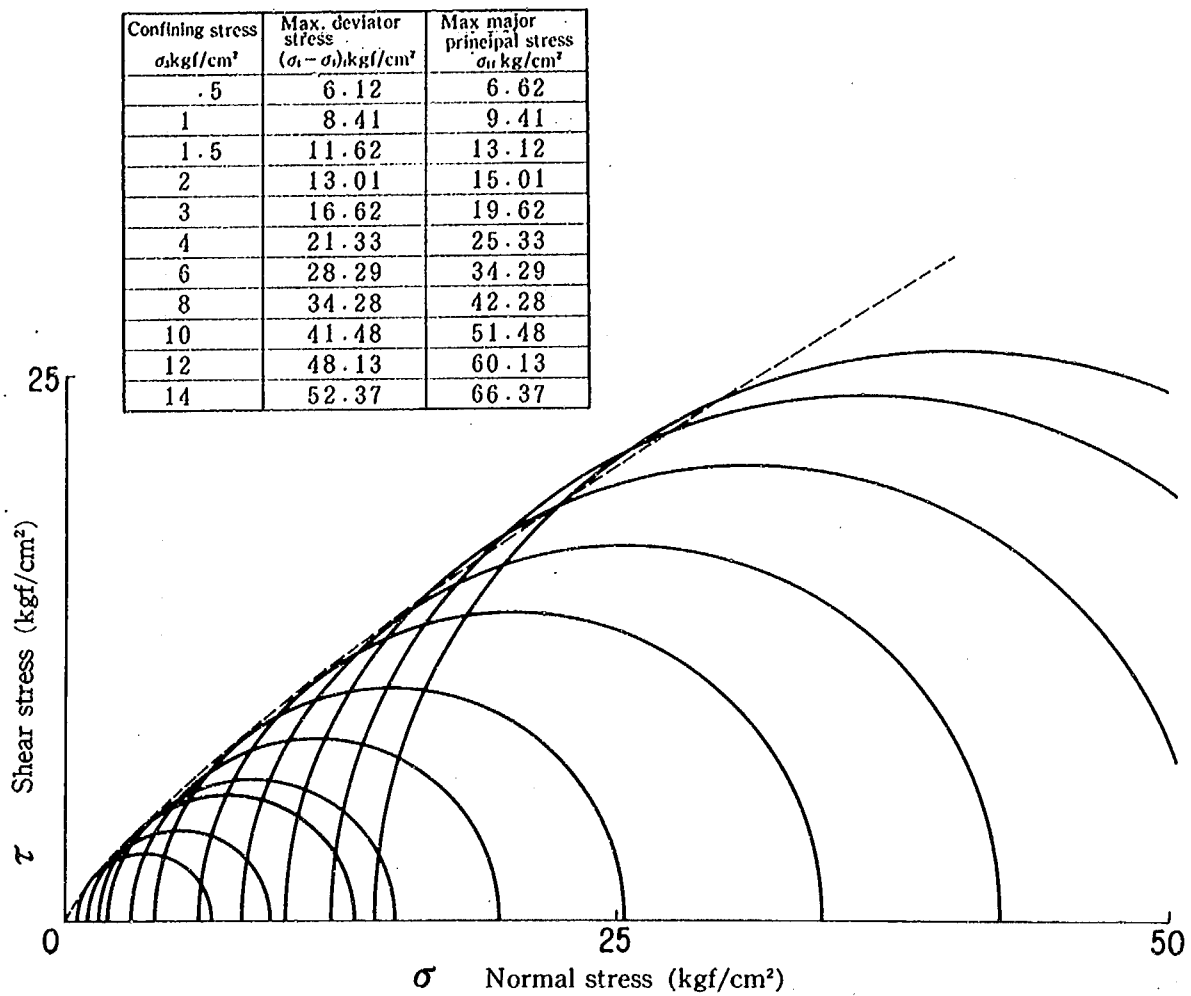


Fig. 2.5 Mohr envelope for Fig. 2.4

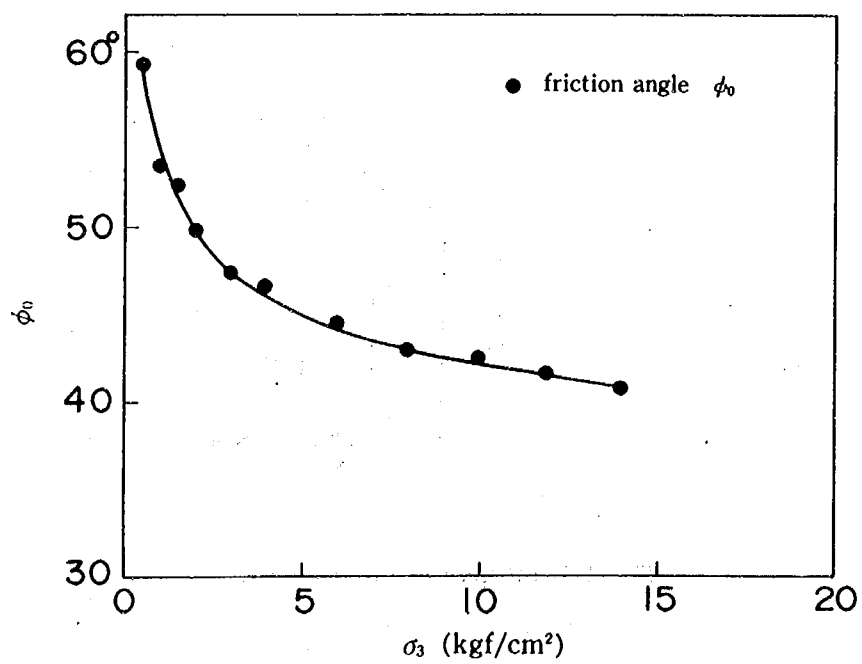


Fig. 2.6 Relations between ϕ_0 and σ_3

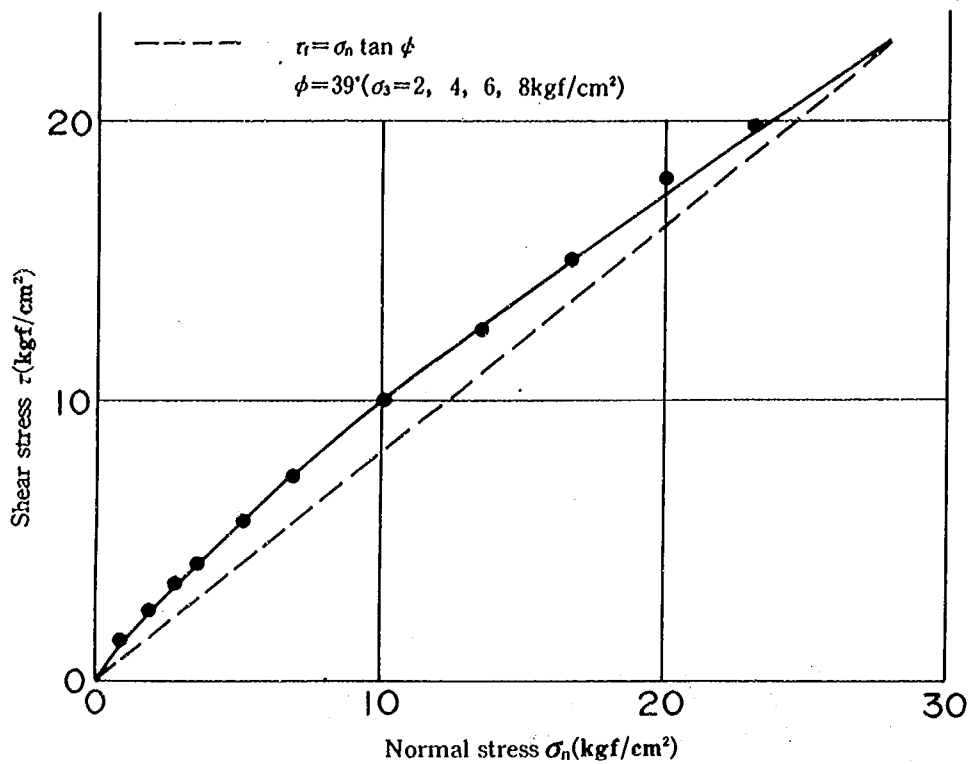


Fig. 2.7 Relations between σ_n and τ

a. Expression for Curved Mohr Envelope

$$\tau = A (\sigma_n)^b \dots \dots \dots (2.8)$$

b. Application of Eq. (2.3) for stability analysis of infinite slope (Fig. 2.8).

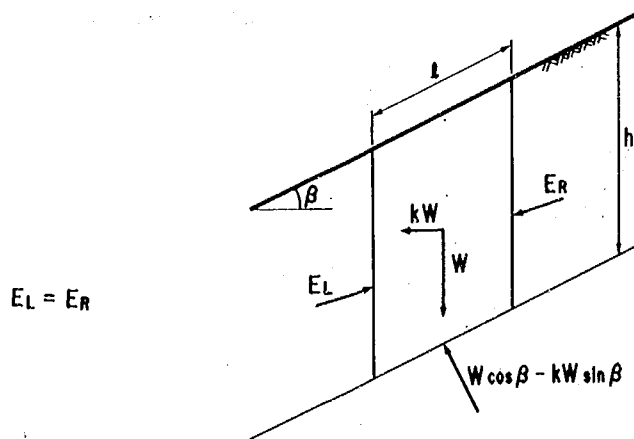


Fig. 2.8 Stability of Infinite Slope

$$F_s = \frac{R}{W \sin \beta + kW \cos \beta} \dots \dots \dots (2.9)$$

where

R = resisting force

W = weight of element

k = seismic coefficient

in case of $R = A(\sigma_n)^b$

$$F_s = A(\gamma h \cos \beta)^{b-1} \times \frac{(\cos \beta - \sin \beta)^b}{\sin \beta + k \cos \beta} \dots\dots\dots (2.10)$$

2.2 Factors causing the Difference of Shear Strength

2.2.1 Initial Void Ratio

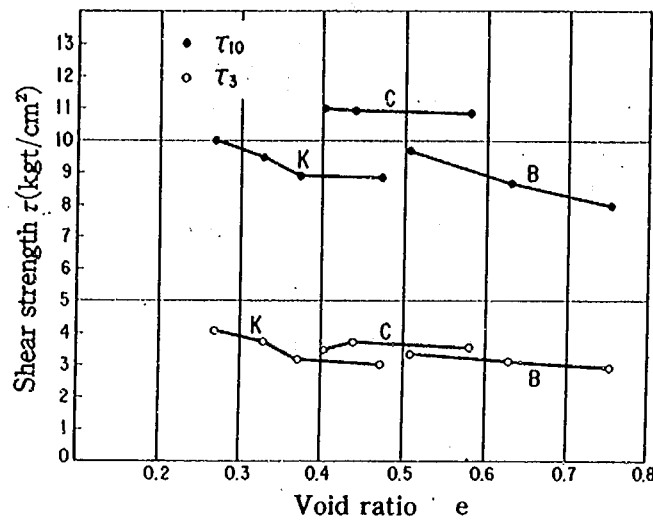


Fig. 2.9 Effect of Initial Void Ratio

Mogami's Equation

$$\sin \phi = \frac{k}{1+e_0} \dots\dots\dots (2.11)$$

where

e_0 : initial void ratio

k : constant

2.2.2 Loading Conditions

a. Comparison of Triaxial and Direct Shear

$$\phi_{TRX} < \phi_{DS} \dots\dots\dots (2.12)$$

b. Comparison of Triaxial and Plane Strain

$$\phi_{TRX} < \phi_{PS} \dots\dots\dots (2.13)$$

c. Intermediate Principal Stress

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \dots\dots\dots (2.14)$$

$b = 0$ for TRX Compression ($\sigma_2 = \sigma_3$)

$b = 0.2-0.3$ for Plane strain

$b = 1.0$ for TRX Extension ($\sigma_1 = \sigma_2$)

2.2.3 Anisotropic Properties

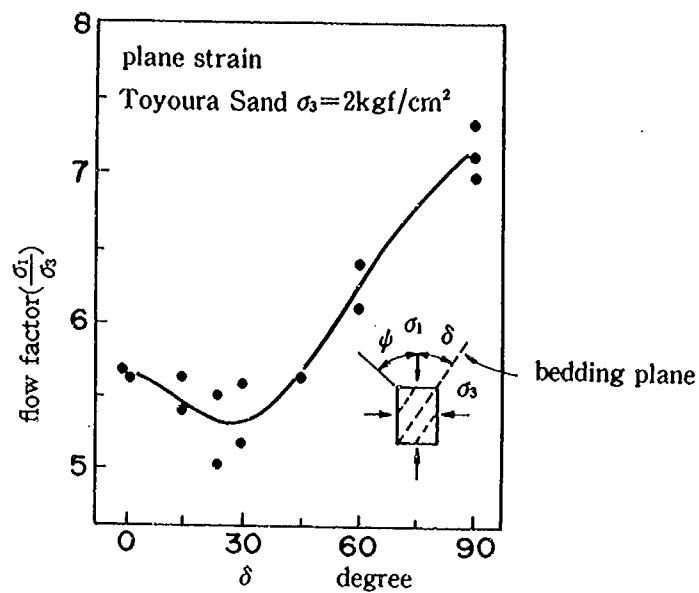


Fig. 2.10 Effect of Bedding Plane (Oda)

2.2.4 Particle Size

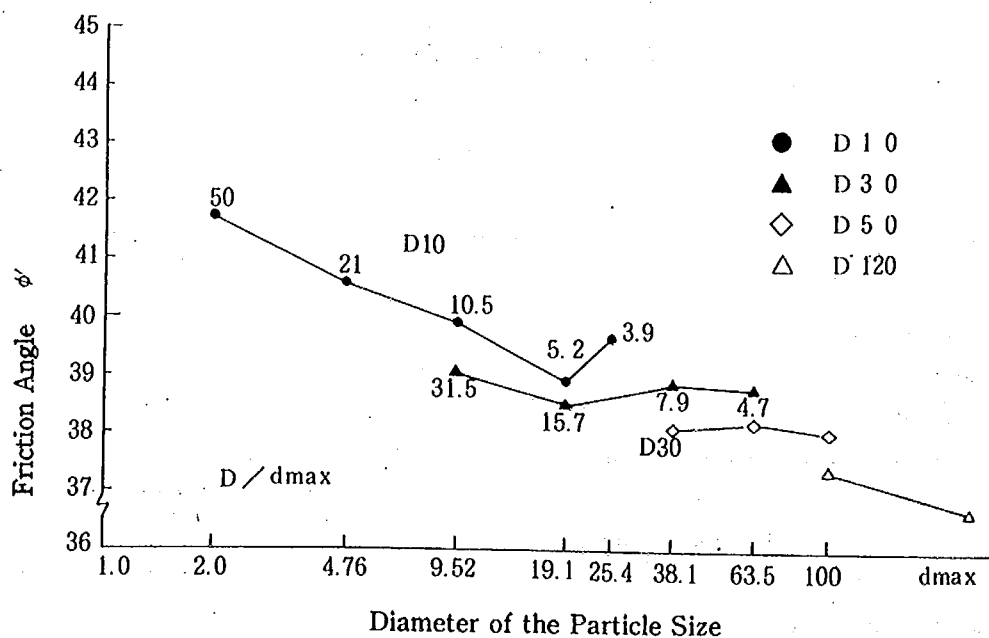


Fig. 2.11 Effect of Particle Size

2.2.5 Gradation

$$e = AU^{-b} \dots \dots \dots (2.15)$$

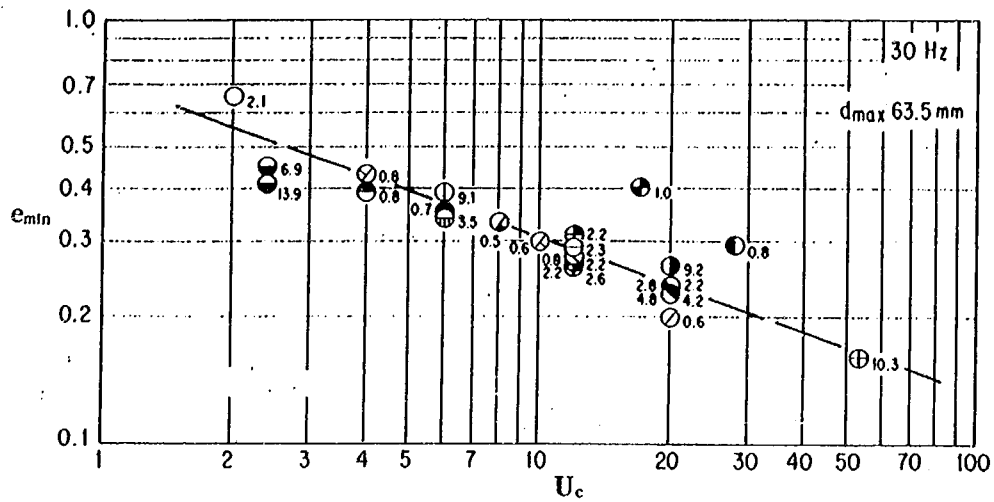


Fig. 2.12 Effect of Uniformity Coefficient to Void Ratio

2.2.6 Parent Rock or Mineral of Sand

2.2.7 Angularity

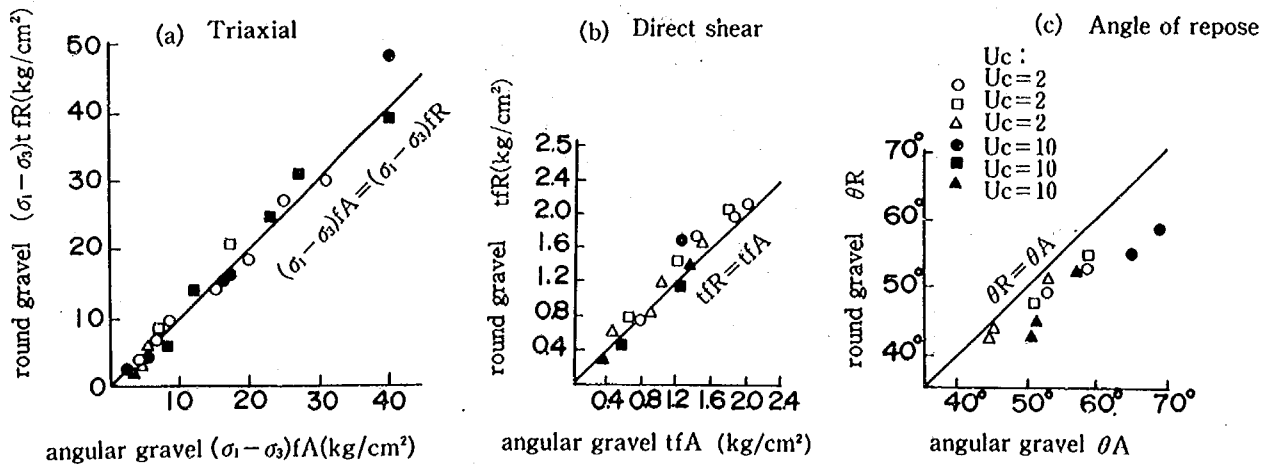


Fig. 2.13 Comparison of Strength between Round Gravel and Angular Rock

2.2.8 Dilatancy Energy Correction

dW = Plastic consumed energy

dU = Elastic energy

$$dE = dU + dW = \sigma_1 d\epsilon_1 + 2\sigma_3 d\epsilon_3$$

$$\epsilon_v = -(\epsilon_1 + 2\epsilon_3)$$

$$\sigma_1 - \sigma_3 = \frac{dW}{d\epsilon_1} + \sigma_3 \frac{d\epsilon_v}{d\epsilon_1} \dots\dots\dots (2.16)$$

or

$$\frac{dW}{d\epsilon_1} = \sigma_1 - \sigma_3 \left(1 + \frac{d\epsilon_v}{d\epsilon_1} \right)$$

2.3 Shear Strength of Saturated Sand and Rockfill in Drained Condition

2.3.1 Drained Shear Strength of Sand

The behaviour of dry sand $\approx$ The drained behaviour of saturated sand.

2.3.1 Drained Shear Strength of Rockfill

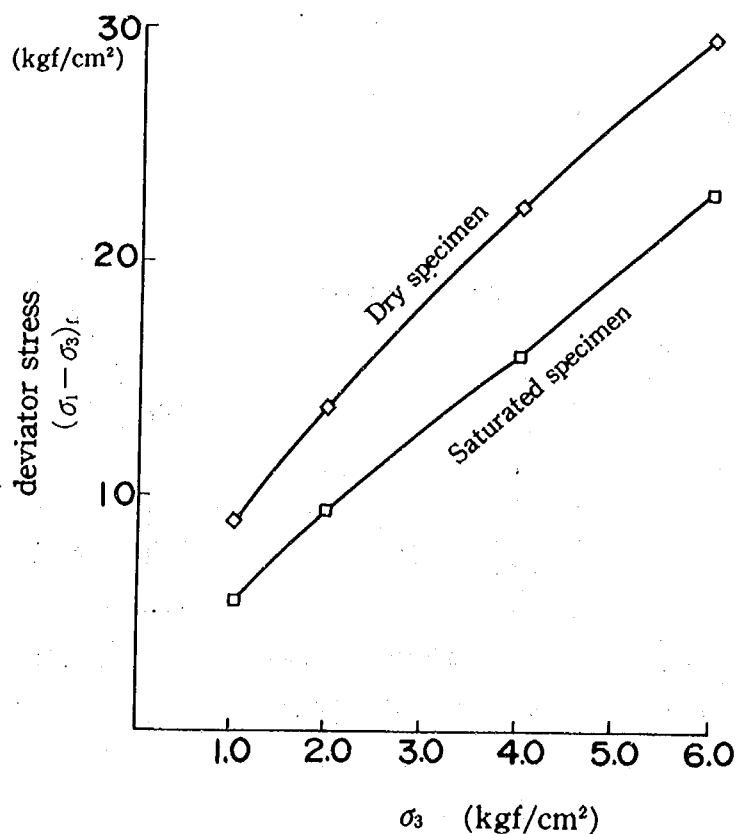


Fig. 2.14 Comparison of Shear Strength between Dry and Saturated Rockfill

2.4 Stress-Strain Behaviour of Sand and Rockfill

2.4.1 Confined Compression

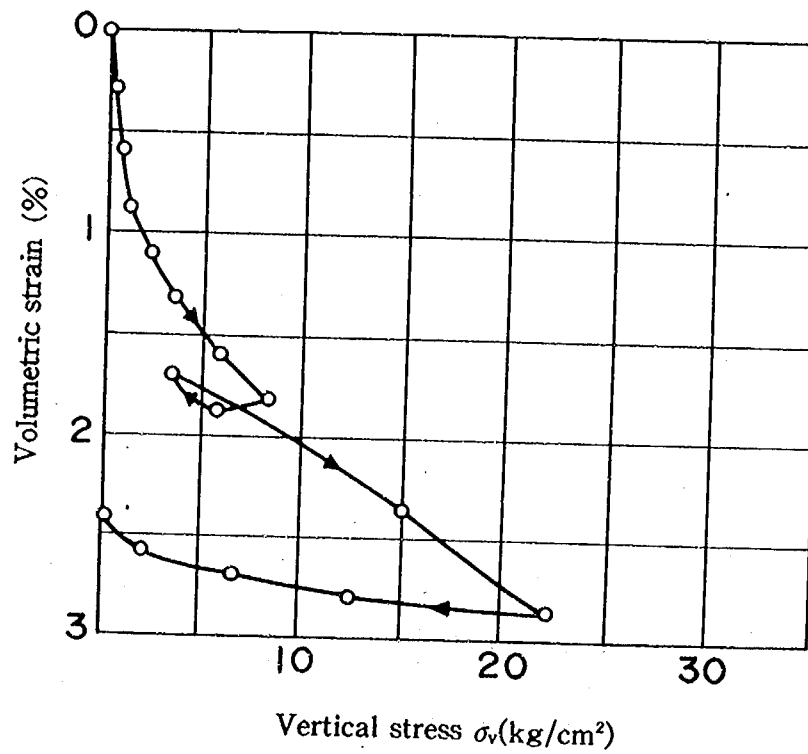


Fig. 2.15 Stress-Strain Curves in Oedometer Test

Jaky's Equation

$$K_0 = 1 - \sin \phi \quad \dots \dots \dots (2.17)$$

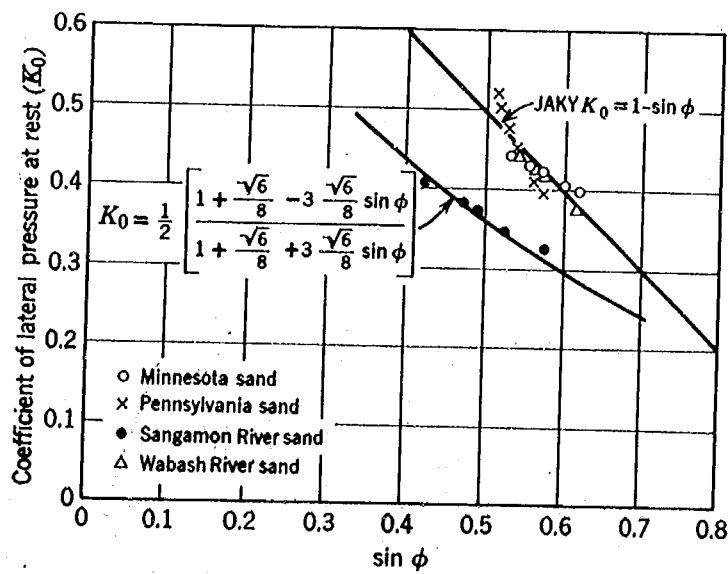


Fig. 2.16 Relations between K_0 and ϕ (Henderson, 1963)

a. Coefficient of Compressibility

$$a_v = -\frac{de}{d\sigma_v} \dots\dots\dots (2.18)$$

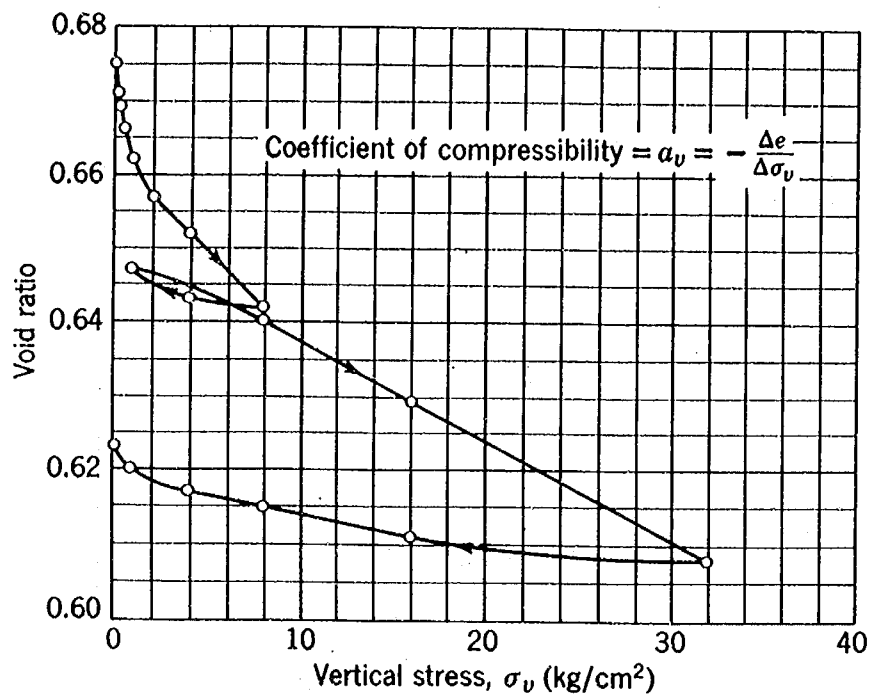


Fig. 2.18 $e-\sigma_v$ Relations in Confined Compression Test (Lambe, 1969)

b. Compression Index

$$C_c = -\frac{de}{d(\log \sigma_v)} \dots\dots\dots (2.19)$$

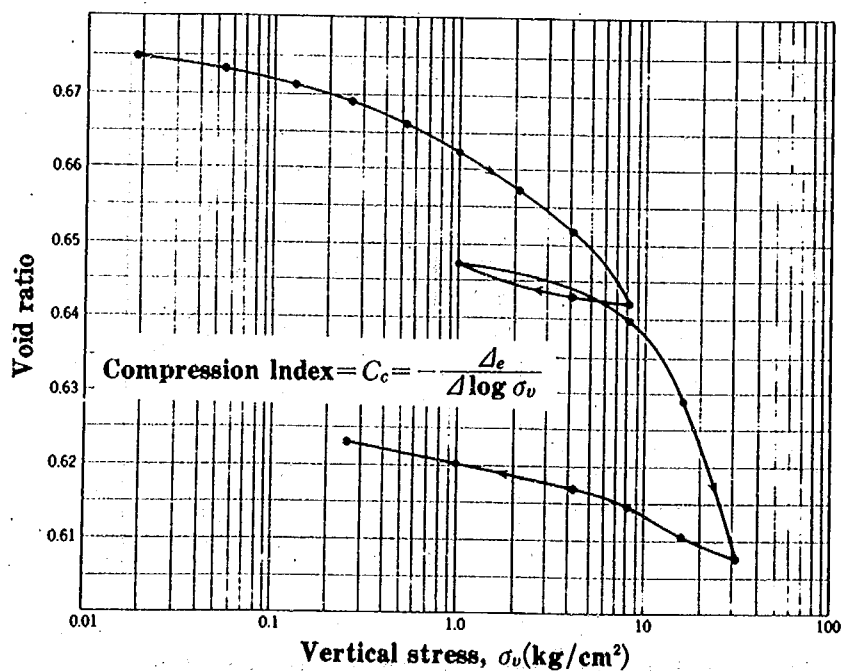


Fig. 2.18 $e-\log \sigma_v$ Relations in Confined Compression Test

c. Coefficient of volume change

$$m_v = \frac{d\varepsilon_v}{d\sigma_v} \dots\dots\dots (2.20)$$

2.4.2 Triaxial Compression Test

a. Secant Modulus

$$E_s = \frac{\Delta\sigma_a}{\Delta\varepsilon_a} \dots\dots\dots (2.21)$$

$$\Delta\sigma_a = (\sigma_a)_2 - (\sigma_a)_1$$

$$\Delta\varepsilon_a = (\varepsilon_a)_2 - (\varepsilon_a)_1$$

b. Tangent Modulus

$$E_t = \frac{d\sigma_a}{d\varepsilon_a} \dots\dots\dots (2.22)$$

$$\Delta\sigma_a = d\sigma_a \quad \text{at} \quad \sigma_a = (\sigma_a) \text{ at point of interest}$$

$$\Delta\varepsilon_a = d\varepsilon_a \quad \text{at} \quad \sigma_a = (\sigma_a) \text{ at point of interest}$$

c. Suggested Stress-Strain Curve

Kondner's Equation

$$\sigma_1 - \sigma_3 = \frac{\varepsilon_1}{a + b\varepsilon_1} \dots\dots\dots (2.23)$$

or

$$\sigma_1 - \sigma_3 = \frac{\varepsilon_a}{\frac{1}{E_t} + \frac{R_t \cdot \varepsilon_a}{(\sigma_1 - \sigma_3)_t}}$$

where

$$R_t = \frac{(\sigma_1 - \sigma_3)_t}{(\sigma_1 - \sigma_3)_{ult}}$$

Janbu's Equation

$$E_t = K \cdot P_a \cdot \left(\frac{\sigma_3}{P_a} \right)^n \dots\dots\dots (2.24)$$

$$E_t = E_i \left\{ 1 - \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_t} \right\}^2 \dots\dots\dots (2.25)$$

from Eq. 2.24

$$\log E_t = \log K + \log \sigma_3$$

if stresses are in kgf/cm²

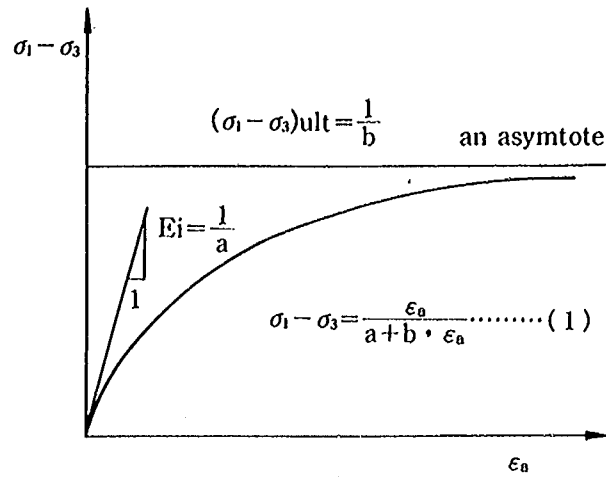


Fig. 2.19 Hyperbolic Approximation

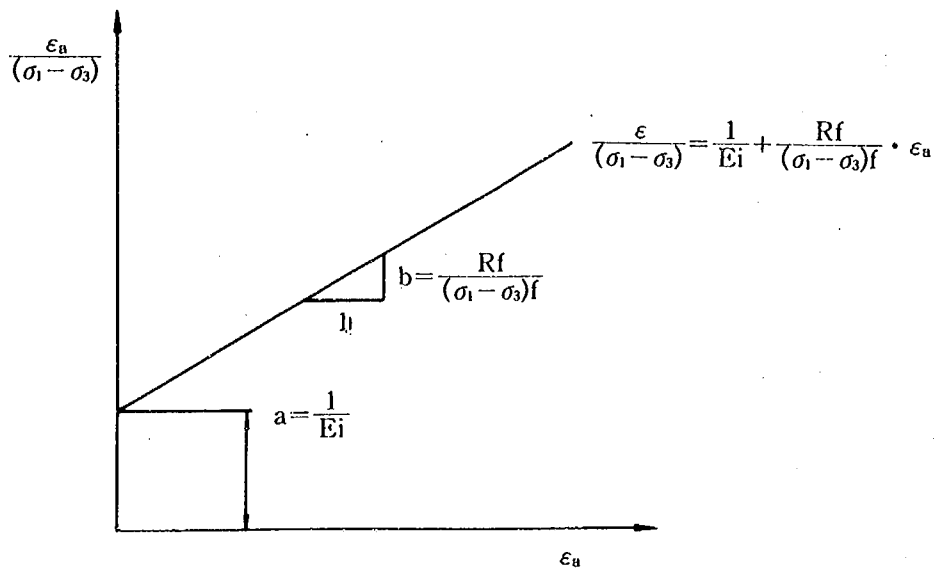


Fig. 2.20 Finding Ei

Exercises

E 2.1

Given. Triaxial testing results of Fig. 2.4.

Find. Stress paths in $p - q$ diagram.

E 2.2

Given. Triaxial testing results of Fig. 2.4.

Find. Apparent c and ϕ by using the solution of E 2.1, assuming straight $p_t - q_t$ relations.

E 2.3

Given. Triaxial testing results of Fig. 2.4.

Find. A and b in Eq.(2.8).

hint: Use relations $\sigma_n = q_t - q_t \sin \phi_0$, $\sigma_t = p_t \cos \phi_0$, and draw $\log \tau_t - \log \sigma_n$ relations.

E 2.4

Given. Triaxial testing results of Fig. 2.4.

Find. Comparison between ϕ_0 and ϕ_{oc} at each confining stress.

ϕ_0 =internal friction angle

ϕ_{oc} =internal friction angle corrected with dilatancy energy by Eq.(2.17).

E 2.5

Given. Triaxial testing results of Fig. 2.21.

Find. E_t and n in Eq.(2.24).

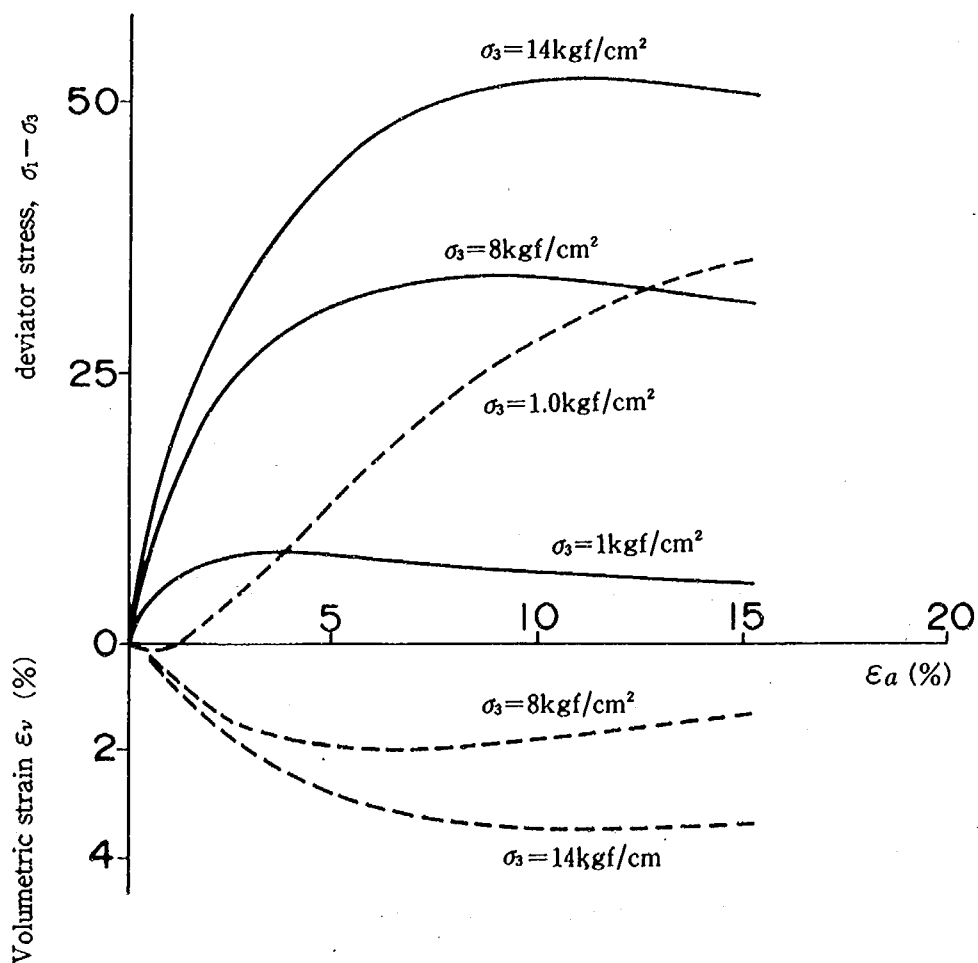


Fig. 2.21a Triaxial Testing Results

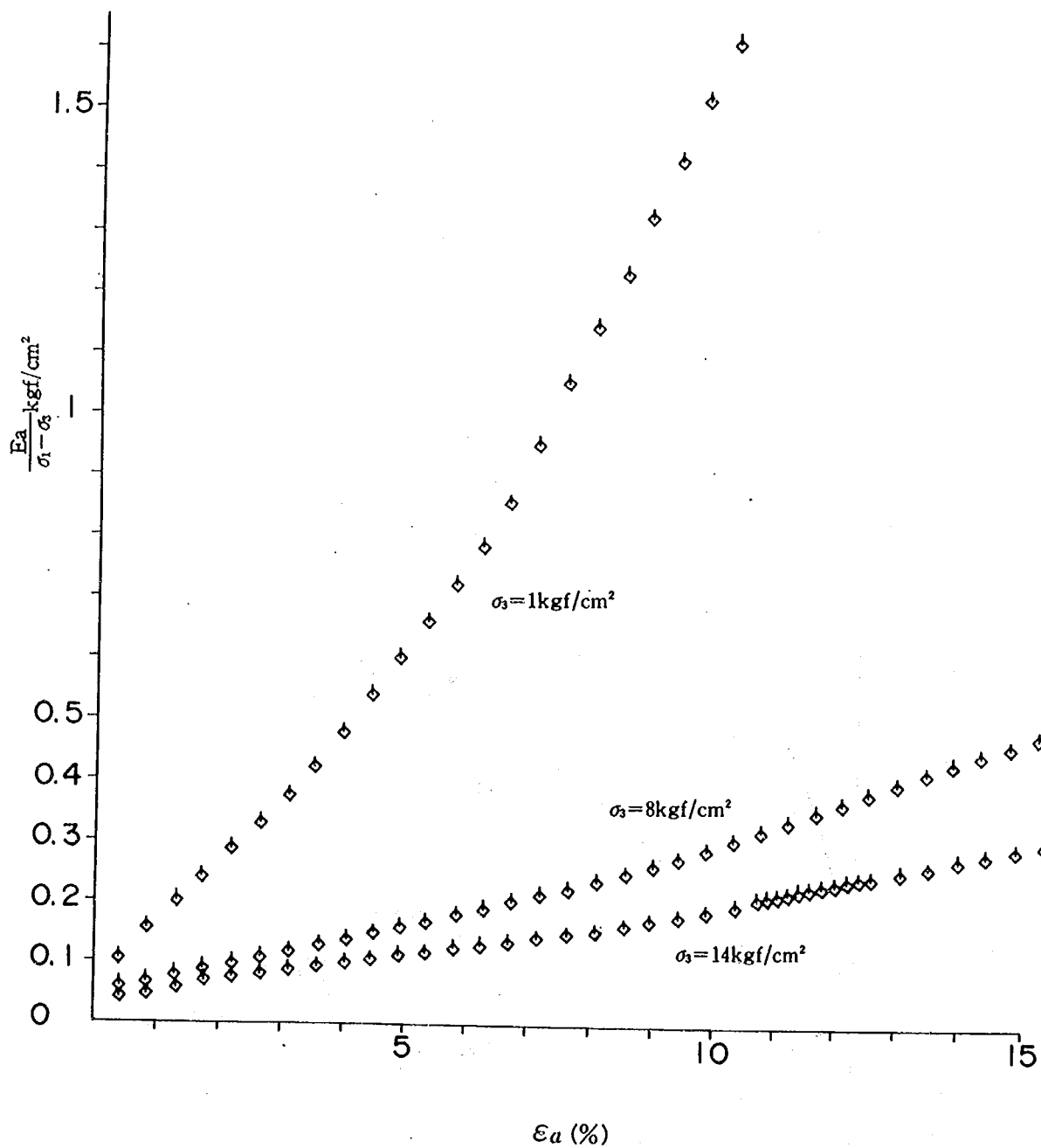


Fig. 2.21b Triaxial Testing Results(cont.)

3. Shear Strength of Clay and Core Materials

3.1 Drained Shear Strength

3.1.1 Normally Consolidated clay

Example 3.1

Given. A $p - q$ diagram of standard drained triaxial test (Fig. 3.1).

Find. ϕ .

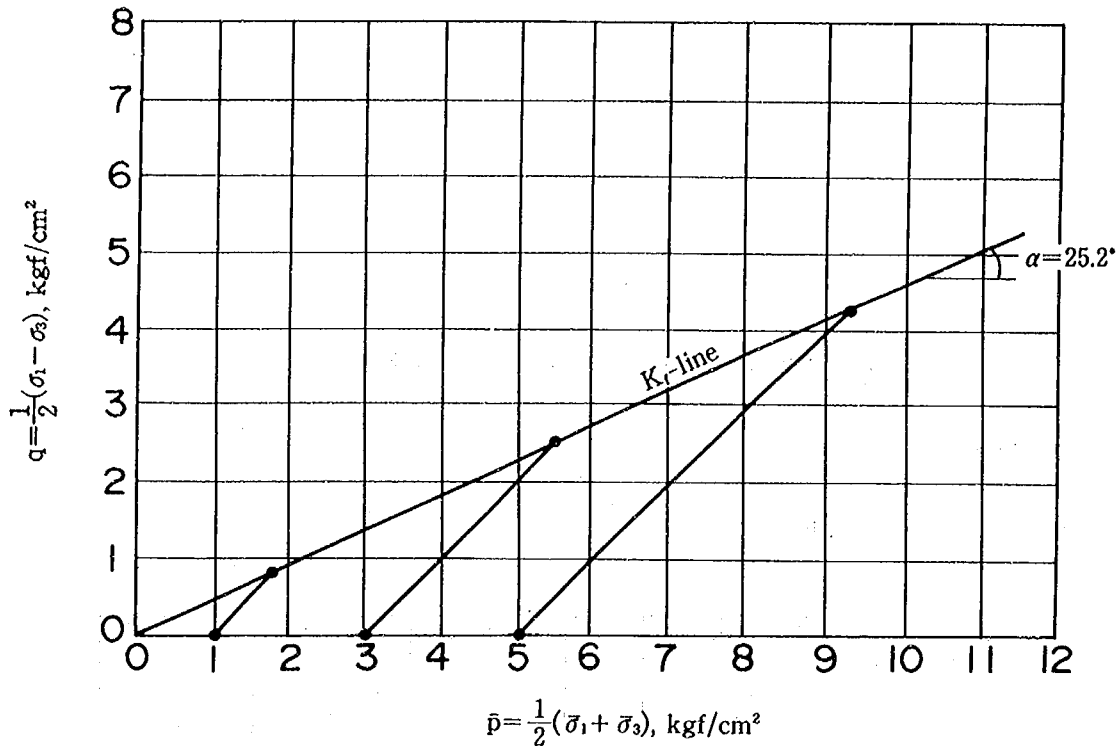


Fig. 3.1 CD Test on Normally Consolidated Clay

Solution.

$$\alpha = \tan^{-1} \frac{q_t}{\bar{p}_t} = \tan^{-1} 0.47 = 25.2^\circ \dots \dots \dots (3.1)$$

$$\phi = \sin^{-1} \tan \alpha = 28.0^\circ \dots \dots \dots (3.2)$$

a. Shear Strength vs Effective Stress for Normally Consolidated Clay

$$\tau_t = \bar{\sigma}_n \tan \phi$$

b. Typical Values for ϕ (Fig.3.2)

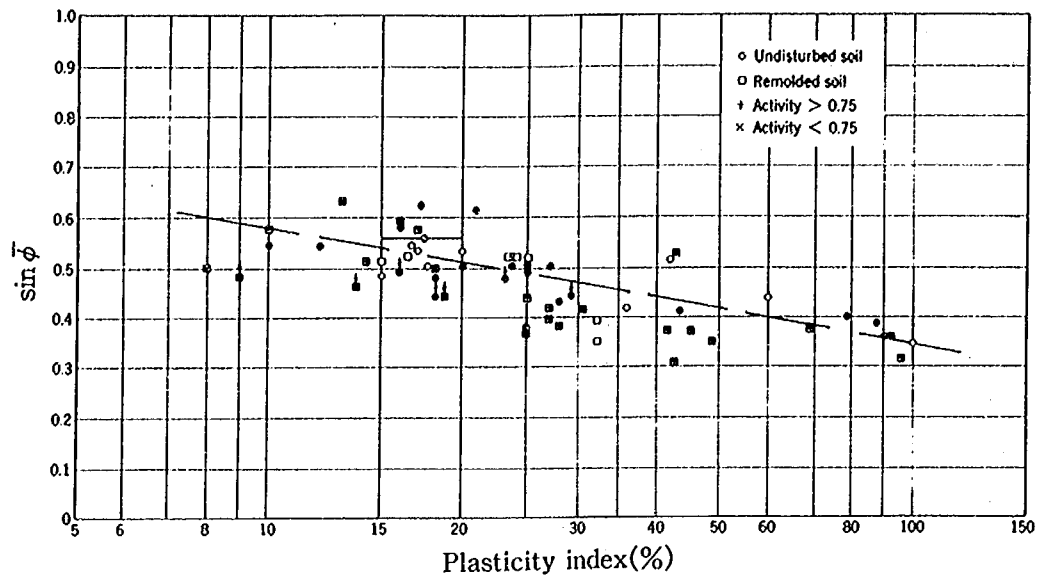


Fig. 3.2 Relations between $\bar{\phi}$ and PI(Kenny, 1959)

3.1.2 Overconsolidated Clay

Example 3.2

Given. A $p-q$ diagram in Fig. 3.3.

Find $\bar{c}$ and $\bar{\phi}$.

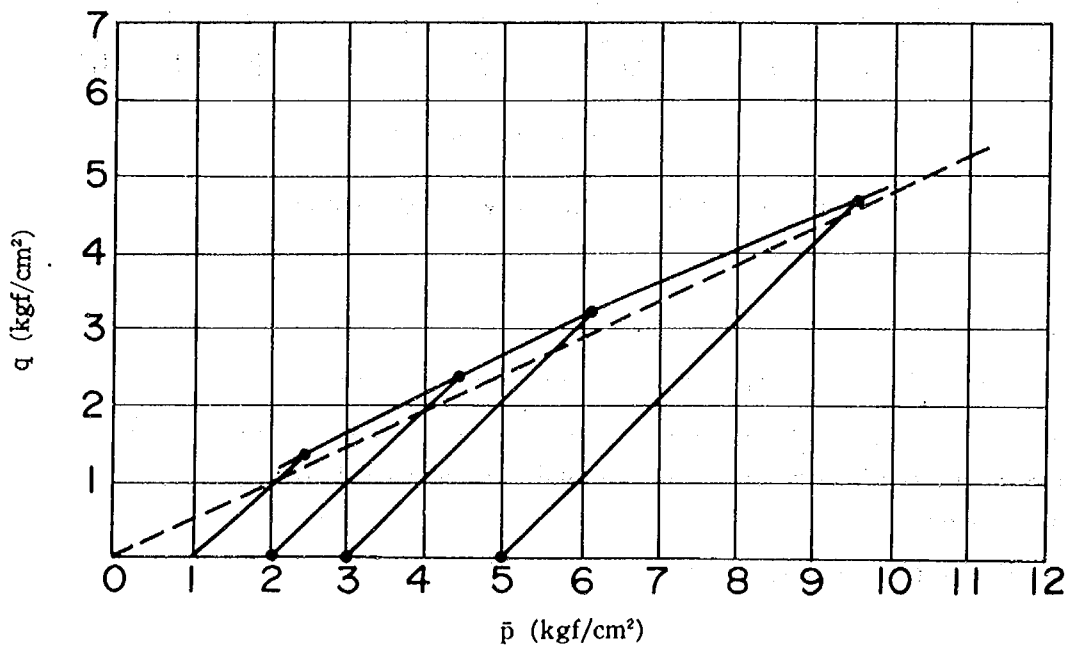


Fig. 3.3 Results of CD Tests on Overconsolidated Clay

a. Shear Strength vs Effective Stress

$$\tau_f = \bar{c} + \bar{\sigma} \tan \bar{\phi}$$

b. $q_f - p_f - w_f$ relations

3.1.3 Comparison of Clay and Sand

NC clay : Loose Sand

OC clay : Dense Sand

$\bar{c} \neq 0$ for OC clay

$\bar{c} = 0$ for Dense Sand

3.2 Pore Pressure Development

Example 3.3

Given. Fig. 3.4.

Find. Pore pressure immediately after loading.

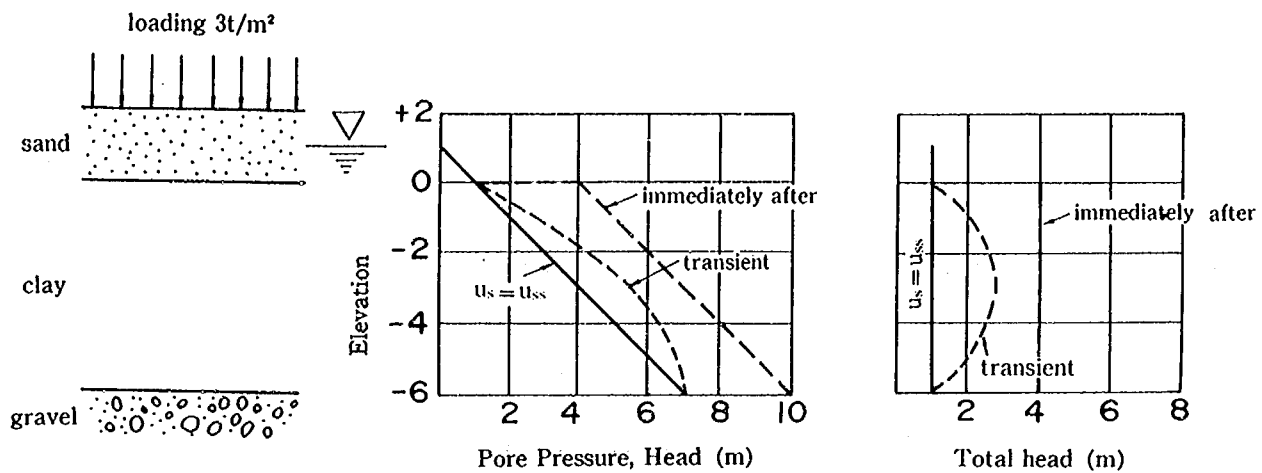


Fig. 3.4 Undrained Surface Loading

u_s : static pore pressure

u_{ss} : steady state pore pressure

u_e : excess pore pressure

$u_e = u - u_{ss}$

pore pressure parameter : C

$$C = \frac{\Delta u}{\Delta \sigma}$$

3.2.1 Confined Test

$$C = \frac{\Delta u}{\Delta \sigma_1} = \frac{1}{1 + n \left(\frac{C_w}{C_{cl}} \right)} \quad (3.5)$$

where

C_w = compressibility of water

n = porosity

$$C_{cl} = \frac{\Delta V}{V_0} \cdot \frac{1}{\Delta \sigma_1}$$

3.2.2 Isotropic Compression

$$B = \frac{\Delta u}{\Delta \sigma} = \frac{1}{1+n\left(\frac{C_w}{C_{c3}}\right)} \quad (3.6)$$

where

$$C_{c3} = \frac{\Delta V}{V_0} \cdot \frac{1}{\Delta \sigma}$$

3.2.3 Triaxial Tests

$$\Delta u = B\Delta\sigma_3 + D(\Delta\sigma_1 - \Delta\sigma_3) \quad (3.7)$$

for a clay saturated with incompressible fluid

$$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \quad (3.8)$$

A : loose fine sand 2--3

NC clay 0.7--1.3

OC clay $\begin{cases} 0.3-0.7 \\ -0.5-0.0 \end{cases}$

3.2.4 Estimation of Pore Pressure

Example 3.4

Given. Surface loading as shown in Fig. 3.5. A in Eq. (3.8) = 0.7.

Find. Δu at z.

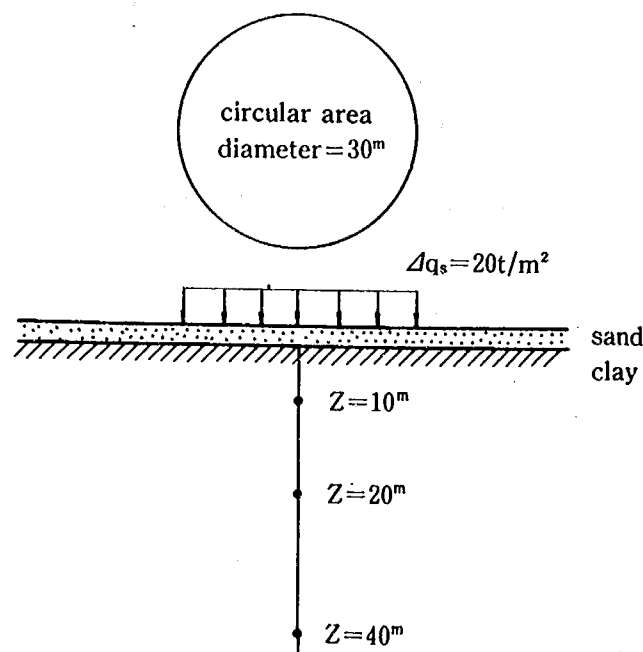


Fig. 3.5 Surface Loading

Solution. From Eqs (1.15) and (1.16), σ_1 and σ_3 are computed as follows. Δu is obtained by using Eq. (3.8).

z(m)	$\frac{\Delta\sigma_1}{\Delta q_s}$	$\frac{\Delta\sigma_3}{\Delta q_s}$	$\frac{\Delta\sigma_3}{\Delta q_s} + A \frac{\Delta\sigma_1}{\Delta q_s} - \frac{\Delta\sigma_3}{\Delta q_s}$	$\Delta u(m)$
10	0.83	0.24	0.65	13.1
20	0.49	0.05	0.36	7.2
40	0.18	0.01	0.13	2.6

3.3 Undrained Tests

3.3.1 Consolidated Undrained Test and Unconsolidated Undrained Test

a. CU Test

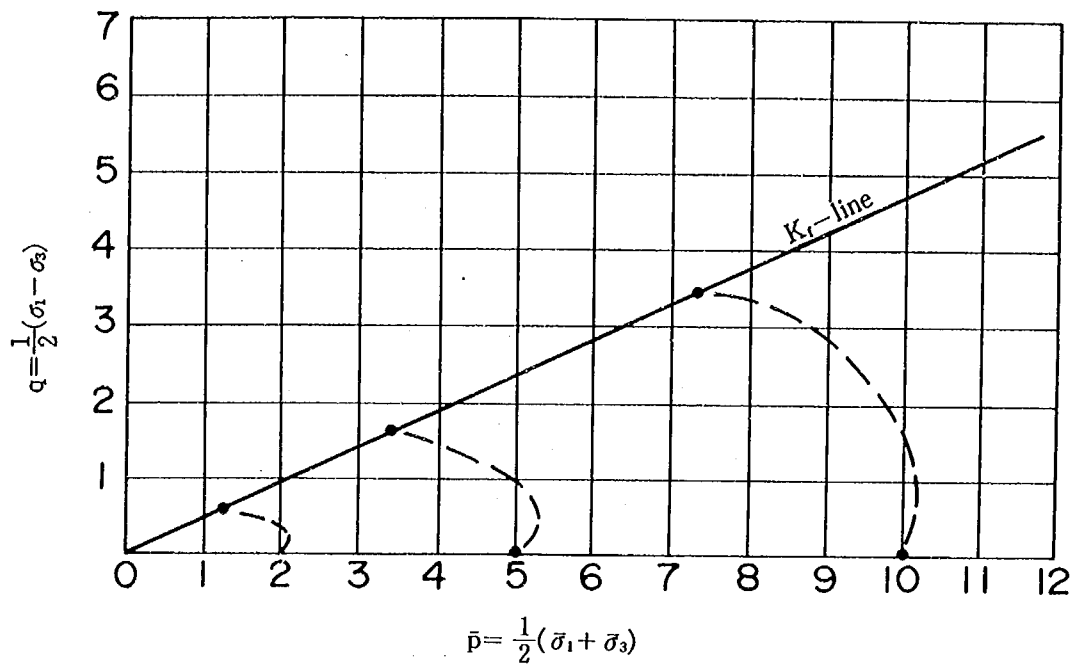


Fig. 3.6 Effective Stress Paths for CU Tests

b. UU Test

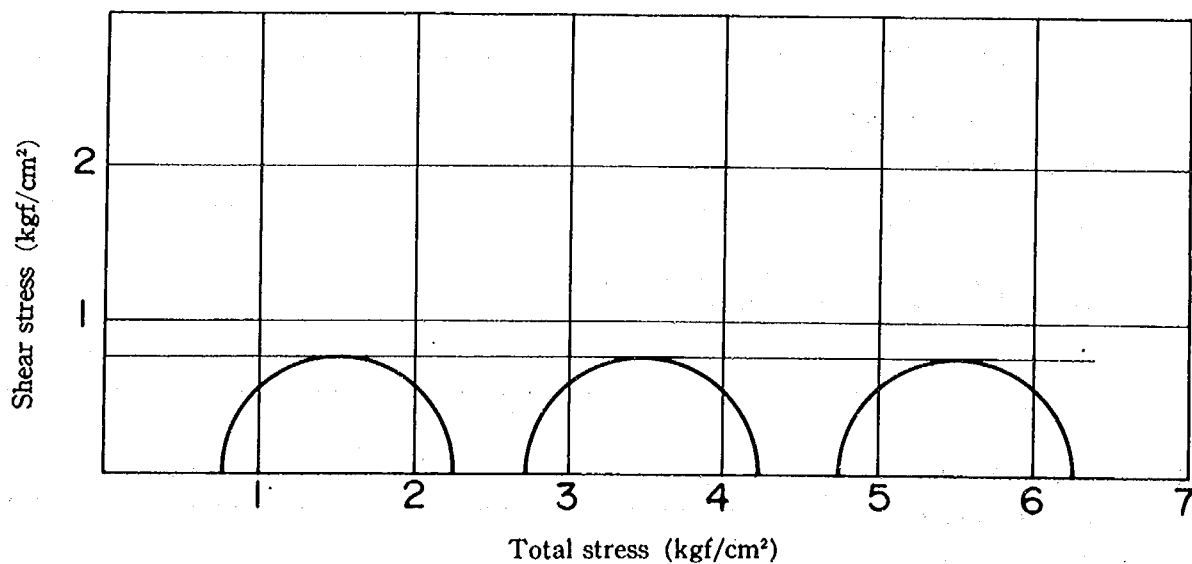


Fig. 3.7 Mohr Circles for UU Test

$$\left. \begin{array}{l} s_u = q_t \\ \phi = 0 \end{array} \right\} \dots\dots\dots (3.9)$$

3.3.2 Undrained Shear Strength

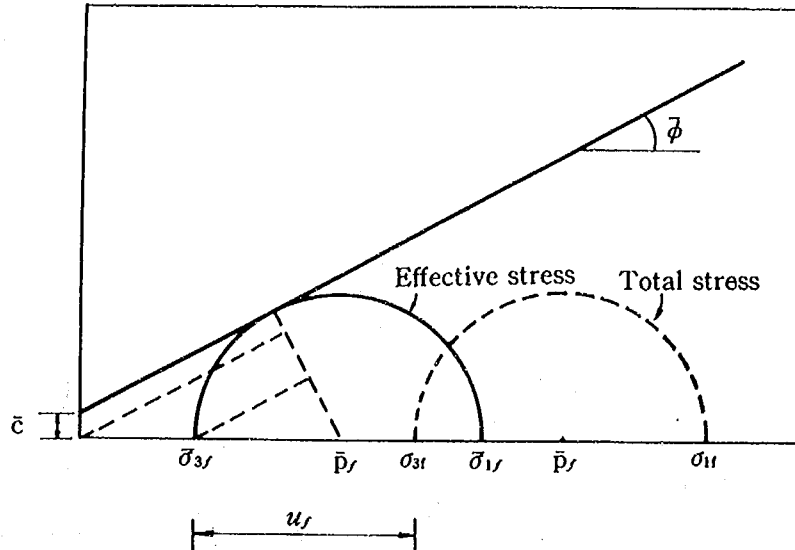


Fig. 3.8 Pore Pressure and Mohr Circle

$$q_t = \frac{1}{2}(\sigma_{1t} - \sigma_{3t}) = \frac{\bar{c} \cos \bar{\phi} + \bar{\sigma}_{3t} \sin \bar{\phi}}{1 - \sin \bar{\phi}} \dots\dots\dots (3.10)$$

Assuming

$$\Delta u = A_r(\Delta \sigma_1 - \Delta \sigma_3) \dots\dots\dots (3.11)$$

$$q_t = \frac{\bar{c} \cos \bar{\phi} + (\bar{p}_0 - 2 A_r q_t) \sin \bar{\phi}}{1 - \sin \bar{\phi}} \dots\dots\dots (3.12)$$

a. Normally Consolidated Clay

In Eq. (3.12), $\bar{c} \neq 0$

$$\frac{q_t}{\bar{p}_0} = \frac{\sin \bar{\phi}}{1 + (2 A_r - 1) \sin \bar{\phi}} \dots\dots\dots (3.13)$$

$$\frac{q_t}{\bar{p}_0} = \frac{q_t}{\frac{1}{2}(\bar{\sigma}_v + \bar{\sigma}_h)} = \frac{q_t}{\frac{1}{2}(1 + K_0)\bar{\sigma}_v} = \text{const.}$$

$$\therefore \frac{s_u}{\bar{\sigma}_v} = \text{const.} (0.2 \sim 0.4) \dots\dots\dots (3.14)$$

b. Overconsolidated Clay

$$s_u = f(\bar{\sigma}_v, \bar{\sigma}_{vm}) \dots\dots\dots (3.15)$$

where

$\bar{\sigma}_{vm}$ = maximum past stress

Exercises

E3.1

Given. Specimen of clay in Fig. 3.1 is consolidated to 2 kgf/cm^2 , and then failed increasing $\bar{\sigma}_1$ and decreasing $\bar{\sigma}_3$ in such a way that $\bar{p}$ remains constant.

Find. q_f

E3.2

Given. Saturated clay specimen is consolidated to 1.0 kgf/cm^2 , and then failed increasing σ_1 while remaining σ_3 constant in undrained condition. A in Eq. (3.8) = 0.7 , and $\tan \bar{\phi} = 0.4$.

Find. Peak σ_1 , $\bar{\sigma}_1$, u_f , and $\bar{\sigma}_3$.

E3.3

Given. A $p-q$ diagram of CU test for overconsolidated clay in Fig. 3.9.

Find. $\bar{\sigma}_1$, $\bar{\sigma}_3$, u_f and A_f at failure.

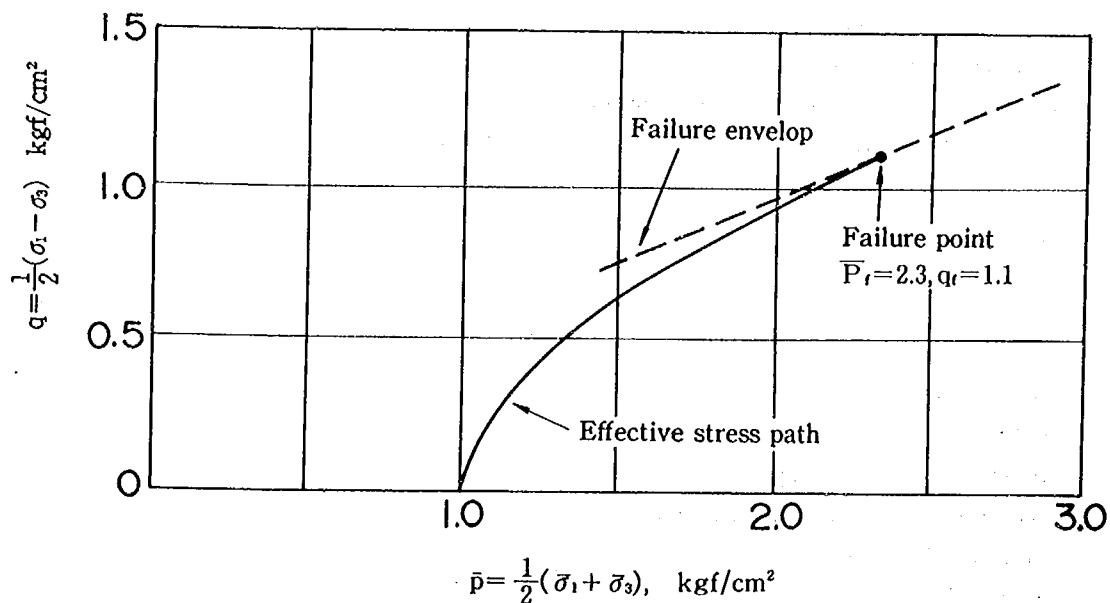


Fig. 3.9. $p-q$ Diagram for Overconsolidated Clay

Symbols for Part I

1. Stresses

σ = normal stress (total stress)

$\bar{\sigma}$ = normal effective stress

τ = shear stress

τ_f = shear stress on failure plane at failure

$$p = \frac{\sigma_1 + \sigma_3}{2}$$

$$q = \frac{\sigma_1 - \sigma_3}{2}$$

σ_v, σ_h = normal stress in the vertical, horizontal direction

σ_n = normal stress on failure plane at failure

$\sigma_x, \sigma_y, \sigma_z$ = normal stress in the x, y, z direction

u_w = pore water pressure

$\sigma_1, \sigma_2, \sigma_3$ = major, intermediate, minor principal stress

σ_a, σ_r = axial, lateral confining stress, in triaxial test

2. Constants

$$K = \sigma_h / \sigma_v$$

$$K_0 = \bar{\sigma}_h / \bar{\sigma}_v \text{ at } \epsilon_x = 0$$

ϕ = friction angle

c = cohesion

s_u = undrained shear strength

$\bar{\phi}$ = friction angle based on effective stresses

$\bar{c}$ = cohesion based on effective stresses

ν = Poisson's ratio

U_c = coefficient of uniformity

E = Young's modulus

e = void ratio

γ = unit weight

S_r = saturation degree

Part II DYNAMIC DEFORMATION PROPERTIES OF FILL MATERIALS

1. Review of Theory of Wave Propagation

1.1 Equations of Equilibrium

Summing all the forces acting on the element in the X-direction

$$\begin{aligned} & [(\sigma_x)_1 - (\sigma_x)_2] \delta_y \cdot \delta_z \\ & + [(\sigma_{xy})_3 - (\sigma_{xy})_4] \delta_x \cdot \delta_z \\ & + [(\sigma_{xz})_5 - (\sigma_{xz})_6] \delta_x \cdot \delta_y \\ & + X \delta_x \cdot \delta_y \cdot \delta_z = 0 \end{aligned}$$

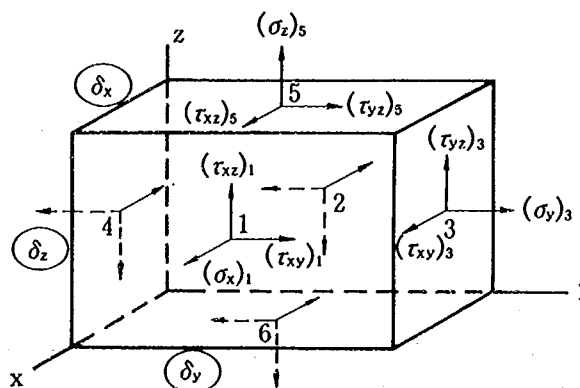


Fig.1

After dividing by $dx \cdot dy \cdot dz$ and proceeding to the limit by shrinking the element down to the point x, y, z , we find

$$\frac{(\sigma_x)_1 - (\sigma_x)_2}{\delta_x} + \frac{(\tau_{xy})_3 - (\tau_{xy})_4}{\delta_y} + \frac{(\tau_{xz})_5 - (\tau_{xz})_6}{\delta_z} + X = 0$$

$$\delta_x, \delta_y, \delta_z \rightarrow 0$$

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X &= 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + Y &= 0 \\ \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + Z &= 0 \end{aligned} \right\} \dots \dots \dots (1)$$

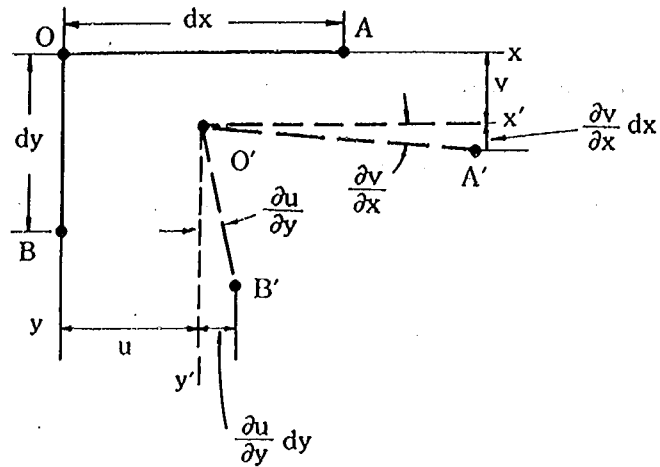
1.2 Components of Strain

$$\left. \begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \epsilon_z = \frac{\partial w}{\partial z} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{aligned} \right\} \dots \dots \dots (2)$$

ϵ_x : unit elongation at point O in the X direction

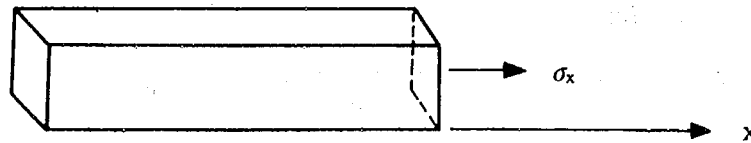
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \angle B'O'y' + \angle A'O'x'$$

shearing strain



1.3 Hook's Law

The relations between components of stress and components of strain have been established experimentally and are known as a Hook's law.



$$\begin{aligned} \sigma_x \text{ only} \quad \epsilon_x &= \frac{\sigma_x}{E}, & \epsilon_y &= -\frac{\nu}{E}\sigma_x, & \epsilon_z &= -\nu\frac{\sigma_x}{E} \\ \sigma_y \text{ only} \quad \epsilon_x &= -\frac{\nu}{E}\sigma_y, & \epsilon_y &= \frac{\sigma_y}{E}, & \epsilon_z &= -\nu\frac{\sigma_y}{E} \\ \sigma_z \text{ only} \quad \epsilon_x &= -\frac{\nu}{E}\sigma_z, & \epsilon_y &= -\frac{\nu}{E}\sigma_z, & \epsilon_z &= \frac{\sigma_z}{E} \\ \epsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)], & \epsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)], & \epsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \dots\dots\dots (3) \end{aligned}$$

$$G = \frac{E}{2(1+\nu)} \text{ modulus of rigidity} \dots\dots\dots (4)$$

$$\gamma = \frac{\tau}{G} \dots\dots\dots (5)$$

1.4 Equations of Equilibrium in Terms of Displacements

$$\begin{aligned} e &= \epsilon_x + \epsilon_y + \epsilon_z & e : \text{Volume Expansion} \\ \theta &= \sigma_x + \sigma_y + \sigma_z & \theta : \text{Sum of Normal Stresses} \end{aligned} \quad \dots\dots\dots (6)$$

Adding equations(3) together we obtain

$$\epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2\nu}{E} (\sigma_x + \sigma_y + \sigma_z) \quad \dots\dots\dots (a)$$

and using the notations (6) we obtain

$$\theta = \sigma_x + \sigma_y + \sigma_z = \frac{E}{1-2\nu} e \quad \dots\dots\dots (b)$$

The first equation of Eqs. (3) is rewritten as,

$$\sigma_x - \nu\sigma_y - \nu\sigma_z = E\epsilon_x \quad \dots\dots\dots (c)$$

$$(b) \times \nu + (c) \rightarrow \nu\sigma_x + \sigma_x = \frac{E}{1-2\nu} e + E\epsilon_x \quad \dots\dots\dots (d)$$

$$\therefore \sigma_x = \frac{\nu E}{(1+\nu)(1-2\nu)} e + \frac{E}{1+\nu} \epsilon_x$$

$\lambda \qquad \qquad \qquad 2G$

Similarly

$$\sigma_y = \frac{\nu E}{(1+\nu)(1-2\nu)} e + \frac{E}{1+\nu} \epsilon_y$$

$$\sigma_z = \frac{\nu E}{(1+\nu)(1-2\nu)} e + \frac{E}{1+\nu} \epsilon_z$$

$$\sigma_x = \lambda e + 2G\epsilon_x$$

$$\sigma_y = \lambda e + 2G\epsilon_y \quad \dots\dots\dots (7)$$

$$\sigma_z = \lambda e + 2G\epsilon_z$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad \lambda : \text{Lame's Constant} \quad \dots\dots\dots (8)$$

From Eq. (7)

$$\begin{aligned} \sigma_x &= \lambda e + 2G\epsilon_x \\ &= \lambda e + 2G \frac{\partial u}{\partial x} \quad \dots\dots\dots (f) \end{aligned}$$

$$\left. \begin{aligned} \tau_{xy} &= G\gamma_{xy} \\ &= G \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \tau_{xz} &= G\gamma_{xz} \\ &= G \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \end{aligned} \right\} \quad \dots\dots\dots (g)$$

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} &= \lambda \frac{\partial e}{\partial x} + 2G \frac{\partial^2 u}{\partial x^2} \\ \frac{\partial \tau_{xy}}{\partial y} &= G \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) \\ \frac{\partial \tau_{xz}}{\partial z} &= G \left(\frac{\partial^2 w}{\partial x \partial z} + \frac{\partial^2 u}{\partial z^2} \right) \end{aligned} \right\} \quad \dots\dots\dots (h)$$

Substituting (f) and (g) in the first of Eqs.(1), we find

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \lambda \frac{\partial e}{\partial x} + G \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + G \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right)$$

$$\downarrow$$

$$G \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \right) \dots\dots\dots (i)$$

$$\therefore \lambda \frac{\partial e}{\partial x} + G \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + G \frac{\partial e}{\partial x} + X = 0 \dots\dots\dots (j)$$

If we introduce Laplace operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \dots\dots\dots (9)$$

$$\left. \begin{aligned} (\lambda + G) \frac{\partial e}{\partial x} + G \nabla^2 u + X &= 0 \\ (\lambda + G) \frac{\partial e}{\partial y} + G \nabla^2 v + Y &= 0 \\ (\lambda + G) \frac{\partial e}{\partial z} + G \nabla^2 w + Z &= 0 \end{aligned} \right\} \dots\dots\dots (k)$$

1.5 Equations of Waves in Isotropic Elastic Media

If the body forces X, Y, Z are only inertia forces,

$$\left. \begin{aligned} X &= -\rho \frac{\partial^2 u}{\partial t^2} \\ Y &= -\rho \frac{\partial^2 v}{\partial t^2} \\ Z &= -\rho \frac{\partial^2 w}{\partial t^2} \end{aligned} \right\} \dots\dots\dots (a)$$

Substituting (a) in (k), we obtain

$$\left. \begin{aligned} (\lambda + G) \frac{\partial e}{\partial x} + G \nabla^2 u - \rho \frac{\partial^2 u}{\partial t^2} &= 0 \\ (\lambda + G) \frac{\partial e}{\partial y} + G \nabla^2 v - \rho \frac{\partial^2 v}{\partial t^2} &= 0 \\ (\lambda + G) \frac{\partial e}{\partial z} + G \nabla^2 w - \rho \frac{\partial^2 w}{\partial t^2} &= 0 \end{aligned} \right\} \dots\dots\dots (10)$$

Equations of Waves in Isotropic Elastic Media Basic Equations

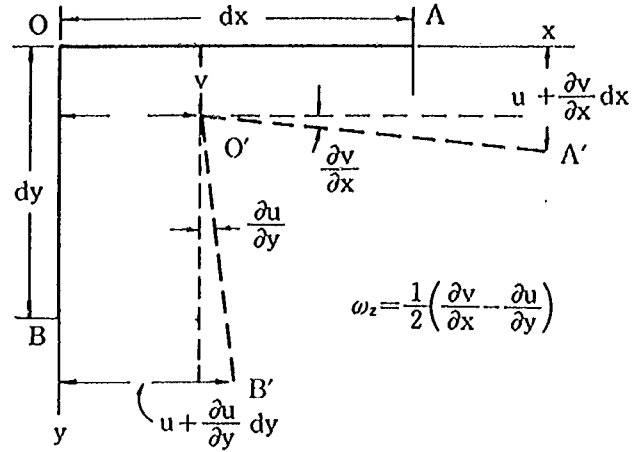
1.6 Waves of Dilation (or irrotational waves)

Rotation of the element with respect to the z-axis

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0$$

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) = 0$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = 0$$



These equations are satisfied, if the u, v, w are derived from a single function ϕ as follows :

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z} \dots\dots\dots (b)$$

Then

$$e = \epsilon_x + \epsilon_y + \epsilon_z = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \nabla^2 \phi \dots\dots\dots (c)$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x} \nabla^2 \phi = \nabla^2 \frac{\partial \phi}{\partial x} = \nabla^2 u \dots\dots\dots (d)$$

Substituting (d) in Ep. (10), we find that

$$(\lambda + G) \nabla^2 u + G \nabla^2 u - \rho \frac{\partial^2 u}{\partial t^2} = 0 \dots\dots\dots (e)$$

Using (d)

$$\left. \begin{aligned} (\lambda + 2G) \nabla^2 u - \rho \frac{\partial^2 u}{\partial t^2} &= 0 \\ (\lambda + 2G) \nabla^2 v - \rho \frac{\partial^2 v}{\partial t^2} &= 0 \\ (\lambda + 2G) \nabla^2 w - \rho \frac{\partial^2 w}{\partial t^2} &= 0 \end{aligned} \right\} \dots\dots\dots (11)$$

General Form of Wave Equations

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \dots\dots\dots (12)$$

$$u = f(x+ct) + f_1(x-ct)$$

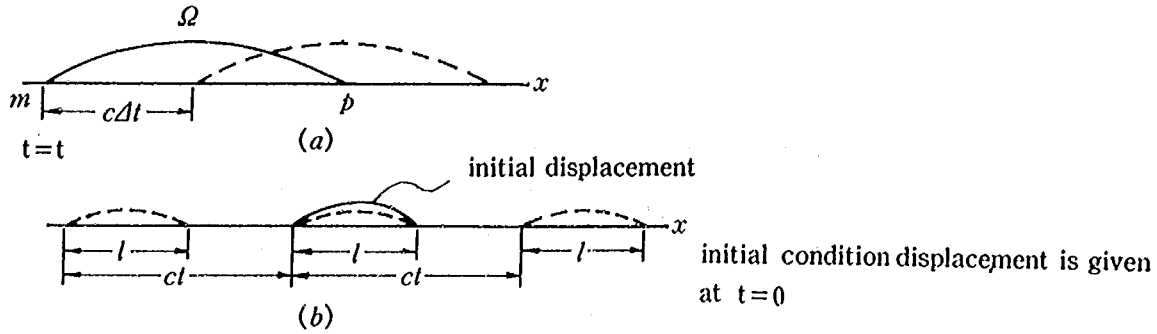


Fig.2

$$c = c_1 = \frac{\lambda + 2G}{\rho} \dots\dots\dots (13)$$

Velocity of Waves of Dilation

For a definite instant t , f is a function of x only and can be represented such as mnp . After an interval of time Δt , argument of f_1 becomes $x - c(t + \Delta t)$.

The function f_1 will remain unchanged provided that simultaneously with the increase of t by Δt the abscissas are increased by an amount Δx equal to $c\Delta t$

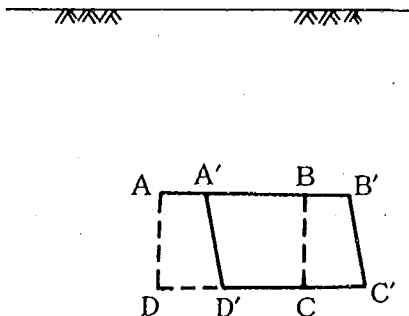
$$x + c\Delta t - c(t + \Delta t) = x - ct$$

This means mnp at $t = t$ can also be used for the instant $t = t + \Delta t$, if it is displaced in the x direction by $\Delta x = c\Delta t$.

1.7 Exercises

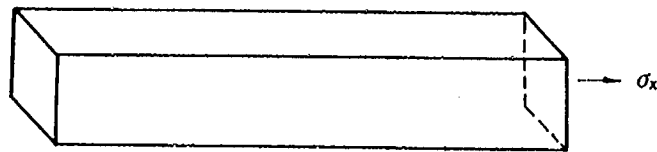
E1. Obtain the velocity of shear waves V_s by using Eq. (10).

hint: In case of shear wave or wave of distortion, the volume expansion is zero.



$$\begin{aligned} AB &= A'B' \\ DC &= D'C' \\ \therefore ABCD &= A'B'C'D' \end{aligned}$$

E2. Obtain the longitudinal wave velocity in a bar.

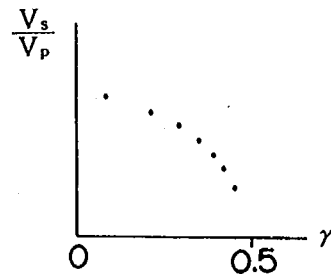


E3. By using the results of E1 and Eq.(3), show how the ratio V_p/V_s is affected by Poisson's ratio ν .

$$V_p = f_p(E, \nu, \rho)$$

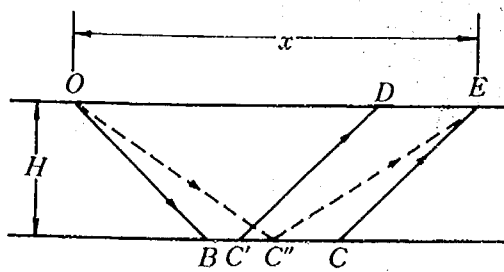
$$V_s = f_s(E, \nu, \rho)$$

$$\therefore V_s/V_p = f(E, \nu, \rho) \rightarrow f'(\nu)$$



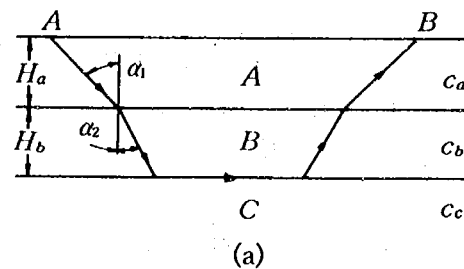
2. Vibration Tests of Soils

2.1 Field Geophysical Tests

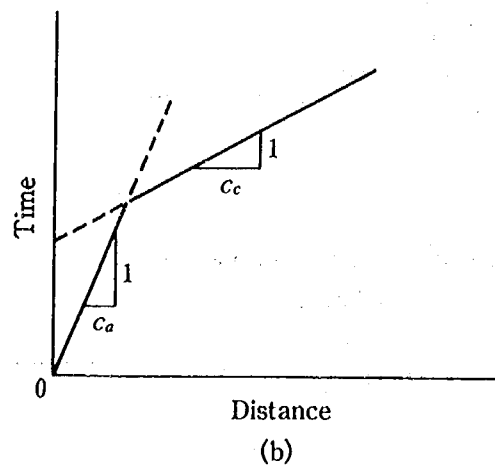


Refraction method.

Fig. 3



(a)



(b)

Refraction method; three-layered medium

2.2 Undamped Vibration of Soil Columns

Longitudinal Vibration

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} \dots\dots\dots (14)$$

assuming the solution has the form

$$u = u(x) \sin \omega t \dots\dots\dots (a)$$

$$\frac{\partial^2 u}{\partial t^2} = -\omega^2 u(x) \sin \omega t$$

$$\frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} = \frac{E}{\rho} \frac{\partial^2 u(x)}{\partial x^2}$$

$$\therefore -\omega^2 u(x) = \frac{E}{\rho} \frac{\partial^2 u(x)}{\partial x^2}$$

$$\frac{\partial^2 u(x)}{\partial x^2} + \frac{\omega^2 \rho}{E} u(x) = 0$$

$$u(x) = A_1 \sin \omega \frac{\rho}{E} x + A_2 \cos \omega \frac{\rho}{E} x$$

$$\therefore u = [A_1 \sin \omega \frac{\rho}{E} x + A_2 \cos \omega \frac{\rho}{E} x] \sin \omega t$$

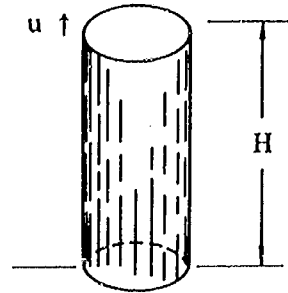


Fig. 4a

Boundary Conditions

$$u = 0, \quad \text{at } x = 0 \quad A_2 \rightarrow 0$$

$$\sigma_x = E \frac{\partial u}{\partial x} = 0, \quad \text{at } x = H$$

$$\frac{\partial u}{\partial x} = [A_1 \omega \frac{\rho}{E} \cos \omega \frac{\rho}{E} x]_{x=H} \sin \omega t = 0$$

$$\therefore \omega_n \frac{E}{\rho} H = \frac{2n-1}{2} \pi$$

$$\therefore \omega_n = \frac{(2n-1)\pi}{2H} \frac{E}{\rho} \dots\dots\dots (14)$$

$n = 1$, for the first mode

$$\omega_1 = \frac{\pi}{2H} \frac{E}{\rho}$$

$$f_1 = \frac{1}{T_1} = \frac{\omega_1}{2\pi} = \frac{1}{4H} \frac{E}{\rho}$$

$$= \frac{V_p}{4H}$$

$$\left. \begin{aligned} V_p &= 4 H f_1 \\ E &= 16 H^2 f_1^2 \rho \end{aligned} \right\} \dots\dots\dots (15)$$

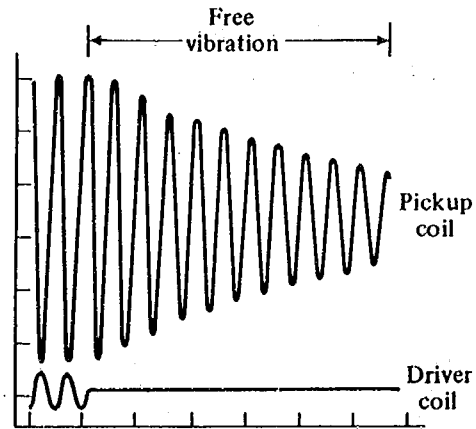


Fig. 4c

In case of torsional Vibration

Torsional Vibration

$$V_s = 4 H f_1$$

$$G = 16 H^2 f_1^2 \rho \dots\dots\dots (16)$$

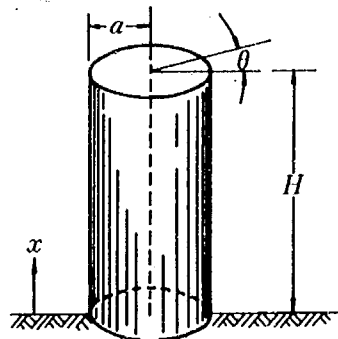


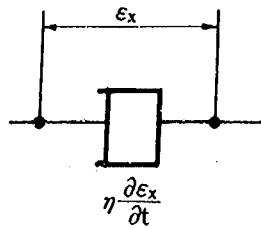
Fig. 4b

2.3 Exercises

- E4. In the field geophysical tests, the measured shear wave velocity of the sand layer was 200 m/s. Find the modulus of rigidity G of the sand, and determine the natural frequency of the shear vibration of the sand, if the density is 2.0 g/cm^3 and the thickness of the layer is 25 m.

2.4 Damped Vibration of the Soil Columns

$$\sigma_x = E\epsilon_x + \eta \frac{\partial \epsilon_x}{\partial t} \dots\dots\dots (17)$$



Viscous damping

Substituting Eq.(17) in Eq.(1)

$$E = \frac{\partial \epsilon_x}{\partial x} + \eta \frac{\partial \epsilon_x}{\partial x} \frac{\partial}{\partial t} - \rho \frac{\partial^2 u}{\partial t^2} = 0 \dots\dots\dots (a)$$

$$\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^3 u}{\partial t \partial x^2} \dots\dots\dots (b)$$

$$\left. \begin{aligned} \rho \frac{\partial^2 u}{\partial t^2} &= \left(E + \eta \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial x^2} \quad \text{longitudinal vibration} \\ \text{torsion} \\ \rho \frac{\partial^2 \theta}{\partial t^2} &= \left(G + \eta \frac{\partial}{\partial t} \right) \frac{\partial^2 \theta}{\partial x^2} \quad \text{torsional vibration} \end{aligned} \right\} \dots\dots\dots (18)$$

2.5 Steady State Vibration

Boundary Conditions

$$(A) \quad \frac{\partial \theta}{\partial X} = 0, \quad \text{at } x = H$$

no mass

$$\theta = \theta_0 e^{i\omega t}, \quad \text{at } x = 0$$

(a)

for Fig.4 b

$$\frac{\theta_1}{\theta_0} = \frac{2}{(2n-1)^2 \pi} \cdot \frac{1}{h} \dots\dots\dots (19)$$

$$h = \frac{\eta}{2G} \cdot \frac{\pi V_s}{2} = \frac{\pi}{4G} \eta V_s \dots\dots\dots (b)$$

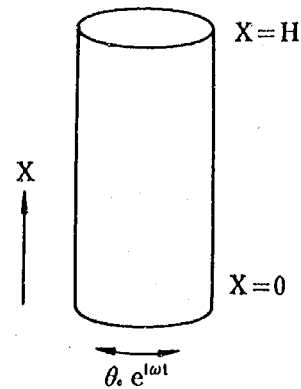


Fig. 4d

(B) with a rigid mass to the free end

Hardin (1965)

ASCE(SM 1)

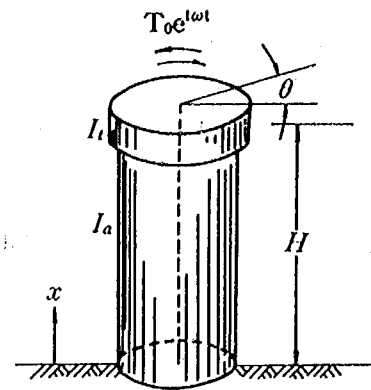


Fig.5a

Ishimoto, et al. 1936

Shannon 1959

Richart 1963

Kuribayashi 1974

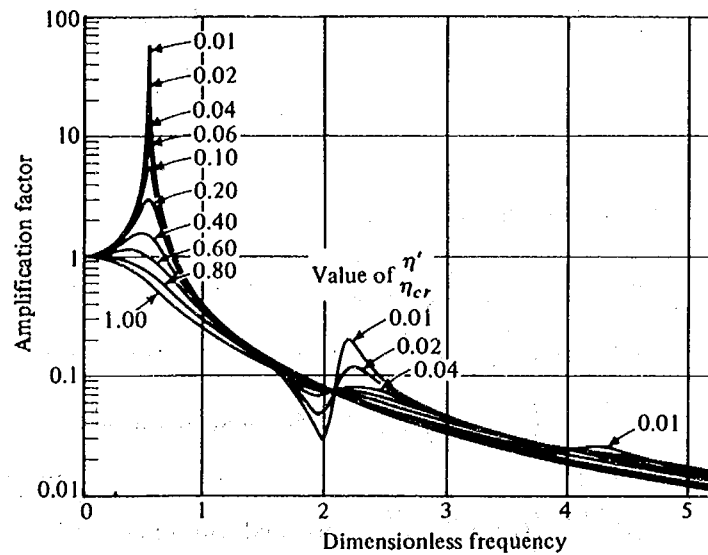


Fig.5b

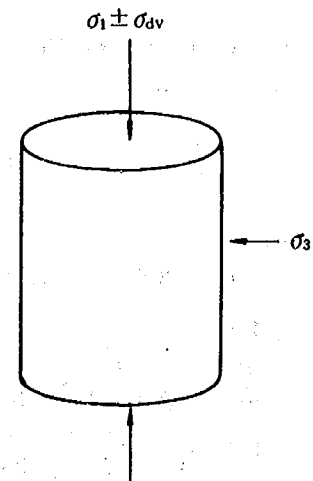
2.6 Cyclic Triaxial Tests

$$\sigma = E\varepsilon + c \frac{d\varepsilon}{dt} \dots\dots\dots (a)$$

$$\sigma = \sigma_a \sin \omega t \dots\dots\dots (b)$$

$$\varepsilon = \varepsilon_a \sin (\omega t - \delta) \dots\dots\dots (c)$$

$$\therefore \frac{d\varepsilon}{dt} = \omega \varepsilon_a \cos (\omega t - \delta) \dots\dots\dots (d)$$



$$\sigma_a \sin \omega t = E \epsilon_a \sin (\omega t - \delta) + c \omega \epsilon_a \cos (\omega t - \delta) \dots\dots\dots (e)$$

$$\therefore \frac{\sigma_a}{\epsilon_a} = \sqrt{E^2 + (c\omega)^2} \dots\dots\dots (f)$$

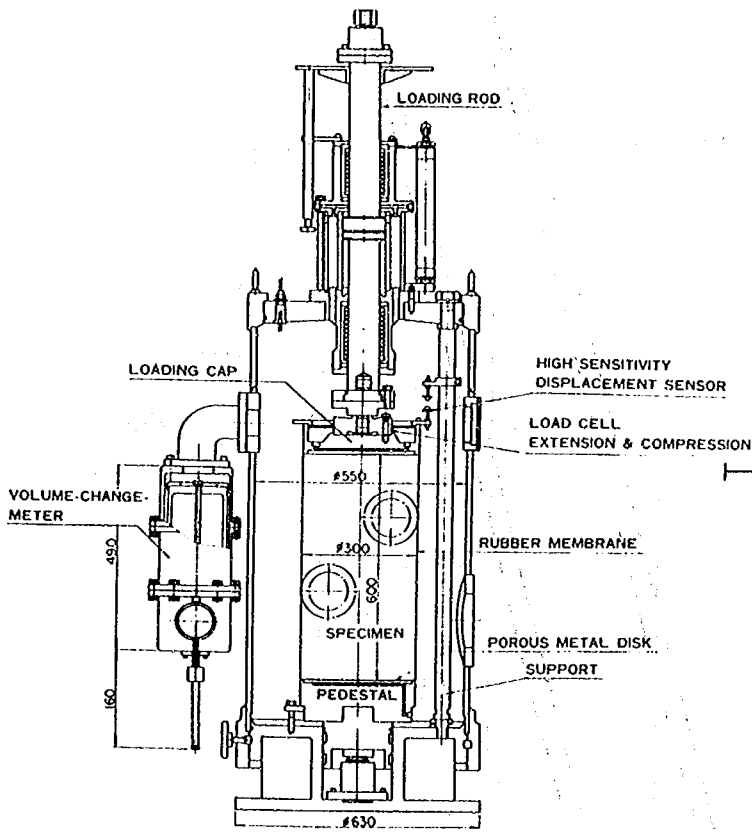
$$\tan \delta = \frac{c\omega}{E} \dots\dots\dots (g)$$

Damping ratio

$$h = \frac{c\omega}{E} = \frac{1}{2} \tan \delta \dots\dots\dots (h)$$

$$h = \frac{1}{2\pi} \frac{\Delta\omega}{\omega} \dots\dots\dots (21)$$

$$\rightarrow \sigma_a = \sin \omega t$$



Cyclic triaxial compression testing equipment

Fig. 6a

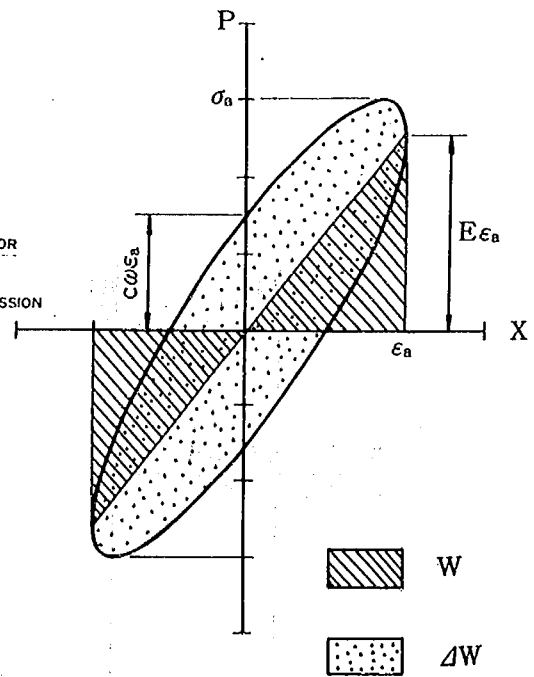


Fig. 6b

2.7 Exercises

E5. Show that the damping ratio h is given by Eq.(21).

hint : use Eq.(h) and $\Delta w = \oint \sigma d\varepsilon = \int \sigma \frac{d\varepsilon}{dt} dt$

E6. Obtain the dynamic elastic modulus E_d , shear modulus G , and damping ratio h from the measured hysteresis loops in Fig.8 and Fig.9.

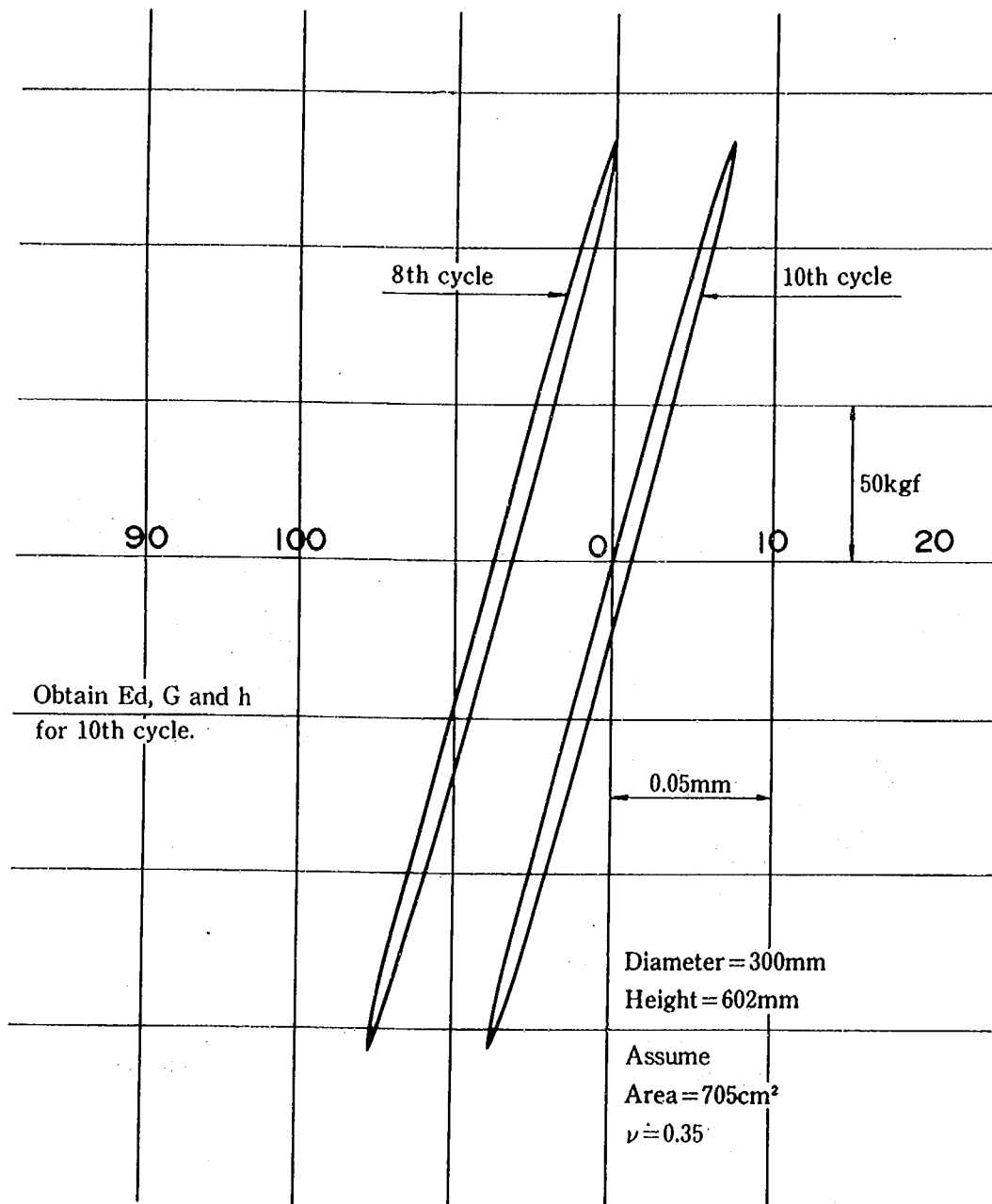
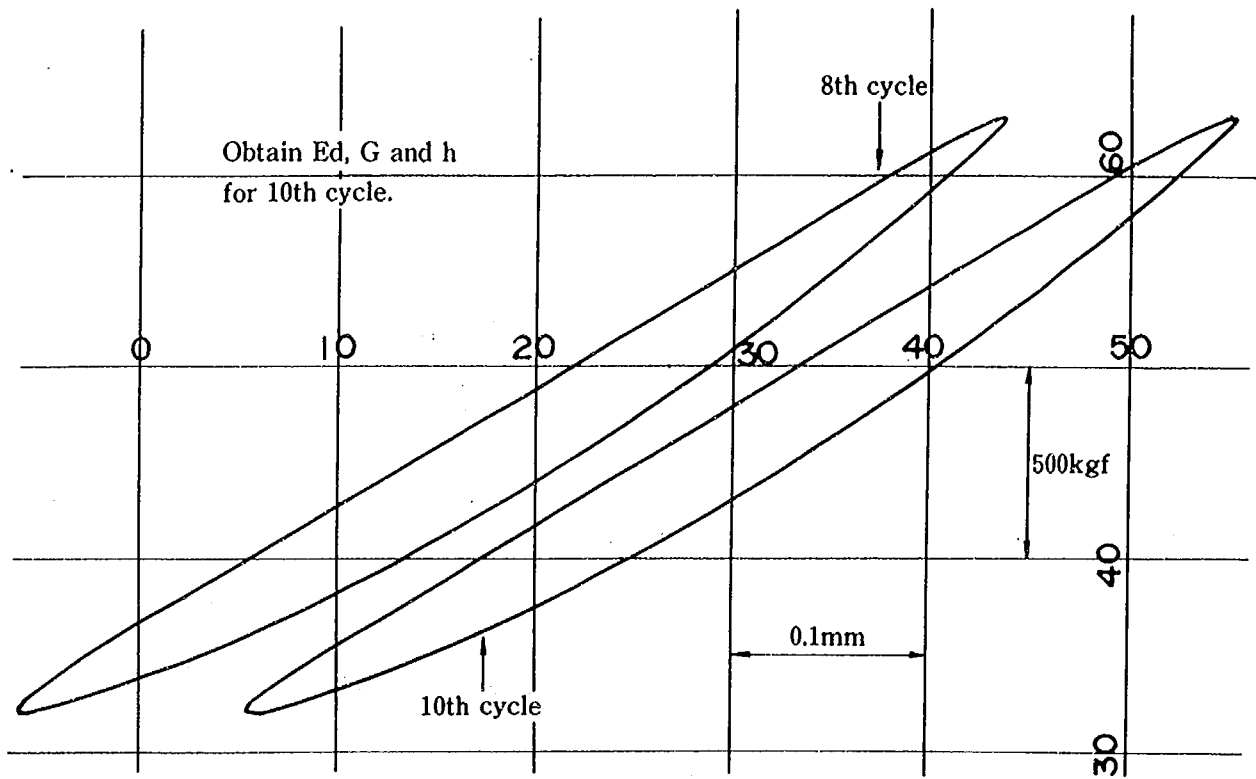


Fig.8



Diameter = 300mm

Height = 602mm

Assume

Area = 705cm²

$\nu = 0.35$

Fig.9

3. Dynamic Deformation Properties of Sands and Coarse Grained Materials

3.1 Effects of Void Ratio

Hardin and F. E. Richart, Jr. (1963),

Richart Fig. 10

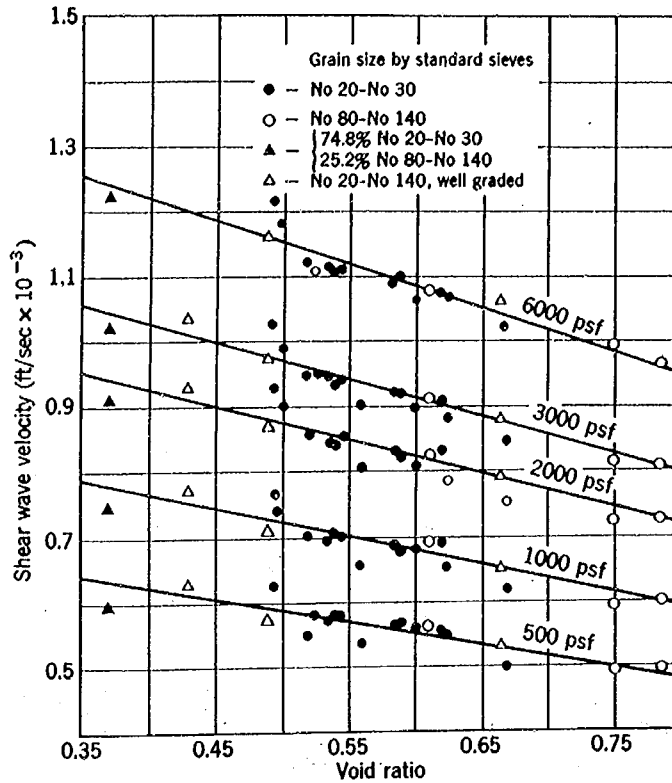


Fig. 10

originally

rounded

$$V_s = (357 - 168e) \sigma^{3/10} \quad \sigma < 1 \text{ kgf/cm}^2$$

$$= (348 - 161e) \sigma^{1/4} \quad \sigma > 1 \text{ kgf/cm}^2$$

angular

$$V_s = (326 - 110e) \sigma^{1/4}$$

for rounded grains

$$\left. \begin{aligned} G &= 700 \frac{(2.17 - e)^2}{1 + e} (\bar{\sigma}_0)^{1/2} \\ \text{for angular grains} \\ G &= 330 \frac{(2.97 - e)^2}{1 + e} (\bar{\sigma}_0)^{1/2} \end{aligned} \right\} \dots\dots\dots (22)$$

3.2 Effects of Amplitude of Strains

$$\frac{G}{G_0} = \frac{1}{1 + \frac{\gamma}{\gamma_0}} \quad \dots\dots\dots (23)$$

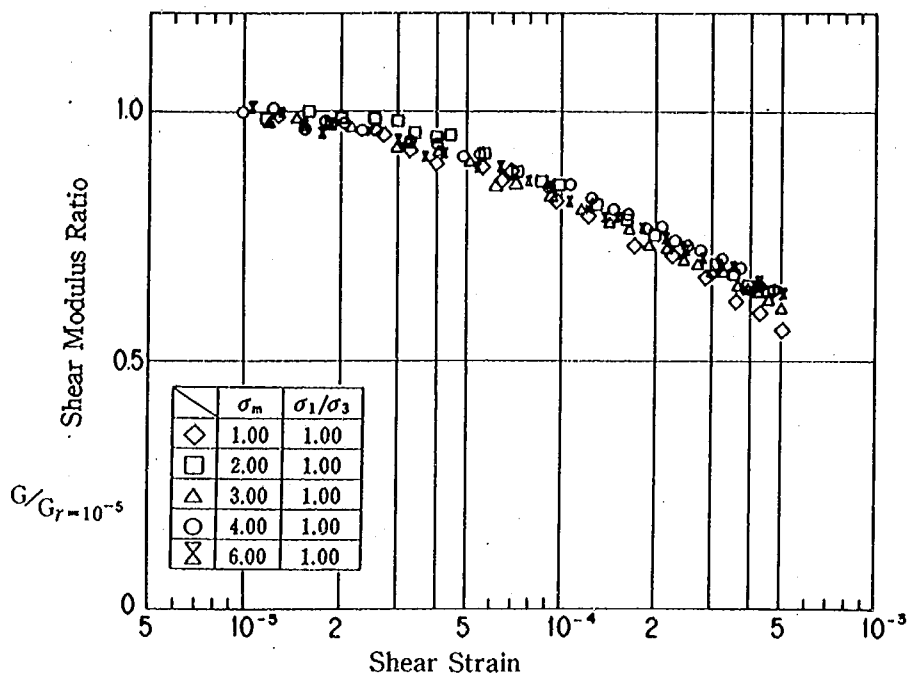


Fig. 11

3.3 Effects of Confining Stresses

$$G = K_1 (\bar{\sigma}_0)^m \quad \bar{\sigma}_0 : \text{mean effective confining stress} \quad \dots\dots\dots (24)$$

$$G = K_2 \left(\frac{\sigma_1}{\sigma_3} \right)^n \quad \frac{\sigma_1}{\sigma_3} : \text{stress ratio} \quad \dots\dots\dots (25)$$

3.4 Dynamic Properties of Rockfill Materials

3.4.1 Shear Modulus

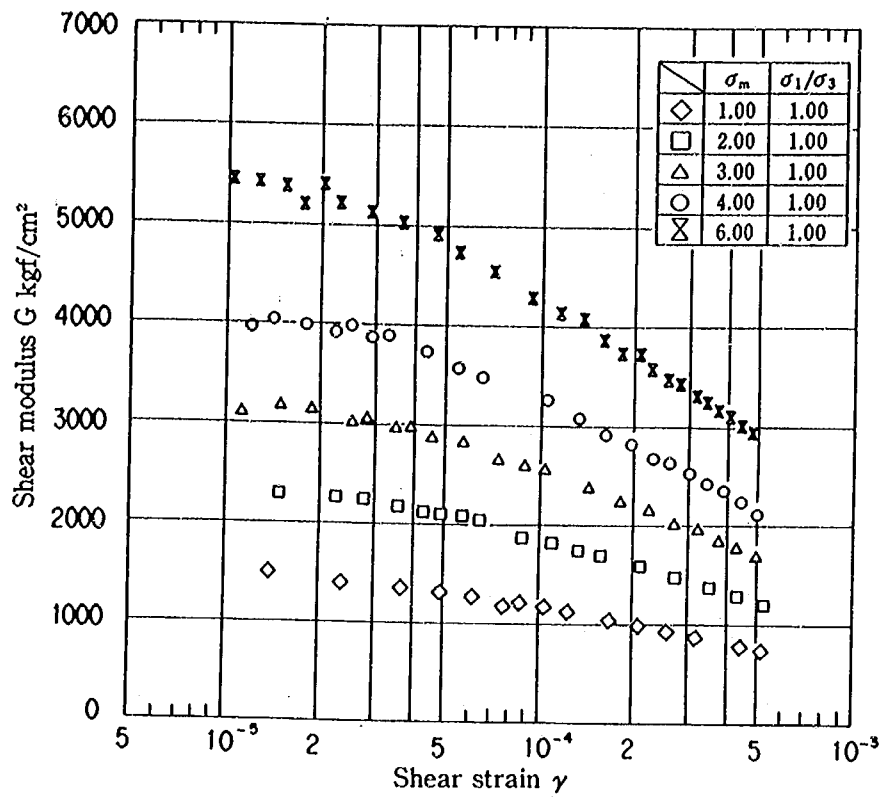


Fig. 12 Shear Modulus

3.4.2 Damping Ratio

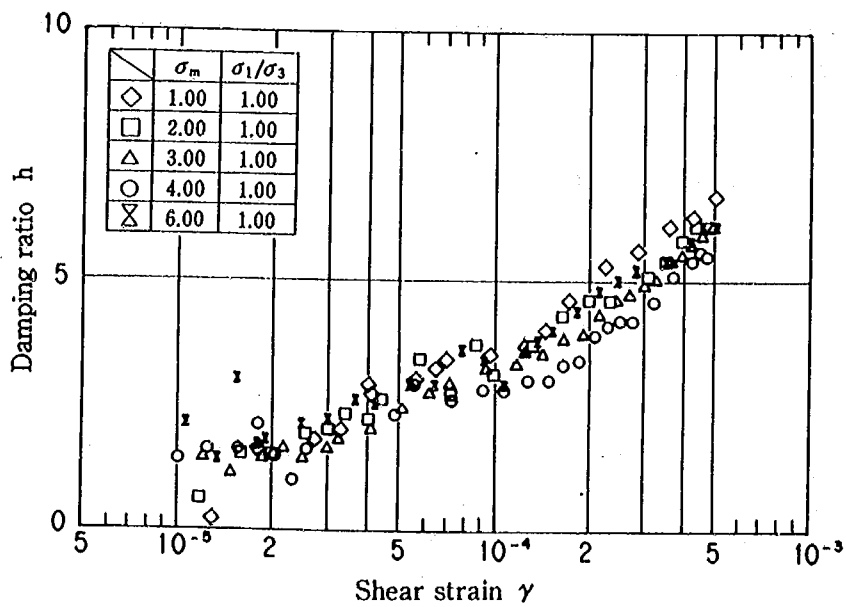


Fig. 13 Damping Ratio

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shear modulus, damping ratio, wave propagation

The aim of this booklet is to present the fundamental concept of soil mechanics applicable to fill dam engineering.

The booklet consists of two parts. Part I deals with the shear strength of fill materials which is one of the most important design values in the stability analysis of fill dams. The shear strength characteristics are explained by the effective stress basis. Dependency of friction angle on confining stresses is explained by using large-scale triaxial compression test data.

Part II describes the fundamentals of soil dynamics, which are the most basic principles in earthquake resistant design of fill dams. Both part I and II include several exercises for use in lecture at Public Works Research Institute, Ministry of Construction. The authors also expect that this booklet will serve as quick reference material for practicing engineers.



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