

BEHAVIOUR OF DAMS DURING EARTHQUAKES

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**Fill Dams Division
Dams Department
Public Works Research Institute
Ministry of Construction
Tsukuba Science City, Ibaraki, Japan**

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BEHAVIOUR OF DAMS DURING EARTHQUAKES

by

Norihisa MATSUMOTO

Nario YASUDA

Synopsis :

The Japanese islands, belonging to the Circum-Pacific Seismic Belt, are one of the most active seismic regions in the world. The safety of dams during earthquakes is extremely important because failure of such a structure may have disastrous consequences on life and property. Therefore, the study of designing dams to resist earthquakes is prerequisite for dam engineers in Japan.

When the seismic phenomenon is regarded as a load acting on a structure or an external disturbance, the influence of the earthquake hazard is divided into two factors, i.e. shaking of the ground and fault movement. Earthquake-resistant design mentioned in this report means design of structures to resist earthquake shaking. While we find a few cases of dams which were designed to withstand fault movement, these design methods are not necessarily commonly accepted. Thus, a conservative attitude would be to avoid any faults which are considered to cause movement during the lifetime of the dam from the geologic evidence. The evaluation of faults is wholly a geological problem and is not discussed in this report.

This report has attempted to explain behaviour of dams during earthquakes based on observed performances of existing structures, analytical procedures, and experimental results to approach the rational earthquake design of dams.

Key Words : Earthquake, Dam, Earthquake motion, Vibration theory, Response

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1. Introduction

The Japanese islands, belonging to the Circum-Pacific Seismic Belt, are one of the most active seismic regions in the world. The safety of dams during earthquakes is extremely important because failure of such a structure may have disastrous consequences on life and property. Therefore, the study of designing dams to resist earthquakes is prerequisite for dam engineers in Japan.

When the seismic phenomenon is regarded as a load acting on a structure or an external disturbance, the influence of the earthquake hazard is divided into two factors, i.e. shaking of the ground and fault movement. Earthquake-resistant design mentioned in this report means design of structures to resist earthquake shaking. While we find a few cases of dams which were designed to withstand fault movement, these design methods are not necessarily commonly accepted. Thus, a conservative attitude would be to avoid any faults which are considered to cause movement during the lifetime of the dam from the geologic evidence. The evaluation of faults is wholly a geological problem and is not discussed in this report.

This report has attempted to explain behaviour of dams during earthquakes based on observed performances of existing structures, analytical procedures, and experimental results to approach the rational earthquake design of dams.

2. Earthquakes and Earthquake Motions

The final goal of this chapter is to describe the characteristics of earthquake motions at damsites. For this purpose, firstly technical terms related to seismology are explained, and secondly the characteristics of earthquake motions are described based on observed acceleration records at damsites.

2.1 Earthquakes

According to current theory of seismology, "earthquake" (particularly shallow earthquake) is caused by fault destruction taking place in the earth's interior¹⁾. When the strain stored in underground rocks exceeds the allowable strain of the rocks for some reason, then minor destruction taking place in a small area in the beginning develops quickly to a large area with a resultant shearing destruction plane. Waves propagated from this destruction are earthquake motions. The size of an earthquake is generally expressed by magnitude M . The definition of M is,

$$M = \log a + 3 \log \Delta - 2.92 \quad \dots (2.1)$$

where

a ; Maximum value of ground motion amplitude (μm)

Δ ; Epicentral distance (km)

Evaluating a size of an earthquake by using magnitude is convenient in many respects, but any magnitude values carry error of 0.2 to 0.3, since this expression is derived from simplifying very complex seismic phenomena. The magnitude of earthquakes which would cause damages to civil engineering structures in Japan is approximately over 6.5 to 7.0. Since the strength of rock constituting the earth's crust is limited and strain energy stored in it is limited, it is assumed that the possible maximum magnitude is around 8.6²⁾.

Meanwhile, the relation shown below exists between energy E (erg) of earthquake and magnitude;

$$\log E = 11.8 + 1.5M \quad \dots (2.2)$$

The equation (2.2) indicates that one unit increase in M results in a $10^{1.5}$ ($=32$) times as much increase in E .

The occurrence of one big earthquake is usually followed by several aftershocks. When an area of the distribution of the aftershock is A (km^2), then the relation between A and M is expressed as follows;

$$\log A = 1.02M - 4.01 \quad \dots (2.3)$$

For instance, Eq. (2.3) gives $1,380\text{km}^2$ and $4,470\text{km}^2$ of A for 7.0 and 7.5 of M , respectively. If we assume this area to be a circle, these areas form circles of 21km and 38km radius, respectively. It is also considered that this area is nearly equal to the area where the land is deformed due to an earthquake.

Furthermore, the relation between a number of earthquakes taking place in a specific area during a certain period of time and magnitude can be expressed as

$$\log N = a + b(8 - M) \quad \dots (2.4)$$

where a and b are constants defined by each local region, and b is about 0.9. Therefore, one increase in M produces decrease of $\frac{1}{10^{0.9}}$ ($\doteq \frac{1}{8}$) in the number.

In designing an earthquake-resistant structure, accurate prediction of the occurrence of earthquakes (or earthquake motion) throughout the life of the structure is most preferable, but the current knowledge to predict an earthquake is not satisfactory in general for the use of engineering design. Thus, in many cases records of historical earthquakes are extended to future to be referred in designing. In such a case, lists of earthquakes to which reference is of made include,

(a) Chronological Table of Science (edited by the Tokyo Astronomical Observatory, and published annually by Maruzen Pub.)

(b) Comprehensive Bibliography of Severe Earthquakes in Japan (Tatsuo Usami, Univ. of Tokyo Press, March 1975)

(c) Earthquake Monthly Report (The Meteorological Agency, Published monthly. The December issue carries a list of the whole year).

(a) and (b) which are books can be available in any book shops, but (c), a monthly journal, should be looked for at a library of a research institute. (a) and (b) consist of lists of strong earthquakes which caused damages during the last 1300 years in Japan, while (c) deals with earthquakes which have been observed at meteorological observatories since 1926, and the number of earthquakes included is about 10 and several thousands. In designing a dam, picking up all earthquakes within, for example, the 100km radius area from the damsite is required, but because of too many earthquakes to single out manually from the above-mentioned materials will take a lot of labor. The Public Works Research Institute has already input these materials into a computer for accurate and quick reference.

As mentioned previously, an earthquake is considered to take place due to fault movement in the earth's interior with resultant shearing plane in the earth's interior. A large size earthquake taking place in a shallow depth causes a shearing plane on the surface. This is a surface fault. Surface faults caused by known historical earthquakes in the past are called earthquake faults. Table-2.1 which is quoted from the Geological Atlas⁵⁾ compiled by the Geological Survey Institute shows earthquake faults in Japan. Important parameters of the earthquake faults are the length L (km) and dislocations D (m) of faults. For the earthquake faults in Japan the following relation was proposed.

$$\log L = 0.6M - 2.9 \quad \dots (2.5)$$

$$\log D = 0.6M - 4.0 \quad \dots (2.6)$$

According to the experiences of the earthquake faults in Japan, no surface faulting has occurred due to the earthquake which magnitude was less than 6.5. Then, when $M=6.5$, the length of fault corresponding to $M=6.5$ becomes $L=10\text{km}$ based on Eq. (2.5). Thus, it can be concluded that any faults with less than 10km in length is unlikely to cause fault movement on the surface. Discussion on the activity of faults should be referred to some other books on geology.

2.2 Earthquake Motions

In case of a strong earthquake, the Meteorological Agency announces its seismic intensity at each place. Table-2.2 shows the definition of intensity scale established by the Japan Meteorological Agency (JMA) in 1949, and Modified Mercalli (MM) Scale

Table-2.1 Earthquake faults (After Geological Atlas)

Date	Earthquake	Magnitude	Fault	Location	Length (Km)	Maximum displacement (m)	
						Vert.	Hori.
1938.5.23	Kussyaro	6.0	South of Kussyaro Lake	East of Hokkaido	(10)	0.85	Left 2.6
1896.8.31	Rikuu	7.2	Senya	East of Akita Pref.	36 (60)	3.5	—
			Kawafune	West of Akita Pref.	6 (15)	2.0	—
1894.10.22	Sakata	7.0	Yanagarezawa	West of Yamagata Pref.	(10)	1.0	—
1965 ~ 66	Matsushiro	5.4	Matsushiro	North of Nagano Pref.	4+	0.2	Left 0.6
1847.7.4	Zenkoji	7.4	Noguchi	North of Nagano Pref.	1.4+(8)	2.7	—
1923.9.1	Kanto	7.9	Enmyoji	South of Bōsō Peninsula	3+	1.9	(Left 1.2)
			Shimoura	South of Miura Peninsula	1	1.5	—
1930.11.26	Kita-Izu	7.0	Tanna	North of Izu Peninsula	30	2.4	Left 3.5
			Himenoyu	North of Izu Peninsula	3+	0.9	Right 1.2
1978.1.14	Izu-Ōshima-kinkai	7.0	Inatori	East of Izu Peninsula	4	0.4	Right 1.2
1974.5.9	Izuhantō-oki	6.1	Irōzaki	South of Izu Peninsula	5.5+	0.2	Right 0.5
1854.12.23	Ansei-Tōkai	8.4	Fujigawa	Middle of Shizuoka Pref.	6+	2~3	(Left 10?)
1945.1.13	Mikawa	7.1	Fukōzu	Middle of Aichi Pref.	19 (28)	2.0	Left, Right 1.3
1891.10.28	Nōbi	8.0	Neodani	Gifu Pref. Fukui Pref.	80	6.0	Left 8.0
1948.6.28	Fukui	7.3	Fukui	North of Fukui Pref.	25	0.9	Left 2.0
1854.9.9	Iga-Ueno	6.9	Kizugawa	Northern West of Mie Pref.	1.5+	1.5	—
1927.3.7	Kita-Tango	7.5	Gōmura	Tango Peninsula	15 (18)	1.2	Left 3.3
			Yamada	Tango Peninsula	3 (8)	0.7	Right 0.8
1925.5.23	Tajima	7.0	Tayui	North of Hyogo Pref.	1.6	0.5	(Left 0.13)
1943.9.10	Tottori	7.3	Shikano	East of Tottori Pref.	8 (14)	10	Right 1.5

**Table-2.2 Relation between maximum acceleration
and maximum velocity**

JMA Scale	Max. acceleration (cm/sec ²)	Max. velocity for vibration of period longer than 1 second (cm/sec)	MM Scale	Max. acceleration (cm/sec ²) (after M. W. McCann.)
O	under 0.8	under 0.13	VI	30 - 80
I	0.8 - 0.8	0.13 - 0.40	VII	80 - 140
II	2.5 - 8.0	0.40 - 1.3	VIII	140 - 260
III	8.0 - 25	1.3 - 4.0	IX	260 - 410
IV	25 - 80	4.0 - 13	X	410 - 570
V	80 - 250	13 - 40	XI	570 - 750
VI	250 - 400	over 40	XII	750 - 890
VII	over 400			

which is widely used in North America. In this table, the relations between JMA and MM scales and maximum acceleration are given. Since a seismic intensity-scale is not necessarily determined from instrumental observation, however, these relations lack consistency. Seismic intensity scale becomes higher with higher magnitude and shorter epicentral distance.

Earthquake motions are vibrations of the ground resulted from the arrival of waves caused by the destruction of the earth's interior. The size of this vibration (amplitude) can be expressed by each of three dimensions, i.e., displacement, velocity and acceleration. If amplitude of any one of displacement, velocity and acceleration of earthquake motion is expressed perfectly as a function of time, then the other two can be known, because these three kinds of amplitude are connected each other in differential and integral relations. However, in this case, "perfectly" raises a question. When many frequency components are involved in earthquake wave, it is not easy to obtain, for instance, accurate acceleration wave from displacement wave. Therefore, in general, it is better to regard that those measured with a displacement seismograph are displacement wave, while those measured with an acceleration seismograph are acceleration wave.

In earthquake-resistant design, acceleration, frequency characteristics, and duration are important factors among the characteristics of earthquake motions, in general. "In general" implies some exceptional cases. For instance, in case of a underground structure, displacement rather than acceleration is more important. Since acceleration can be easily converted to inertia force by multiplying it by mass, design load can be immediately estimated if we know acceleration. Frequency characteristics are an important element in the sense that the response of a structure

to earthquake motion is determined in relation to the natural frequency of the structure. With respect to duration, especially when a strain range beyond the elastic limit has to be considered, it is important in understanding how many repetitions of the load exposure take place.

The characteristics of earthquake motions vary according to the nature of the ground. Therefore, the motion characteristics are defined for the bedrock, or a rock which is not influenced by weathering on the surface, and if sediment is placed on it, earthquake motion on the surface is obtained taking account of the amplification due to the sediment.

Earthquake motions at a certain point is determined by the characteristics of the earthquake itself and the propagation characteristics of the wave up to the point. Therefore, when the earthquake mechanism and the propagation characteristics on the way are known, then it is possible to obtain theoretically the earthquake motion at the point. Research in this field is currently under progress. At present, empirical formulae obtained from summarizing measured records are very often used in designing. Several examples of them are introduced below.

2.2.1 Maximum Acceleration

A. Kanai's Formula⁶⁾

This formula had been proposed based on the observed records at 300m depth in the Hitachi Mine in Ibaraki Prefecture and later it was revised taking account of the data of the Matsushiro Earthquakes.

When displacement, velocity, and acceleration of the ground are d_o (cm), v_o (cm/sec), and a_o (cm/sec²), respectively, then

$$d_o = T \times 10^{0.61M - (1.66 + \frac{3.60}{x}) \log_{10} r - (1.43 + \frac{1.83}{x})} \quad \dots (2.7)$$

$$v_o = 10^{0.61M - (1.66 + \frac{3.60}{x}) \log_{10} r - (0.631 + \frac{1.83}{x})} \quad \dots (2.8)$$

$$a_o = \frac{1}{T - 0.3} \times 10^{0.61M - (1.66 + \frac{3.60}{x}) \log_{10} r + (0.167 - \frac{1.83}{x})} \quad \dots (2.9)$$

where x =epicentral distance (km), M =magnitude, and T =period of the ground earthquake motion (sec).

B. Okamoto's Formula⁷⁾

Okamoto proposed the following formula to indicate the relation among measured values of maximum acceleration a_m (gal), epicentral distance Δ (km) and magnitude M , based on the accelerograms recorded in a shaft in the Kinugawa Power Plant;

$$\log_{10} \frac{a_m}{640} = \frac{(\Delta + 40)}{100} \cdot (-7.604 + 1.7244M - 0.1036M^2) \quad \dots (2.10)$$

C. Gutenberg-Richter Formula⁸⁾

Guthenberg and Richter expressed the relation between maximum acceleration a_o in an energy discharging area and magnitude M for the California Earthquake. It is assumed that the average focal center of the California Earthquake was nearly 15km.

$$\log a_o = -2.1 + 0.81M - 0.027M^2 \quad \dots (2.11)$$

Furthermore, they proposed Fig.-2.1 for the average damping property of acceleration with distance from the energy discharging area. Based on Eq. 2.11 and Fig.-2.1, maximum amplitude of base rock acceleration at a certain point can be given.

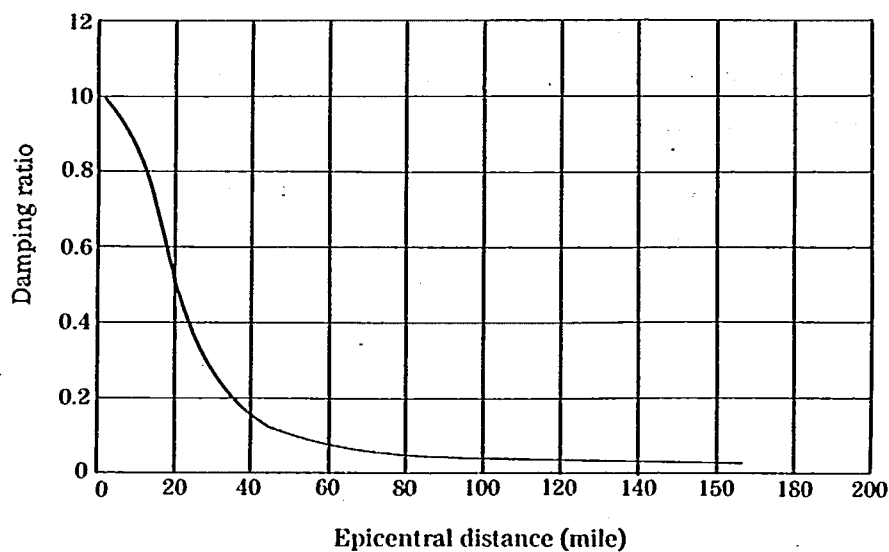


Fig.-2.1 The relation between damping of acceleration at rock base and epicentral distance (after Gutenkerg and Richter)

D. Esteva⁹⁾

Mainly based on the data of strong earthquakes in U.S.A., Esteva proposed the following relation among maximum acceleration Y_c (cm/s²), maximum velocity Y_v (cm/s), magnitude M and epicentral distance R (km);

$$Y_c = b_1 e^{b_2 M} R'^{-b_3} \quad \dots (2.12)$$

where b_1 , b_2 , b_3 , and R' are;

as to acceleration, $b_1 = 5600$, $b_2 = 0.8$, $b_3 = 2.0$, and $R' = R + 40$,

as to velocity, $b_1 = 32$, $b_2 = 1.0$, $b_3 = 1.7$, and $R' = R + 25$ (2.13)

E. Blume's Damping Ratio¹⁰⁾

Blume showed the ratio of damping of the maximum ground acceleration according to epicentral distance and focal depth by the formula below:

$$F_a = \frac{1}{\left(1 + \frac{D}{b}\right)^2} \quad \dots (2.14)$$

where F_a = damping ratio, b = focal depth, and D = epicentral distance.

F. Seed¹¹⁾

Seed plotted values of peak acceleration obtained from the Gutenberg-Richter formula, the Esteva-Rosenblueth formula and the Kanai formula and obtained a diagram shown in Fig.-2.2 by setting a weighted average for earthquakes of 10 ~ 15km focal depth.

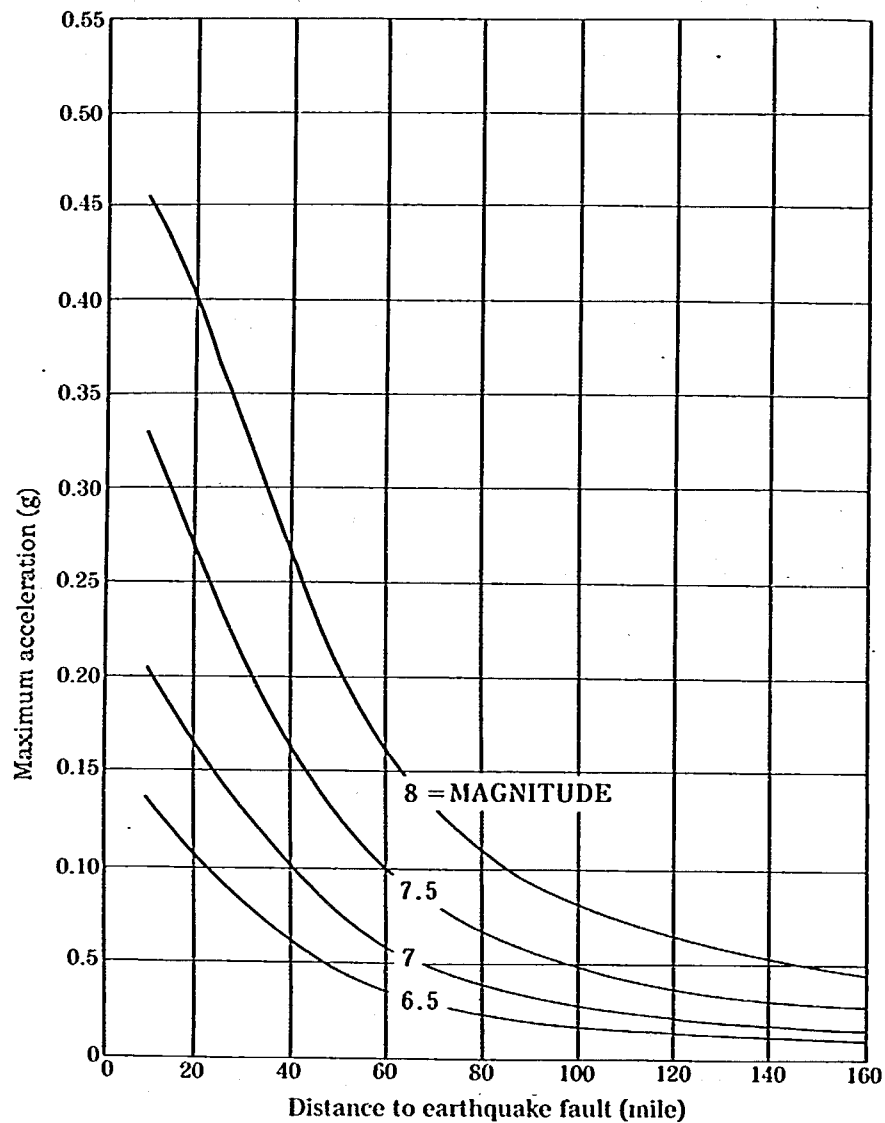


Fig.-2.2 The relation between epicentral distance to earthquake fault and maximum acceleration (after Seed)

G. Housner¹²⁾

Housner obtained for maximum acceleration illustrated in Fig.-2.3 by considering the fact that earthquake intensity is roughly uniform for each earthquake within the range of several miles away from a fault. The damping of acceleration with distance was also indicated.

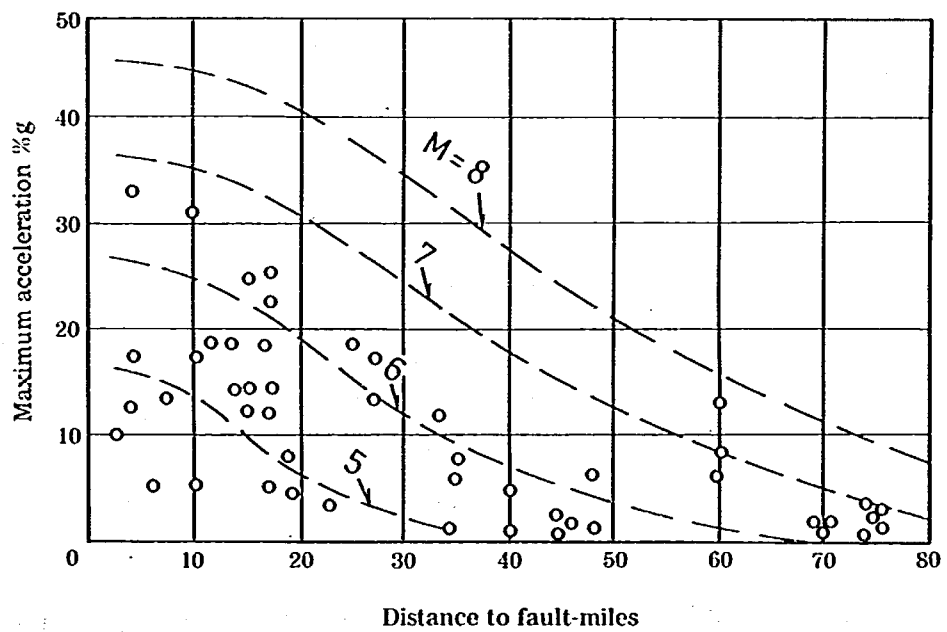
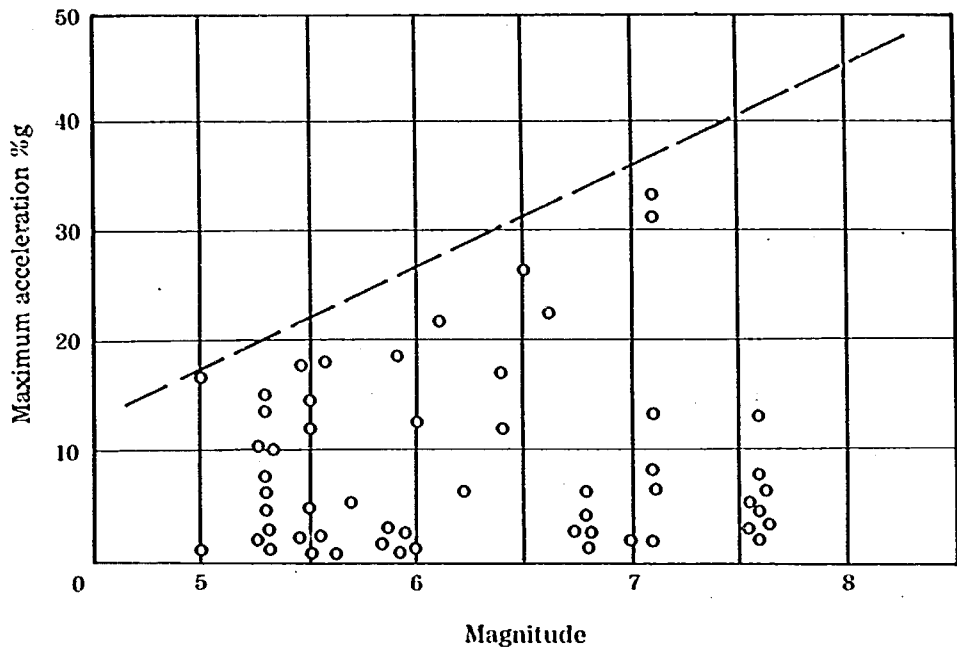


Fig.-2.3 The relations between maximum acceleration and distance to faults, and magnitude (after Housner)

H. Cloud, et al.

Cloud (1970), et al. studied the relation between maximum acceleration of earthquake motion and epicentral distance Δ (mile) based on the records of 19 earthquakes, showing by the equation below: (Fig.-2.4)

$$\log \left(\frac{a}{g} \right) = 3.0 - 2 \log (\Delta + 43) \quad \text{average value} \quad \dots (2.15)$$

$$3.5 - 2 \log (\Delta + 80) \quad \text{slightly overestimated}$$

where,

a ; Peak acceleration of earthquake motion (gal)

g ; The acceleration of gravity (980 gal)

Δ ; Distance to fault or epicentral distance (mile)

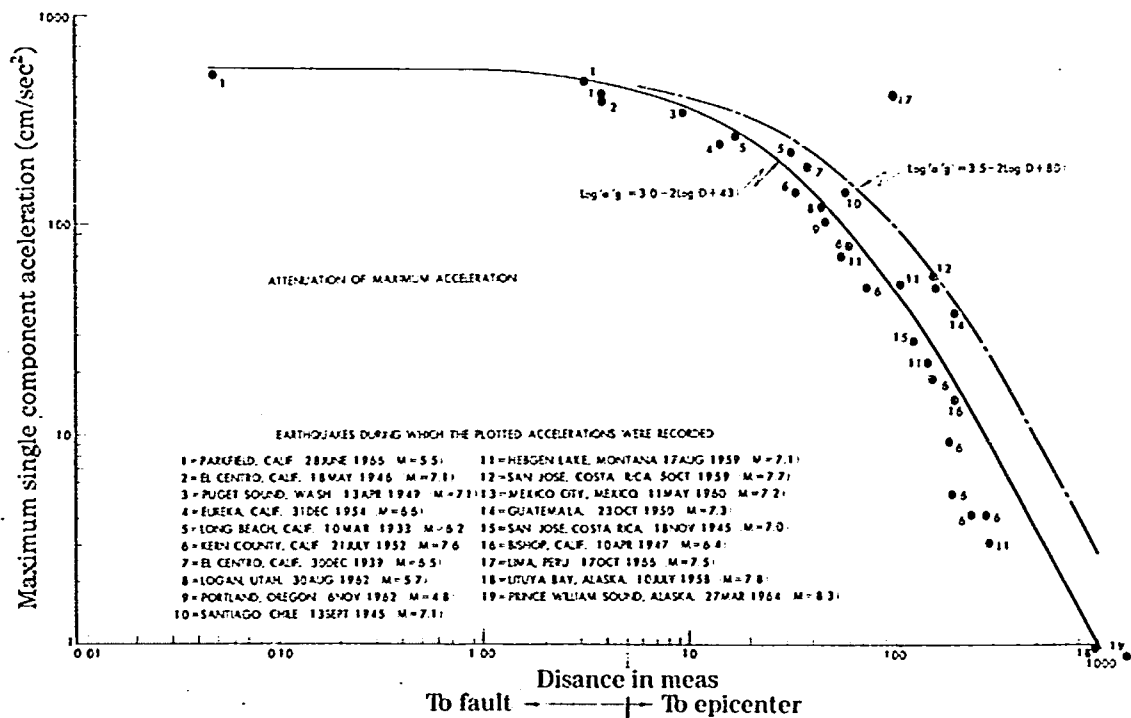


Fig.-2.4 The relation between maximum acceleration and distance to fault (or to epicenter) (after Cloud)

I. Katsumata¹⁴⁾

Katsumata (1972) proposed a method to estimate maximum acceleration of earthquake motion assuming that a focus area is a spherical body. According to his proposal,

$$\log a = \log a_o - \beta \log R' \quad \dots (2.16)$$

where,

a ; Maximum acceleration of earthquake motion at an arbitrary point

a_o ; Maximum acceleration on the surface of a spherical body with radius of r

R' ; Effective distance defined by the formula below

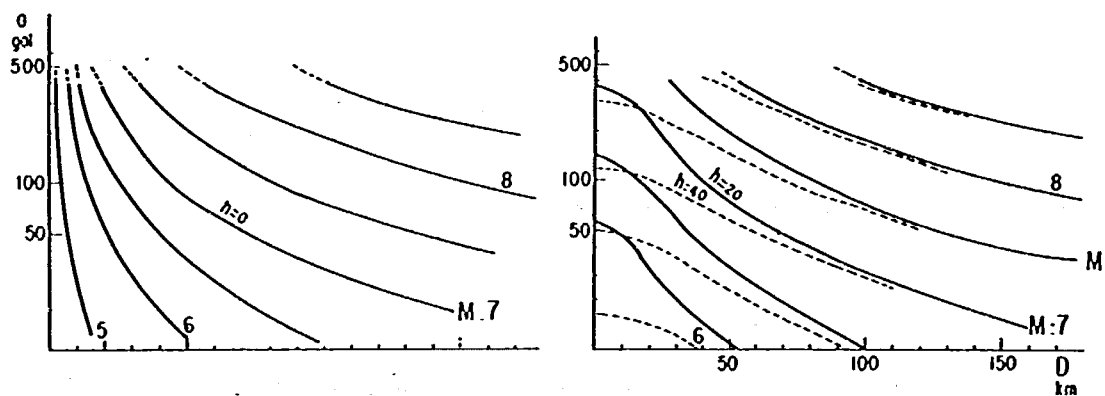
$$R' = R - r \quad \dots (2.17)$$

r ; Radius of a sphere indicating a focus area, defined by the following formula.

$$\log r = 0.5M - 2.25 \quad \dots (2.18)$$

$$\beta = -0.125M + 2.4 \quad \dots (2.19)$$

Here, the maximum acceleration on the surface of the sphere of the radius of r is assumed to be 400 gal. The degree of damping of maximum acceleration with distance is shown in Fig.-2.5.



Variation of Maximum Acceleration with Earthquake Magnitude and Epicentral Distance.

a ; Maximum acceleration in gal

D ; Epicentral distance in kilometer

h ; Focal depth in kilometer

left: a D curves for $h=0$, or a hypocentral distance curves

Right: solid curves, a D four $h=20$, dashed curves, a D for $h=40$

Fig.-2.5 Variation of maximum acceleration with earthquake magnitude and epicentral distance (after katsumata)

J. New Earthquake-resistant design method (Comprehensive Program by the Ministry of Construction)¹⁵⁾

The Comprehensive Program categorized records of strong motion into class 1 through class 4 according to the kinds of the ground. The classification of the ground is the same with Guidelines of Roads and Bridges Earthquake-resistant Design (the Road Association of Japan, 1971). The proposed relations among maximum acceleration of earthquake motion, epicentral distance and magnitude for each class of the ground are given by the following equations.

$$a = 108 \times 10^{0.202M} \times (\Delta + 30)^{-0.813} \quad (\text{Class 1 ground}) \quad \dots (2.20)$$

$$a = 170 \times 10^{0.260M} \times (\Delta + 30)^{-1.06} \quad (\text{Class 3 ground}) \quad \dots (2.21)$$

where,

a ; Maximum acceleration (gal)

M ; Magnitude

Δ ; Epicentral distance (km)

K. Iwasaki, et al.¹⁶⁾

Iwasaki, et al. evaluated many records measured by the Ministry of Construction and proposed the formula shown below for maximum acceleration a_{BR} on the bedrock at the location of Δ km distance from the epicenter.

$$a_{BR} = \frac{9}{10} \times 108 \times 10^{0.202M} \times (\Delta + 30)^{-0.813} \quad \dots (2.22)$$

L. Department of Dams of the Public Works Research Institute

The Department of Dams of the Public Works Research Institute rearranged records of 20 earthquakes observed with accelerometers installed at the foundations of dams of the Ministry of Construction¹⁷⁾. The summary of the results is indicated below;

For $30 \leq x \leq 200$ km

$$\log a = 0.078M^2 - 0.483M - 1.32 \log x + 3.67 \quad \dots (2.23)$$

For $x < 30$ km, it is difficult to summarize due to a few observatory data, but 200 ~ 600 gal seems to be expected.

Formulae to estimate maximum acceleration $a_{\max}$, having magnitude M and epicentral distance x km, have been described from A to L . Performing calculation for $a_{\max}$ from each formula results in a fairly large scattering of values of peak acceleration. The reasons for this scattering include:

1. Acceleration, when it contains short period components, tends to become large, and acceleration is influenced not only by M and x , but also by frequency characteristic of seismic wave. Thus, scattering is unavoidable.
2. A variety of instruments with different sensitivity were used.
3. Types of the ground and location (on the ground or under the ground) where instruments were installed differ.
4. Data for large M and small x are very rare and so values are extrapolated from data for large x .
5. A certain amount of errors are unavoidable for M and x themselves. Therefore, in determining the peak acceleration of design earthquake motions, it is recommended to consult a man of expertise.

2.2.2 Frequency Characteristics

Fig.-2.6 shows accelerograms recorded on the base rock of Naruko Dam and Yuda

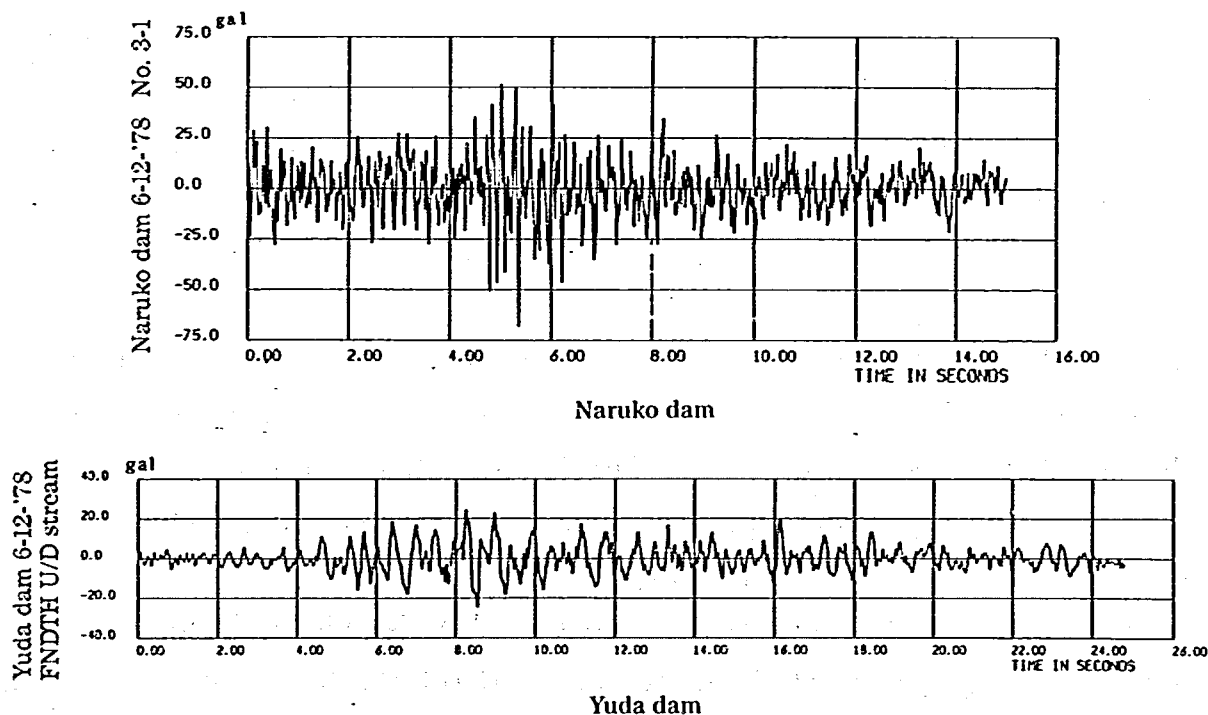


Fig.-2.6 Observed earthquake wave motion (the Miyagi-ken-oki Earthquake of 1978)

When the amplitude of an earthquake motion (acceleration, velocity or displacement) at time t is $x(t)$, then

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \quad \dots (2.24)$$

$X(\omega)$ given by this equation is called a complex Fourier transformation and $X(\omega)$ is a complex number. $X(\omega)$ is divided into amplitudes and phase angles, the plots of the relation between amplitudes and frequency is usually called a Fourier spectrum diagram.

In addition, $R(\tau)$, which is given by

$$R(\tau) = \frac{1}{T} \int_{-\infty}^{\infty} x(t) \cdot x(t+\tau) dt \quad \dots (2.25)$$

is an autocorrelation coefficient, and $S(\omega)$ is a power spectrum function in

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau \quad \dots (2.26)$$

An computation of $X(\omega)$ or $S(\omega)$ of earthquake motions indicates which frequency component is dominant. In actual calculations, since $x(t)$ is given by dispersed values of time scale with equal intervals, a modified formula for dispersed values is applied instead of the formula (2.24). For further explanation, readers who are interested in this point should refer to another technical books (for instance, reference (18)).

The Fourier spectrum and the power spectrum of the motions shown in Fig.-2.6 are illustrated in Fig.-2.7. In this figure, the fluctuation of the spectrum amplitude values to the frequency is rounded by filtering the calculated spectrum amplitude. In general, a power spectrum expression tends to emphasize frequency characteristics compared with a Fourier spectrum expression.

According to Fig.-2.7, the acceleration record at Naruko Dam indicates that components ranging from 2 to 12Hz are contained (this is called wide range). On the other hand, the record at Yuda Dam shows concentration in a narrow frequency range from 1 to 3Hz (this is called narrow range), and since the peak near 2.3Hz is remarkable, this 2.3Hz can be regraded as the predominant frequency.

Dam foundations during the Miyagi-ken-oki Earthquake of 1978. Both data look like random wave, but more careful examination indicates that the record at Naruko Dam contains more waves per unit time than the record at Yuda Dam. In other words, this means that there is a difference in the frequency characteristics of the motions at each site. Fourier analysis gives us a powerful means for understanding random time history data by the use of periodic functions.

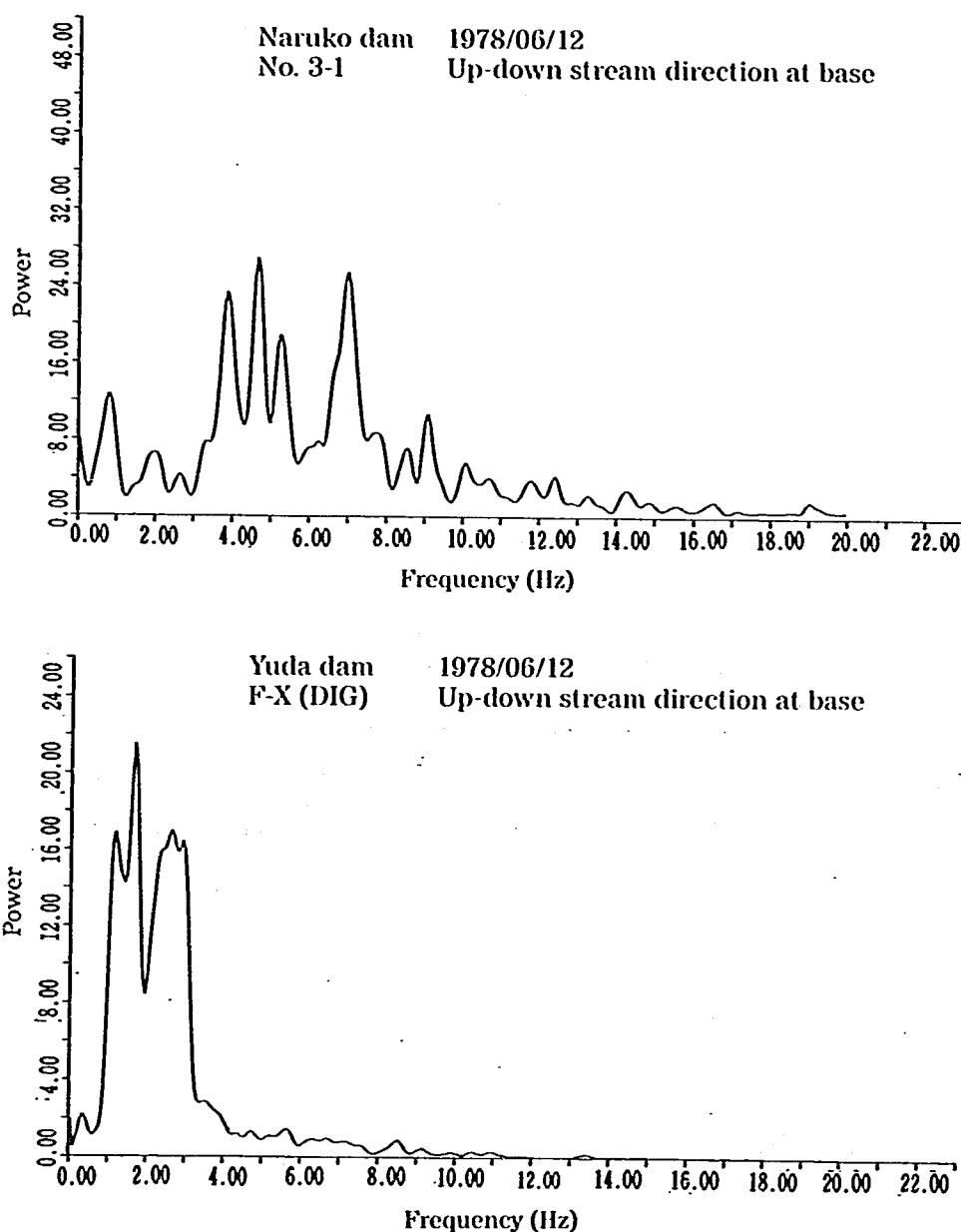


Fig.-2.7 Power spectra

The predominant frequency of acceleration on a base rock is affected by magnitude M and epicentral distance x of an earthquake. For example, Seed Proposed Fig.-2.8¹⁹⁾.

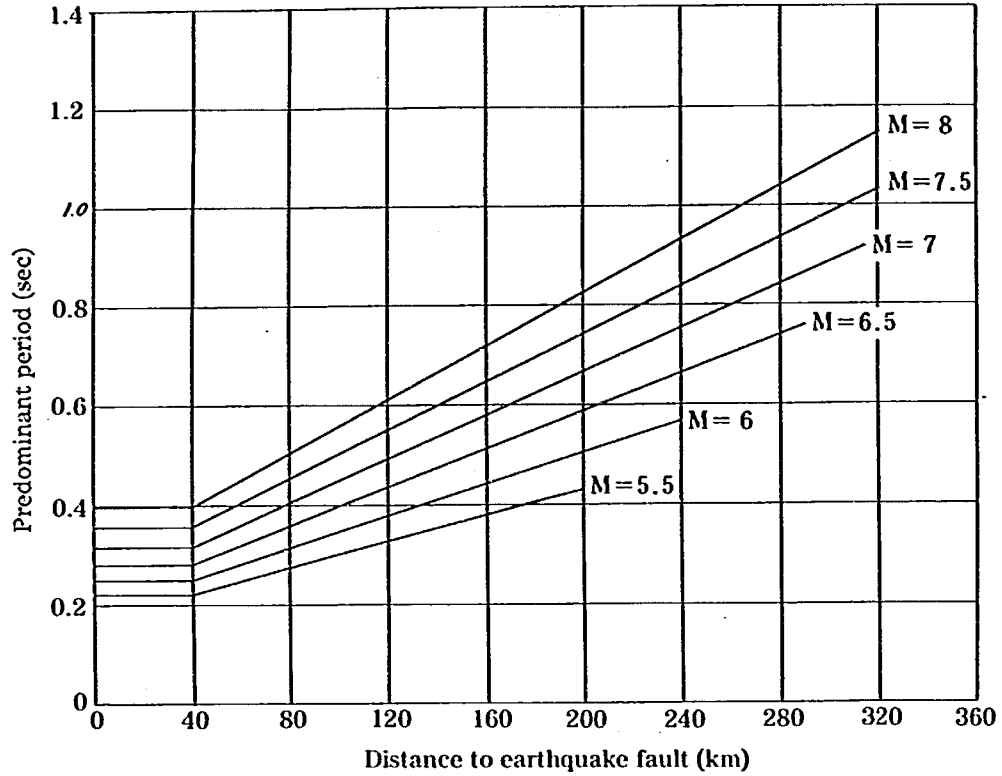


Fig.-2.8 Predominant period of maximum acceleration at base (after Seed)

Measured data obtained by the Department of Dams of P.W.R.I.²⁰⁾ shown in Fig.-2.9 indicate very clearly a tendency that an increase in epicentral distance x (km) leads to a decrease in the predominant frequency f_p (Hz). The range is roughly,

$$2.0 - \frac{2}{300}x < f_p < 7.0 - \frac{5}{300}x \quad \dots (2.27)$$

and the average value is,

$$f_p = 4.0 - 0.01x \quad \dots (2.28)$$

where, f_p is the predominant frequency for horizontal motion.

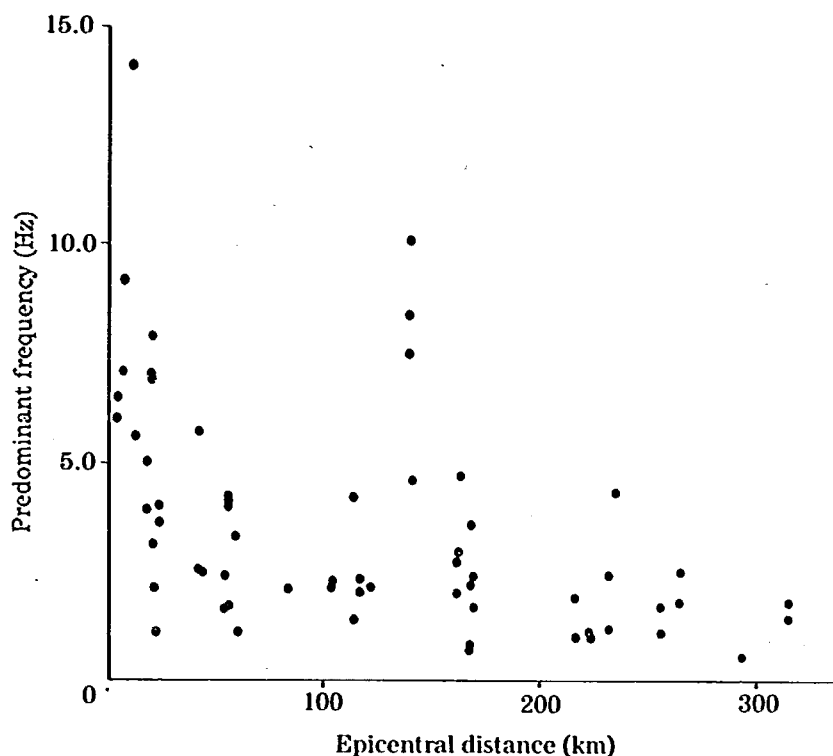


Fig.-2.9 The relation between predominant frequency and epicentral distance (Horizontal motion)

Frequency characteristics of earthquake accelerations on damsite base rock can be characterized as follows;

1. They are usually wide ranged, it is not so often to have a single predominant frequency.
2. The frequency characteristics of accelerograms recorded at the same damsite sometimes show completely different characteristics with the different earthquakes. Frequency characteristics tend to reflect the characteristics of earthquake itself more than the characteristics of the ground.
3. In general, fast components are contained in near field events, while slow components are predominant in far field events.

2.2.3 Duration

There can be several definitions of duration of earthquake motion. When duration is defined as the time length from a time when 20% acceleration value of maximum acceleration appears for the first time to a time when it appears for the last time, then Fig.-2.10 is given, according to the results summarized by the Department of Dams of P.W.R.I. When duration T (sec) is approximated to a exponential function

of magnitude M , then

$$T = 10^{0.027M^2 - 0.147}$$

..... (2.29)

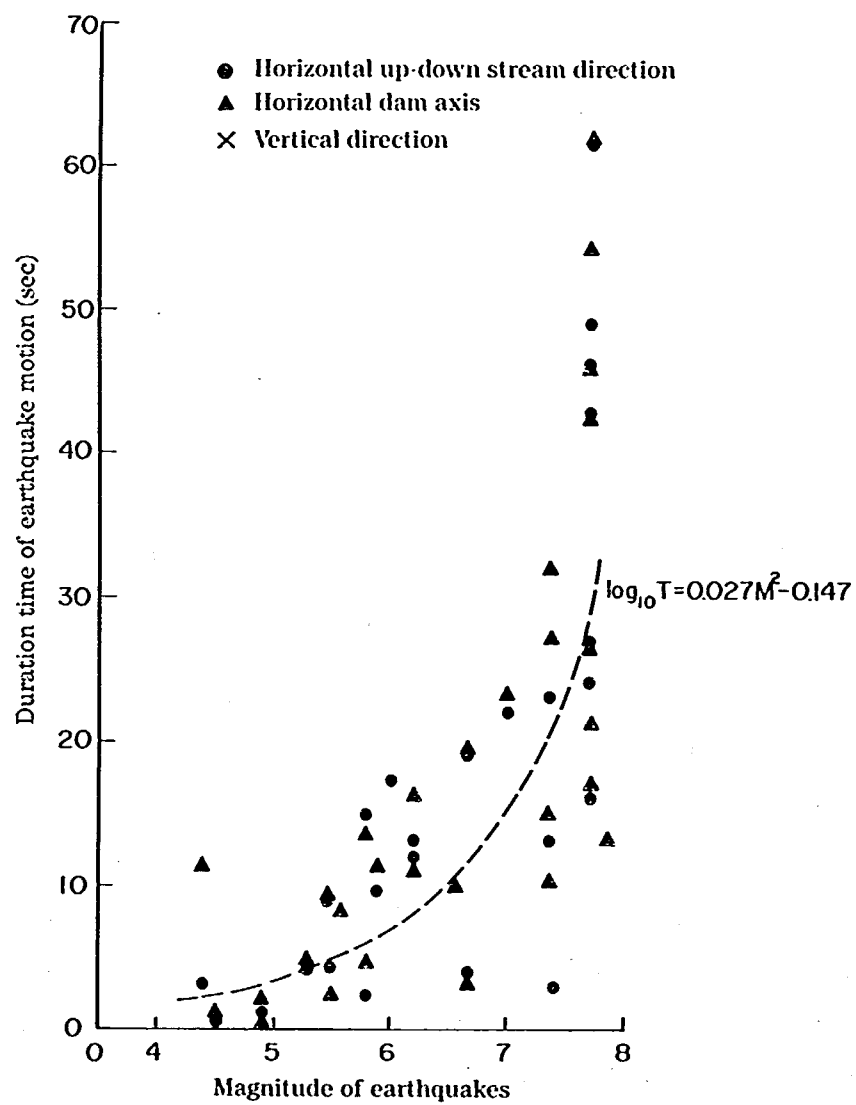


Fig.-2.10 Duration time of earthquake motion

3. Elementary Theory of Vibrations

In designing an earthquake-resistant structure, it is very helpful to understand elementary theory of vibrations. The majority of readers of this volume can be a specialist on dams, but not all of them might be quite familiar to earthquake-resistant designing and vibrations. This chapter aims at better understanding of technical terms and techniques commonly used in earthquake engineering, and explains elementary theory of vibrations (For further discussions of vibration theory, see the references 21) through 24).

3.1 Simple Vibrating Systems

In daily life we can see many vibration phenomena. One of them is a pendulum of a clock. Motion of a pendulum is schematically shown in Fig.-3.1. The reason why a pendulum has been used for a clock is that movement of a pendulum is very periodic. Then, the equation of motion for the pendulum shown in Fig.-3.1 is made to represent its movement.

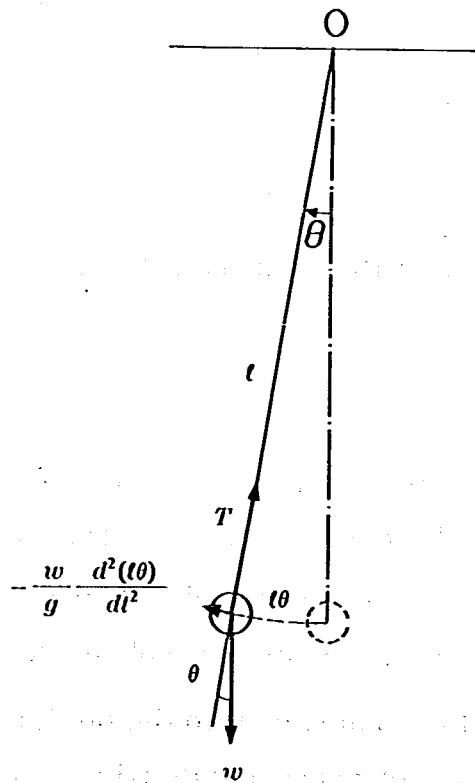


Fig.-3.1 Forces acting on a pendulum

The pendulum moves on the circumference of a circle of radius of ℓ , the length of the string, around the fixed end of the string O. When the weight of the pendulum is W , the location of the pendulum can be expressed by the angle θ (clock-wise direction is positive) which forms the angle between the direction of pendulum and the vertical (stationary position). Here, the distance of the pendulum from the stationary position is $\ell \cdot \theta$. Therefore, the equation of the pendulum of equilibrium of the motion in the circumferential direction is assuming the weight of the string to be negligible,

$$-\frac{w}{g} \frac{d^2(\ell\theta)}{dt^2} - w \sin\theta = 0 \quad \dots (3.1)$$

The reason why the equation (3.1) expresses the equilibrium is explained below.

Firstly, with respect to the first term, $-\frac{w}{g}$, a division of a weight, w , by the acceleration of gravity, g , is a mass, while $\frac{d^2(\ell \cdot \theta)}{dt^2}$ is an acceleration in the circumferential direction. In general, when a substance with a mass m is subjected to force f , an acceleration of α will be generated. m , f , and α are connected in the relation below, in accordance with the Newton's Second Law,

$$f = m\alpha \quad \dots (3.2)$$

Then, the transposition of the right-hand side makes,

$$f - m\alpha = 0 \quad \dots (3.3)$$

This formula can be regarded as an equation of equilibrium between a force, f , and a force of $-m\alpha$ which direction is reverse to that of acceleration. This is called D'Alembert's principle, which provides us a convenient idea to handle dynamic problems along the same lines of static problems. Namely, $-\frac{w}{g} \cdot \frac{d^2(\ell \cdot \theta)}{dt^2}$ of the Eq. (3.1) means that a force equivalent to the product of mass and acceleration is acting in the reverse direction of acceleration (in accordance with the D'Alembert's principle). The second term of Eq. (3.3), $-w \sin \theta$, is a component of circumferential direction of weight w , as shown in Fig.-3.1, and this negative sign means that the

direction is counter clockwise.

In Eq. (3.1), when θ is small, then $\sin \theta = \theta$,

$$-\frac{w}{g} \frac{d^2(\ell\theta)}{dt^2} - w\theta = 0 \quad \dots (3.4)$$

Since ℓ is constant, Eq. (3.4) can be reduced to

$$\frac{d^2\theta}{dt^2} + p^2\theta = 0 \quad \dots (3.5)$$

where,

$$p^2 = \frac{g}{\ell} \quad \dots (3.6)$$

Since Eq. (3.5) is a linear differential equation of second order with constant coefficients, the solution can be given by,

$$\theta = A \cdot e^{\lambda t} \quad \dots (3.7)$$

By using the Euler relation, this becomes

$$\theta = A \cdot \sin(pt + \psi) \quad \dots (3.8)$$

where the arbitrary constant A and ψ are evaluated from the initial conditions of the pendulum. The constant A is called the amplitude of the motion, and ψ is called the phase. If the pendulum is displaced an angle θ_0 from the rest position and then released at $t=0$, the initial condition is,

$$\left. \begin{aligned} [A \sin(pt + \psi)]_{t=0} &= \theta_0 \\ \left[\frac{d}{dt} A \sin(pt + \psi) \right]_{t=0} &= 0 \end{aligned} \right\} \quad \dots (3.9)$$

From these equations, the following expressions are obtained for A and ψ

$$\left. \begin{aligned} A &= \theta_0 \\ \psi &= \frac{\pi}{2} \end{aligned} \right\} \quad \dots (3.10)$$

Namely, the solution of Eq. (3.5) in the initial condition of Eq. (3.9) is,

$$\theta = \theta_0 \sin \left(pt + \frac{\pi}{2} \right) \quad \dots (3.11)$$

Fig 3.2 shows the relation between t and θ . From Fig.-3.2 we can see that for free vibration of an undamped single-degree-of-freedom system, the motion is harmonic or sinusoidal and occurs at a natural period T , which is given by the following equation.

$$T = \frac{2\pi}{p} = 2\pi \sqrt{\frac{\ell}{g}} \quad \dots (3.12)$$

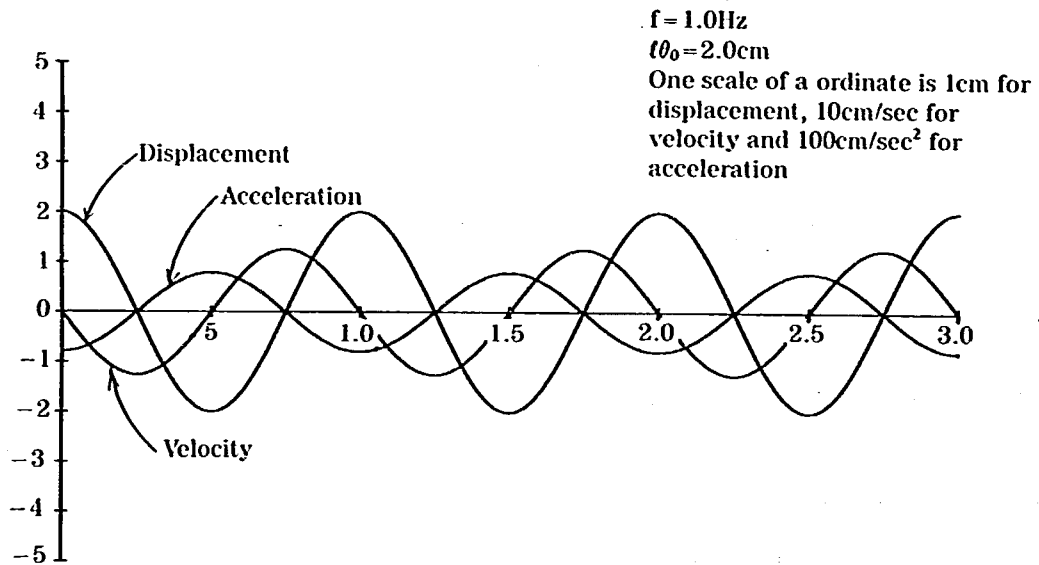


Fig.-3.2 Motion of a pendulum

The period of the pendulum, T is determined only by the length and the gravity of the earth, as shown by Eq. (3.12), but has nothing to do with the weight of the pendulum. For instance, in order to make a pendulum with 1.0 second period, $\ell = 24.8 \text{ cm}$, because

$$1.0 = 2\pi \sqrt{\frac{\ell}{980}} \quad \dots (3.13)$$

In reverse, it is possible to measure a local change in the acceleration of gravity with a pendulum using the property that a period changes with the acceleration of gravity.

Furthermore, the acceleration of the pendulum in the circumferential direction $\frac{d^2(\ell)}{dt^2}$ can be obtained from Eq. (3.11),

$$\frac{d^2(\ell\theta)}{dt^2} = -\ell\theta_0 \cdot p^2 \sin\left(pt + \frac{\pi}{2}\right) \quad \dots (3.14)$$

This relation is also illustrated in Fig.-3.2. This figure indicates that the displacement is 180° out of phase with the acceleration.

The frequency of oscillation in terms of cycles per unit time is given by $f = \frac{1}{T}$.

The unit of cycles per second are called Hz. The quantity p of Eq. (3.11) is called the circular frequency or angular frequency. As shown in Fig.-3.2, $2\theta_0$ represents the peak-to-peak amplitude, sometimes referred to as double amplitude and θ_0 represents the amplitude from the mean position, sometimes referred to as single amplitude. The double amplitude is most often measured from vibration records.

The motion of the pendulum shown in Fig.-3.1 has been so far explained. This discussion is entirely applicable to the system shown in Fig.-3.3. Here, the quantity k is a spring constant which is defined as the change in force to displace the spring or the column by a unit length and m is a mass (if instead of mass, weight w is used, then $m = \frac{w}{g}$). The circular frequency p of Eq. (3.6) or (3.11) should be replaced with

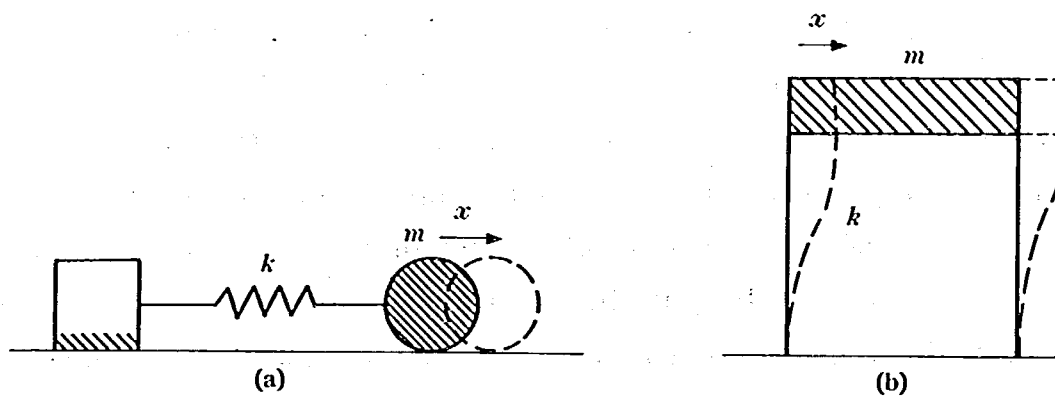


Fig.-3.3 Vibration system with a mass supported by an elastic spring

$$p^2 = \frac{k}{m} \quad \dots (3.15)$$

In Fig.-3.1 or 3.3, the position of all parts of the system can be described by a single variable at any instant of time. Such a system is called the one-degree-of-freedom system. In general, the number of degrees of freedom of a system is equal to the number of independent variables required to define the position of all parts of the system at any instant of time. In Fig.-3.1, the circumferential component of gravitational force acts as the restoring force. While in Fig.-3.3, the spring force acts as the restoring force.

Any actual structure is not a single-degree-of-freedom system. However, many systems can be adequately modeled using a single degree-of-freedom system. As explained later, vibrations of n degrees of freedom system of a linear elastic body can be decomposed into a combination of motions of n single degree-of-freedom systems. Therefore, it is very helpful for understanding of vibrations of complex structures to examine the characteristics of the single-degree-of-freedom system.

3.2 Damping

The spring-mass systems in the above discussion have no element to dissipate energy, so that, once vibration starts, the motion continues permanently, as seen from Eq. (3.8) or as shown in Fig.-3.2. However, in reality this never happens. Actually the energy of motions of the vibration systems is consumed during the motion and finally the motion will come to a stop. This consumption of energy is represented by damping in vibration theory.

When we think of the case of concrete dams during earthquake, some of the incident-wave energy will be reflected and some will be transmitted at joints of blocks or at an interface with air and water. The damping of concrete itself is considered to be not so much. Damping resulting from the transmission of the incident-wave into the base rock is especially called dissipation damping. Thus, damping is caused through a variety of mechanisms. On the other hand, in the case of fill-dams, much damping will exist in the dam body itself. For instance, static triaxial tests for fill materials indicate that there is nonlinear relationship between stress and strain even in the small strain range. Thus, it is considered that the elastic energy to be stored in the fill dam body will be lost through the internal material damping or hysteresis

damping.

Thus, causes of damping are very complex, but here two types of damping will be introduced in the following paragraph.

3.2.1 Viscous Damping

One of useful expressions for a mechanism of energy dissipation in a vibration system is viscous damping. This is based on the idea that the force of damping is directly proportional to velocity. Fig.-3.4 is a schematic representation of viscous damping in which the dashpot is added to the spring-mass system in Fig.-3.3. Let us consider that vibration takes place in the vibration system of Fig.-3.4 for some reason and no external force will act after the vibration begins. This type of vibration is called free vibrations. In this vibration system, the forces acting on the mass include inertial force $-m\ddot{x}$ according to D'Alembert principle, damping force $-c\dot{x}$, (c is called viscous damping coefficient), and restoring force due to spring $-kx$. We find the differential equation of motion to be

$$-m\ddot{x} - c\dot{x} - kx = 0 \quad \dots (3.16)$$

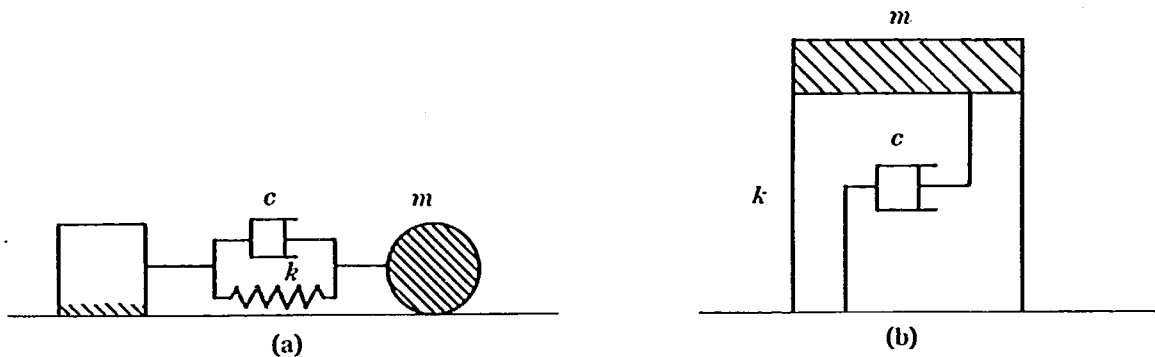


Fig.-3.4 Vibration system with a dash pot

The dot over the quantity indicates derivatives with respect to time. Rewriting Eq. (3.16), we obtain

$$\ddot{x} + 2h\dot{x} + p^2x = 0 \quad \dots (3.17)$$

where

$$\left. \begin{aligned} p^2 &= \frac{k}{m} \\ k &= \frac{c}{2\sqrt{km}} \quad (h: \text{damping constant}) \end{aligned} \right\} \dots (3.18)$$

Since Eq. (3.17) is a linear differential equation of 2nd order with constant coefficients, the solution may be written in the form.

$$x = e^{\lambda t} \dots (3.19)$$

Substituting the assumed form of the solution in Eq. (3.17), we obtain,

$$\lambda^2 + 2hp\lambda + p^2 = 0 \dots (3.20)$$

$$\therefore \lambda = -hp \pm p\sqrt{h^2 - 1} \dots (3.21)$$

The nature of the solution depends on the nature of the roots. There are three possible cases to consider, depending upon whether the roots are real, complex, or equal.

CASE 1: $h > 1$. In this case the two roots of Eq. (3.20) are real as well as negative.

$$\text{If we let } p_1 = p\sqrt{h^2 - 1} \dots (3.22)$$

$$\begin{aligned} \text{then (3.21) becomes } \lambda_1 &= -hp + p_1 \\ \lambda_2 &= -hp - p_1 \end{aligned} \dots (3.23)$$

Therefore, the solution to Eq. (3.17) is,

$$x = e^{-hpt} \cdot (c_1 e^{p_1 t} + c_2 e^{-p_1 t}) \dots (3.24)$$

Where c_1 and c_2 are arbitrary constants, and will be determined from the initial conditions. Fig.-3.5 is an illustration of the solutions of Eq. (3.24) on the assumption of certain initial conditions, circular frequency p and h . This shows that no oscillation will occur the displacement and x will decrease exponentially without change in sign.

CASE 2: $h = 1$. In this case, Eq. (3.21) becomes.

$$\lambda = -p$$

and the solution to Eq. (3.16) is

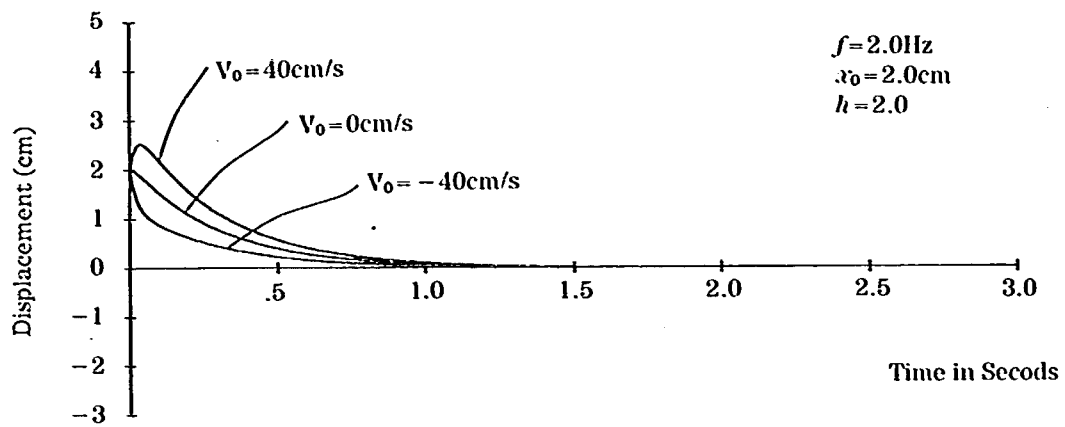


Fig.-3.6 Free vibration of one degree of freedom system with damping $h > 1$

$$x = e^{-pt}(c_1 + c_2 t) \quad \dots (3.25)$$

c_1 and c_2 are also the arbitrary constants. This case is similar to the above overdamped case except that it is possible for the sign of x to change once.

CASE 3: $h < 1$. In this case Eq. (3.21) will be complex conjugates.

$$\begin{aligned} \lambda_1 &= -hp + ip_1 \\ \lambda_2 &= -hp - ip_1 \end{aligned} \quad \dots (3.26)$$

Letting

$$p_1 = p\sqrt{1-h^2}, \quad \dots (3.27)$$

then we obtain

$$\left. \begin{aligned} x_1 &= e^{\lambda_1 t} \\ x_2 &= e^{\lambda_2 t} \end{aligned} \right\} \quad \dots (3.28)$$

By using the Euler's relation, this will be converted to the trigonometric form

$$\begin{aligned} \frac{c_1}{2}(x_1 + x_2) &= \frac{c_1}{2}(e^{\lambda_1 t} + e^{\lambda_2 t}) \\ &= c_1 e^{-hpt} \cos p_1 t \end{aligned}$$

$$\begin{aligned} \frac{c_2}{2i}(x_1 - x_2) &= \frac{c_2}{2i}(e^{\lambda_1 t} - e^{\lambda_2 t}) \\ &= c_2 e^{-hpt} \sin p_1 t \end{aligned}$$

Here, the Euler's relation is

$$\begin{aligned}\cos x &= \frac{e^{iu} + e^{-iu}}{2} \\ \sin x &= \frac{e^{iu} - e^{-iu}}{2i}\end{aligned}\quad \dots (3.29)$$

Thus, the general solution becomes

$$x = e^{-hpt}(c_1 \cos p_1 t + c_2 \sin p_1 t) \quad \dots (3.30)$$

We may rewrite the general solution in the form,

$$x = A \cdot e^{-hpt} \cos(p_1 t - \psi) \quad \dots (3.31)$$

where

$$\begin{aligned}A &= \sqrt{c_1^2 + c_2^2} \\ \tan \psi &= \frac{c_2}{c_1}\end{aligned}\quad \dots (3.32)$$

A and ψ or c_1 and c_2 are two new arbitrary constants which will be determined from the initial conditions. Fig.-3.6 shows the motion of this case.

The solution given by Eq. (3.30) or (3.31) describes the damped free vibrations of the system shown in Fig.-3.4 when no external force is present.

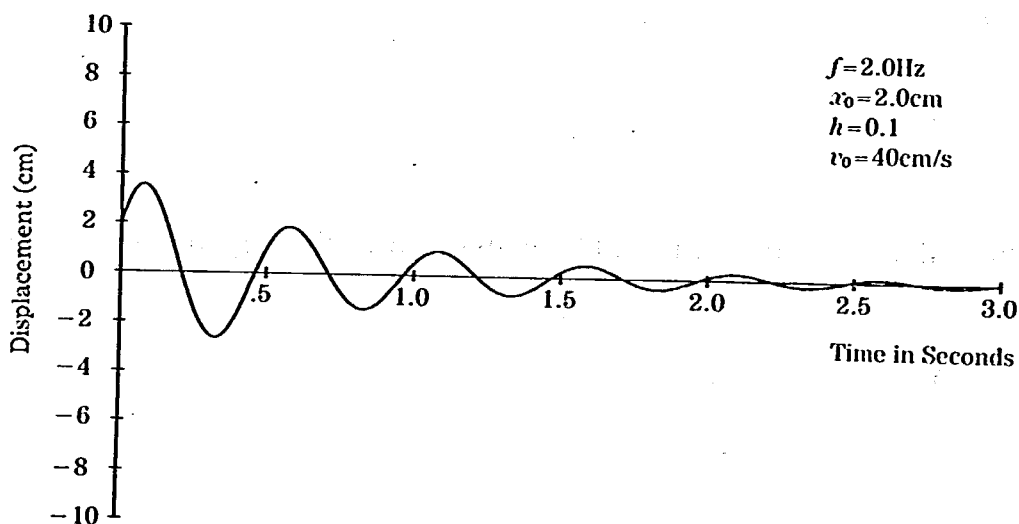


Fig.-3.6 Free vibration of one degree of freedom system with damping $h < 1$

Referring to Fig.-3.6, the amplitudes of two successive peaks will occur at times t_1 and t_2 . The interval $T = t_1 - t_2$ is given by

$$T = \frac{2\pi}{p_1} = \frac{2\pi}{p} \cdot \frac{1}{\sqrt{1-h^2}} \quad \dots (3.33)$$

T is called the damped natural period. In right-hand side of Eq. (3.33), $\frac{2\pi}{p}$ is the undamped natural period T_0 .

If h is small, the difference between T and T_0 will be very minor.

The amplitude ratio of two successive peaks at times t_1 and t_2 is given by Eq. (3.34) or Eq. (3.35) is often used for determining the damping constants from experiment. If the peak amplitude of vibration is plotted on a logarithmic scale against the cycle number on an arithmetic scale, the points will fall on a straight line in the case of viscous damping.

$$\frac{x_{t=t_1+T}}{x_{t=t_1}} = e^{\frac{2\pi h}{\sqrt{1-h^2}}} \quad \dots (3.34)$$

Namely, this ratio is not dependent on t_1 . The natural logarithm of this ratio is called the logarithmic decrement.

$$\Delta = \frac{2\pi h}{\sqrt{1-h^2}} \approx 2\pi h \quad \dots (3.35)$$

Although the expression $-c\dot{x}$ is used to represent the damping force in Eq. (3.16), the quantity h was introduced in Eq. (3.17) and then the damping ratio h has been instead of c , used to discuss the equations of motion. This is because h is more easily obtained through experiment than c and as described below, h , a nondimensional number, represents directly a size of damping. c is a constant indicating the characteristics of dashpot, with a unit of [force/velocity]. In Eq. (3.17) no vibration occurs when $h = 1$, and the value c corresponding to this critical condition is called the critical damping coefficient c_c .

$$c_c = 2\sqrt{km} \quad \dots (3.36)$$

$$\text{and } h = \frac{c}{c_c} \quad \dots (3.37)$$

The damping ratio h is a ratio of damping coefficient to the critical damping coefficient. For free damped vibrations of the system in Fig.-3.4, the ratio of two peak amplitudes is given by damping ratio h according to Eq. (3.34). If we use damping coefficient c instead of damping ratio h in Eq. (3.34), however, not only c but also k and m are included in Eq. (3.34). This indicates that damping of a vibration system depends upon the rigidity and mass of the system as well as c . This is because it is assumed that the damping force is proportional to the velocity, and the velocity in the free damped vibration is determined from the natural period of the system.

Next, instead of the vibration system of Fig.-3.4, we will consider the system shown in Fig.-3.7 which is consisted of the spring and the dashpot under the application of a harmonic force P .

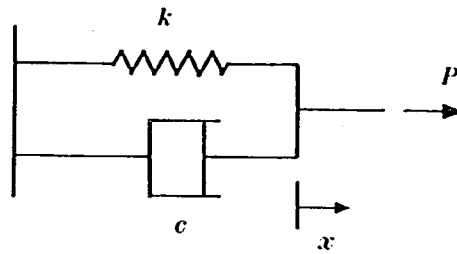


Fig.-3.7 Modified Voigt model

The differential equation of motion will be, based on the same idea with that for Eq. (3.16),

$$P - c\dot{x} - kx = 0 \quad \dots (3.38)$$

When P is given by the form of $P_o \cdot \cos \omega t$, or by the form of complex number $P_o e^{i\omega t}$, for the convenience in calculation, the displacement x will be assumed as $x = x_o e^{i(\omega t - \psi)}$.

Then we get P_o by substituting x into Eq. (3.38).

$$P_o = x_o (k + i\omega c) e^{-i\psi} \quad \dots (3.39)$$

If we partition Eq. (3.39) into the real part and the imaginary part, we obtain ψ ,

$$\left. \begin{aligned} x_o &= \frac{P_o}{c\omega \sin\psi + k \cos\psi} \\ \tan\psi &= \frac{c\omega}{k} \end{aligned} \right\} \dots\dots (3.40)$$

The phase angle ψ is given by the second equation of Eq. (3.40), that is the product of deformation characteristics of the system c/k , and exciting frequency ω . The relation between $P_o \cos \omega t$ and $x_o \cos(\omega t - \psi)$ can be expressed by a elliptical loop, as shown in Fig.-3.8. This figure is plotted for two different ω ($\omega = \pi$ and 2π rad./sec), and we can see that the vibration systems with the same k and c produce different loops to the different exciting frequency. The relation shown in Fig.-3.8 is used for obtaining the damping constant from cyclic material testing.

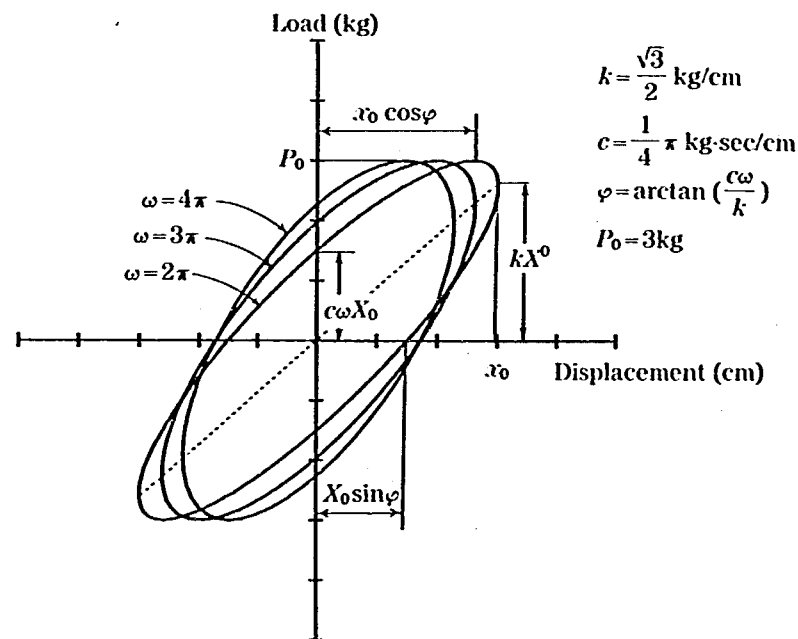


Fig.-3.8 Hysteretic damping loop

3.2.2 Hysteretic Damping

In the previous section, the viscous damping expressed by the dashpot has been explained in regard to energy dispersion of the vibration system. In this section, a different kind of damping is considered. This damping originates from the inelasticity of the material itself. This damping is called the hysteretic damping.

Fig.-3.9a is a schematic illustration of the hysteretic damping material. When a force

P is applied on the movable end A and the displacement x occurs, then the relation between P and x should be linear, if the material is a linear elastic body. However, let us consider the case that the time delay occurs as shown in Fig.-3.9b. In this case we assume that x and P in Fig.-3.9b are expressed by the formulae below.

Then

$$\left. \begin{aligned} x &= x_0 e^{i\omega t} \\ P &= P_0 e^{i(\omega t + \psi)} \end{aligned} \right\} \dots (3.41)$$

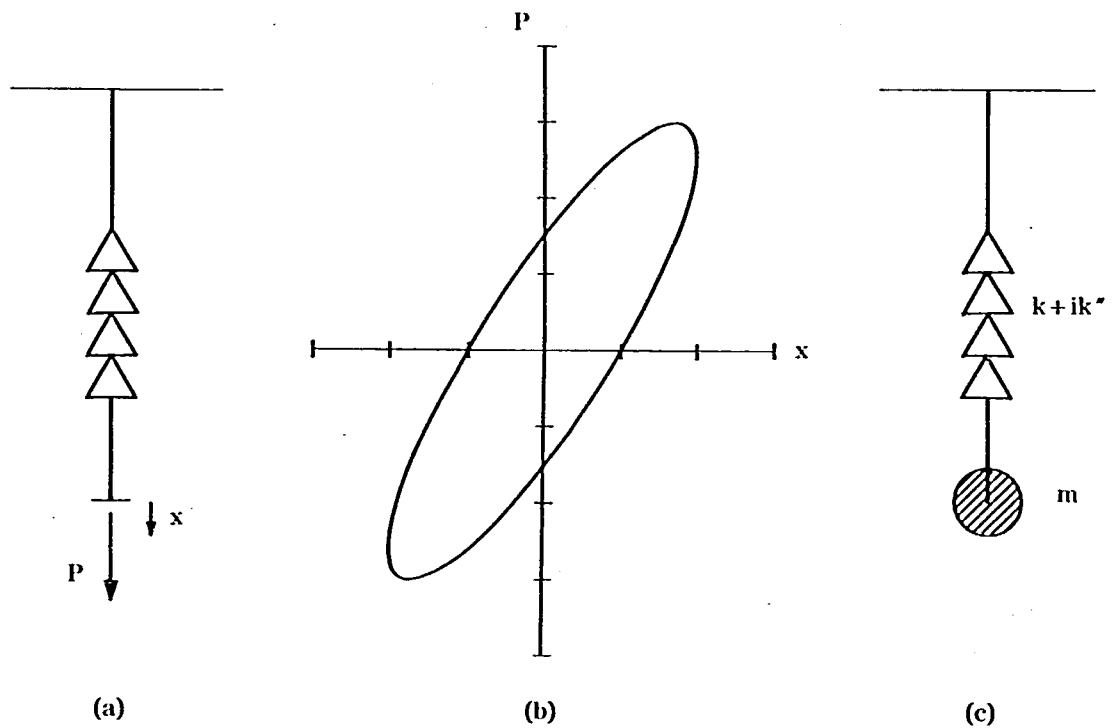


Fig.3.9 Hysteretic damping materials

Here, in Eq. (3.41) x and P are expressed by complex numbers, and it is assumed that each real part and imaginary part of both x and P correspond each other in such a way that when the displacement is $x_0 \sin \omega t$, the force is $P_0 \sin(\omega t + \psi)$, while the displacement is $x_0 \cos \omega t$, the force is $P_0 \cos(\omega t + \psi)$. The use of complex numbers makes the calculation easier.

When $e^{i\omega t}$ is eliminated from Eq. (3.41), we obtain

$$P = \frac{P_o}{x_o} e^{i\psi} \cdot x = \frac{P_o}{x_o} (\cos\psi + i\sin\psi)x \quad \dots (3.42)$$

It is convenient to introduce the notation

$$\left. \begin{aligned} k &= \frac{P_o}{x_o} \cos\psi \\ k' &= \frac{P_o}{x_o} \sin\psi \end{aligned} \right\} \quad \dots (3.43)$$

then Eq. (3.42) can be written in the form

$$P = (k + ik') x \quad \dots (3.44)$$

Now, the spring system of Fig.-3.9 is mathematically expressed by a spring with complex number, which is called the complex spring.

As shown in Fig.3.9c, the equation of motion for the free vibration of mass-complex spring system may be written in the form,

$$-m\ddot{x} - (k + ik')x = 0 \quad \dots (3.45)$$

This is the equation of motion for hysteretic damping, corresponding to the equation of motion for viscous damping (Eq. (3.16)). Substitution of $x = e^{\lambda t}$ into Eq. (3.45) gives

$$m\lambda^2 + (k + ik') = 0 \quad \dots (3.46)$$

Since λ is a complex number, let us place $\lambda = X + iY$ to solve the equation.

Then we get

$$\left. \begin{aligned} mX^2 - mY^2 + k &= 0 \\ i(2mXY + k') &= 0 \end{aligned} \right\} \quad \dots (3.47)$$

These two equations give

$$\begin{aligned}
p^2 &= \frac{k}{m} \\
h &= \frac{k'}{2k} \\
h' &= \left(\frac{-1 + \sqrt{1 + (2h)^2}}{2} \right)^{\frac{1}{2}} \quad \dots (3.48)
\end{aligned}$$

Then the displacement x will be of the form

$$x = e^{-h'pt} \left[A_1 \cos \frac{h}{h'} pt + A_2 \sin \frac{h}{h'} pt \right] \quad \dots (3.49)$$

If h is small, $h' = h$ will be assumed and Eq. (3.49) will be rewritten in the form

$$x = e^{-hpt} [A_1 \cos pt + A_2 \sin pt] \quad \dots (3.50)$$

Eqs. (3.50) and (3.30) are the same form. Therefore, the decay in amplitude with time in the hysteretic damping material is the same form with Fig.3.6. However, the property of h differs greatly. As given by Eq. (3.48), h is determined from only k and k' , not from m , unlike in the case of the viscous damping.

Using Eqs. (3.43) and (3.48), h will be obtained in the form

$$h = \frac{1}{2} \tan \psi \quad \dots (3.51)$$

Since Eq. (3.50) is introduced from the assumption of $h' = h$ for small h , Eq. (3.49) has to be applied for large h . One of the characteristics of Eq. (3.49) is that the vibration occurs even in $h > 1$, in comparison with the Eq. (3.30) for viscous damping. In addition, the period T in Eq. (3.49) is,

$$T = \frac{2\pi}{\left(\frac{h}{h'} \cdot p \right)} \quad \dots (3.52)$$

3.3 Forced Vibrations

In the vibration systems described previously, the characteristics of free vibrations

have been discussed. Next we will consider the response of the single-degree-of-freedom model to a harmonic force Q with amplitude Q_o as shown in Fig.-3.10.

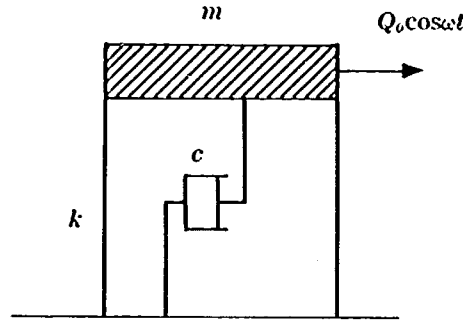


Fig.-3.10 One degree of freedom system with viscous damping subjected to cyclic external force, $Q_o \cos \omega t$

The differential equation of motion may be in the form,

$$-m\ddot{x} - c\dot{x} - kx + Q_o \cos \omega t = 0 \quad \dots (3.53)$$

In this equation, the external force $Q_o \cos \omega t$ is equal to the real part of $Q_o e^{i\omega t}$. Assuming $x = X_o e^{i\omega t}$ to be the particular solution, and substituting it into Eq. (3.53), we obtain

$$X_o = \frac{Q_o}{(-m\omega^2 + k + ic\omega)} \quad \dots (3.54)$$

Rearranging this equation by introducing the undamped natural period p and damping ratio h , the amplitude of the displacement X_o is given by

$$X_o = \frac{Q_o}{m} \frac{1}{(p^2 - \omega^2 + 2ihp\omega)} \quad \dots (3.55)$$

$$\therefore x = X_o e^{i\omega t} = \frac{Q_o}{m} \frac{e^{i\omega t}}{(p^2 - \omega^2 + 2ihp\omega)}$$

$$= \frac{Q_o}{mp^2} \cdot \frac{1}{[1 - (\frac{\omega^2}{p})]^2 + 4h^2 (\frac{\omega}{p})^2} \cdot e^{i(\omega t - \psi)} \quad \dots (3.56)$$

Where

$$\psi = \tan^{-1} \cdot \frac{2h (\frac{\omega}{p})}{1 - (\frac{\omega}{p})^2} \quad \dots (3.57)$$

That is, the displacement of the vibration system shown in Fig.-3.10 is given by Eq. (3.56) and the external force will be ψ degrees ahead of the displacement. Furthermore, although in the case of free vibrations no oscillations occur when $h > 1$, in the case of forced vibrations oscillation takes place even when $h > 1$. From the general form of x given by Eq. (3.56), we have the amplitude of the displacement X_d of the form

$$X_d = \frac{Q_o}{mp^2} \cdot \frac{1}{\sqrt{[1 - (\frac{\omega}{p})^2]^2 + 4h^2 (\frac{\omega}{p})^2}} \quad \dots (3.58)$$

If a static force of amplitude Q_o is applied to the system, the static displacement X_s of the system will be

$$X_s = \frac{Q_o}{k} = \frac{Q_o}{mp^2} \quad \dots (3.59)$$

The ratio M of the dynamic displacement to the static one is

$$M = \frac{X_d}{X_s} = \frac{1}{\sqrt{[1 - (\frac{\omega}{p})^2]^2 + 4h^2 (\frac{\omega}{p})^2}} \quad (3.60)$$

This is called the dynamic magnification factor. Fig.3.11 illustrates the relation between M and $\frac{\omega}{p}$ using h as a parameter.

The quantity $(\frac{\omega}{p})$ which gives an extreme value for M may be obtained from the condition of $\frac{dM}{d(\frac{\omega}{p})} = 0$ and hence

$$\frac{w}{p} = \sqrt{1 - 2h^2} \quad \dots (3.61)$$

When $\frac{\omega}{p}$ takes the value given by Eq. (3.61), M becomes the maximum value M_{max} giving

$$M_{max} = \frac{1}{2h\sqrt{1 - h^2}} \quad \dots (3.62)$$

$$\text{If } h \text{ is small, } M_{max} \approx \frac{1}{2h} \quad \dots (3.63)$$

The discussion, however, is based on the assumption of $h < \frac{1}{\sqrt{2}}$. Thus, in the case when $h > \frac{1}{\sqrt{2}}$, the term inside the root of the right hand side of Eq. (3.61) becomes negative, and so no extreme values of M exist.

Fig.-(3.12) shows phase angle between force and of displacement given by Eq. (3.57). Using the relation $Q_0 e^{i\omega t}$ = external force, Eq. (3.56) may be written in the form

$$(\text{displacement}) = F(i\omega) \times \text{external force}$$

where

$$F(i\omega) = \frac{1}{m} \cdot \frac{1}{(p^2 - \omega^2 + 2ihp\omega)} \quad \dots (3.64)$$

Here, $F(i\omega)$ is the called frequency response function. The absolute value of $F(i\omega)$ in Eq. (3.64) indicates the response of the displacement to the harmonic external force of the unit amplitude, and the argument indicates a phase angle between the displacement and external force. The frequency response function can be obtained for acceleration, velocity, stress or strain as well as displacement. This function provides us with a very effective method to express the characteristics of a vibration system in single equation, as to be mentioned later.

The discussion above is for forced vibrations of single-degree-of-freedom system with viscous damping. Next, we will consider forced vibrations of single-degree-of-

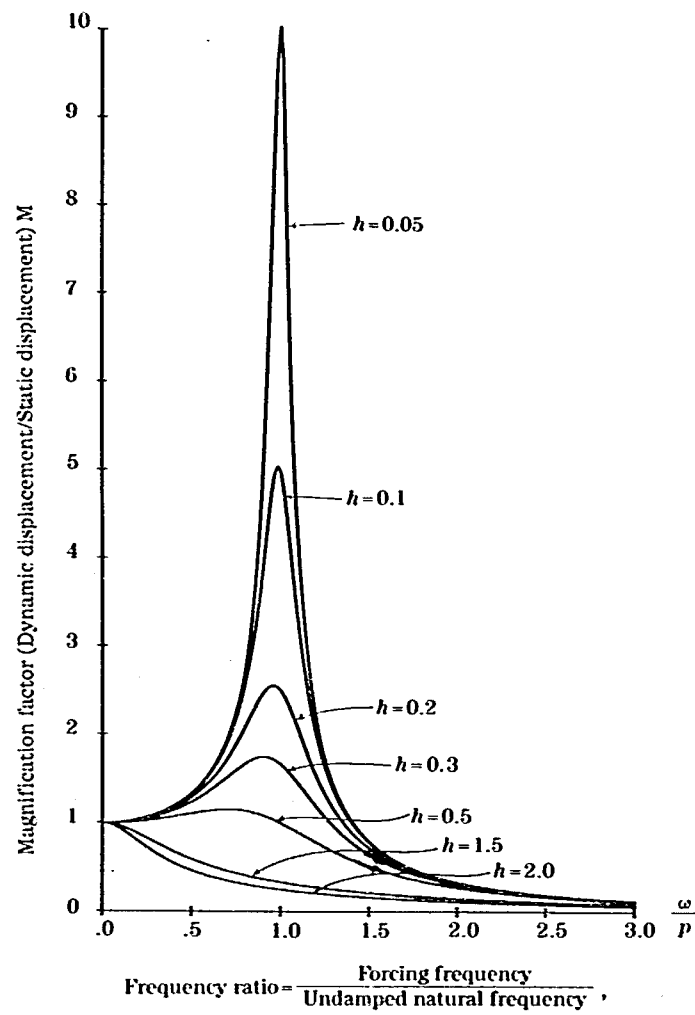


Fig.-3.11 Magnification factor vs. frequency ratio for displacement (Viscous damping)

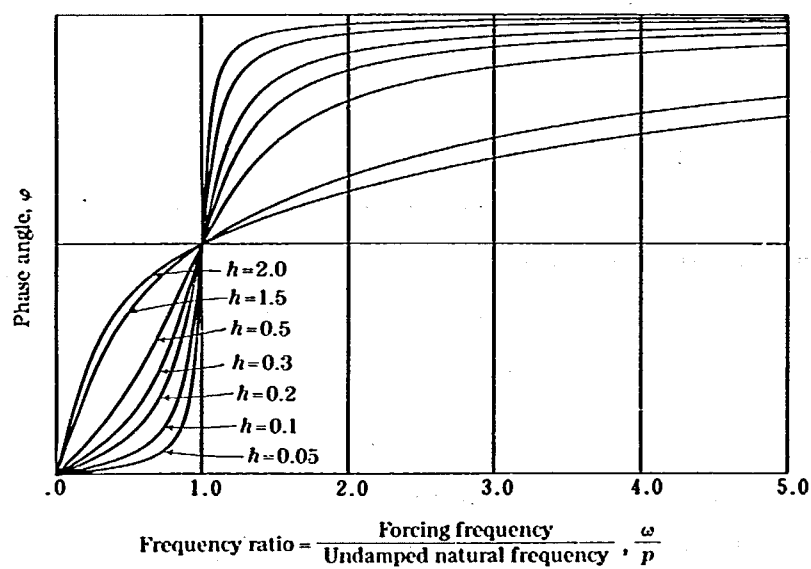


Fig.-3.12 Phase angle vs. frequency ratio (Viscous damping)

freedom system with hysteretic damping (complex spring). When an external force, $Q_o \cos \omega t$ is acting on the mass shown in Fig.-3.9c, the equation of motion corresponding to Eq. (3.53) for viscous damping is;

$$-m\ddot{x} - (k + ik')x + Q_o e^{i\omega t} = 0 \quad \dots (3.66)$$

Again using the solution assumed for the viscous damping,

$$x = X_o e^{i\omega t}$$

We may obtain the particular solution to Eq. (3.66) in the form.

$$X_o = \frac{Q_o}{mp^2} \cdot \frac{1}{1 - (\frac{\omega}{p})^2 + 2ih} \quad \dots (3.67)$$

$$x = \frac{Q_o}{mp^2} \cdot \frac{1}{\sqrt{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2}} e^{i(\omega t - \psi)} \quad \dots (3.68)$$

Here,

$$\psi = \tan^{-1} \frac{2h}{1 - (\frac{\omega}{p})^2} \quad \dots (3.69)$$

From Eq. (3.68), the dynamic magnification factor for the case of hysteretic damping is;

$$M = \frac{1}{\sqrt{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2}} \quad \dots (3.70)$$

The relations between M and $\frac{\omega}{p}$, and ψ and $\frac{\omega}{p}$ are shown in Figs.-3.13 and 3.14, respectively. An extreme value of M will be always exist regardless of h , unlike in the case of viscous damping. From the condition of $\frac{dM}{d(\frac{\omega}{p})} = 0$, M becomes the

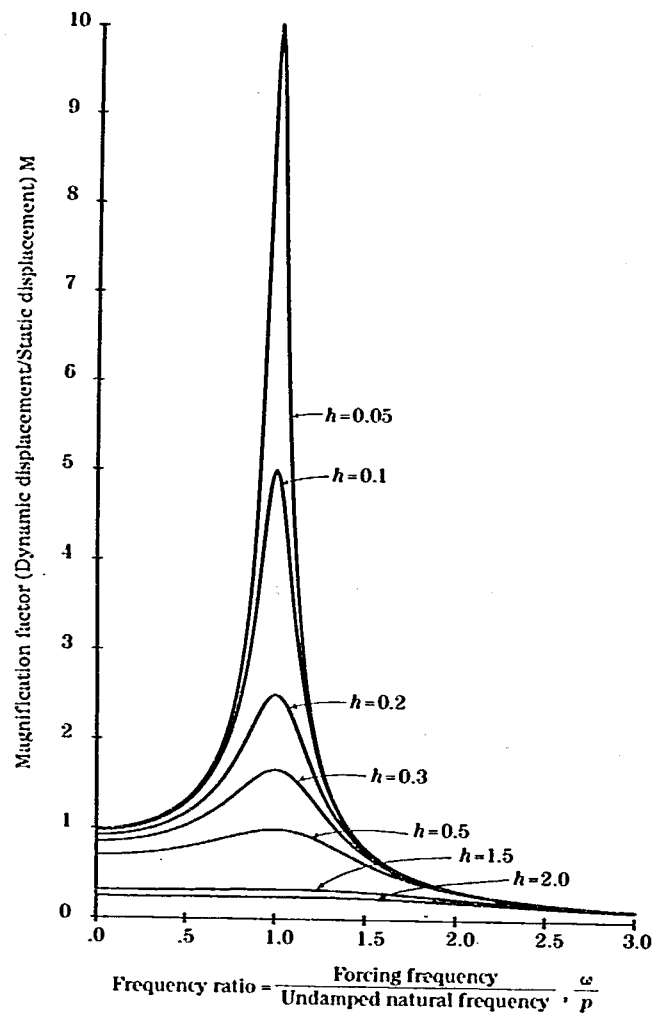


Fig.-3.13 Magnification factor vs. frequency ratio for displacement (Hysteretic damping)

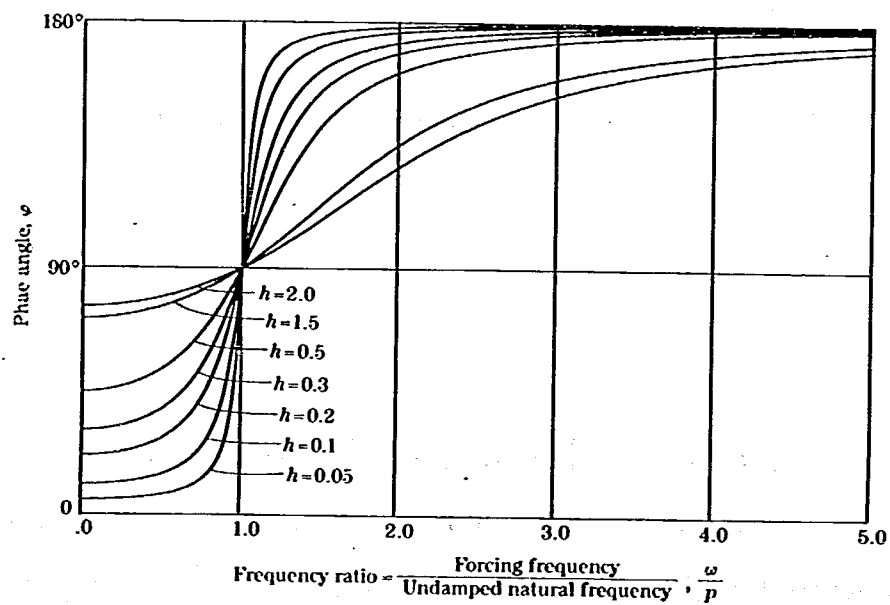


Fig.-3.14 Phase angle vs. frequency ratio (Hysteretic damping)

maximum when

$$\frac{\omega}{p} = 1 \quad \dots (3.71)$$

The maximum value of M corresponding to the M_{max} is;

$$M_{max} = \frac{1}{2h} \quad \dots (3.72)$$

We should note that when $\frac{\omega}{p} > 2$, M is always under 1.0 irrespective of h value.

3.4 Forced Vibrations due to Harmonic Motion of the Support.

In the previous section, harmonic external force acted directly on the mass. However, especially in the case of earthquake motions, the vibration of a system is not due to forces on the mass but from the motion of the support.

Fig.-3.15 illustrates a model in which the structure in Fig.-3.4a is placed on the moving support. If this support is regarded as the earth's surface, then the motion of the support is to correspond to an earthquake motion. As shown in Fig.-3.15, denoting z to be the displacement of the support from the stationary position and y to be the relative displacement of the mass relative to the support, we then have

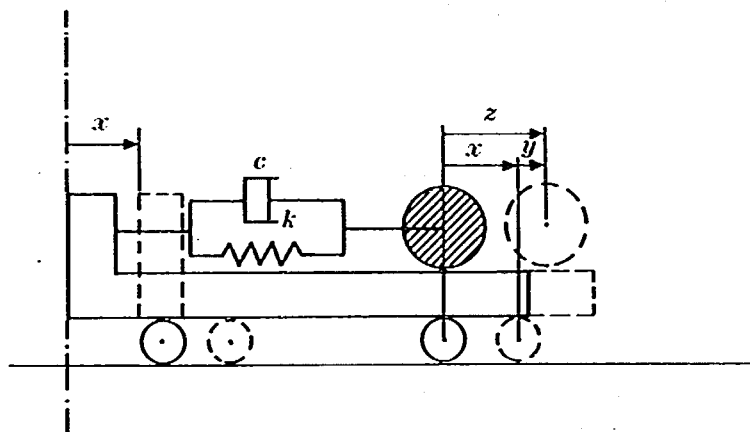


Fig.-3.15 One degree of freedom system on the truck

$$z = x + y \quad \dots (3.73)$$

The equation of motion of the mass is,

$$-m\ddot{z} - c\dot{y} - ky = 0 \quad \dots (3.74)$$

For simplification, the displacement of the support x , is assumed to be sinusoidal, and then

$$x = x_o \cos \omega t = R_c (x_o e^{i\omega t}) \quad \dots (3.75)$$

Eqs. (3.73) and (3.75) are substituted into Eq. (3.74), giving

$$m\ddot{y} + c\dot{y} + ky = m\omega^2 x_o e^{i\omega t} \quad \dots (3.76)$$

Again, the expression of Eq. (3.18) is introduced for rearrangement, and then,

$$\ddot{y} + 2hp\dot{y} + p^2y = \omega^2 x_o e^{i\omega t} \quad \dots (3.77)$$

Letting $y = y_o e^{i\omega t}$, y_o is given by,

$$y_o = \frac{\omega^2 x_o}{p^2 - \omega^2 + 2ihp\omega} \quad \dots (3.78)$$

$$\begin{aligned} \therefore y &= \frac{\omega^2 x_o}{p^2} \cdot \frac{e^{i\omega t}}{1 - (\frac{\omega}{p})^2 + 2ih(\frac{\omega}{p})} \\ &= \frac{\omega^2 x_o}{p^2} \cdot \frac{1}{\sqrt{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2(\frac{\omega}{p})^2}} e^{i(\omega t - \psi)} \quad \dots (3.79) \end{aligned}$$

Where,

$$\psi = \tan^{-1} \frac{2h(\frac{\omega}{p})}{1 - (\frac{\omega}{p})^2} \quad \dots (3.80)$$

A comparison of Eq. (3.79) with Eq. (3.56) indicates that the relative displacement y in Fig.-3.15 is equivalent to the displacement in Fig.-3.10 under the condition of

$$w^2 x_o = \frac{Q_o}{m}.$$

Next, the absolute acceleration of the mass is given by;

$$\ddot{z} = \ddot{x} + \ddot{y} = - \frac{1 + 2ih \left(\frac{\omega}{p} \right)}{1 - \left(\frac{\omega}{p} \right)^2 + 2ih \left(\frac{\omega}{p} \right)} x_o \omega^2 e^{i\omega t} \quad \dots (3.81)$$

Obtaining the ratio of the acceleration of the mass to that of the support, Eq. (3.81) reduces to

$$F_a(i\omega) = \frac{\ddot{z}}{\ddot{x}} = \frac{1 + 2ih \left(\frac{\omega}{p} \right)}{1 - \left(\frac{\omega}{p} \right)^2 + 2ih \left(\frac{\omega}{p} \right)} \quad \dots (3.82)$$

Eq. (3.82) is the frequency response function of an absolute acceleration of the structure against the support motion. Modification of Eq. (3.82) to the general form of the complex number yields the following:

$$F_a(i\omega) = \sqrt{\frac{1 + 4h^2 \left(\frac{\omega}{p} \right)^2}{\left\{ 1 - \left(\frac{\omega}{p} \right)^2 \right\}^2 + 4h^2 \left(\frac{\omega}{p} \right)^2}} e^{-i\psi} \quad \dots (3.83)$$

Where,

$$\psi = \tan^{-1} \frac{2h \left(\frac{\omega}{p} \right)^3}{1 - \left(1 - 4h^2 \right) \left(\frac{\omega}{p} \right)^2} \quad \dots (3.84)$$

The modulus, or the absolute value of $F_a(i\omega)$ gives the magnification ratio M for acceleration, and ψ in Eq. (3.84) gives the phase angle. The magnification ratio and phase angle have been plotted in Figs.3.16 and 3.17, respectively for various values of h . As seen in Fig.-3.16 the acceleration magnification ratio of the structure subjected to the support motion is similar to the dynamic magnification factor in Fig.-3.11. However, when $\frac{w}{p} = \sqrt{2}$, $M=1$ regardless of h value, while when $\frac{w}{p} > \sqrt{2}$, M becomes larger with larger h . Furthermore, when ω is small compared to p , M can be nearly equal to 1 regardless of h value. This corresponds to the basic assumption

of the quasistatic method or the seismic coefficient method. That is, if the frequency of an earthquake motion is small compared to the natural frequency of the structure, the motion of the structure almost coincides with the earthquake motion.

The value $\frac{\omega}{p}$, which gives the maximum value of the acceleration magnification ratio can be obtained from the condition of $\frac{dM}{d(\frac{\omega}{p})} = 0$;

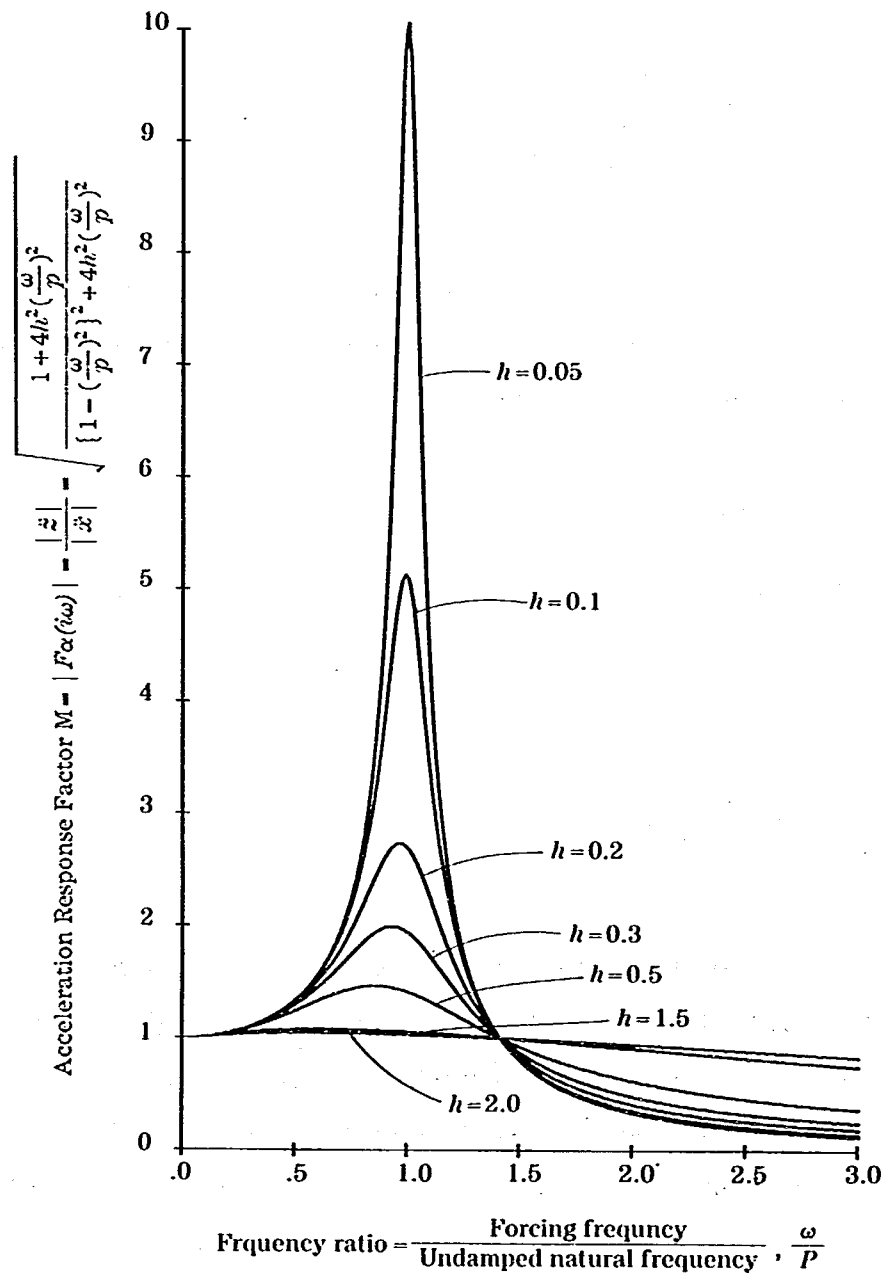


Fig.-3.16 Magnification factor vs. frequency ratio for acceleration (Viscous damping)

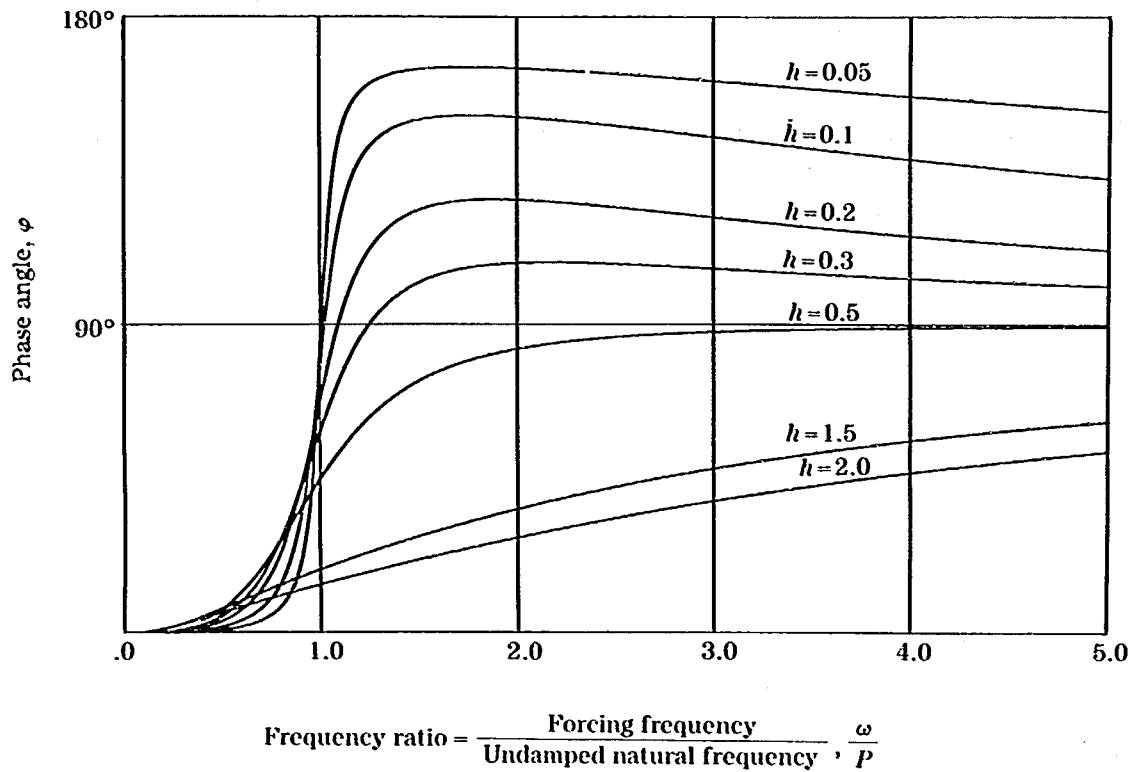


Fig.-3.17 Phase angle vs. frequency ratio (Viscous damping)

$$\frac{\omega}{p} = \frac{1}{2h} \sqrt{-1 + \sqrt{1 + 8h^2}} \approx 1 \text{ (when } h \text{ is small)} \quad \dots (3.85)$$

The value of M at this resonant frequency is given by,

$$M_{\max} \approx \frac{1}{2h} \sqrt{1 + 4h^2} \approx \frac{1}{2h} \text{ (when } h \text{ is small)} \quad \dots (3.86)$$

The discussion above is of the response of the single-degree-of-freedom system with viscous damping to the sinusoidal displacement of the support. Next, we will consider the response of the vibration system with complex spring in Fig.-3.15.

The equation of motion in this case is;

$$-m\ddot{z} - (k + ik')y = 0 \quad \dots (3.87)$$

Substitution for the expression for z and x by using Eqs. (3.73) and (3.75), respectively, and rearrangement gives,

$$m\ddot{y} + (k + ik') y = m\omega^2 x_o e^{i\omega t} \quad \dots (3.88)$$

Rewriting Eq. (3.88) by introducing the relation of Eq. (3.48), we obtain,

$$\ddot{y} + p^2 (1 + 2ih) y = \omega^2 x_o e^{i\omega t} \quad \dots (3.89)$$

When $y = y_o e^{i\omega t}$, y_o is;

$$y_o = \frac{\omega^2 x_o}{p^2 - \omega^2 + 2ihp^2} \quad \dots (3.90)$$

where

$$\begin{aligned} y &= \frac{\omega^2 x_o}{p^2} \frac{e^{i\omega t}}{1 - (\frac{\omega}{p})^2 + 2ih} \\ &= \frac{\omega^2 x_o}{p^2} \frac{1 + 4h^2}{\sqrt{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2}} e^{i(\omega t - \psi)} \end{aligned} \quad \dots (3.91)$$

Where,

$$\psi = \tan^{-1} \frac{2h}{1 - (\frac{\omega}{p})^2} \quad \dots (3.92)$$

The frequency response function $F_a(i\omega)$ of the absolute response acceleration z of the structure to the ground motion acceleration x is;

$$F_a(i\omega) = \frac{\ddot{z}}{\ddot{x}} = \frac{1 + 2ih}{1 - (\frac{\omega}{p})^2 + 2ih} = \sqrt{\frac{1 + 4h^2}{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2}} e^{i\psi} \quad \dots (3.93)$$

Where,

$$\psi = \tan^{-1} \frac{2h(\frac{\omega}{p})^2}{1 - (\frac{\omega}{p})^2 + 4h^2} \quad \dots (3.94)$$

The absolute value of Eq. (3.93) means a magnification of the acceleration of the structure to the input acceleration. Fig.-3.18 shows this relation, while Fig.-3.19 indicates the phase angle (Eq. (3.94)). As seen in Fig.-3.18., in case of hysteretic damping, unlike viscous damping, the amplitude rate is always maximum at $\frac{\omega}{p} = 1$, and the maximum value at this frequency is;

$$M_{max} \doteq \frac{1 + 2h^2}{2h} \doteq \frac{1}{2h} \quad \dots (3.95)$$

Also, when $\frac{w}{p} = \sqrt{2}$, $M=1$ regardless of h value, as the case of viscous damping. In addition, when $\frac{w}{p} > \sqrt{2}$, the value M increases with increasing the value of h .

Figs.-3.16 and 3.18 illustrate the acceleration response magnification of the vibration system with viscous damping and hysteretic damping, respectively. A comparison of these two figures indicates that the difference between them is very small in the case of single-degree-of-freedom system.

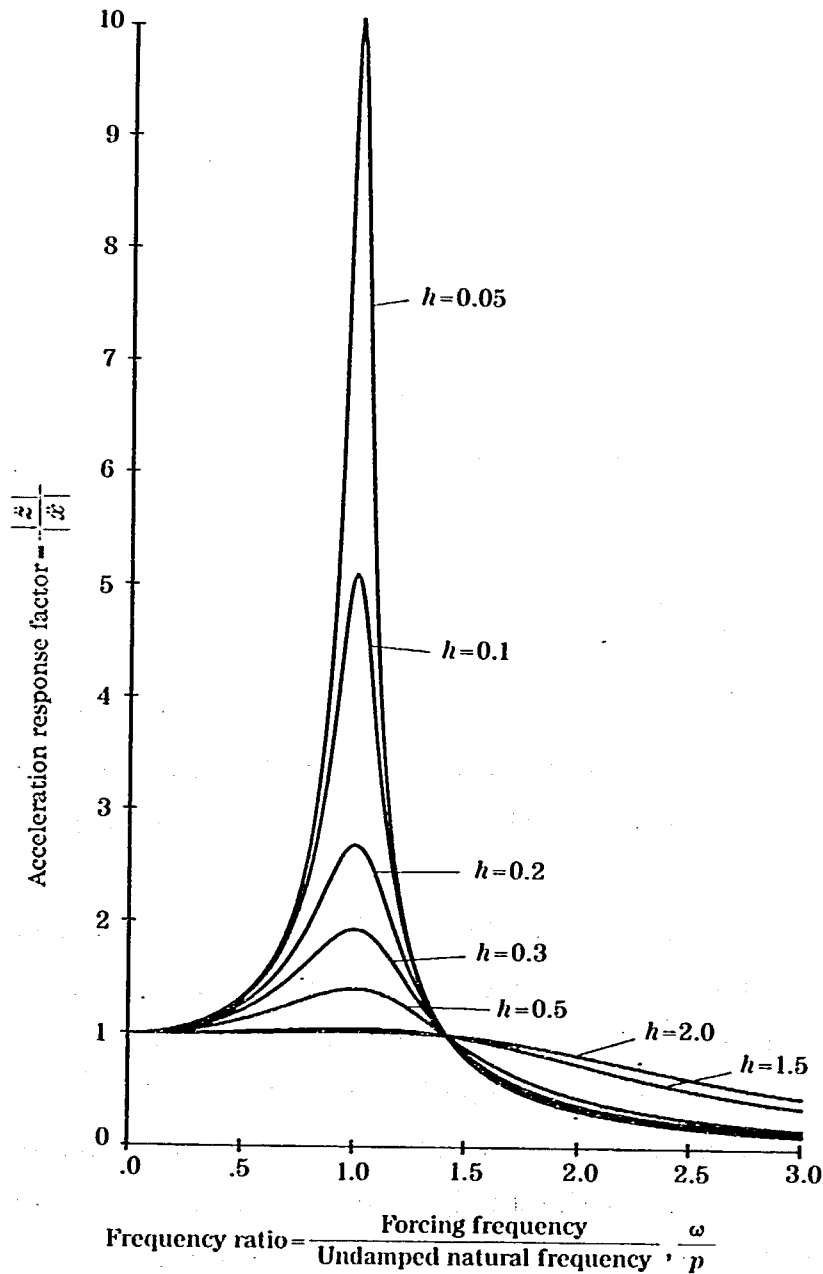


**Fig.-3.18 Magnificaiton factor vs. frequency ratio
(Hysteretic damping)**

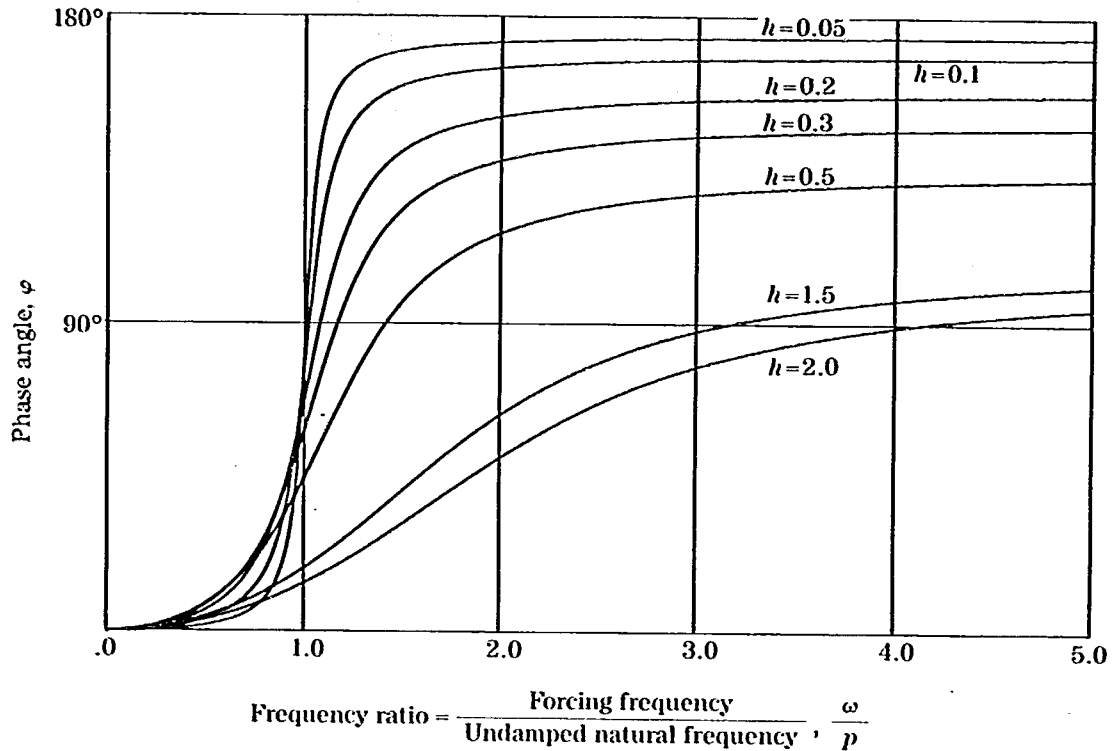


Fig.-3.19 Phase angle vs. frequency ratio (Hysteretic damping)

3.5 Vibrations due to Random Motions.

The vibrations described previously were of free vibrations or of forced vibrations due to harmonic excitement. The earthquake ground motions, however, have the amplitude of motion which magnitude varies irregularly, with time. Hence, single-degree-of-freedom system subjected to irregular external forces or displacements will be examined.

The solutions of single-degree-of-freedom system subjected to sinusoidal external forces or displacements discussed in the previous sections were the solutions corresponding to particular solutions for differential equation, which are the solutions for a steady state when a sufficient time has passed after the external force or displacement was given, and effects of the initial conditions exist no more. Vibrations from the initiation of vibration to the steady state are called the transient vibration, which corresponds to the solution containing a general solution for a differential equation. Since vibrations of a structure caused by earthquake motions can never reach a steady state due to irregularity of the earthquake motions, these vibrations are of continual transient vibrations.

Therefore, first, we will consider the characteristics of transient vibrations.

Fig.-3.10 illustrates the case where a periodic external force $Q_o \cos \omega t$ acts on the mass of single-degree-of-freedom system. The solution for the steady state was given by Eq. (3.56). That is, Eq. (3.56) provides the solution for the state where the mass vibrates at the same frequency with the frequency of the external force ω (however, phase delay occurs) after the force $Q_o \cos \omega t$ has acted for a sufficient period of time.

The solution for the transient motion will be given by adding the solution for free vibration, i.e, Eq. (3.30).

$$x = e^{-hpt} (C_1 \cos p_1 t + C_2 \sin p_1 t) + \frac{Q_o}{mp^2} \frac{\cos(\omega t - \psi)}{\sqrt{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2(\frac{\omega}{p})^2}} \quad \dots\dots (3.96)$$

The arbitrary constants c_1 and c_2 are determined from the initial conditions, and here we assume the displacement and the velocity to be zero at $t=0$, and then, we get

$$\left. \begin{aligned} C_1 &= -\frac{Q_o}{mp^2} \frac{\cos \psi}{\sqrt{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2(\frac{\omega}{p})^2}} \\ C_2 &= -\frac{Q_o}{mp^2} \frac{1}{\sqrt{\{1 - (\frac{\omega}{p})^2\}^2 + 4h^2(\frac{\omega}{p})^2}} \\ &\quad \times \left(\frac{hP \cos \psi + \omega \sin \psi}{p_1} \right) \end{aligned} \right\} \quad \dots\dots (3.97)$$

Then, when $p=2\pi$, $\frac{Q_o}{mp^2}=1.0$, and $\frac{\omega}{p}=0.5, 1.0$, and 2.0 , the displacement of the mass will be shown in Fig.-3.20 with various damping constants h ranging from 0.05 to 0.1 . Fig.-3.20 suggests that the duration of the transient vibration becomes shorter with larger h .

For the preparation to obtain the response of single-degree-of-freedom system subjected to irregular external forces, the response to an impact is first examined. As shown in Fig.-3.21, let us consider that a force F is exerted during a very short period of time $\Delta\tau$. In this case, $F \cdot \Delta\tau$ is an impulse, which should be equal to the change in momentum. Thus, when the initial velocity at the mass of the single-degree-of-

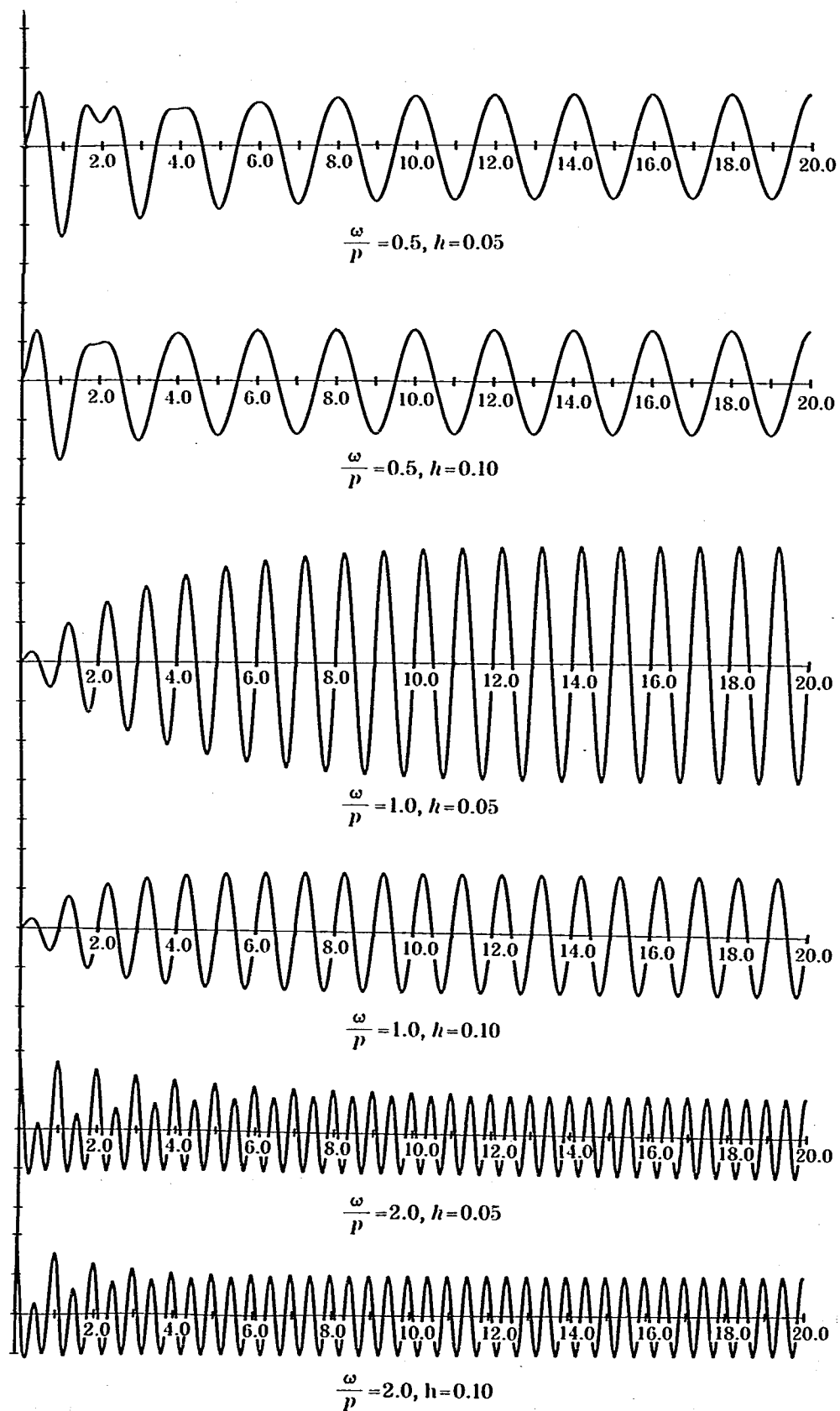


Fig.-3.20 Transitional vibration

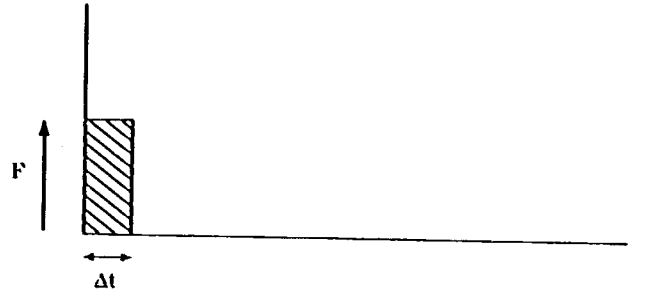


Fig.-3.21 Pulse acting during short time Δt

freedom system is x_o , then we find the equation of motion to be

$$m\dot{x}_o = F \cdot \Delta t$$

giving, $\dot{x}_o = \frac{F}{m} \cdot \Delta t$

Since the initial displacement can be regarded as zero for short $\Delta\tau$, the motion of the system after the impulse will be expressed by the solution of the free vibration in Eq. (3.30) or (3.31) under given initial conditions. From the condition of $x=0$ at $t=0$ in Eq. (3.30), we obtain,

$$C_1 = 0$$

From the condition of $\dot{x} = \frac{F}{m} \cdot \Delta t$ at $t=0$, we obtain

$$C_2 = \frac{F\Delta t}{mp_1}$$

$$x = \frac{F\Delta t}{mp_1} e^{-hpt} \sin p_1 t \quad \dots (3.98)$$

When the impulse $F\Delta t$ equals to 1, then

$$x(t) = \frac{1}{mp_1} e^{-hpt} \sin p_1 t \quad \dots (3.99)$$

where for $t < 0$,

$$x(t) = 0$$

Eq. (3.99) is called the impulse response function.

Next, when an aperiodic force $F(t)$ as shown in Fig.-3.22 continues to act to the mass, the impulse $F(\tau)\Delta\tau$, which acts during time from $t=\tau$, to $t=\tau+\Delta\tau$, causes the displacement $dx(t)$, after $t=\tau+\Delta\tau$. The displacement $dx(t)$ will be expressed in the form from Eq. (3.99)

$$dx(t) = \frac{F(\tau)\Delta\tau}{mp_1} e^{-h_p(t-\tau)} \sin p_1(t-\tau)$$

From this we get the following equation by integration.

$$x(t) = \int_0^t \frac{F(\tau)}{mp_1} e^{-h_p(t-\tau)} \sin p_1(t-\tau) d\tau \quad \dots\dots (3.100)$$

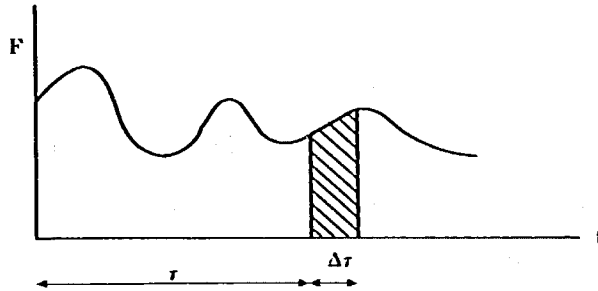


Fig.-3.22 Force acting continuously

The equation above is referred to as Duhamel's integral, indicating a response of single-degree-of-freedom system to an irregular motion.

Finally, let us consider a response to an earthquake motion. The equation of motion will be expressed by Eq. (3.74), and substitution of z into Eq. (3.74) by using Eq. (3.73) gives,

$$-m(\ddot{x} + \ddot{y}) - c\dot{y} - ky = 0 \quad \dots\dots (3.101)$$

This equation will be rewritten as follows,

$$m\ddot{y} + c\dot{y} + ky = -m\ddot{x} \quad \dots\dots (3.102)$$

Eq. (3.102) means that an irregular force $-m\ddot{x}$ continues to act on the mass, and its solution is already given by Eq. (3.100). That is,

$$y(t) = \int_0^t \frac{-\ddot{x}(\tau)}{p_1} e^{-hp(t-\tau)} \sin p_1(t-\tau) d\tau \quad \dots (3.103)$$

Where, the relations below are again used.

$$p^2 = \frac{k}{m}, \quad k = \frac{c}{2\sqrt{km}}, \quad \text{and} \quad p_1 = p \sqrt{1-h^2}$$

Eq. (3.103) gives the relative displacement $y(t)$ at the mass, when acceleration of an irregular earthquake motion $x(t)$ is given.

The relative velocity response is given by,

$$\dot{y}(t) = \int_0^t \frac{-\ddot{x}(\tau)}{p_1} e^{-hp(t-\tau)} \{ -hp \sin p_1(t-\tau) + p_1 \cos p_1(t-\tau) \} d\tau$$

The absolute acceleration response is given by,

$$\begin{aligned} \ddot{x}(t) + \dot{y}(\tau) = p_1 \int_0^t \ddot{x}(\tau) e^{-hp(t-\tau)} \left\{ \left(1 - \frac{h^2}{1-h^2}\right) \sin p_1(t-\tau) \right. \\ \left. + \frac{2h}{\sqrt{1-h^2}} \cos p_1(t-\tau) \right\} d\tau \quad \dots (3.105) \end{aligned}$$

In general, acceleration values of an earthquake motion are given by digital values at every $\Delta\tau$ second (for instance, 0.01 second). An example of acceleration time history was replotted in Fig.-3.23, in which accelerogram was digitized at every 0.01 second. Using this diagram as an earthquake motion input, the response of single-degree-of-freedom system can be numerically obtained from Eqs. (3.103) through

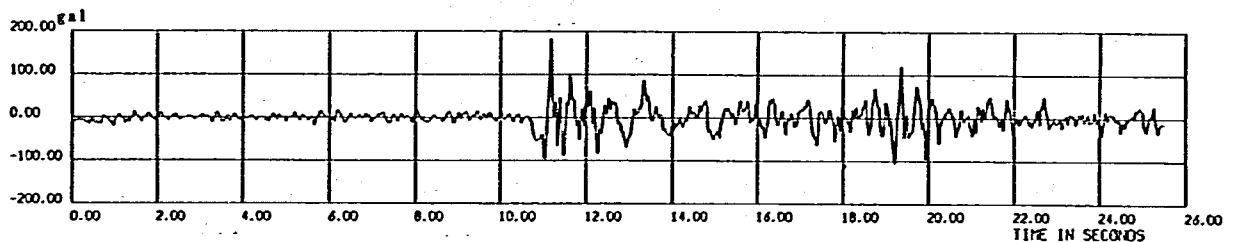


Fig.-3.23 Time history of acceleration of earthquake motion (Record observed at the base of Tarumizu dam during the Miyagi-ken-oki Earthquake of 1978)

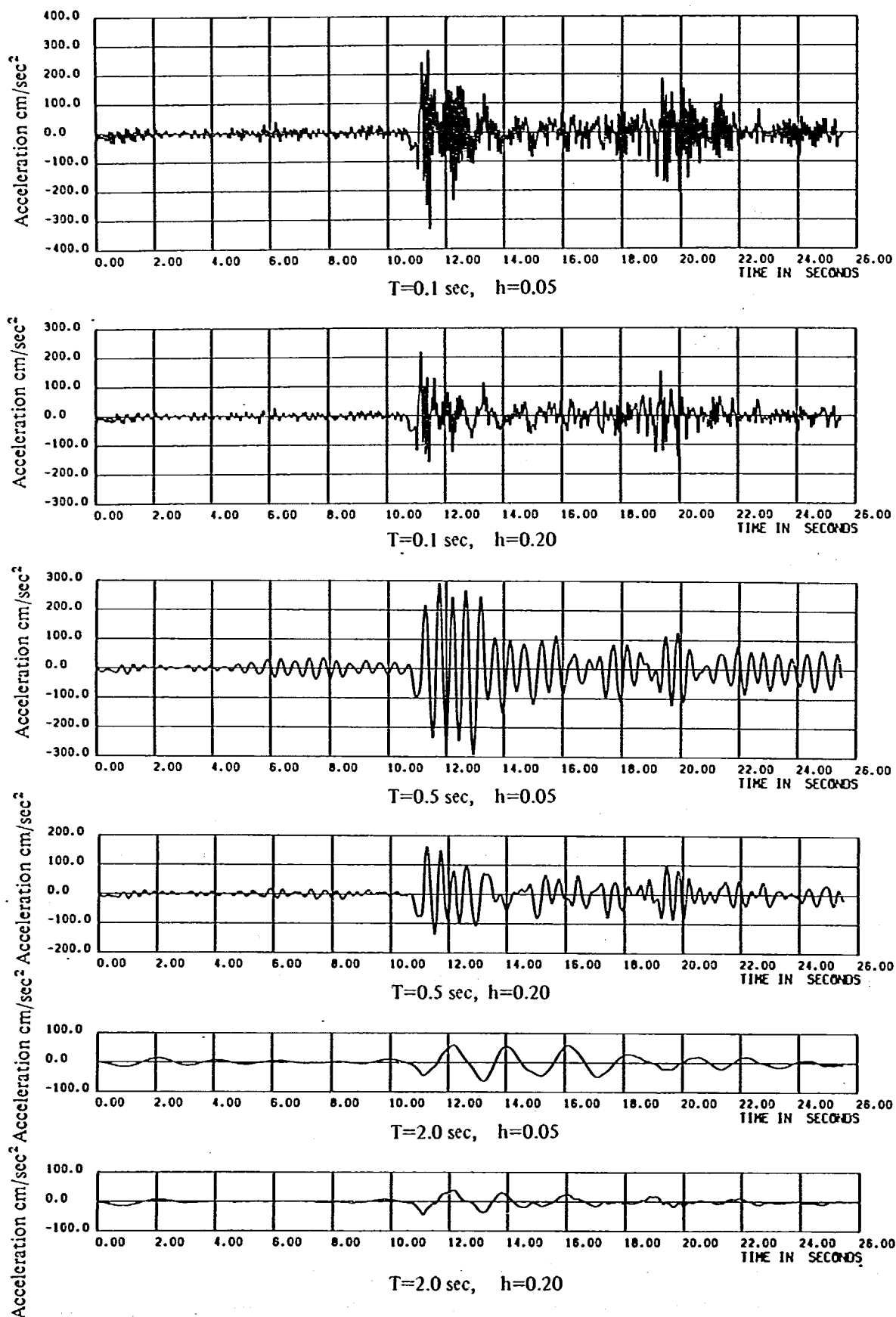


Fig.-3.24 Time history of response acceleration

(3.105). However, actually, numerical integration of Eqs. (3.103) to (3.105) needs long time of computation. Therefore, direct integration of Eq. (3.102) or Fourier transformation is often used to obtain the response.

Fig.-3.24 indicates a time history of the absolute acceleration response to the input motion shown in Fig.-3.23, for various damping ratio h and circular frequency p . From engineering point, the maximum displacement, velocity or acceleration is of importance. These maxima will be a function of natural circular frequency p and damping ratio h , or natural period T and damping ratio h (Housner, G.W. 1959)

$$\begin{aligned} S_a &= S_a(T, h) \\ S_v &= S_v(T, h) \\ S_d &= S_d(T, h) \end{aligned} \quad \dots (3.106)$$

Here, S_a , S_v and S_d are called the absolute acceleration response spectrum, relative velocity response spectrum, and displacement response spectrum, respectively.

Fig.-3.25 indicates S_a for the acceleration shown in Fig.-3.23.

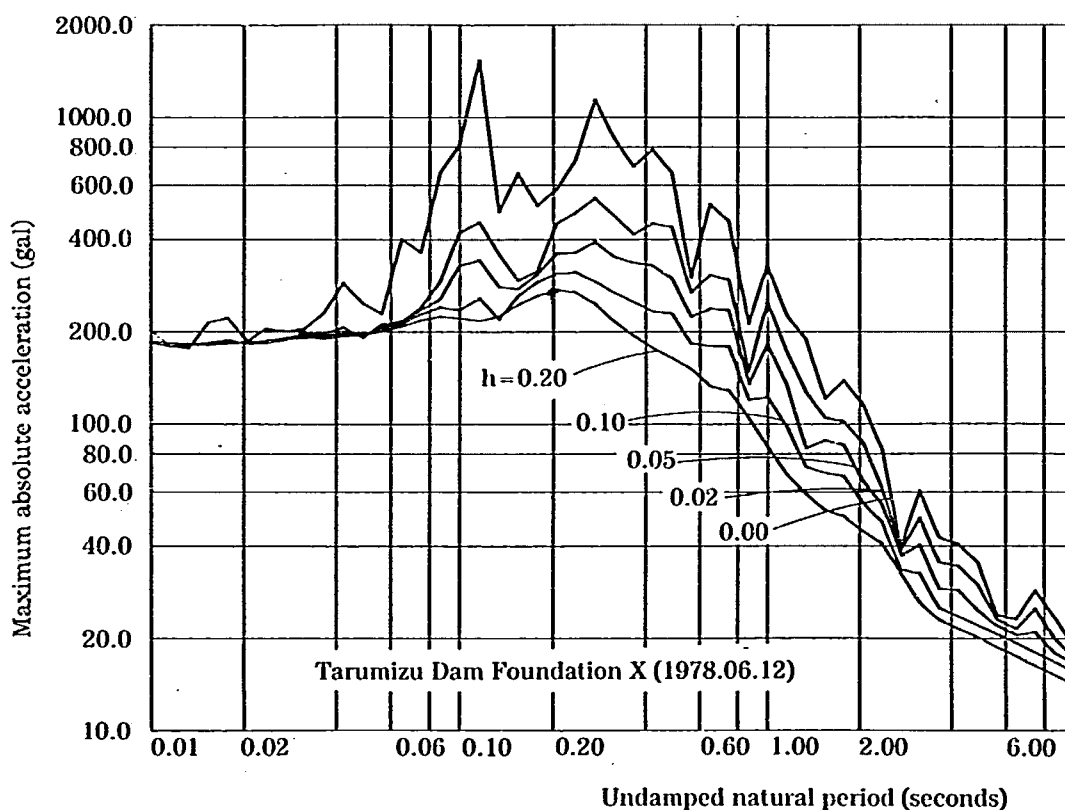


Fig.-3.25 Absolute response acceleration spectra

These response spectra are very useful technique for the study of the earthquake effects on structures.

Since S_a is absolute acceleration response we will be able to evaluate the maximum inertia force of the structure from S_a . The quantity S_v is closely related to energy which the structure takes from an earthquake motion and S_d , a relative displacement, is closely connected to the maximum strain in the structure.

The response spectrum shown in Fig.-3.25 does not have high precision for $T > 5.0$ sec. This is because the input acceleration has low accuracy for longer period components in recording. However, since a natural period T of dam structures ranges from $T=0.1$ to 1.0 sec, the longer period components are not important when Fig.-3.25 is applied to dams.

4. Behaviors of Dams during Earthquakes

This section deals with behaviors of dams during earthquakes based on the case studies and measured records at dams.

Dense stationing of strong motion seismographs at dams and efforts taken by dam control offices have made it possible for Public Works Research Institute, the Ministry of Construction to collect a large number of earthquake records. Analysis of earthquake records in prototype dams provides us with the surest source of information to understand behaviors of dams during earthquakes.

4.1 Behaviors of Concrete Dams during Earthquakes

As for behaviors of concrete dams during earthquake, it may be expected that the stress-strain characteristics of concrete is elastic to a fairly large strain. If it is the case of the elastic body, then natural period and damping constant of a dam body play an important role in the response to earthquake motions. Thus, firstly natural periods of a dam body are estimated from measured accelerograms. Calculation of frequency response function from measured accelerograms gives us useful tools for obtaining natural frequency of dams. When the time records of acceleration at the dam foundation and the dam body are $f(t)$ and $g(t)$, respectively, Fourier spectra, $F(\omega)$ and $G(\omega)$, can be given by

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad \dots (4.1)$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-i\omega t} dt \quad \dots (4.2)$$

Here, a frequency response function $Z(\omega)$ for the foundation of the dam body is given by,

$$Z(\omega) = \frac{G(\omega)}{F(\omega)} \quad \dots (4.3)$$

In actual records, $g(t)$ is given as a result of the response for three components—two horizontal components and one vertical component of the foundation, but since it is difficult to get absolute $Z(\omega)$ by decomposing this coupling, $Z(\omega)$ is obtained simply from the Eq. (4.3). Fig.-4.1 is an example of the frequency response function

of a dam body obtained in this manner. It shows a very clear peak at a certain frequency (=proper frequency) and indicates the behavior of the dam body as an elastic body. The fact that $g(t)$ and $G(\omega)$ are closer to a sine wave than $f(t)$ and $F(\omega)$ suggests elasticity of the dam body.

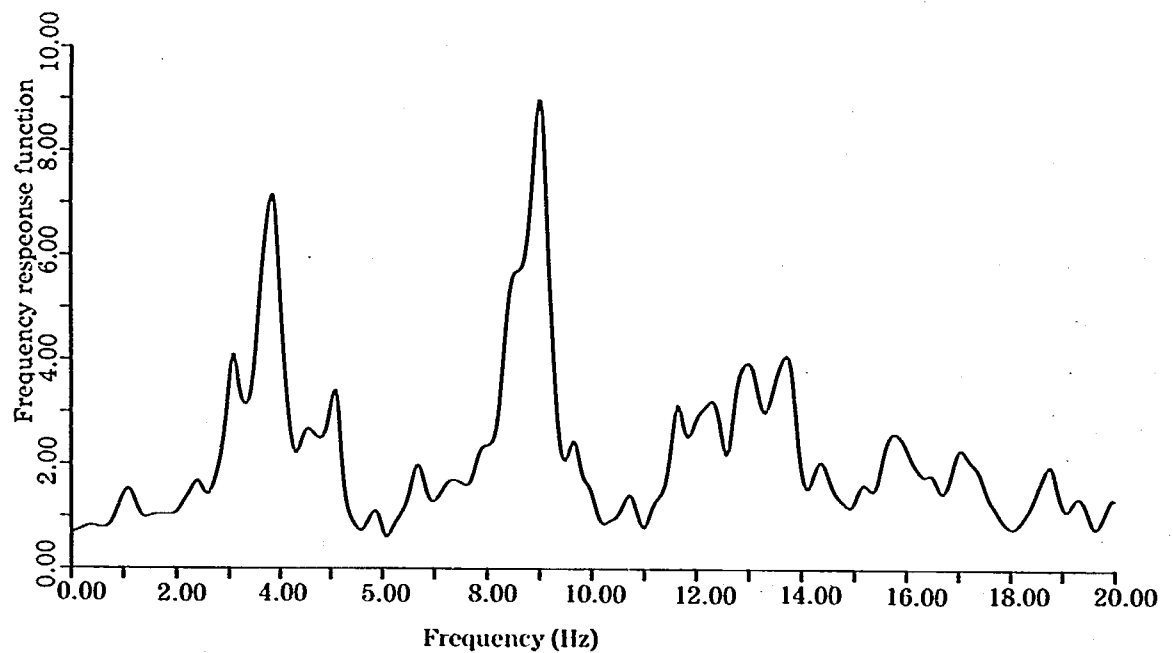


Fig.-4.1 Observed frequency response function (Naruko Dam)

Next, natural periods of dam bodies are shown in Figs.-4.2 and 4.3 by gravity type and arch-type, respectively.

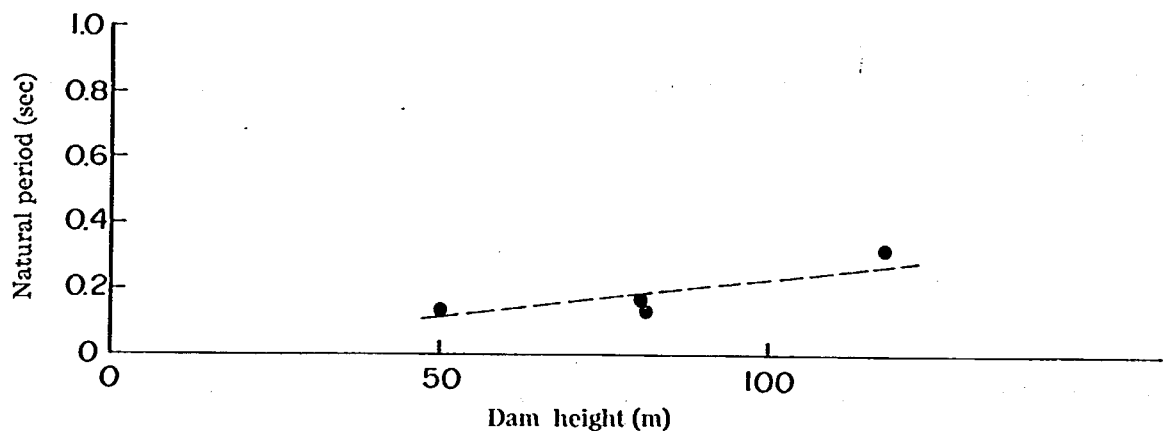


Fig.-4.2 Natural period of gravity dam (Vp-down stream direction)

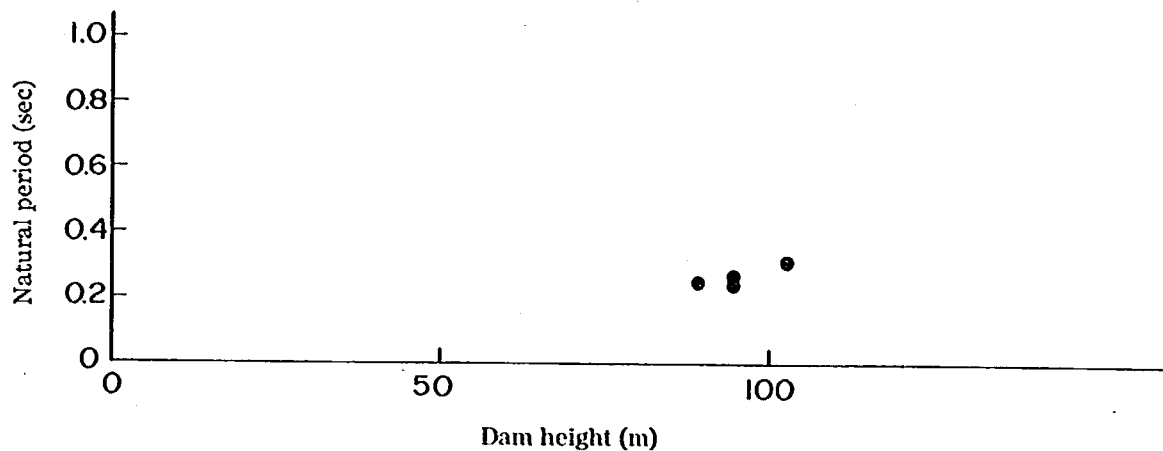


Fig.-4.3 Natural period of arch dam (Up-down stream direction)

With respect to a gravity dam, which standard cross section is usually triangular, the proper period of bending vibration of a cantilever T under the presence of water upstream is given by the following equation;

$$T = 1.695 \sim 1.637 \frac{H^2}{B} \sqrt{\frac{12\gamma}{Eg}} \quad \dots (4.4)$$

In the Eq. (4.4), when $\frac{H}{B} = \frac{1}{0.8}$, $\gamma = 2.35 \text{ t/m}^3$, and $E = 2.5 \times 10^6 \text{ t/m}^2$ are assumed as a standard gravity-type dam, $T = \frac{0.22}{100} H$ is obtained. This relation is shown in Fig.-4.2, which suggests elasticity of the gravity dam.

Next, a case of arch dam is shown in Fig.-4.3. This case also indicates a tendency of coincidence with the equation of proper period T for arch dam, shown below;

$$T = 0.1 + 0.2 \left(\frac{H}{100} \right) \quad \dots (4.5)$$

Next is an example of comparison between actually measured response and calculated response through a dynamic analysis²⁵⁾. That is, acceleration waves measured at the foundation of Yuda Dam were used as an input motion into the base of an analytical model to compare the measured response acceleration at the crest with the estimated acceleration. In the dynamic analysis of a concrete dam, the method of evaluating the effect of hydraulic pressure on the dam body during

earthquake is important; here, two methods proposed by Westergaard and Chopra are applied. The calculation case and the analytical model are shown in Table-4.1 and Fig.-4.4, respectively. Strictly speaking, Yuda Dam is an concrete curved gravity dam of three dimensional structure, but it is treated as a two-dimensional model because it is fairly thick. It is assumed that Young's modulus, Poisson's ratio and damping constant are $2.5 \times 10^5 \text{kgf/cm}^2$, 0.2 and 3%, respectively. Table-4.2 is a summary of the comparison between the calculated values and the observed ones. Fig.-4.5 is a comparison between calculated acceleration and measured response one. Based on these results, the calculated and measuerd results are compared as listed below;

Table4.1 Analyzed cases

Case	Notation	Foundation motion	Hydrodynamic pressure
1	H-W	Horizontal wave	Westergaard
2	H-C	Horizontal wave	Chopra
3	HV-W	Horizontal and vertical waves	Westergaard
4	HV-C ($\alpha^* = 0.0$)	Horizontal and vertical waves	Chopra
5	HV-C ($\alpha^* = 0.85$)	Horizontal and vertical waves	Chopra

* Reflexibility of wave at the bottom of reservoir.

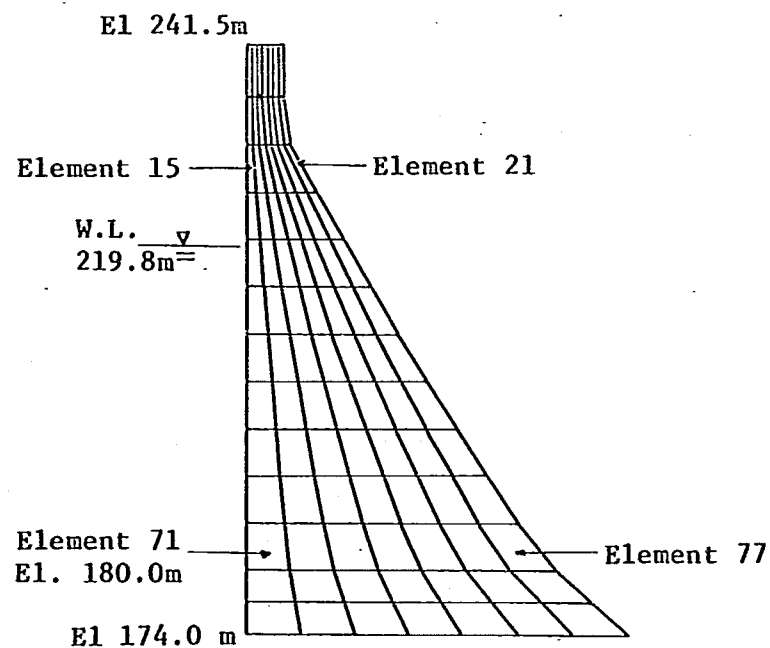


Fig.4.4 Analyzed model

Table 4.2 Comparisons between observed and calculated values

		Observed value	Calculation				
			H-W	H-C	HV-W	HV-C ($\alpha=0.0$)	HV-C ($\alpha=0.84$)
Horizontal maximum acceleration (gal)	Downstream	152 (8.10)	168 (8.33)	135 (8.23, 13.59, 13.83)	194 (8.70)	143 (13.59)	159 (9.04)
	Upstream	183 (8.02)	169 (7.91)	139 (8.53)	176 (7.92)	130 (13.21)	170 (9.12)
Vertical maximum acceleration (gal)	Up	29 (9.69)	25 (7.99)	23 (13.6)	35 (10.7)	37 (12.84)	40 (9.04)
	Down	31 (9.78)	30 (8.63)	22 (12.76)	44 (8.62)	37 (7.9)	38 (9.13)
Horizontal maximum displacement (cm)	Downstream	0.211 (18.13)	0.173 (16.31)	0.183 (12.77)	0.200 (7.67)	0.195 (13.21)	0.216 (17.45)
	Upstream	0.187 (18.25)	0.186 (16.17)	0.177 (13.83)	0.198 (7.53)	0.179 (6.46)	0.178 (9.03)
Vertical maximum displacement (cm)	Up	0.066 (8.14)	0.030 (6.85, 16.31)	0.032 (12.77)	0.034 (7.67)	0.034 (13.21)	0.038 (17.45)
	Down	0.074 (8.29)	0.033 (16.17, 8.30)	0.030 (18.83)	0.039 (7.53)	0.031 (6.46)	0.030 (9.02)

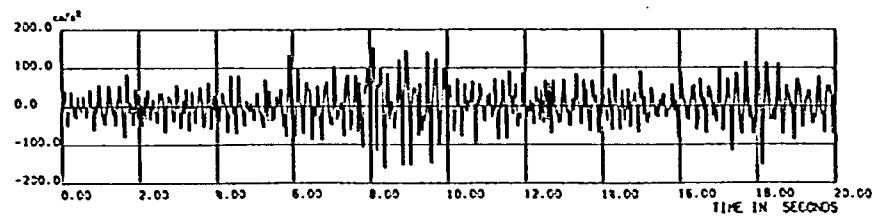
* Value within parenthesis is the occurred time.

1. Up and downstream response acceleration

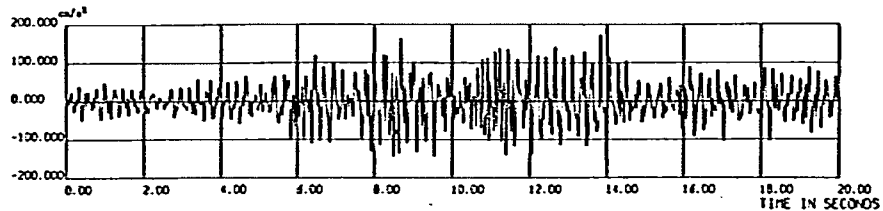
According to the record at the rock foundation, *S* wave arrived around $t=4.5$ sec, with a high acceleration amplitude around $t=6.0 \sim 9.0$ sec. Acceleration was rather low till $t=16$ sec, and the second main motion arrived at $t=16.0 \sim 19.0$ sec. The dam body responded remarkably at $t=6.0 \sim 10.0$ sec, and $t=17.0 \sim 19.0$ sec according to input wave at rock foundation. The analytical result, which shows strong vibration at $t=6.0 \sim 10.0$ sec in every case, coincides with the observed one, but the vibration becomes damped in the measurement later. In the section of $t=10.0 \sim 17.0$ sec, all cases except Case 1 show greater response than the measurement. Case 1 and Case 3 resemble in wave form, but Case 3 with vertical motion shows slightly higher acceleration. Case 2 and Case 3 can be characterized by being similar to each other in wave form and amplitude of acceleration and resonant condition at $t=12.0 \sim 14.0$ sec. Assumedly, this is because the section of $t=12.0 \sim 14.0$ sec of the input acceleration at the rock foundation contains a high frequency component similar to the proper period of the dam body, but since the Chopra method assumes a reservoir as a kind of elastic body, first resonant mode tends to be emphasized in this calculation. A comparison between Case 4 and Case 5 in which vertical motion and Chopra method are taken into consideration suggests a very minor effect of the wave reflectance at the reservoir bottom.

2. Vertical response acceleration

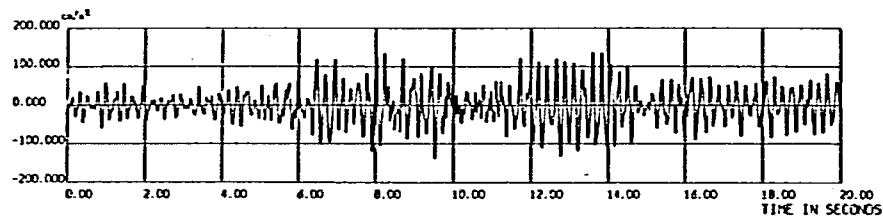
S wave component of the measured vertical motion at the foundation is not so clear unlike horizontal motion, and the acceleration amplitudes for both the foundation and the dam body are flat. The vertical motion in Case 1 and Case 2 is caused by only a horizontal input motion without vertical one, while in Case 3 and Case 4 the vertical



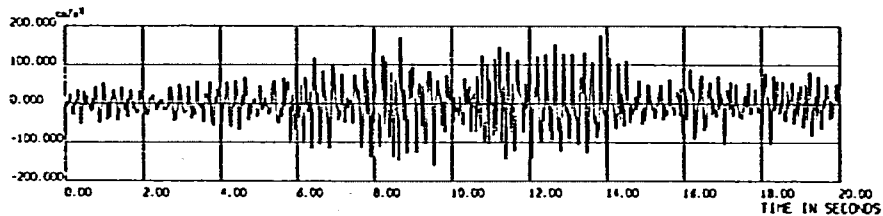
Measured response



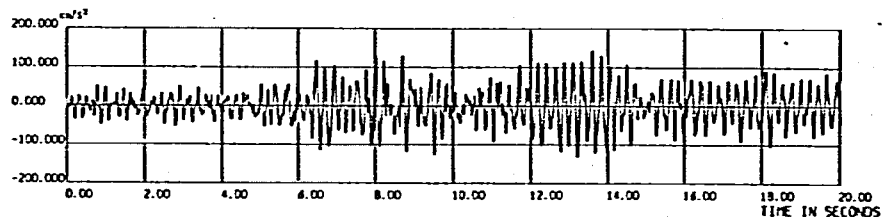
Case No. 1; reservoir by Westergaard's theory, horizontal input



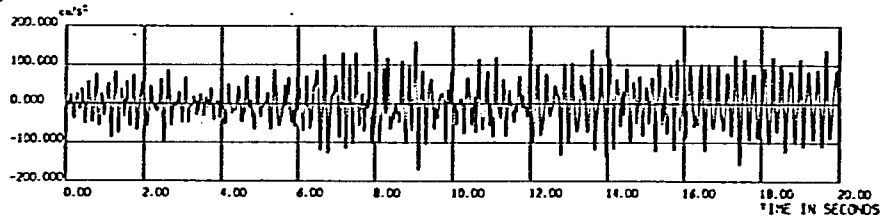
Case No. 2; reservoir by Chopra's theory, horizontal input



Case No. 3; reservoir by Westergaard's theory, horizontal and vertical input



Case No. 4; reservoir by Chopra's theory ($\alpha=0.0$), horizontal and vertical input



Case No. 5; reservoir by Chopra's theory ($\alpha=0.85$), horizontal and vertical input

Fig.-4.5 The comparisons between observed and analyzed response acceleration of crest

motion is taken into consideration in the input, it is a sum of the responses caused by the horizontal and vertical input; Case 1 and Case 3 are relatively near to the measured values, while Case 2 has a less high frequency component than the measured one. In contrast, Case 4 has a high frequency component overlapped with low frequency. This is due to the assumption that 18Hz of resonant frequency of the vertical motion enters the dam body from the reservoir bottom.

Above-mentioned these trends are in common with the results observed in the analysis of an earthquake in 1976. Two methods proposed by Chopra and Westergaard were adopted to evaluate hydrodynamic pressure. With respect to the frequency, characteristic of the response acceleration, the Chopra method tends to yield predominant period, while the Westergaard method gives the closer result to the measured values. The Chopra method assumes that a reservoir is a kind of half-infinite elastic body, and as a result a coupled vibration of the dam body and the reservoir is taken into account in calculation. On the other hand, since the Westergaard method also assumes a reservoir as an elastic body, the period of an earthquake motion is assumed to be single when the reservoir is dealt as an added mass. Therefore, higher resonant frequency of the dam body tends to appear. Furthermore, although the upstream face of a dam is actually three-dimensioned and hydrodynamic pressure might escape towards the dam-axis direction, two-dimensional analysis is carried out. These are also causes of differences between calculated and measured values.

Next, extremely high acceleration observed at concrete dam is mentioned.²⁶⁾ At Tsuruta Dam, one seismometer was set on the base rock of the shore of the reservoir 200m upstream away from the dam. A seismometer for the dam body was placed at the crest of the standard cross section of the dam. As the epicentral distance was only 7km, although 4.9 of earthquake magnitude, earthquake motion which has high frequency components and short duration was recorded as shown in Fig.-3.6. The maximum acceleration at the foundation was 360 gal, while that at the dam body was 273 gal. This must be attributable to two reasons; the predominant period of the input earthquake motion was much faster than the resonant period of the dam body, and the acceleration at the foundation was recorded on the rock surface away from the dam, instead of just under the dam body. Therefore, the acceleration observed just under the dam body might have been much lower because of the effect of the dam.

Then, the damping constant of 30% is assumed, including the dispersion energy

due to the interaction between the dam body and the foundation, to conduct the response analysis as the case of Yuda Dam. The result is shown in Fig.-4.7.

A comparison between Fig.-4.7 and Fig.-4.6 suggests that the peak values do not accord with each other, while the levels of other average responses are relatively coincident. 0.5cm of the maximum displacement and 10kgf/cm² of the maximum dynamic stress at the high response acceleration shown in Fig.-4.7 suggest that there is no problem about safety. In any case, short earthquake motion with such a high frequency component rarely causes large stress, but several questions such as precision of the recorded peak values, the location of meters and the characteristics of the analytical model in a high frequency region make the comparison difficult.

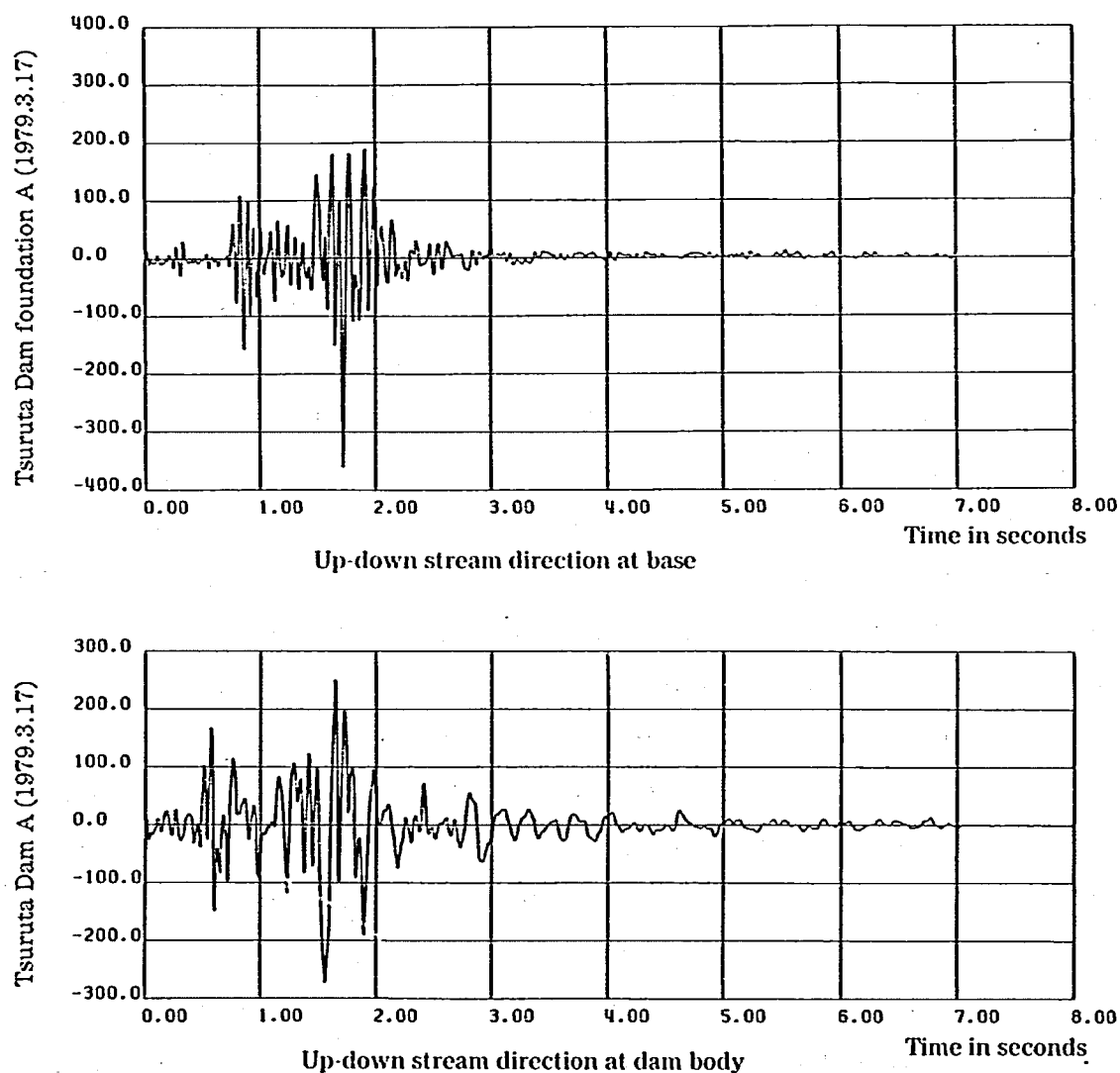


Fig.-4.6 Observed acceleration record at Tsuruta Dam

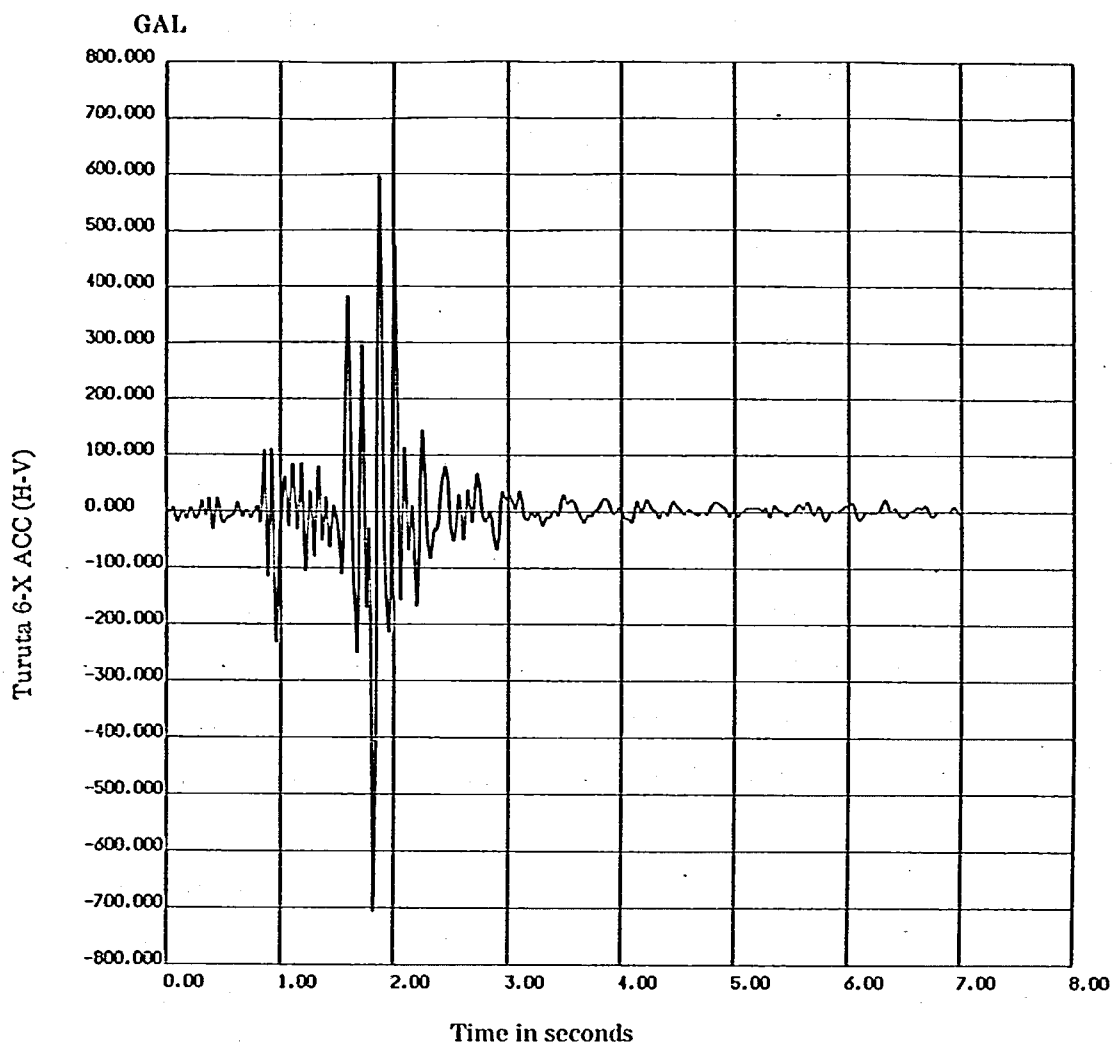


Fig.-4.7 Calculated response acceleration at the crest of Tsuruta Dam

Thus, further examination is required for better explanation of observed value by the analytical model.

As the final discussion on the response characteristics of a concrete dam, response magnification of a dam body is examined. The response magnification here is obtained from dividing the maximum acceleration at the crest of the dam body by the maximum acceleration at the foundation. Since this magnification is dependent on the frequency characteristic of the input earthquake motion, the maximum value $Z(\omega)$ for a certain ω in Eq. (4.3) is the response magnification as an elastic body. The response magnification from Eq. (4.3) yields a extremely high scattering for ω . Since $Z(\omega)$ is dependent on the window which is used to diminish scattering, the response magnification is tentatively defined by a ratio of peak values for convenience. The response magnification for gravity dams in accordance with this definition varies from

4 to 8 times, while that for arch dams varies from 6 to 12 times. These response magnifications, if dam bodies behave as an elastic body, are determined by damping, and correspond to damping constants of 5~10% for gravity dams, while approximately 3% for arch dams taking account of other calculation cases.

It is confirmed at the response of a gravity dam subjected to a strong earthquake motion at the foundation that although the maximum tensile stress occurring on the dam body is less than 1/10 of the compressive strength and the response magnification is very high, there is no problem about the safety factor of shearing friction because the inertia force acting on the dam body becomes small due to phase difference.

4.2 Behaviors of Fill Dams during Earthquake

Accelerations of earthquake motions were observed at 8 fill dams related to the control of the Ministry of Construction. They are Shijushida; Miho; Hirose; Shitoki; Shirakawa; Gosho; Minase; Ishibuchi Dams. Shijushida Dam is an earth dam, Minase and Ishibuchi Dams are concrete face rockfill dams, and the others are zoned type dams.

In case of fill dams like concrete dams, a dam body can be considered as an elastic body within a certain shear strain range. In addition, the stress-strain relation of a fill material is not linear, but is influenced by size of shear strain and especially hysteresis. However, in dynamic analysis, an equivalent linear model, which approximates stress-strain at the maximum strain, is often used based on the concept of "equivalent linearity". Then, firstly, a fill dam is examined through an elastic model, like concrete dams to get its natural period.

Fig.-4.8 illustrates the relation between heights of fill dams and their natural periods. These results are compared with Eq. (4.6) given by Okamoto;

$$\begin{aligned} T &= \frac{1.0}{100} H \text{ (earth dam)} \\ T &= \frac{0.5}{100} H \text{ (rockfill dam)} \end{aligned} \quad \dots (4.6)$$

These measured records are all for rockfill dams except Shijushida Dam, and so they are on the line of Eq. (4.6) for rockfill dams, it is known that an elastic constant inside rockfill is greatly dependent upon a size of confining stress and becomes higher with

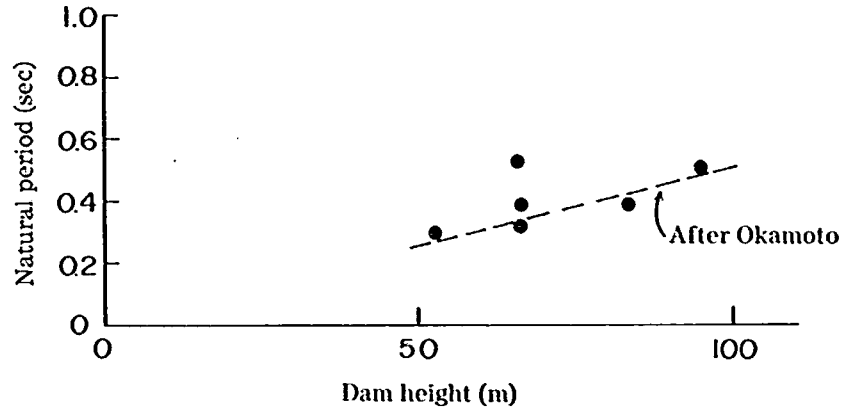


**Fig.-4.8 Natural period of fill dam
(Up-down stream direction)**

depth. When the rigidity G of the interior dam body is constant and the dam body behave as shearing vibration of a two-dimensional triangular cantilever, then the natural period T is given by the following equation (Matsumura)²⁷⁾:

$$T = 2.61 \sqrt{\frac{\rho}{G} H} \quad \dots (4.7)$$

Where ρ : density of the dam body

H: Dam height

Then, the rigidity of an general fill dam obtained from Fig.-4.9 and Eq. (4.6) on the assumption of $\rho = 2.0 \text{ t/m}^3$ becomes:

$$G = 5,500 \text{ kgf/cm}^2 \quad \dots (4.8)$$

This value can be helpful in conducting a dynamic analysis as an average rigidity of rockfill dams when the occurred strain is expected to be rather small.

In addition, response magnification is obtained in the same manner with concrete dams. The result is shown in Fig.-4.9.

Since records during 2 earthquakes are available for Miho Dam, an example of analysis is introduced below²⁸⁾.

Fig.-4.10 is a maximum cross section of Miho Dam, a center core type rock fill dam of 95m-height. It was completed in 1978, and its main features are shown in Table-4.3.

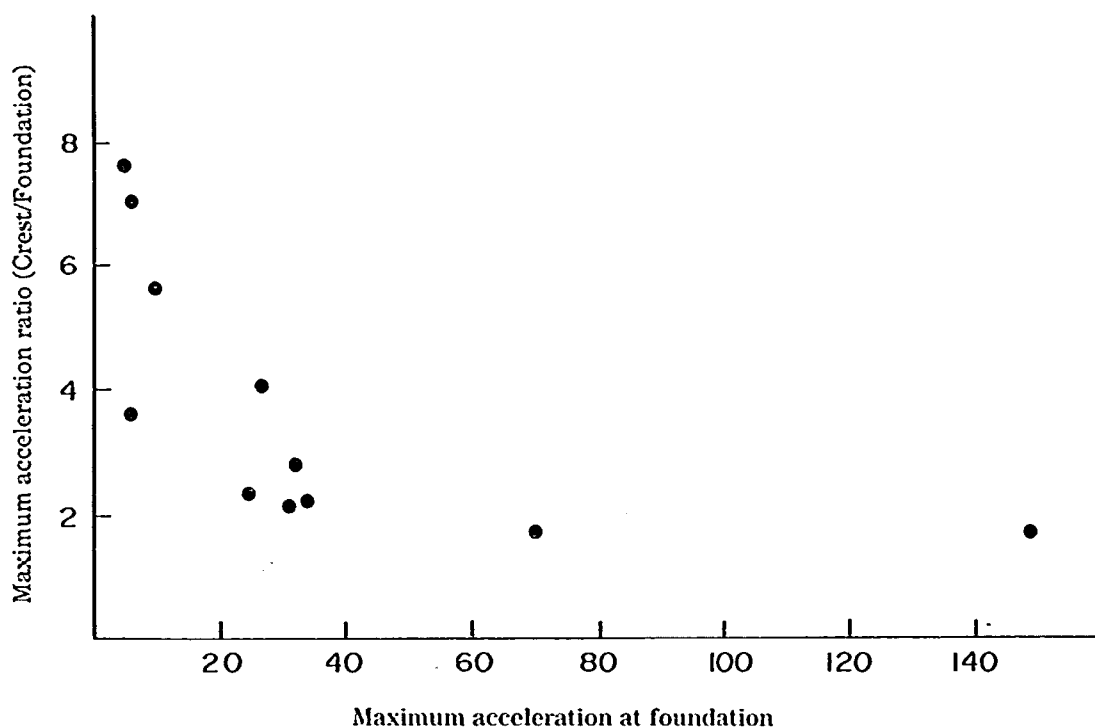


Fig.-4.9 Maximum acceleration ratio (Crest/Foundation)

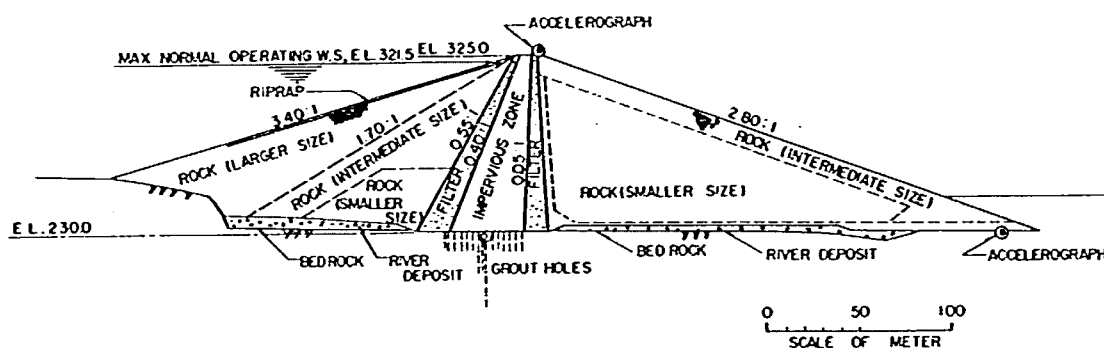


Fig.-4.10 Standard cross section of Miho Dam

The earthquake of magnitude 6.7 with the epicenter at the east Izu Peninsula off, at longitude, $139^{\circ}14'E$ and latitude $34^{\circ}55'N$ occurred on June 29, 1980. The seismometers placed at Miho Dam (SMAC-Q type and electromagnetic type, Fig.-4.10) recorded the earthquake motion. The epicentral distance and the focal depth were 56km and 10km, respectively. In addition, an earthquake of magnitude 4.5 with epicenter at the eastern part of Yamanashi Prefecture at longitude, $138^{\circ}55'E$ and

Table-4.3 Generals of Miho Dam

Reservoir		Dam	
Catchment area	158.5km ²	Height	95.0m
Reservoir area	2.18km ²	Crest length	587.7m
Reservoir capacity	64.9 × 10 ⁶ m ³	Volume	5.816 × 10 ⁶ m ³
Effective storage capacity	54.5 × 10 ⁶ m ³	Upstream slope	1 : 3.4
Normal water level	EL. 321.5m	Downstream slope	1 : 2.8
Controlled water level in summer	EL. 316.8m	Crest width	15m
Spillway		Geology of foundation	
Radial gate	4	Tuff Breccia during Oligocene and Miocene of Neogene	
Roller gate	1		

latitude 35°28'N occurred on April 14, 1981. The epicentral distance and the focal depth were 13km and 20km, respectively.

The maximum accelerations of these two earthquakes were nearly equal; approximately 30 gal horizontally and 10 gal vertically. However, the frequency characteristics of them were different; 4.0Hz in the earthquake of Izu Peninsula off, while 7.0Hz in the earthquake of east Yamanashi Prefecture.

The acceleration waves recorded at the dam foundation and at the crest are shown in Figs.-4.11 and 4.12.

The frequency response function at the crest to the dam foundation is shown in Fig.-4.13.

According to Fig.-4.13, it is assumed that the natural frequency in up-down stream direction is 2.0, 3.8, 4.7, 5.2 and 6.1Hz from the frequency response function at the crest and the first natural frequency is assumed to be 2.0Hz. On the other hand, the natural frequency toward the up-down stream direction of the dam body in the earthquake of eastern Yamanashi Prefecture is assumed to be 1.85, 2.8, 3.5 and 5.2Hz. The most predominant frequency is 3.5Hz, and disagrees with that of the earthquake of Izu Peninsula off. However, it is obvious that Miho dam body has predominant response at least near 2.0, 3.8 and 5.2Hz.

In order to obtain the elastic wave velocity-distribution inside the Miho Dam body, geophysical exploration and velocity probing were conducted for the dam body. Shear modulus, Poisson's ratio and elastic modulus were estimated from the obtained elastic wave velocity.

The velocity distribution of the dam body is resulted as shown in Fig.-4.14. Since no monitoring line is provided in upstream face because of water storage, no velocity

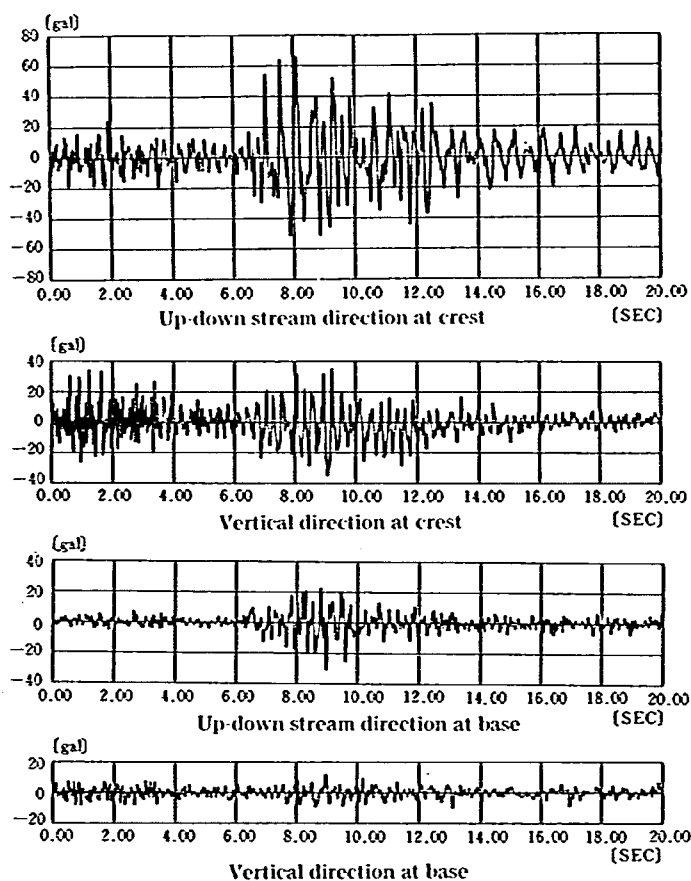


Fig.-4.11 Observed Earthquake at Miho Dam during East Izu Peninsula Off Earthquake (June 29,1980)

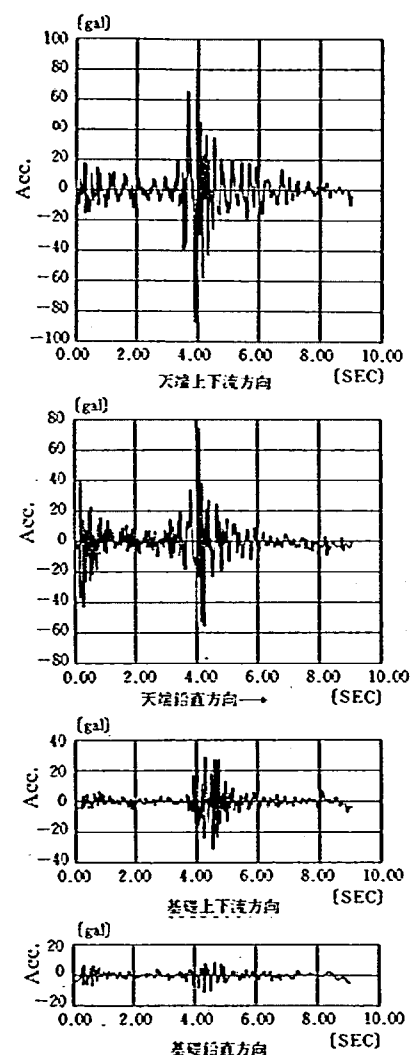
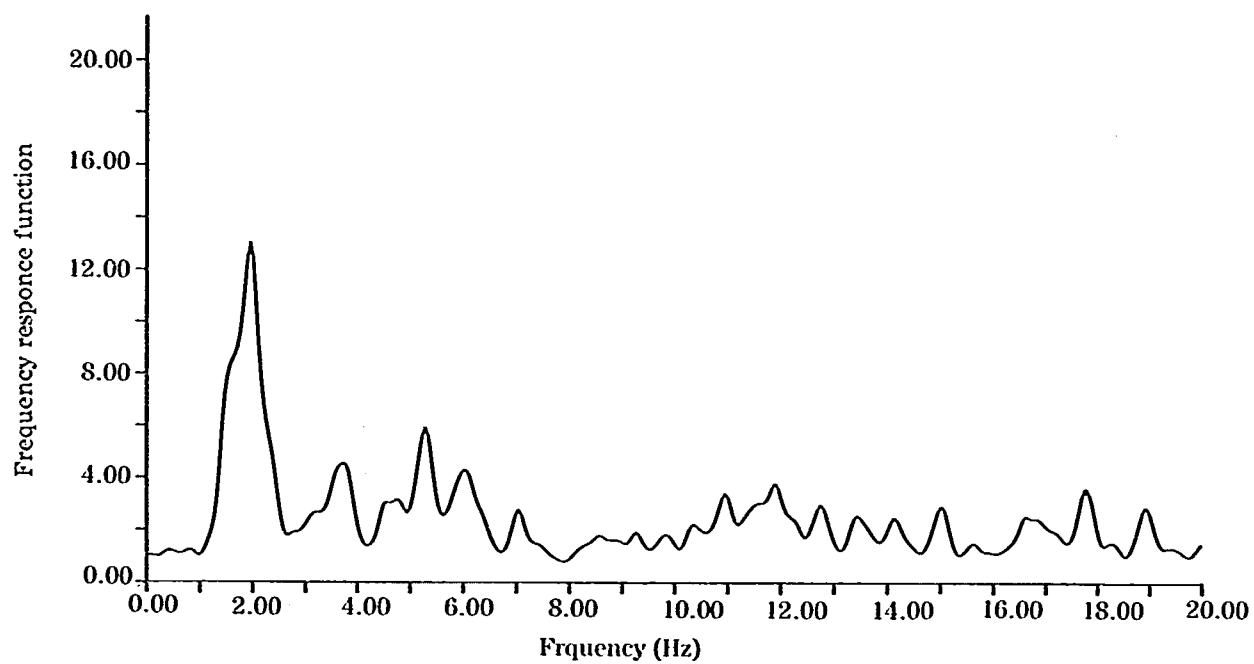


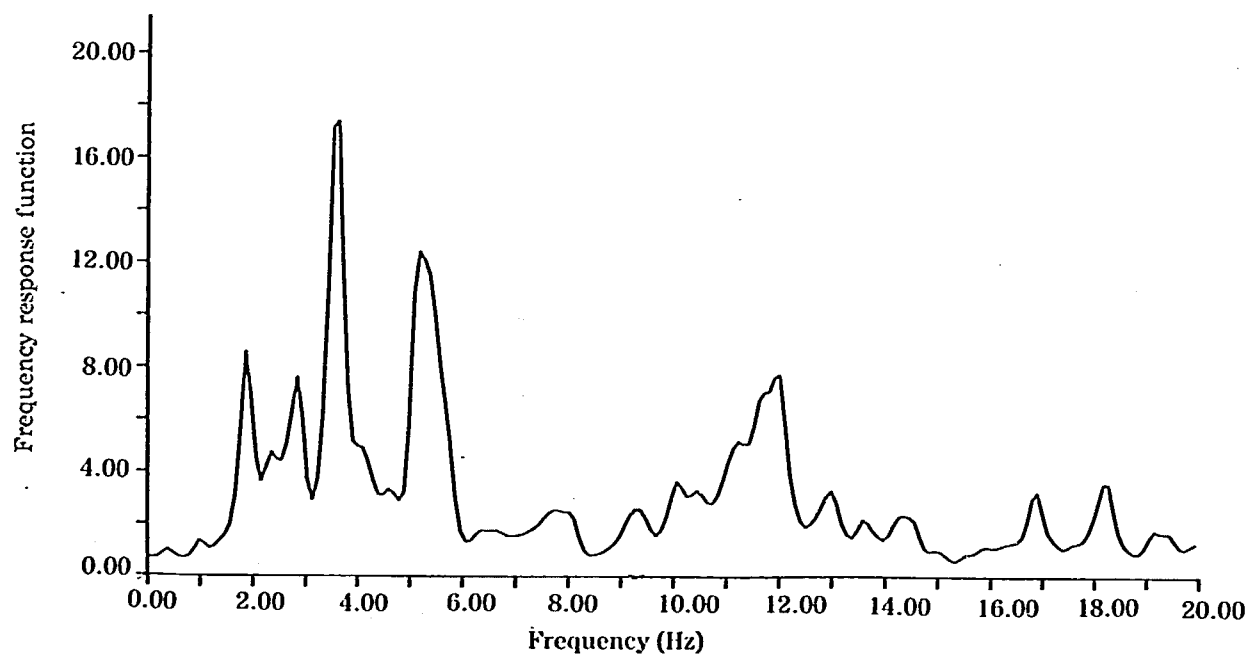
Fig.-4.12 Observed Earthquake at Miho Dam during East Yamanashi Prefecture Earthquake (April 14, 1981)

distribution is obtained.

A cyclic triaxial test was conducted for the materials of Miho Dam. The rock material transported from quarry site and adjusted to the grain size distribution equal to that of the fine grained rock of Miho Dam. The specimen is 30cm in diameter and about 60cm long. Fig.-4.15 indicates the relation between shear modulus and shearing strain resulted from the test. Here, the shear modulus and the shearing strain were estimated from the elastic modulus and the axial strain on the assumption of a certain Poissen's ratio. Fig.-4.16 also shows the shear moduli by depth resulted from



East Yamanashi Prefecture Earthquake (April 14, 1981)



East Izu Peninsula Off Earthquake (June 29, 1980)

Fig.-4.13 Frequency response function at crest

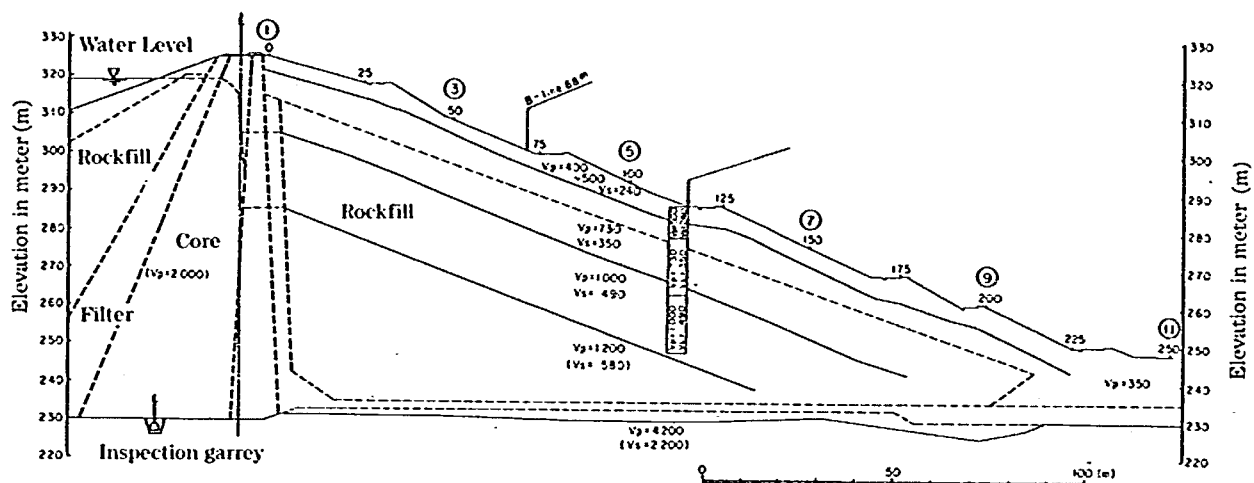


Fig.-4.14 Velocity distribution observed by geophysical exploration

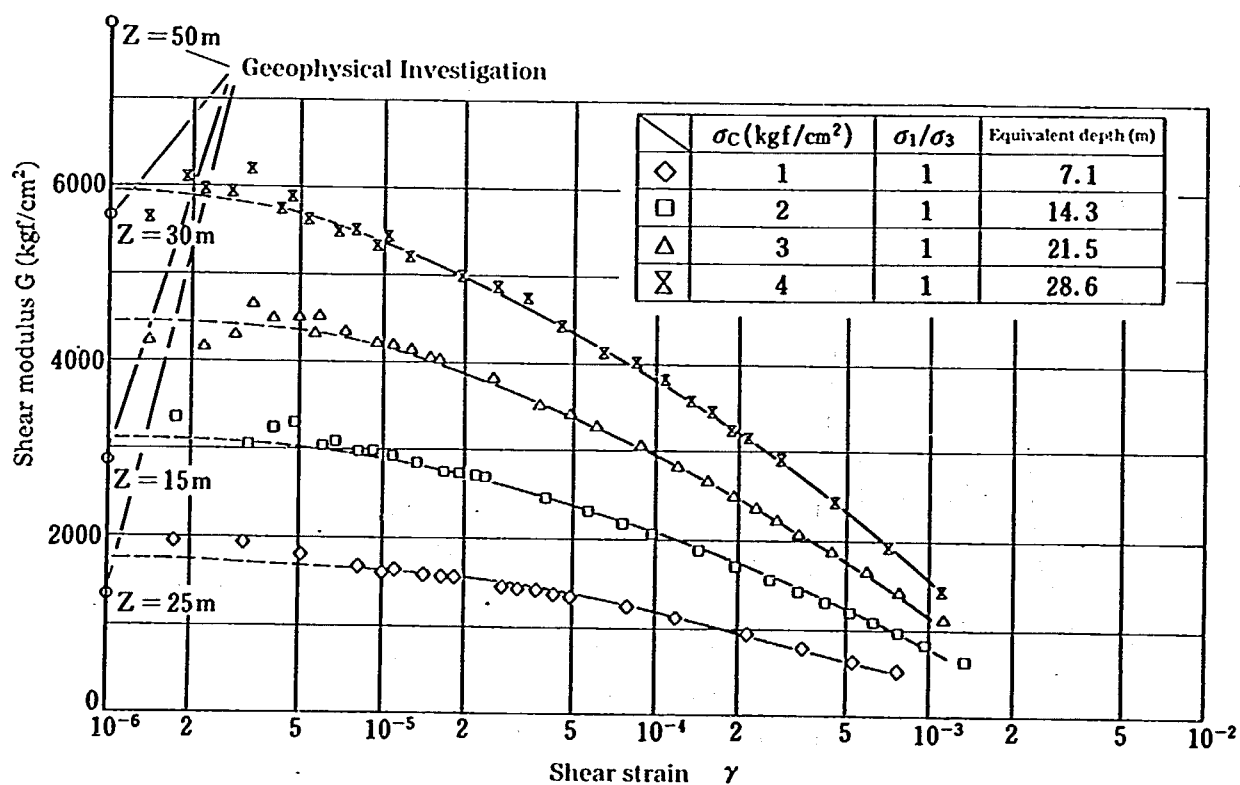


Fig.-4.15 Shear moduli obtained from cyclic triaxial test and geophysical exploration

geophysical exploration. In general, it is said that a range of strain due to geophysical exploration is between 10^{-6} and 10^{-5} .

The confining pressure $\sigma_3 = 4.5 \text{ kg/cm}^2$ and the mean principal stress $\sigma_m = 6.0 \text{ kg/cm}^2$ in the cyclic triaxial test correspond to the stress condition at about 40m depth of the dam. Therefore, the assumed strain-dependency of the shear modulus at 40m depth becomes the broken line in Fig.-4.15.

The shear modulus of Miho Dam was estimated from the S wave velocity distribution obtained by geophysical exploration to conduct a dynamic analysis using this value as the initial physical value. The initial damping constant was assumed to be 15% for all elements. The strain dependent curve used for the analysis is already shown in Fig.-4.16. The analytical model using finite elements with 236 elements and 257 nodal points is illustrated in Fig.-4.17. The input earthquake waves are recorded at Miho Dam site in the earthquakes at east Izu Peninsula off June 29, 1980 and eastern Yamanashi Prefecture April 14, 1981 and then corrected.

The frequency response function at the crest for the dam foundation after the convergence of the calculation is shown in Fig.-4.18. The first natural frequency in

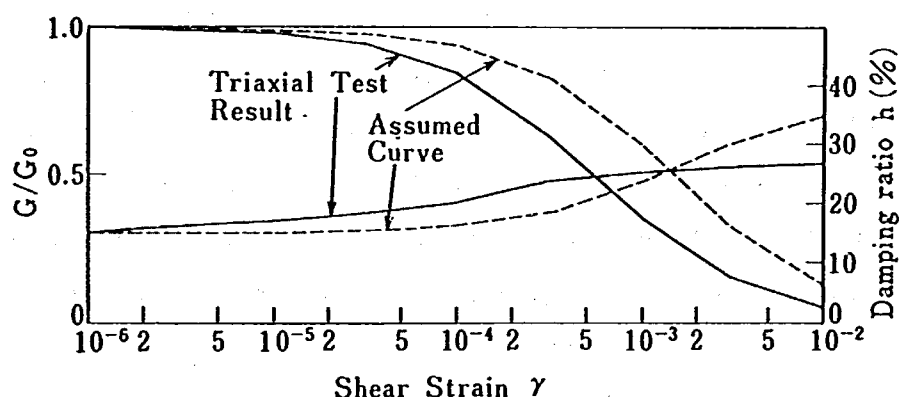


Fig.-4.16 Strain dependencies of shear moduli G and damping ratio h

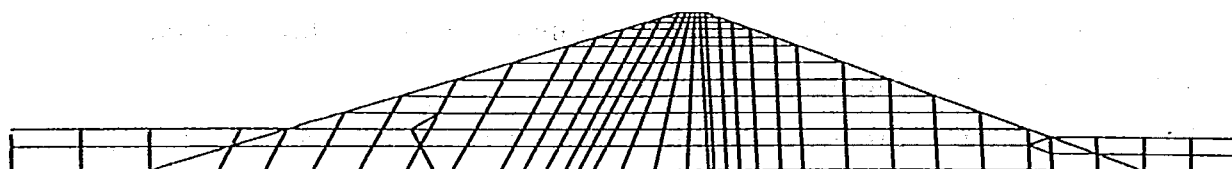


Fig.-4.17 Finite element mesh

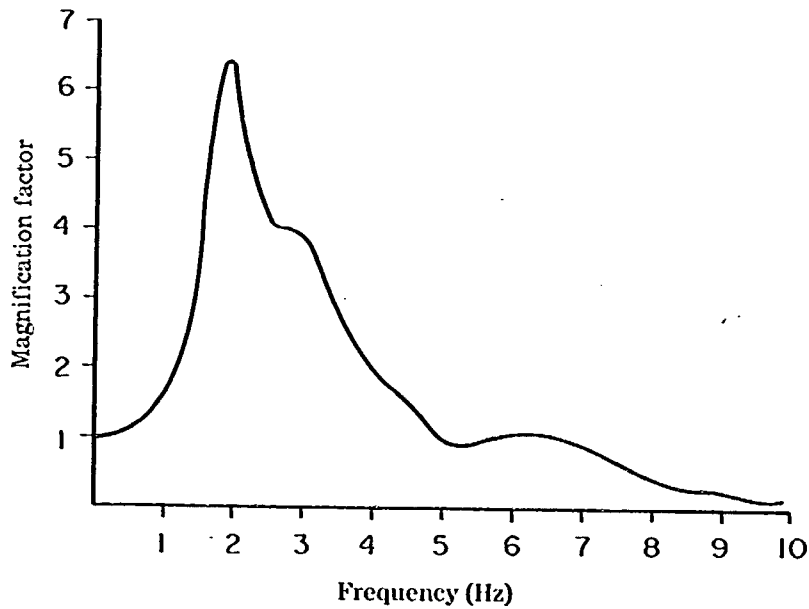


Fig.-4.18 Frequency response function

the analytical model is obviously 1.86Hz, but the natural frequency after secondary is unclear. A single predominant response appears at 6.3Hz.

The frequency response function at the crest for the foundation obtained from actual measurement has a predominant response near 2.0Hz. In particular, in case of the earthquake of east Izu Peninsula off, this response gives the first natural frequency, and nearly equals to the first natural frequency in the analytical model.

The first natural frequency in the analytical model, 1.85Hz is practically lower than that in the measurement, but as the analysis was performed for a infinite two-dimensional cross section, although the prototype dam is three-dimensional, the effect of the side restraint is conceivable. The solution of shearing vibration of a triangular cantilever was obtained to examine this effect. The ratio of a natural frequency f_L of a dam with crest length L to a natural frequency $f_{L=\infty}$ of a dam with infinite crest length, $f_L/f_{L=\infty}$ is affected by the ratio of dam height H to crest length L . In case of Miho Dam, $H/L=0.4$, and so $f_L/f_{L=\infty}=1.12$. The slight difference between the measured natural frequency and the analyzed natural frequency is attributed to the fact that the three-dimensional measured value got higher by 10% due to the side restraining effect.

The calculated up and down stream, and vertical response acceleration wave at the crest are shown in Fig.-4.19. For the earthquake of Izu Peninsula off, the measured maximum response acceleration is 63cm/sec^2 , while the estimated value is

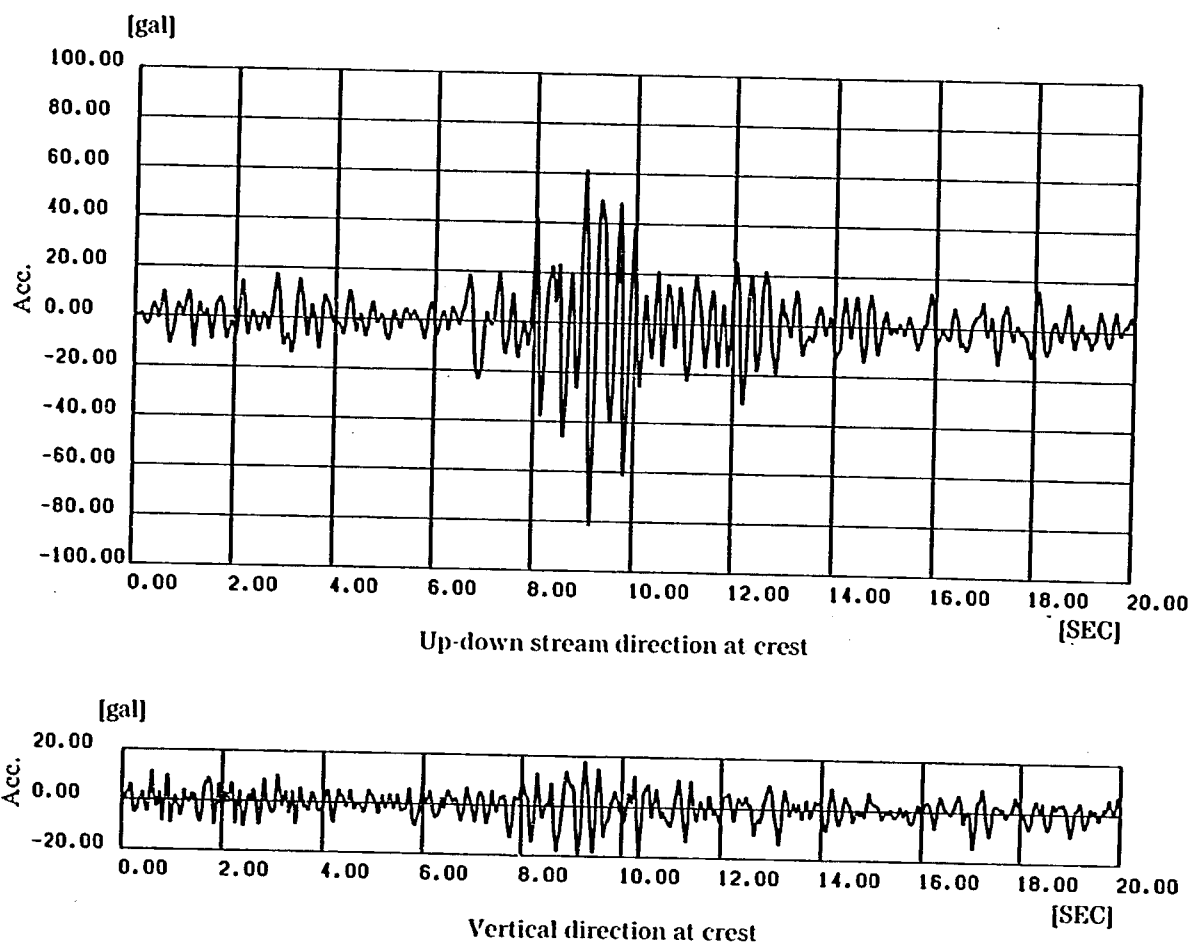
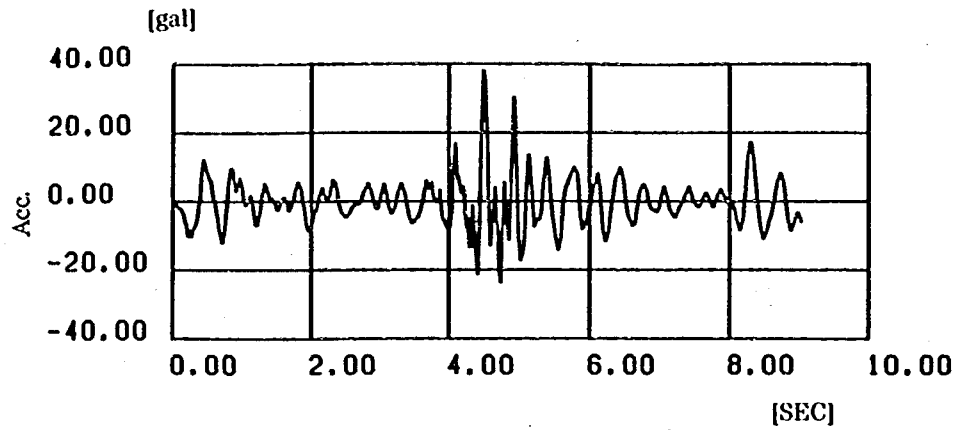
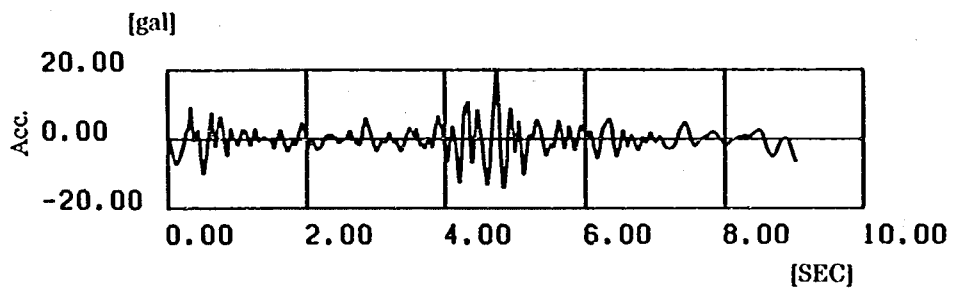


Fig.-4.19 Calculated response acceleration at at dam crest, East Izu Peninsula Off Earthquake (June 29, 1980)

81cm/sec², with an 30% increase. However, the measured and the estimated values of average levels of main motion from 7 sec. to 12 sec. are almost equal. On the other hand, in case of the earthquake of eastern Yamanashi Prefecture, the estimated maximum response acceleration is 40cm/sec² for horizontal direction with a 50% higher value than the measured one, while the measured value for vertical direction, 70cm/sec² is much higher than the estimated value, 20cm/sec². With respect to the acceleration wave form, the both cases are similar for horizontal direction, and fairly good accord was recognized for both level and the acceleration wave form, when a damping curve with dispersion damping of 5% was adopted (See Fig.-4.20(a) and (b)).

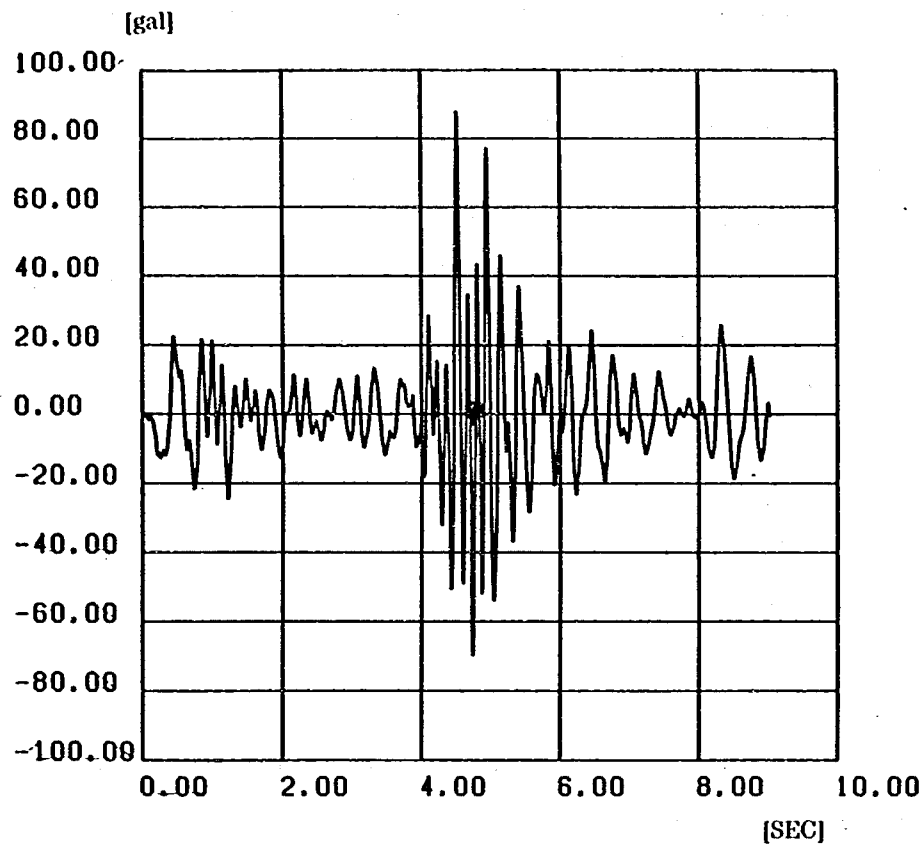


Up-down stream direction at crest

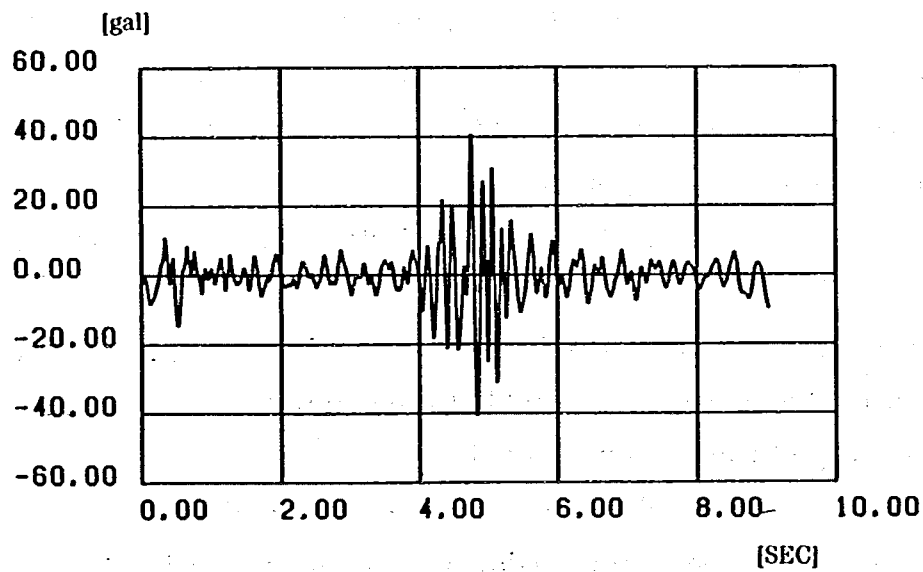


Vertical direction at crest

Fig.-4.20(a) Calculated response acceleration at dam crest, 15% dissipation damping, East Yamanashi Earthquake (April 14, 1918)



Up-down stream direction at crest



Vertical direction at crest

Fig.-4.20(b) Calculated response acceleration at crest, 5% of dissipation damping, East Yamanashi Earthquake (April 14, 1981)

4.3 Damages of Dams in Japan

The first summary report on damages of dams in Japan is a report on damage on small-scale earth dam caused by the Oga Peninsula Earthquake in 1939. The area damaged by this earthquake of magnitude 7.0 was limited only near the Oga Peninsula, but the rather severe earthquake motion caused damage on 74 small-scale earth dams and destroyed 12 earth dams. The dams collapsed after several to 24 hours, although a few of them collapsed during the earthquake. The majority of these dam bodies were made of sandy soil.

Information on damages of dams collected by the Public Works Research Institute was compiled in reports after the Niigata Earthquake in 1964. Damages caused by major earthquakes after the Niigata Earthquake are summarized below based on the Annual Reports of the Public Works Research Institute and the Annual Reports of the Society of Civil Engineer.

Niigata Earthquake (1964)

The Niigata Earthquake occurred on June 16, 1964. The epicenter was near Awashima and the magnitude was 7.5. The damaged area extended to 9 prefectures centering around Niigata and Yamagata.

There were approximately 8,700 earth dams within the region of more than IV seismic intensity in the Niigata Earthquake (Yamagata, Niigata, Akita, Fukushima, Iwate and Miyagi). Some 1.5% of them (146 dams) were damaged. Among these 146 dams, 7 dams collapsed and 7 dams could no longer store water. The causes can be roughly grouped into two categories²⁰⁾.

1) The dam bodies were eroded and came to collapse because of water leakage due to cracking or sliding of the dam bodies.

2) The dam bodies were eroded and came to collapse because of water leakage due to cracking or buckling of inclined and bottom conduits.

A brief summary of the survey results is:

1. The higher the dam height is, the more serious the damage is. (Table-4.4)
2. With respect to gradient of up and downstream slopes, the damage becomes more serious in the order of 1. steep upstream and gentle downstream, 2. same at the up and downstream, and 3. gentle upstream and steep downstream (Table-4.5).
3. Dams with steep down and upstream slope have a lower damage rate. This seems to be related with the lower height for steep slopes and the propriety of materials.

Table-4.4 Dam height and damage

Dam height (m)	Dam	Damage	Percentage (%)
1 ~ 4	310	23	7
5 ~ 9	410	43	10
10 ~ 14	94	14	15
15 ~ 19	28	9	32
20 ~	60	4	6.7
Total	848	93	11

Table-4.5 Gradient of up stream and down stream slopes and damage

	Shape	Dam	Percentage (A)	Percentage of damage (B)	B/A
Loose upstream slope Steep downstream slope		178	69%	61%	0.89
Same gradient		46	18	18	1.02
Steep upstream slope Loose downstream slope		34	13	21	1.54
Total		258	100	100	1.00

4. As to water level during the earthquake, when it is 50 ~ 60% and 90 ~ 100% of full water level, the damage rate becomes highest, while the rate gets lower when water level is 60 ~ 80% and below 40%. This clearly indicates the relation of water level during an earthquake in a stability calculation.

5. Rather new dams less than 10 years after completion suffered more damages, and dams with repairment record show still higher damage rate.

At that time, there were a few high rockfill dams in the Uetsu Region. At the damsite of Minase Dam about 145km from the focus, the seismic intensity was V. This dam, a concrete facing rockfill dam, was exposed to the Niigata Earthquake after 1 year and 4 months from the completion. Although leakage increased temporarily due to the earthquake, it turned back to the original state after 2 days. The crest settled by 26cm and displaced by 15cm towards the downstream because of the earthquake.

Behaviors of the concrete dams are shown in Table-4.6³⁰⁾.

Tokachi-oki Earthquake (1968)

The Tokachi-oki Earthquake on May 16, 1968 had magnitude 7.9 with the focus at the eastern offing of Aomori Prefecture.

The number of completely and half destroyed buildings and houses reached approximately 4,000³¹⁾.

Unusual rainfall was recorded totalling 100~211mm for 4 days before the earthquake. Thus, earth structures were in a condition of high water content. In particular, earth structures including banking of the Tohoku Railroad Line which had just completed were seriously damaged. Table-4.7 summarizes the damage on earth dams by cause. As shown in this table, sliding towards upstream and cracking account for a high percentage. The cause of the collapse of these dams is assumed to be the

Table-4.6 Behaviors of concrete dams during the Niigata Earthquake

Dam	Hight (m)	Water level during earthquake	Epicentral distance (Km)	Intensity	Body	Leakage	Remarks
Sugano	42.5	—	74	IV	Each joint seemed to open	Brown muddy water 0.3l/min. before earthquake 1.7l/min. after earthquake	Falling stones along the route Waves on the surface of reservoir
Kijiyama (Hollow gravity)	46.0	41	67	IV	① Crack on the footing of diamond head and web ② Each joint seemed to open	Decrease and increase of leakage are partially confirmed. Total volume of leakage was unchangeable.	Falling stones along the route. No collapse around the reservoir.
Miomote	87.5	80	47	V		42l/min. before earthquake. 230l/min. after earthquake. 60l/min. 4days after earthquake. New leakage from grouting pipe in the middle level.	Falling stones
Hana-yama	47.8	37	148	V		5.1l/min. before earthquake 14l/min. after earthquake	Temporary filthy water
Arazawa	61.0	53	51			Changeless. Partial disappearance of seepage water.	Devotion of the house of transmitting station

Table-4.7 Disaster of earth dam caused by the Tokachi-oki Earthquake

Phenomenon	Dam Height		More than 10m		Less than 10m	
	Disaster	%	Disaster	%	Disaster	%
Collapse by sliding	9	10.5	1	12.5		
Sliding of upstream slope	21	24.7	4	50.0		
Sliding of downstream slope	10	11.7	0	0		
Sliding of upstream and downstream slopes	4	4.7	0	0		
Cracks in body	24	28.2	1	12.5		
Settlement of body	7	8.2	1	12.5		
Damage of intake facility	22	25.8	2	25.0		

bodies loosened by the earthquake motion resulting in sliding and subsidence which lead to the decrease of freeboard and the overflow of reservoir water in full level over the crest due to 4 days precipitation before the earthquake. The collapse occurred at the lowest place of the gorge of the old river bed, where the weak stratum of the bedrock was thicker than others with the maximum cross section of dam body. In case of the Niigata Earthquake, cracking led to piping and the dams collapsed after a while, but in the Tokachi-oki Earthquake, sliding took place in a moment during the earthquake.

The earthquake record of the Tokachi-oki Earthquake was measured at Sannokai Dam and Shijushida Dam, where the maximum accelerations at crests were 158 gal and 93 gal, respectively.

Oita-ken-chubu Earthquake (1975)

This Oita-ken-chubu Earthquake with magnitude 6.4 occurred on April 21, 1975 in the middle part of Oita Prefecture. Behavior of two dams located in the middle part of Oita Prefecture is explained³²⁾.

Yamashita Ike Dam, an earth type dam with the cross section shown in Fig.-4.21, was completed in 1918. The damage caused by the earthquake included a vertical crack along the dam axis of the crest, a horizontal crack from the right abutment, and a crack on the stone block of upstream slope. The gradient of the upstream slope changes suddenly at about 1.5m down from the crest.

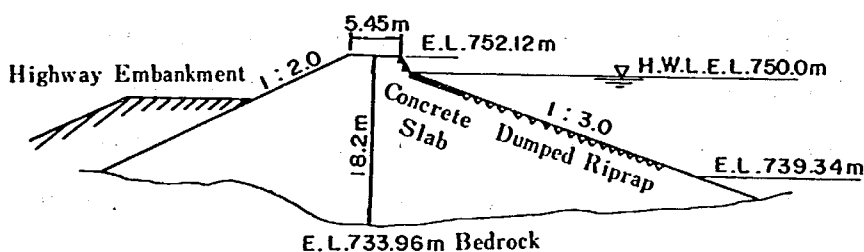


Fig.-4.21 Standard cross section of Yamashita-Ike

It is assumed that the vertical crack was mainly created by the influence of this sudden change in cross section. The width and the depth of the vertical crack must be 20cm at maximum and less than 1.5m, respectively. The horizontal crack from the right abutment is considered to be caused by the differences in vibration and deformation between the damsite and the dam body. The width and the depth of

the crack were about 10cm and 1.5m, respectively. The reservoir was under full water level (2m below the top) during the earthquake, and no leakage through the horizontal crack was observed. The upstream face is stone block, where concrete slab is partly placed. Cracks were created in all over the slab. Assumedly, since the stone block and the slab are more rigid than the dam body, they could not follow the deformation of the dam body. The material of the dam body is the loam in the neighborhood, and the dam is homogeneous type without core zone. The dam foundation is slightly weathered lava of rather new age.

The major direction of the earthquake motion is assumed to be close to the dam axis judging from the state of distruction of a collapsed hotel in the neighborhood.

After the earthquake, the water level was lowered down by 3m at the rate of 20~30cm/day for repairing the cracks on the body. Boring was conducted to investigate the distribution of the bedrock, the seepage line, and the materials of the dam body. The result indicates no abnormality on the seepage line.

Serikawa Dam completed in 1956 in Seri River of the Oita River System is a concrete gravity dam with 52.2m height. No abnormality was recognized on the dam body, but the lighting lamps in the garden of the Dam Control Office (RC2F) on the left bank were damaged.

As grave stones on the grave yard on the right abutment turned over, the acceleration is assumed to be 200~300 gals. The water level during the earthquake was 43.1m, 7.5m lower than the design maximum water level.

There is an inspection gallery on the left abutment, where leakage of 3.3 ℓ /min. before the earthquake increased to 8.1 ℓ /min. after the earthquake, and since then it gradually diminished to a stable stage. The left abutment is a lean ridge on a sedimentary formation, and a rim tunnel to stop water passes through the ground. Leakage took place through this tunnel, and the earthquake is assumed to influence the infiltration route on the natural ground.

Yamashita Ike Dam and the Serikawa Dam are located about 8km and 20km away from the focus, respectively.

Miyagi-ken-oki Earthquake (1978)

At the time of the Miyagi-ken-oki Earthquake, the number of existing dams (more than 15m in height) by type in the Tohoku Region is³³⁾ as follows:

Total number of dams: 198 (in the jurisdiction of the Tohoku Regional Construction

Bureau. Namely, dams located in Aomori, Akita, Iwate, Yamagata, and Miyagi prefectures and dams excluding those of Agano River system in Fukushima Prefecture).

Concrete dam: 62 (gravity type 57, hollow gravity type 2, and arch type 3)

Fill dam: 133 (earth 127, rockfill 4, and surface cut-off wall type 2).

Partial concrete dam and partial earth dam (combined dam): 3

After the earthquake, the supervisors conducted a temporary check of the dams, and the result was summarized by the Tohoku Regional Construction Bureau. The checked items include:

(a) Dam body

- a. The presence of leakage and changes in the amount of leakage.
- b. Cracking on the concrete surface.
- c. Cracking on the crest.
- d. Changes on the surface cut-off walls and riplaps.
- e. Others

(b) The natural ground in the neighborhood of the abutment and the natural ground around the reservoir.

- a. Leakage
- b. Cracking
- c. Breakdown
- d. Land sliding

The result of this check confirmed that there was no abnormality in 192 dams, and comments were made on 6 dams listed in Table-4.8. The comments were made in details in the followings.

Table-4.8 Commented dams

Dam	Type	Epicentral distance (km)	Max. acceleration at base (gal)
Ushino	Rockfill	122	(93)*
Hidatani	Earth	122	(93)
Magosawa	Earth	128	(87)
Minase	Rockfill with impervious surface	167	48 (54)
Hiruzawa	Earth	170	(53)
Tōbe	Concrete gravity	288	(13)

* Parenthesis means estimated value after Tamura and Okamoto.

(1) Ushino Dam

The standard cross section of the dam is shown in Fig.-4.22. With respect to the deformation of the dam, the cracks extended to 72m towards the dam axis on the upstream slope and the stone pitching section.

This dam is inclined core type rockfill dam. The upstream slope is built with stones in order to protect the core, and unscreened gravel is placed by 20 ~ 30cm between the stone pitching and the core as a filter. It is assumed that cracking took place because the rigidity of the dam body and the stone section, namely the stone pitching could not follow the deformation of the dam body. The Ojojihara Compensation Construction Office of the Miyagi Prefecture turned over a part of the stones for survey and dug the unscreened gravel to expose the core. Cracking did not reach the core with high density and no influence on the waterproof property is recognized.

(2) Hirutataki Dam

The cross section obtained through actual measurement (conducted by the Nanakita Minamikawa Dam Construction Office of Miyagi Pref.) is shown in Fig.-4.23.

The deformation of the dam includes cracking towards the dam axis at the crest (near the center of the dam) and cracking towards the up and downstream (the part of the dam body near both right and left abutments). A sketch of these cracks is shown in Fig.-4.24. As shown in Fig.-4.24(a), the longest crack towards the dam axis is 17m in length and 4.5cm in width, while the crack towards the up and downstream is 4.5m in length and about 1.5cm in width. In order to trace the depth of this crack, the Nanakita Minamikawa Dam Construction Office poured cement milk in the crack and then trenched to observe the distribution of the cement milk using phenol phthalain. As a result, the depth of the crack was approximately 40 ~ 60cm, as shown in Fig.-4.24(b).

An increase or a decrease in leakage before and after the earthquake cannot be judged because of too low accuracy of the measurement.

The base rock is tuff of the Yumoto Formation in the Miocene and the base rock observed at the abutment is the low welded tuff.

The distance between joints is approximately 1m. Judging from the construction stage of this dam and a field investigation, the material of this dam is in low quality and compacted insufficiently, unlike recent fill dams.

(3) Minase Dam

In this dam, the amount of leakage increased by about 5% before and after the

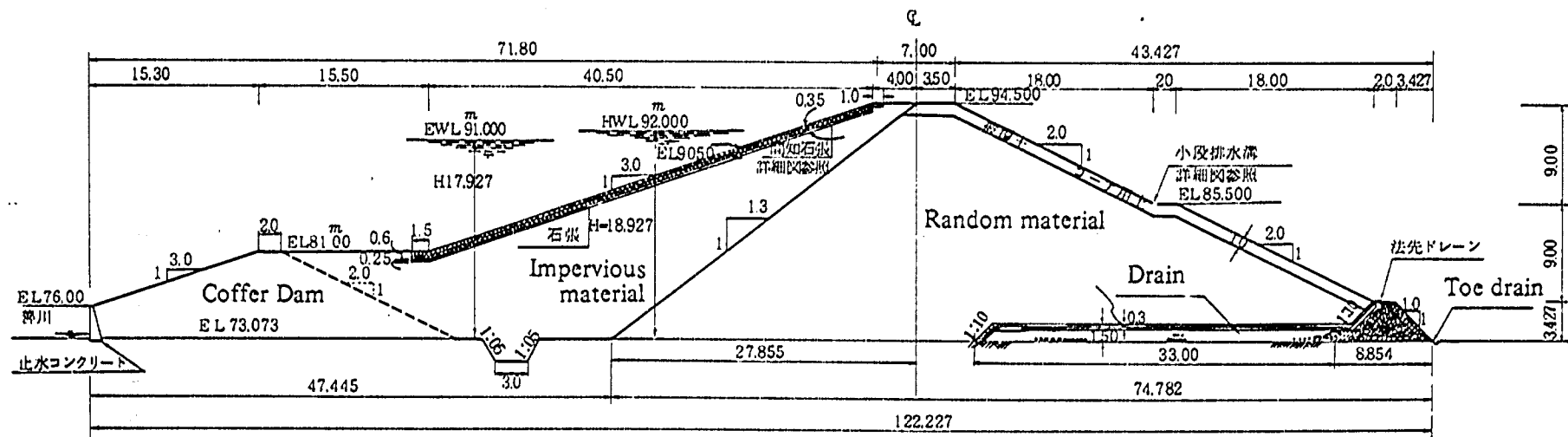


Fig.-4.22 Standard cross section of Ushino Dam

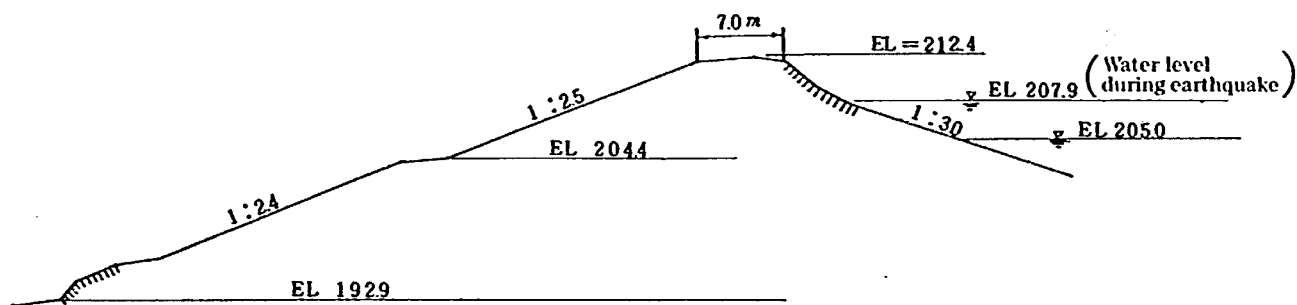


Fig.-4.23 Surveyed cross section of Hidataki Dam

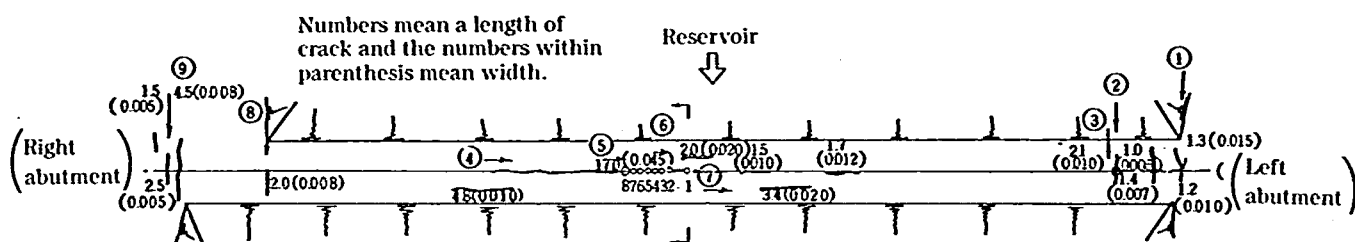


Fig.-4.24(a) Cracks at crest

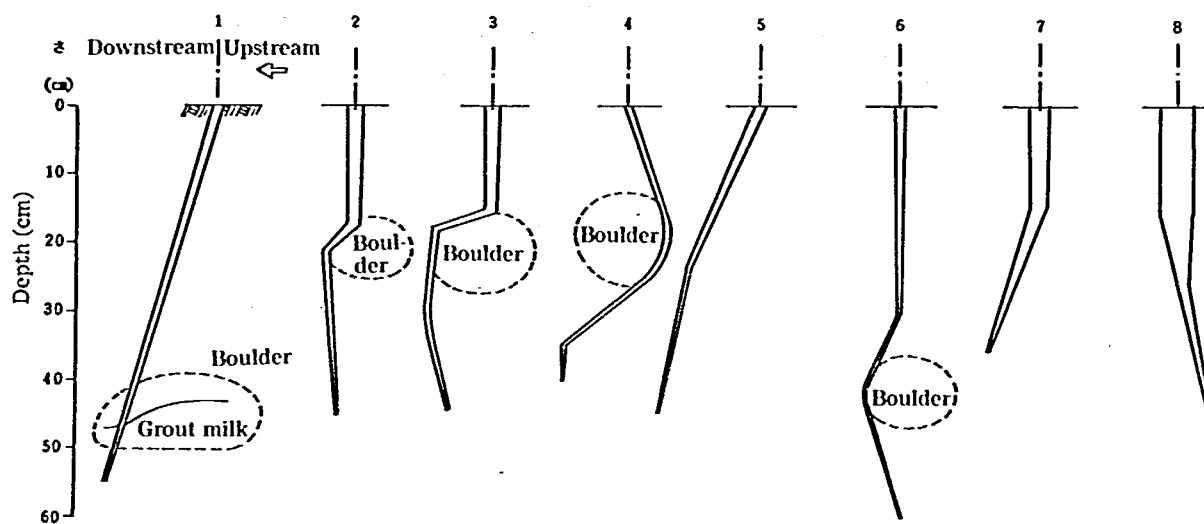


Fig.-4.24(b) Sketch of cracks of Hidataki dam

earthquake. Surface cut-off wall dam with a concrete slab is generally high in leakage, and has an inclination to be repaired regularly.

(4) Hirusawa Dam

The base rock is comprised of tuff of the Furuguchi Formation in the Miocene and sediment of about 15m thickness deposits on the river bed. The construction of this dam was started in 1935 by the Ministry of Agriculture and Forestry, and completed in 1948. The river bed sediment within reservoir area was used as the fill material, and fine grained materials in the sediment were selectively used for the cut-off section. The dam was built through compaction by wooden tools after transportation of the materials by straw baskets. Grouting was conducted on the core foundation which was very rare at that time (1937 ~ 1938).

Reservoir filling after the completion of the dam in 1948 disclosed high leakage, and thus grouting mainly by water-glass was conducted to the cut-off zone.

The Niigata Earthquake in 1964 (the epicentral distance of 100km) caused cracks along the dam axis nearly all over the crest length of about 200m. After this earthquake, grouting by pozolis cement was made within the core for repair.

During this earthquake, the water level of the reservoir was EL270.0m, and the reservoir was almost empty (about 10% of the total storage, 200,000 tons). The earthquake caused cracking, which extended to the nearly whole length of the crest along the dam axis, 1.5 ~ 2.0m towards the center from the upstream end.

After the earthquake, the Regional Office of Yamagata Pref. dug a pit (6.5m deep) to investigate the depth of the cracks and confirmed the depth up to 5.25m. The width of the cracks was about 10cm at 2 ~ 3m depth.

The reasons for cracking in the Niigata Earthquake and the earthquake of this time must be: the loose sediment is left in place other than the core zone; the material of the dam body is not uniform; the dam material has low plasticity and compaction is not sufficient; and the ability of deformation of the dam body is rather low due to grouting inside the dam body. However, these reasons are only assumptions, and correct reasons for the damage should be judged after testing and analyzing.

(5) Tōbe Dam

The changes of this dam include that the leakage from joints between 12 ~ 13 blocks temporarily increases after the earthquake, and draining through the drains within the dam body temporarily stopped. However, both of the leakage and the stoppage turned back to the previous stage at 8.00 a.m. on June 13. Leakage before and after

an earthquake at a gravity dam (through drain holes and joints) is very often observed, and the change of leakage of this amount does not cause any serious problem. However, it is rather rare that the change in leakage was recognized inspite of the long epicentral distance of 300km.

The earthquake-resistance in this earthquake is examined by dam types.

i) Fill dam

One of the conditions below or a combination of them might cause damage by an earthquake (Hirusawa, Hirutataki)

a. When roller compaction is not conducted with a heavy machine, b. When a material is not good in particular, a core material near the crest has low plasticity, and c. Loose bed sediment is left in the foundation. When a fill dam is covered with surface facing, stone pitching and concrete block which have rigidity different from the dam body, these covers cannot follow the deformation of the dam body during the earthquake, resulting in the possible occurrence of cracks (Magosawa, Minase, Ushino and Kawarago).

There were a few rockfull dams (4 dams) in the Tohoku Region at that time, but like Tarumizu Dam the acceleration of 236 gal did not cause any abnormality in the foundation (seismic coefficient=0.15), which might indicate the high stability of rockfill dams during an earthquake.

ii) Concrete dam

There were 62 concrete dams in total, of which 61 dams observed no change. Tōbe Dam experienced a temporary increase in leakage through the transversal joints, but this leakage did not affect the stability of the dam body.

Nihon-kai-chubu Earthquake (1983)³⁴⁾.

This earthquake occurring on May 26, 1983, was magnitude of 7.7 with its epicenter at 100km offshore Noshiro, Akita Prefecture. The earthquake motion at the dam site had a long period (about 1.0 sec) and the duration was 80 seconds.

With respect to dams, damage was relatively minor as a whole. As shown in Table-4.9, the damage and the changes mainly include cracking on crests or slopes of earth dams. Here, a dam means the structure built for water storage with the height of more than 15m. (Therefore, sabo dam and tailing dam are excluded). Among the dams shown in Table-4.9, the Narakan reservoir in Aomori Prefecture with 14.6m-

height does not fall into this category, but was included in the table due to nearly 15m-height. Table-4.9 was prepared based on the reports made by Aomori and Akita Prefectures. Shinkodoroku, Yazurazawa, Yunosawa and Subari Dams are described in details.

(1) Shinkodoroku Dam

The crest length of this dam is 294m, and its upstream slope is covered with riprap at the lower altitude, and with concrete block at the higher altitude. The damage includes three cracks almost parallel to the dam axis within the area of 10m width and 115m length of the upstream riprap section. The depth of the cracks is shallow only in the riprap section. According to a personnel at the control office, a very big wave occurred on the surface of reservoir immediately after the earthquake and the filling stones were sucked out from the cracks in the riprap. A part of the block cover on the higher altitude was broken into pieces through running against (about 1cm). The change in leakage before and after the earthquake was from 6l/min. to 8l/min., but the leakage decreased partly because of lowered water level. Measurements conducted before and after the earthquake confirmed to permanent deformation (conducted on June 2).

(2) Yunosawa Dam

This dam with 87m in crest length is a small-scale earth dam built in a fairly narrow valley. The damage includes cracks of 34m length, 15cm width and more than 15m depth on the crest, and cracks of 30~50cm depth on the upstream slope riprap section. The riprap is tuff and, tuff sand stone of 30~50cm in diameter and the dam material is weathered tuff. The downstream slope is in a saturated condition extending 1m in height and 13m in width. This part was covered by moss, which suggests that the saturated portion had been already exposed at the slope end before the earthquake. Water intake is performed through inclined conduits and bottom conduits, but no change in appearance was recognized.

(3) Yazurazawa Dam

This dam with 267m in crest length is located in a fairly wide valley compared with the dam height.

This dam was improved in 1970 in the framework of the prefectural large-scale old reservoir repair project. The details of the improvement are:

1. Heightening of the dam (to improve the insufficient freeboard)
2. Curtain grouting of the dam body (to prevent leakage)

3. Block covering on the upstream slope.
4. Complete repair of the spillway and intake facilities.

The damage includes cracks extending to 34m long with maximum 5cm width on the crest.

At the downstream slope end, three vinyl chloride pipes were installed in a piping hole caused by leakage in order to collect leakage water. This indicates the existence of concentration of leakage before and the repair of the dam body by grouting in 1971 was unable to stop the leakage, thus the vinyl chloride pipes were used to collect leakage. It is necessary to investigate the relation between the spots of the cracks on the crest and the location of grouting on the dam body.

(4) Subari Dam

This dam is a only concrete dam among dams listed in Table-4.9. The change includes leakage from the joint between block No. 4 and 5. The observation from the inspection gallery suggested that this joint was opened and closed slightly during the earthquake (a new crack was observed on the surface where free lime adheres). The amount of leakage from this joint was about 6 l/min. The cause of the leakage must be either the damage of waterstop caused by the opening and closing of the joint, or holing of the percolation conduit clogged with some material. Since a gravity dam is originally in two-dimensional design, leakage from a transversal joint does not relate to the stability. Thus, it is good enough to observe for a while.

Table-4.9 Disasters caused by the Nihon-kai-chubu Earthquake

Dam	Location	Type	Height (m)	Storage ($\times 10^3 \text{m}^3$) capacity	Management	Remarks
Shin-kodo roku	Aomori	Homogeneous earth	21.8	1,750	Department of agriculture and forestry of prefectural authority	Cracks on upstream riprap block
Narakan	Aomori	Homogeneous earth	14.6	360	Federation of land improvement	Cracks on crest
Hongou	Aomori	Homogeneous earth	18.5	200	Federation of land improvement	Settlement at crest. Cracks on upstream slope
Yazurazawa	Akita	Homogeneous earth	16.5	681	Federation of land improvement	Cracks on crest
Yunosawa	Akita	Homogeneous earth	19.4	200	Federation of land improvement	Cracks on crest and upstream slope
Subari	Akita	Concrete gravity	72.0	39,500	Department of public works of prefectural authority	Lakage from lateral joint

(5) Others

As described above, this earthquake did not cause overflow of reservoir water into the downstream, and it can be concluded that there was no serious damage, although partial repair might be necessary.

In some earth dams of less than 15m height, the damage near to collapse was observed (such as Kansukenuma, Aomori Prefecture), but since the design and construction criteria of these structures are different from those of dams, damages cannot be treated equally.

Nagano-ken-seibu Earthquake (1984)

This earthquake occurred on September 14, 1984, with magnitude of 6.8 and caused serious damage in the southern part of Mt. Ontake. The Makio Dam (rockfill type) located near the epicenter was subjected to the strong earthquake motion, which caused cracks of about 1.5m deep on the crest and loosening on the top of slope.

However, the stability was basically kept. The observatory data during the aftershock suggest Nagano-ken-seibu Earthquake has a very short period and the maximum acceleration of 0.5g or more with a short duration of 10 ~ 15 seconds.

4.4 Damages of Dams in Other Countries

In the previous section, the behaviors of dams in Japan during the past earthquakes have been discussed. In this section, the behaviors of dams in other countries during big earthquakes are quoted from the references shown below. When we try to learn a lesson on earthquake-resistant design of Japanese dams from the earthquake damages in foreign dams, it is necessary to take two factors into consideration; first, foreign dams are designed on different criteria, and materials and construction methods are different from those in Japan, and second, investigation and analysis of earthquake damages are apt to depend on the judgement of authors who wrote reports.

4.4.1 Concrete dams

Dams which were exposed to acceleration of more than 0.1g and recorded behaviors during the earthquakes in papers are 17 dams in 9 countries of the world as listed in Table-4.10. Almost all types of concrete dams: arch type; buttress multiple arch; and gravity type are listed in Table-4.10. 8 dams in Table-4.10 are more than 100m

Table-4.10 Concrete dams received strong earthquake motion (Hansen-Roehm)

Dam Type—Year of Completion	Nation	Dam Height	Crest length	Earthquake	Epicentral distance (Km)	Magnitude	Observed Maximum Acceleration	Remarks
Lower Crystal Springs Curved gravity—1890	USA	47	183	1906.4.18 San Francisco Earthquake	0.4	8.3		No crack
Hoover Curved gravity—1936	USA	221	379	Many Earthquakes after 1936	<8	Max. is 5.0		No damage
Honen-ike Multiple Arch—1930	Japan	30	145	1946.12.21 Nankai Earthquake		8.1		Crack at arch near buttress, Improvement by Grout
Blackbrook Gravity—1900	England	30.5	147	1857.2.11	6.4			Crack on the surface of downstream masonry
Hsintengkiang Buttress—1959	China	105	440	1962.3.19	5.0	6.1	0.54g at the after shock	Crack at crest
Monteynard Arch—1962	France	155	215	1963.4.23		Max. is 4.9		No damage
Kariba Arch—1959	Rhodesia	128	617	1963.9.23		Max. is 6.1		No damage
Kurobe Arch—1960	Japan	186	475	1963 ~ 1970 Many Earthquakes	<10	Max. is 5.0		No damage
Koyna Straight—1963	India	103	835	1967.12.11	3	6.5	0.63g (Dam axis) 0.49g (Up and Down stream)	Cracks on up and downstream slope
Crandval Multiple arch—1959	France	88	400	Many Earthquakes after 1969		Intensity is V		No damage
Pacoima Arch—1927	USA	113	180	1971.2.9 San Fernando Earthquake	5	6.6	1.25	No crack at arch, opening of joint between arch and thrust block
Santa Anita Arch—1927	USA	76	204	1971.2.9 San Fernando Earthquake	27	6.6	0.17	No damage
Big Tujunga Arch—1931	USA	76	122	1971.2.9 San Fernando Earthquake	32	6.6		No damage
Ambicsta Arch—1956	Italy	59	145	1976.5.6 Gemona- Friuli Earthquake	22	6.5	0.33	No damage
Maina di Sauris Arch—1952	Italy	136	138	1976.5.6 Gemona- Friuli Earthquake	43	6.5		No damage
Barcis Arch—1955	Italy	50	84	1976.5.6 Gemona- Friuli Earthquake	48	6.5		No damage
Poiana Usului Buttress—1969	Roman	80	527	1977.3.4	60	7.2		No damage

high. Investigations and analysis were conducted on 6 dams after earthquakes. Results were recorded, and briefly explained in the table.

(1) Lower Cristal Spring Dam³⁵⁾

This concrete dam is a curved gravity type with 194.2m curvature. Its standard cross section is 13.1m wide at the top and 53.6m at the foundation in thickness. The area of the reservoir is $67 \times 10^6 \text{m}^3$. The dam completed in 1887 to 1890 is for water supply to San Francisco City. The San Francisco Earthquake of magnitude 8.3 in 1906 did not cause any damage on the dam located about 340m away from the San Andreas Fault. The San Andreas Fault in this area slid by 2.4m.

Since then, a dynamic analysis was conducted by the dam manager about dam safety evaluation, which indicated that when the maximum acceleration of 0.6 was input, the maximum tensile stress on the dam body was 41kg/cm^2 , but this value was lower than the allowable tensile strength of concrete.

(2) Blackbrook Dam³⁶⁾

This dam completed in 1900 is a gravity type with 30.5m in height, which is a single dam damaged by earthquake in England. The upstream slope is covered with bricks, while the downstream slope is covered with stones. In 1957 an earthquake occurred at 6.4km to the epicentral distance, and the dam was exposed to an earthquake motion of English seismic intensity 8 degree (maximum 10), but no damage was caused. The changes include falling down of the stone balustrade at the crest, cracking on the stone arch of the spillway, and cracking on the stones on the downstream slope. Whether the cracks on the downstream slope stones reached the concrete body or not is unknown.

Leakage from the draining holes increased after the earthquake, but then continued to decrease to the original stage for several months after the earthquake. It was judged that the increase in leakage was attributable to the foundation but not to the dam body.

(3) Koyna Dam³⁶⁾

This dam with design seismic coefficient of 0.05g was subjected to an earthquake motion with nearly ten times as large acceleration as the design value during the Koyna Earthquake in 1967. This dam constructed in 1954 to 1963 has a cross section failing the middle third principle for usual gravity dams, as shown in Fig.-4.10. The magnitude of the Koyna Earthquake was 6.5, and a fault passes 3km away from the dam. Its epicentral distance and its focal depth are assumed to be 13km and 8~13km,

respectively. In Koynanagar Town near the dam, the killed and the injured amounted to 180 and 2,200 persons, respectively.

The maximum acceleration of the earthquake motion recorded at the inspection gallery located 10m above the dam foundation was 0.63g towards the horizontal dam axis, 0.49g towards horizontal up-down stream, and 0.34g towards vertical direction. No overflow to the downstream area due to lowered water level of the reservoir was caused by this earthquake. However, 18 and 7 marked horizontal cracks were caused on the upstream slope and the downstream slope, respectively, in the non-overflow section. On the contrary, cracking did not take place in the overflow section. The result of stress analysis during the earthquake proved that the location of the cracks corresponded to the location where strong tensile stress was generated. This indicates that the analysis can explain the phenomenon. After the earthquake, buttress was installed at the downstream section in the non-overflow section for reinforcement.

(4) Pacoima Dam³⁷⁾

This dam completed in 1929 is an arch type with 113m in height and 180m in crest length. This dam was exposed to the strongest earthquake motion considering the past recorded accelerations at other dam sites. The San Fernando Earthquake in 1971 with magnitude 6.6 caused thrust fault running 5km below the dam. The epicenter was 6.4km north of the dam and the focal depth was 13km.

Acceleration data measured at the abutment 16m higher than the crest indicated 1.25g horizontally and 0.7g vertically. Even with such a strong earthquake motion, no crack and opening between blocks were recognized in the dam body itself, but the only change was opening of the joints between the thrust block and the dam body on the left abutment by 1cm extending 16m.

According to a dynamic analysis on the assumption of the San Fernando Earthquake in 1971, the maximum tensile stress is $53 \sim 75 \text{ kg/cm}^2$ on the dam body. In order to relate this analytical result to the fact of no damage on the dam body, the hypotheses described below have to be met.

- With respect to a stress with fast strain velocity in the earthquake, the tensile strength of concrete has to be high.
- The base rock absorbs a part of the energy generated by the earthquake motion.
- The concrete produces much higher final strength than the assumed one from the elastic theory due to its elastic property.

Since the vertical joints is expected to have lower tensile strength, the joint release

tension towards the arch.

(5) Hsinfenkiang Dam³⁶⁾

Originally, this dam was so designed that it could bear VI of the Modified Mercalli Scale and the dam was reinforced in 1961 to resist VII of the seismic scale (since there had been occasional earthquakes after the initiation of water filling in 1959). In 1962, the dam was subjected to an earthquake of magnitude 6.1 and the aftershock of magnitude 4.5 made the record of 0.54g acceleration at the crest. Although the dam body was not damaged to cause overflow, a horizontal crack was extended to 82m long on the downstream slope where the gradient changes sharply. A stress analysis conducted after the earthquake proved this phenomenon in calculation. The location of the crack coincided with where strong tensile stress was created. This part was reinforced later.

4.4.2 Fill dams

(1) Behaviors of fill dams during the San Francisco Earthquake in 1906³⁸⁾

This earthquake resulted in sliding of the San Andreas Fault extending 435km. 33 earth dams, which height ranged from 4.5m to 42m, existed within 60km from the fault. Since the magnitude of the earthquake is $8\frac{1}{4}$, the earthquake motion at the bedrock was assumed to have 0.4 seconds of predominant period, more than 0.6g of maximum acceleration and about 60 seconds of duration. It should be noted that the exposure to such a strong motion did not cause any serious damage (cracking at 5 dams). The primary reason for no serious damage on 32 dams in the strong earthquake must be the cohesive soil used as a dam body material.

(2) Behaviors of fill dams during the San Fernando Earthquake of 1971.

In the San Fernando Earthquake with magnitude 6.6, 44 fill dams (9~57m in height) located within 40km from the epicenter were exposed to the maximum acceleration of 0.2~0.7g. The reservoirs of 13 dams among 44 were empty (not in use) and the remaining 31 dams stored water and functioned at that time. No damage was caused on the 26 "rolled fill" dams, while 2 of 5 "hydraulic fill" dams were damaged. The reason for the damage was that the strength of the earthquake motion and its duration increased pore water pressure inside the dam body and thus caused liquefaction at a part of the dam body. However, the fact that no damage was caused on 3 dams of 5 hydraulic fill dams suggests that hydraulic fill dams are safe against an earthquake motion of approximately 0.2g.

4.4.3 Summary

(1) Concrete dams

Historically, there was no serious damage on all types of concrete dam including gravity type, arch type and buttress type in the past strong earthquakes. This fact shows that concrete dams are very strong structures against earthquake. Among of them, it should be noted that the Lower Crystal Springs Dam located only 340m from the earthquake fault (magnitude 8.3) was not damaged. In order to explain the fact that no damage was caused in spite of the large calculated dynamic tensile stress obtained from the dynamic analysis of concrete dams exposed to the strong earthquake, the hypotheses listed below have to be met:

- Concrete dams, like fill dams, show nonelastic property during the strong earthquake, and the actual stress is smaller than the calculated value.
- The dispersion energy at the base rock is larger than the hypothetical value used in the calculation.
- The dynamic tensile strength of concrete is fairly large.
- The vibration energy is dissipated at joints.

The above-mentioned points have to be further investigated as a research subject in dynamic analysis of concrete dams.

Concrete dams which experienced changes include the Koyna Dam, Hsintengkiang Dam and others. In case of the Koyna Dam, the primary problem lies in the design where the basic triangular cross-section was not provided by the middle third, a basic and original design concept for a gravity dam was not met in the non-overflow section. As for the horizontal cracks on the sharply changing cross section of the high altitude (part), concentration of stress on that part is confirmed by the calculation.

For the Hsintengkiang Dam, the cracks occurred on the sharply changing cross section on the high altitude, and the reason confirmed by both calculation and experiments was the concentration of stress resulted from the sudden change in the cross section as the Koyna Dam.

(2) Fill dams

When fill dams are classified into 4 types by materials and construction methods; a. hydraulic fill dam, b. cohesive soil earth dam, c. central core type rockfill dam and d. surface face rockfill dam, the behaviors of each type can be summarized as follows:

- a) Hydraulic fill dam: Since the body material is usually silt, the pore pressure

increases during an earthquake and the material tends to lose its strength, thus it is subject to damages. In many cases, however, no damage has been resulted from an earthquake motion of about 0.2g maximum acceleration. In Japan, there is no fill dam built by this method.

- b) Cohesive soil earth dam: The San Francisco Earthquake in 1906 did not cause any serious damage on this kind of earth dams with the height up to 30m, except small cracks. Cracking on the crest was reported for this type of dams located within 100km from epicenter in two earthquakes of magnitude 7.1 and 7.5 in Chili. In any case, no serious damage such as overflow occurred.
- c) Central core type rockfill dam: There are few cases of damages on this type in foreign countries. No serious damage was caused during the earthquake in Chili.
- d) Surface face rockfill dam: In the San Francisco Earthquake in 1906, earth dams with surface cut-off wall of low height showed safe behaviors, but a part of the dam body of the Cogoti Dam in Chili (surface cut-off wall type rockfill dam with 82.7m in height) located 89km from the epicenter slid with 60cm subsidence in an earthquake of magnitude 7.9. However, this damage was not so serious as to cause overflow.

As a conclusion, it can be said that fill dams excluding hydraulic fill dams show fairly satisfactory behaviors during an earthquake. However, it is in a foreign country that discharge pipes burried within the dam body were broken by the earthquake and thus caused piping, which led to the eventual collapse of the dam body. In Japan, it is prohibited to install discharge pipes and spillways on a fill dam body, and scrupulous care has been given to the earthquake-resistant structures conclusively.

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BEHAVIOUR OF DAMS
DURING EARTHQUAKES

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要旨

ダムの耐震設計のために、地震および地震動の特性、1
自由度系の振動理論、地震時におけるダムの挙動をとりま
とめた。

キーワード：地震、地震動、ダム、振動理論、応答



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建設省土木研究所

フィルダム研究室 御中

茨城県つくば市大字旭一番地

No. 4

No. 4

配布先氏名

配布先住所



土木研究所資料第2683号 部

上記のとおり受領しました。

所 属(住所)

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