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**Pavements on Swelling and Shrinking
Clay Subgrades**

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**Pavement Division
Road Department
Public Works Research Institute
Ministry of Construction
1, Asahi, Tsukuba-shi, Ibaraki-ken, 305 Japan**

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**Pavements on Swelling and Shrinking
Clay Subgrades**

by

**Binod Sapkota*, STA Fellow, Pavement Division
Toshiyuki Nakamura, Head, Pavement Division
and
Takaharu Kiriya, Researcher, Pavement Division**

Synopsis:

This is a study on the response of a seal coat surfaced unbound granular pavement when subjected to subgrade volume change. Swelling and shrinkage of Subgrade due to monotonic wetting and drying are considered. In the numerical analysis, thermal analogy is employed in which free swell/shrink is considered as analogous to isotropic thermal expansion/contraction. The result of analyses showed that the numerical procedure is capable to describe the response of pavement to subgrade volume change.

**Key words: Seal Coated Pavements, Swelling Clays, Soil Shrinkage,
Expansive Soils, Thermal Analogy**

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PREFACE

This comprehensive report presents the summary of theoretical, numerical and experimental investigations conducted to study the response of thin seal coated road pavement subjected to the seasonal subgrade volume change. The study was conducted at two stages. At first the effect of subgrade swelling on overlying pavement was studied. Later, it was extended to cover the effect of subgrade shrinkage. By combining these two, the work could produce a complete study on the mechanical behaviour of reactive soils subjected to the seasonal moisture variation, and the characteristic response of seal coated pavement to swelling and shrinking of subgrade upon wetting and drying.

The report is written with the idea of providing necessary information to a road design engineer to understand the problem of distortion and cracking of pavements on reactive clay subgrades. Hence significant literature review, substantial description on the theoretical and numerical analysis, and the summary of results of the earlier conducted soil and pavement tests including the test set-ups and testing procedure are covered. It is expected that these information will be sufficient to develop a background for the analysis of response of pavement to subgrade volume change.

Also, the report is written with the use of common terminology of unsaturated moisture flow and volume change characteristic of reactive soils. However, some readers may not be familiar with these terminology. For them, the writers would like to suggest to read the glossary of terms included in the preliminaries of this report.

ABSTRACT

A study on the response of a seal coat surfaced unbound granular pavement when subjected to subgrade volume change is conducted. Monotonic wetting of subgrade due to an infiltration of rain water, and monotonic drying due to an evaporation of existing moisture in subgrade are considered with the moisture flow taking place through an unsealed shoulder or sides. The study initially reviews literature on moisture flow and volume change phenomena of reactive clays. A simplified mechanical model of swell and swell suppression is applied to the response analysis of the swelling subgrade. Later this simplified model is extended to analyse the subgrade shrinkage behaviour.

Available results of lateral swell pressures measurements in the laboratory for wetting of an expansive soil from a controlled initial dry condition are used to validate the theoretical model. Also, available test results on the tensile strength characteristics of an unbound granular base with seal coat are used to assess the mechanical property of the pavement.

The result of these studies are then applied in the development of an appropriate numerical procedure to study the subgrade/pavement interaction. Thermal analogy is employed in which free swell/shrink is considered as analogous to isotropic thermal expansion/contraction. An alternating analysis of moisture flow and volume change of subgrade is considered such that the solution steps through time as series of analysis of thermal transients and stresses and strains. The subgrade strength and stiffness are modified at each interval of mechanical analysis.

The result of analyses showed that the numerical procedure is capable to describe the response of pavement to subgrade volume change. The magnitude of pavement surface heave under swell analysis, or subsidence under shrinkage analysis computed for two months wetting or drying of an extremely reactive clay subgrade are comparable with that observed in the field, and the model highlights critical parameters affecting pavement behaviour.

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Glossary of Terms

Fundamental terms used in this report to describe the moisture flow and volume change in reactive soils:

Reactive soil: any of a group of clay soils that have the tendency of volume increase with the increase in moisture content, and volume decrease with the decrease in moisture content. Also mentioned as reactive clays, these soils are usually gray brown to black in colour, and are rich in montmorillonite and illite clay minerals. Typical examples are expansive clays of the States of Texas, Colorado, etc. in the U.S., black cotton soil of India, black earth, and gray and brown clays of different parts of Australia.

Expansive soil: same as reactive soil, and also mentioned as expansive clay, swelling clay.

Shrinking soil: same as reactive soil, and also mentioned as shrinking clay.

Suction pressure: a negative pore water pressure in an unsaturated soil that develops due to the capillary action of an unsaturated soil-water system and due to the presence of solute in the soil-water. In this study the terms suction pressure and suction potential are identical and interchangeable.

Matrix suction: suction pressure developed solely due to the capillary action of soil matrix in an unsaturated soil-water system. Water in the soil-water system is the pure water. In this study the terms matrix suction and matrix potential are identical and interchangeable.

Osmotic suction: suction pressure developed due to the presence of solute in the soil-water system. Physically, it is the negative gauge pressure to which a pool of pure water must be subjected in order to be in equilibrium through a semi impermeable (i.e. permeable to water molecules only) membrane with a pool containing a solution identical in composition with soil water. In this study the terms osmotic suction and osmotic potential are identical and interchangeable.

- Total suction:** is the sum of above described matrix suction and osmotic suction. In this study the terms total suction and total potential are identical and interchangeable.
- Log total suction:** is the magnitude of total suction after taking logarithm (with base 10). In this study the terms log total suction and simply log suction are identical and interchangeable.
- Wetting front:** is a descriptive term used to refer the contour of moisture flow that wets the dry soil in a wetting process.
- Moisture flow:** is the flow of water in an unsaturated soil. In the study of unsaturated soil characteristics it is often preferred to use moisture in place of water to describe wetness and dryness. It may be because, the unsaturated soil contains relatively small quantity of water than saturated soil, and it is usually moist only not wet.
- In this study moisture flow is described in term of change in total suction rather than in crease in moisture content. Further, as the flow as well as the volume change behaviour of reactive soil is linearly related with the log off total suction, the log total suction is considered as the flow variable rather than total suction only.
- Wetting process:** in general, is the process of increasing moisture content. In terms of this study, it is the process of decreasing log suction.
- Drying process:** in general, is the process of decreasing moisture content. In terms of this study, it is the process of increasing log suction.
- Monotonic wetting/drying:** In this study, it is the wetting or drying processes that occurs with only one process wetting or drying, at a time until the final wet or dry stage is reached. For example monotonic wetting implies a continuous wetting process without stoppage and without intermediate drying.
- Hydraulic conductivity:** is same as permeability of soil in general soil mechanics term. In this report, it may be described by the term moisture

- conductivity. Both terms are considered identical and interchangeable in this study.
- Specific water capacity:** is the quantity of moisture (or water) a soil absorbs when it undergoes a unit decrease in suction or discharges when it undergoes a unit increase in suction.
- Diffusivity:** describes the variation of hydraulic conductivity with the degree of wetness. Numerically it is the ratio of hydraulic conductivity to the specific water capacity
- Swell:** is the increase in volume of reactive soil with decrease in suction (i.e. an increase in moisture content). It is general term and is applied in this study, to describe swelling under all type of boundary conditions.
- Swelling potential:** in term of volume change, it is the maximum possible volume increase a reactive clay can display under 3-dimensionally free boundary with a decrease in suction (i.e. an increase in moisture content). In term of swell pressure development, it is the maximum pressure the same reactive clay can exert against a boundary when the clay decrease suction (increases moisture content) under a confined boundaries against volume change in any direction.:
- Swell suppression:** is a reduction in potential volume increase during the wetting of a reactive soil by application of external pressure or because of confinement against deformation.
- Swell suppression modulus:** is the Young's modulus of linear deformation when the reactive soil during the wetting process undergoes swell suppression. It is often shortened as swell modulus.
- Shrinkage:** is the decrease in volume of reactive soil with an increase in suction (i.e. decrease in moisture content). It is a general term and is applied in this study, to describe shrinkage under all type of boundary conditions.
- Shrinkage potential:** in term of volume change, it is the maximum possible volume decrease a reactive clay can display in a 3-

dimensionally free boundary condition with an increase in suction (i.e. decrease in moisture content). In term of shrinkage stress development, it is the maximum tensile stress the same reactive clay can exert against a boundary when the clay increases suction (i.e. decrease moisture content) under a confined boundaries against volume change in any direction.

Shrinkage

compression: is an increase in potential volume decrease during the drying of a reactive soil by application of external pressure.

Shrinkage

compression modulus: is the Young's modulus of linear deformation when the reactive soil during the drying process undergoes shrinkage compression.

Heave:

is the upward vertical movement of ground surface or pavement surface as a result of swelling of underlying reactive clay. In terms of oedometer test condition with lateral confinement, heave is the measure of upward vertical deformation of clay sample under load free or applied load condition.

Shrinkage subsidence: similar to heave but in the downward direction due to the shrinkage of soil.

Oedometer test: this term in general is used to indicate the swelling and shrinkage tests conducted in a consolidation apparatus resulting a vertical one dimensional volume change (under lateral confinement). In this report, the term oedometer test condition is used to refer the swell and shrinkage measurement in vertical one dimension under lateral confinement. The terms consolidation test and consolidometer are used to identify the test apparatus and procedure.

CHAPTER I

INTRODUCTION

1.1 Background

A typical low cost road pavement consists of a relatively thin granular layer surfaced by a thin layer of bitumen and cover stone as a seal coat. Such pavement when constructed over a reactive clay subgrade, undergoes distress and deterioration when the subgrade: (1) wets up and swells due to infiltration of rain water or (2) dries out and shrinks due to evaporation of existing moisture from the edge of pavement. These phenomena have been widely observed in arid and semi-arid areas of the world where reactive clays of high swelling and shrinking potential are widely spread, and have resulted millions of dollars in annual cost of extra maintenance and replacement of pavement. For example, Steinberg (1985) estimated that the cost of expansive soil damage to road pavements in the United States in 1984 probably exceeded \$5 billion U.S.. Similar are the situations in Australia, South Africa, the Middle East and India where thousands of kilo metres of road run over such soils. Recognising the dimension of the problem and the need for an appropriate model that would provide a consistent description of the mechanics of the pavement/subgrade system, this research was initiated.

The assessment of strength and stiffness of reactive clays during its volume change under the change in moisture environment is extremely complicated work. Because, these soils display all the difficulties associated with their constitutive behaviour. For example, they are non-linear and hysteretic and both stress path and suction path dependent (Richards, 1992). Despite these complexities and lack of data, decisions are often made in the design of pavements in such soil profiles. In such situation there is considerable scope for relatively simple models to characterise both moisture movement and volume change behaviour of reactive soils. In this research, available simplified models are refined and extended, and used to describe the subgrade moisture flow and volume change behaviour.

1.2 Aim and Scope of Study

This research concentrates on the study of the response of seal coated unbound granular pavements to subgrade volume change. Figure 1.1 illustrates a typical situation of the problem under investigation. The work is based on the phenomenological approach to obtain the desired solution, as commonly applied in

geotechnical engineering in which material properties such as swell and shrinkage, and strength measured in the laboratory or from the field tests representing actual field conditions, are used in the analysis.

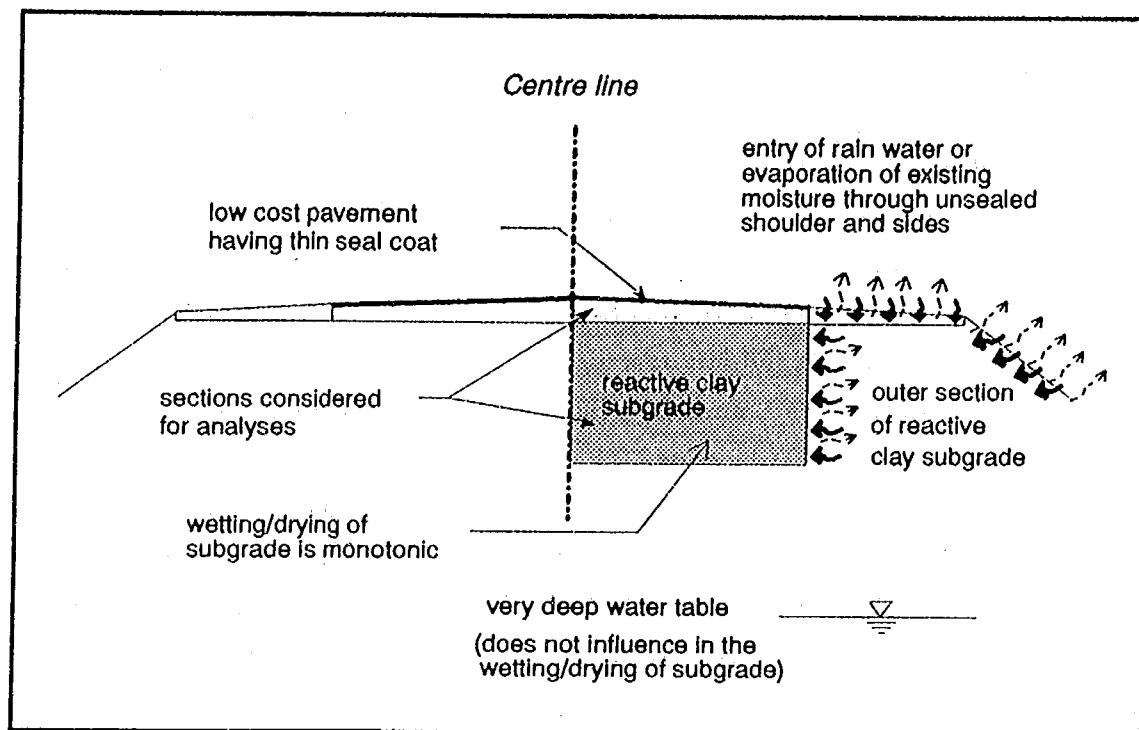


Figure 1.1 Typical situation analysed in this research

This research aims to use a simplified and consistent theoretical description of swelling and shrinkage development in the subgrade. It studies the response of overlying pavement to such subgrade volume changes. Emphasis is given to the situations commonly encountered with low cost road pavements that distort and crack as a result of wetting and swelling of subgrade due to water infiltration, as well as drying and shrinking of subgrade due to water evaporation from the unsealed shoulders and/or sides of a road section.

Consistent material coefficients related to the strength and stiffness of clay subgrades and pavement materials required in the present parametric study are available from existing data. Therefore, extensive laboratory and field measurements on swell and shrinkage properties are excluded. A set of typical laboratory measurements of swell pressure from oedometer tests performed earlier is considered to analyse the swell pressure parameters and to calibrate the mechanistic model of swell and swell suppression.

In this study, the pattern of subgrade wetting is defined to the extent in terms of the degree and the rate of change of suction potential as commonly observed in

semi-arid environment. This definition is applied in the numerical analysis while studying the response of pavement. The study is not concerned with the control of moisture change in subgrades.

1.3 Research Objectives and Methodology

The primary objective of this research is to carry out parametric studies of soil/pavement interaction for pavements over a wide range of reactive soils in order to find the critical parameters that are responsible for distortion and cracking of overlying thin seal coated low cost road pavement. To achieve this it was found necessary to develop the theoretical concept in the following areas by conducting review and theoretical analysis of available information on:

- 1) Subgrade wetting and drying processes
- 2) Mechanism of subgrade volume change upon wetting and drying
- 3) Characterisation of stress/strain, and strength behaviour of a seal coated unbound granular pavement.

Relationships describing the moisture flow and volume change behaviour of unsaturated reactive clay subgrade, and the stress strain behaviour of thin seal coated unbound gravel pavement, that are suitable to apply to the numerical analysis are developed. Using these relationships with the available data on subgrade and pavement properties and parameters, a comprehensive numerical analysis of typical field situation covering both the effect of subgrade wetting and swelling as well as drying and shrinkage are carried out. Finally, parametric studies on the results of analyses are conducted to identify the critical parameters in a given situation.

1.4 Report Organisation

The coverage of this research work can be divided mainly into two stages. The initial stage covers the theoretical study on wetting and drying and consequent swelling and shrinkage of subgrade. In the later stage, a numerical analysis is conducted to study the response of overlying low cost road pavement to subgrade volume change. The writing of this report also follows a similar path, and is described below.

Chapter I presents the aims of the study. How to achieve these aims is outlined in the listed objectives. The scope and delimitation of the study are also mentioned.

Chapters II, III, IV and V respectively present reviews of underlying theories and concepts of unsaturated moisture flow, characteristics of reactive soils, swelling and swell pressure development on wetting, and shrinkage subsidence and shrinkage stress development on drying of reactive clay subgrade. Relationships of unsaturated moisture flow, swell and swell pressure development, and shrinkage and shrinkage stress development that are suitable for application in numerical analysis are also developed in the later part of the respective chapters.

Chapter VI provides a background knowledge on the mechanistic description of stress and strain behaviour of pavements. This is applied in the numerical analysis to study the distortion and cracking in low cost road pavements overlying a reactive clay subgrade.

Chapters VII, VIII, and IX, present the numerical analysis part. Chapter VII deals on the development of procedure for numerical analysis. The application of this numerical procedure to study a typical situation of a reactive clay subgrade/pavement interaction is carried out in Chapter VIII. The detailed discussion on the results of numerical analysis carried out in Chapter VIII are presented in Chapters IX and X.

Finally, Chapter XI presents the conclusions of the research work, and the recommendations for further research.

CHAPTER II

MOISTURE FLOW IN AN UNSATURATED SUBGRADE

2.1 Introduction

Unsaturated soils have the tendency of absorbing moisture due to their capillary action and/or osmotic action of salt present in the soil water. Such tendency of moisture absorption is described here in after as the suction pressure or suction potential. In the wetting and drying processes of an unsaturated soil, suction potential is often preferred as the flow variable rather than the moisture content of soil. Because, the suction pressure is an absolute measure of energy state of soil and is independent of soil type. Therefore, the wetting and drying processes of subgrade described in this chapter and applied in the study, uses the concept of decreasing suction in the wetting and increasing suction in the drying processes. Such description is suitable for reactive clays as these clays undergo significant change in volume as a result of variation in moisture potential.

In this chapter parameters involved in the analysis of unsaturated moisture flow are discussed. Existing theories and available relations for unsaturated flow are briefly reviewed. Finally, an alternative simplified relation of moisture flow is developed and extended to describe the monotonic wetting and drying pattern of reactive clay subgrade. Such alternative description is desirable for simplification of numerical analysis procedure described in Chapter VII and VIII. The analysis is carried out on wetting phase and assumed to be applicable to the drying process by reversing the flow direction. The assumption is commonly applied in practice (Thomas and Rees, 1990, and Rees and Thomas 1993). Therefore, the effect of hysteresis which will be described latter, is not included.

2.2 Unsaturated Soil-Water Characteristics

Suction pressure of a soil results primarily from its capillary action of soil matrix and is termed as matrix suction. In some soils having salt concentration, suction pressure is increased by presence of salt in soil water. This increased part of suction is called osmotic suction. As this study deals with reactive clays, due to its relatively higher chemical activity, the total suction that includes both matrix and osmotic suctions is considered. Gravitational potential, when compared to the very high magnitude of suction potential, is generally very small. Therefore, it plays

little role in unsaturated flow in an intact soil mass free from major cracks and fissures.

In this study, the magnitude of suction pressure will be expressed in terms of log kPa. This unit of measurement is preferred because the moisture flow and the mechanical response of soil is more linearly related to log suction than to suction only.

In dealing with unsaturated flow condition, the soil-water potential is preferred to water content. Because water content is largely dependent on soil type. It has been observed that some reactive clays are very dry even at 15 % water content. However, if only one type of soil is considered, the suction potential ' ψ ' can be related with the volumetric moisture content ' θ '. Several empirical relationships between suction and moisture content are available in the literature (Gardner et al., 1970) but mostly applicable only for individual soil type. Further, the relation between moisture content and suction is not unique even for a single soil. The water content at a given suction is greater in drying than in wetting. This dependence in the direction of process is called hysteresis which is the result of soil micro-structures. In this study, the wetting and drying of subgrade are considered as separate, and monotonic, and hence the effect of hysteresis is not included.

2.3 Parameters of Unsaturated Flow

The detailed discussions on the derivation of unsaturated flow equations are readily available in soil physics text books, and also lies outside the scope of this study. In brief, the equation of moisture flow in an unsaturated soil in one dimension, in the absence of gravity head appears as:

$$\frac{\partial \theta}{\partial t} = - \frac{\partial}{\partial x} \left[K(\psi) \frac{\partial \psi}{\partial x} \right] \quad (2.1)$$

where θ is the volumetric moisture content, ψ is the suction potential, and K is the hydraulic conductivity with $K(\psi)$ indicating K as the function of ψ .

In the derivation of flow equations the volume of water is normalised with the total volume of the soil element. This leads to the assumption that the volume of soil element remains constant during the flow of water. But normally, most of the unsaturated clay soils swell or shrink when moisture movement takes place. Philip and Smiles (1969) considered the volume change phenomenon during moisture

movement. However, the selection of total volume or volume of soil particles to normalise the above relation depends on the degree of swell/shrinkage during the flow of moisture. Since the soil under consideration for the present study exhibits free volume change of less than 8 %, the changes in volumetric water content associated with the heave or shrinkage of reactive soil may be fairly small, in the order of less than 5 % of volumetric water content of non reactive soil. Therefore, the total volume of soil element is selected as the normalising parameter.

In an unsaturated flow, due to the dependency of hydraulic conductivity on suction potential, other two flow parameters are involved, namely: specific water capacity, and moisture diffusivity. Specific water capacity 'C' determines the volumetric water content per unit suction head and signifies moisture holding capacity of soils. Moisture diffusivity D defined as the ratio of the hydraulic conductivity to the specific water capacity, shows the dependency of conductivity on wetness. Both these terms are function of suction potential ' ψ ' or water content ' θ '.

Using chain rule, the gradient of suction potential can be expressed as

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial x} \quad (2.2)$$

Defining the term $\frac{\partial \psi}{\partial \theta}$ as the reciprocal of specific water capacity, $C(\theta)$ (i.e., volumetric water content per unit suction head), and substituting Equation (2.2) in Equation (2.1), the relation for unsaturated flow in one dimension becomes

$$\frac{\partial \theta}{\partial t} = - \frac{\partial}{\partial x} \left[K(\theta) \frac{1}{C(\theta)} \frac{\partial \theta}{\partial x} \right]$$

or

$$\frac{\partial \theta}{\partial t} = - \frac{\partial}{\partial x} \left[D(\theta) \frac{\partial \theta}{\partial x} \right] \quad (2.3)$$

In shorthand matrix form for three dimensions, the flow equation can be written as

$$q = -D(\theta) \nabla \theta \quad (2.4)$$

where 'q' is the rate of flow through unit area.

This equation is in a form analogous to Ficks' Law of Diffusion. The term $D(\theta) = \frac{K(\theta)}{C(\theta)}$ in the above equation is called moisture diffusivity. Physically, moisture diffusivity relates the variation of conductivity with volumetric water content.

As will be discussed in more detail in later chapters (Chapter III, IV, V, VII and VIII) both volumetric moisture content and swell or shrink are linearly related to logarithm of suction rather than absolute suction pressure. Therefore, in this analysis log suction is considered as the common variable for moisture flow and swell/swell suppression and shrinkage/shrinkage compression processes. Therefore, in the moisture analysis discussed in Section 2.4.2 and in Chapters VII and VIII, the diffusivity $D(\theta)$ is further described in terms of log suction $D(\log \psi)$.

2.4 Selection of Diffusivity Relationship for Numerical Analysis of Subgrade Moisture Movement and Volume Change

A numerical method is used in the analysis of moisture movement and volume change behaviour of an unsaturated reactive clay soil. The method uses finite difference technique to solve the diffusion equation. An available micro-computer program, FLAC (Fast Lagrangian Analysis of Continua - a two dimensional explicit finite difference program, Itasca, 1992) is employed. In the analysis, the moisture movement is analysed in terms of thermal diffusion heat. (For thermal analogy, refer Table 7.1 in Section 7.4 A).

FLAC's thermal option simulates the transient flow of heat in materials and the induced thermal stresses/strains. This option is used to solve the moisture movement problem in partially saturated reactive clay soil associated with swell/shrink stresses and volume changes. Also, as the program allows the use of non-linear diffusivity, it is useful to describe the non-linear relation between diffusivity and moisture content or suction potential. Such non-linear diffusivity relation results to a sharper wetting front resembling closely to what is commonly observed in the field. (Further details on application of this program is presented in Chapters VII and VIII).

In the following sections the outlines of the development of such moisture dependent diffusivity relationship based on wetting phase is presented. The applicability of this relationship has already been verified for the wetting phase

(Sapkota, 1994) and applied in a similar analysis of wetting and swelling subgrade by Wallace and Sapkota (1994).

2.4.1 Typical Values of Flow Parameters

The initial and final values of volumetric moisture content were acquired by considering the extreme dry and wet limits of soil in the field. Typical values were obtained from the published literature, e.g., by Wallace and Leonardi (1976), Gupta et al., (1974) and van Grinsven et al., (1985). The following values of volumetric moisture content were adopted:

$$\theta_{\text{initial}} = \theta_{\text{air dry}} = 0.15, \text{ and}$$

$$\theta_{\text{final}} = \theta_{\text{saturated}} = 0.45$$

These values were for high plasticity clay soils with no allowance made for the effect of swelling and shrinking. When these effects were considered for example, a volume increase of 8 % at saturation, the value of saturation water content θ_{sat} becomes 0.486 . This demonstrated that the contribution of swelling at saturation is relatively small. Thus its effect in the calculation of diffusivity parameters could be considered as negligible.

In the subsequent study of parameters describing the non-linearity of diffusivity, a value of $D_{\text{final}} = D_{\text{saturated}} = 0.0002 \text{ m}^2/\text{day}$ will be used. While in modelling the actual field situations in the present numerical analysis, the diffusivity value will be varied to give a typical rate of wetting that has been observed in the field (Section 7.4.2).

The boundary conditions of moisture profile (Figure 2.1) described by Wallace and Lytton (1992b) is applied. This highlighted some of the features of extreme total suction for uncovered sites in semi-arid areas. The boundary values of suction potential as $\log \psi = 1.5 \log \text{ kPa}$ at saturation and $\log \psi = 3.5 \log \text{ kPa}$ at dry condition, and dry and saturated moisture contents as 0.15 and 0.45 respectively were considered.

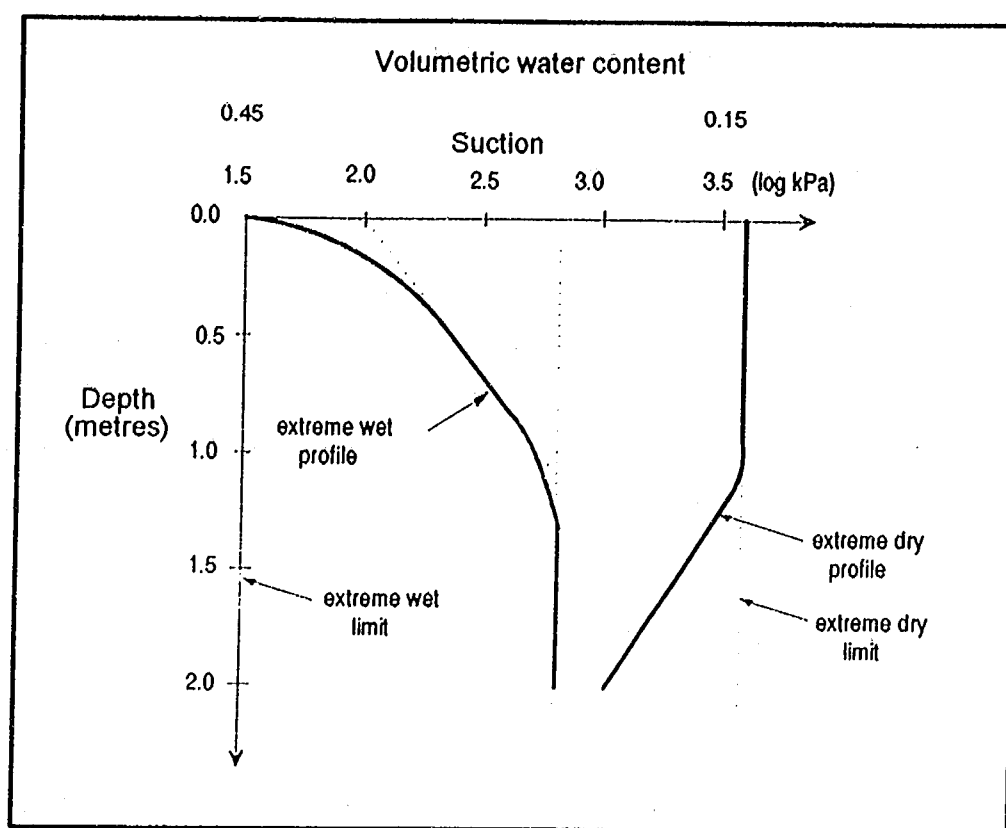


Figure 2.1 Seasonal variation in soil moisture conditions

2.4.2 Relationships for Soil Moisture Diffusivity

The relation of diffusivity 'D' to moisture content ' θ ' is often expressed in empirical forms. The objective of developing such relations is to get a convenient form that describes the shape and rate of advancement of a wetting front in an unsaturated soil profile. Several such empirical relations have been proposed by different investigators e.g., Wagner (1952), Gardner and Mayhugh (1958), and Miller and Bresler (1977). They have attempted to verify their relations with results from experimental investigations on particular soils and/or with computer simulations. Since the actual relationship between diffusivity and moisture content is non-linear, and some of the parameters such as hydraulic conductivity 'K' and volumetric moisture content ' θ ' are affected by hysteresis, any proposed empirical or semi-empirical relation may be suitable only for a particular soil type under consideration.

A. Use of Constant Diffusivity

Wallace and Lytton (1992a) carried out numerical analysis of the wetting process in a small block of soil. To avoid the uncertainty associated with the effect of the soil structure on wetting, they adopted a constant value for diffusivity of 0.0002

m^2/day throughout the range of moisture contents. However, it was found that the application of constant diffusivity is not justifiable for a detailed analysis of an overall subgrade section simply because diffusivity varies rapidly with the length of flow (Sapkota 1994).

B. Application of Available Non Linear Diffusivity Relationships

Sapkota (1994) reviewed the application of different available non linear relationships such as

the exponential form of a closed form relation of diffusivity equation expressed as

$$D(\theta) = D_i \exp(\theta - \theta_i) \quad (2.5)$$

where D_i is the diffusivity at the initial moisture content ' θ_i ', and $D(\theta)$ is the diffusivity corresponding to moisture content ' θ ',

and Gardner's empirical relation (Gardner and Mayhugh, 1958) expressed as

$$D(\theta) = a \exp(b\theta) \quad (2.6)$$

where $D(\theta)$ is diffusivity as function of moisture content, and a and b are constants.

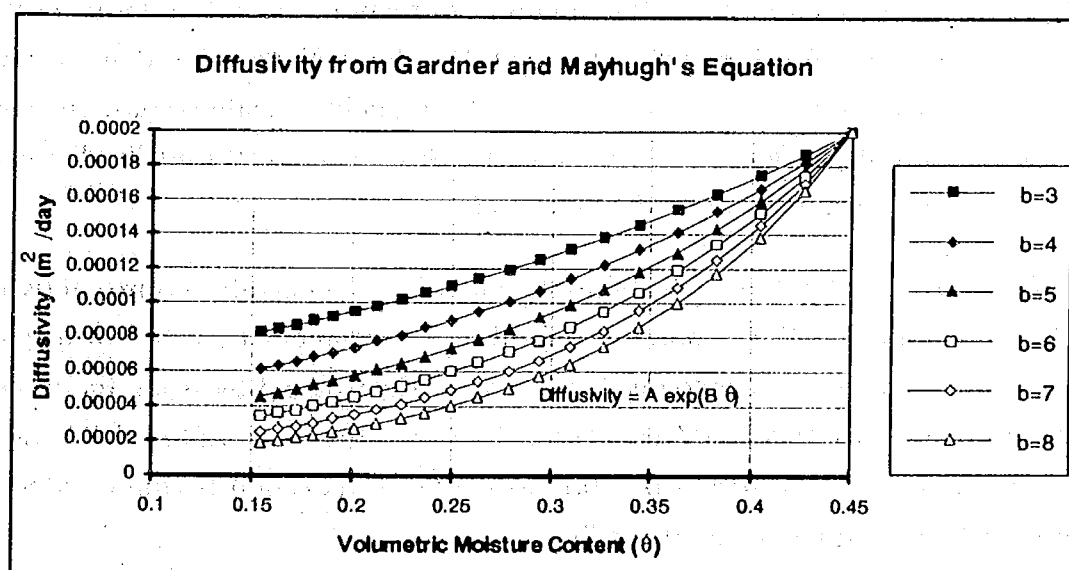


Figure 2.2 Non-linear variation of diffusivity with Gardner & Mayhugh's equation

With the application of the first relationship the variation of diffusivity with moisture content ' θ ' was found to be almost linear. While the second relationship provided a sharper non linearity as shown in Figure 2.2, with increasing value of ' b '. The slope of curve was flatter at the dry side and became steeper, giving sharper

curve at the wet side. The figure also indicated that the change in the shape of curve is less significant after the value of b greater than 6.

Different investigators (Reichardt et al, 1972, Miller and Bresler, 1977, Clothier and White, 1981, and Smiles et al. 1978) used this relationship and estimated the value of constant ' b ' by using the results from laboratory study. Reviewing their studies, Sapkota (1994) found that the value of b should normally fall within 5 to 8. Thus the relation with $b=6$, expressed as $D(\theta) = 1.3441 \times 10^{-5} e^{6\theta}$ has therefore been selected to represent a typical D versus θ relationship for comparison with the alternative non-linear relationship investigated in Section C below.

C. Alternative Non-linear Relationship

For the study of moisture flow in an unsaturated subgrade, an alternative empirical relation between diffusivity and moisture content and/or suction was proposed. The relation was non-linear and in a convenient form to apply to the thermo-mechanical option of FLAC program. The relation in power form was expressed as

$$D(\theta) = A + B \theta^n \quad (2.7)$$

where $D(\theta)$ is diffusivity as function of moisture content, and A , B , and n are constants. It was desired to have description of diffusivity directly in terms of log suction i.e., $D(\log \psi) = A + B (\log \psi)^n$. However, to validate the proposed alternative diffusivity relationships by comparison with available relationships, it was necessary to describe in terms of volumetric moisture content θ initially. Because the available relationships were in the function θ .

For different values of n , the relationship (Equation 2.7) will result to constant (with $n = 0$), linear (with $n = 1$) and non-linear (with $n > 1$) relations between diffusivity and moisture content. The constants A and B were determined by using the upper value of diffusivity as $0.0002 \text{ m}^2/\text{day}$ at saturation moisture content ($\theta = 0.45$) and a diffusivity value $D = 0.000033$ at $\theta = 0.15$ as deduced in Section B with Gardner's empirical relation (Equation 2.6) using ' b ' = 6.

In the above alternative relation (Equation 2.7), the most sensitive component was the value of ' n ' which determines the shape of the wetting front. The analysis was carried out to determine the typical n values that could produce a similar shape of diffusivity versus water content plot obtained by Gardner and Mayhugh (1958), and a similar wetting front determined by Bruce and Klute (1956), Gardner and Mayhugh (1958), and others. The value of n was varied from 0 to 3 as shown in Figure 2.3. The plot shows that the nature of curve remains almost similar after the

index value n is greater than 2.5. Also, it is found to be the closest curve to the one obtained by using exponential relation of Gardner and Mayhugh (1958) with $b=6$. Thus, the equation with $n=2.5$ expressed as

$$D(\theta) = 1.8343 \cdot 10^{-5} + 1.3377 \cdot 10^{-3} \theta^{2.5} \quad (2.8)$$

was considered for the analysis of subgrade wetting process.

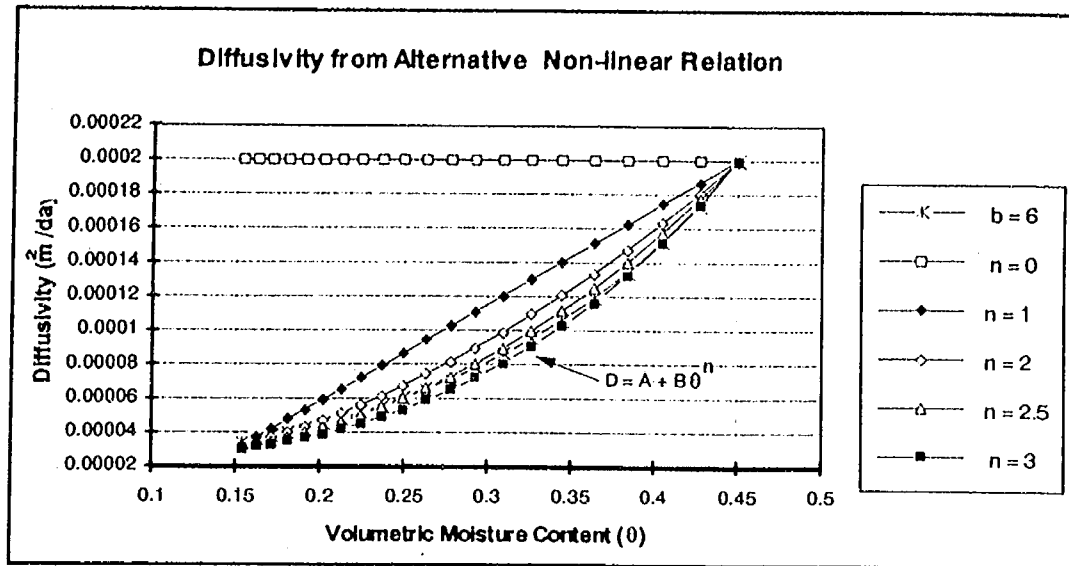


Figure 2.3 Non-linear variation of diffusivity from alternative description

The above mentioned moisture content dependent diffusivity relationship can be described into suction dependent form. For this, well established Gardner's relationship for moisture-suction characteristics $\psi = A \theta^{-b}$ may be used. Applying the typical initial and final values of θ and ψ , and as $b = 4.3$ for loam soil, relationship modifies to $\psi = 1.113 \theta^{-4.192}$. Then, the diffusivity relation described in terms of volumetric moisture content ' θ ' can be redescribed in terms of suction pressure ' ψ '. Thus, for $n=2.5$ Equation (2.8) becomes

$$D(\psi) = 1.8343 \cdot 10^{-5} + 1.426 \cdot 10^{-3} \psi^{-0.59625} \quad (2.9)$$

2.4.3 Review of Results of Subgrade Wetting on the Application of the Alternative Non Linear Difusivity Relationship

A preliminary analysis of the wetting process in an unsaturated reactive clay subgrade using constant and non-linear moisture content or suction dependent diffusivity relations expressed in power form (similar to Equation 2.7 and 2.8) was conducted. In the analyses, all other soil properties were kept constant. The thermal diffusion analysis of FLAC program was used with varying degrees of non-

linearity of the $D-\theta$ relationship in order to study the shape of the wetting front under a monotonic wetting up of a reactive clay subgrade.

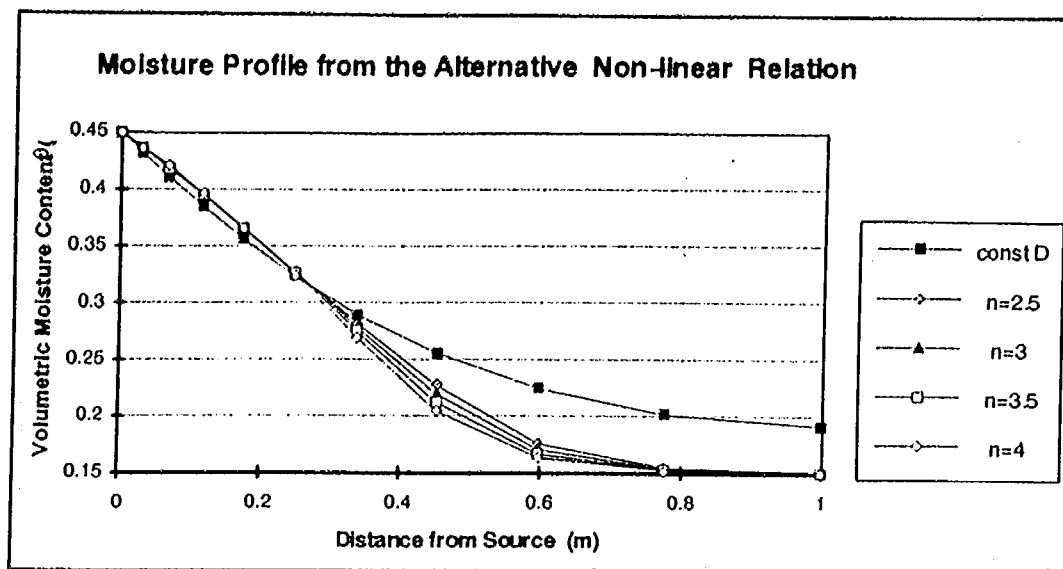


Figure 2.4 Moisture profile from alternative non-linear description of diffusivity

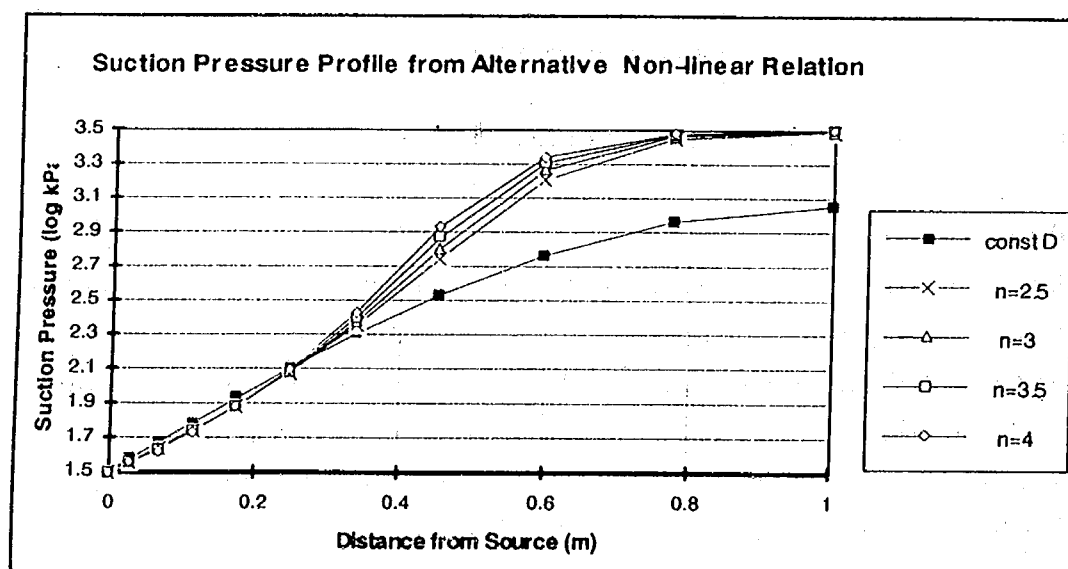


Figure 2.5 Suction profile from alternative non-linear description of diffusivity

Detail of program-runs for analysing the unsaturated moisture flow in a reactive clay subgrade is presented in Chapters VII and VIII, and in Appendix C. The program is run for an equal wetting period for different degrees of non-linearity. Profiles describing the variation of moisture content and suction pressure with respect to distance from source are obtained from the results of analyses and are presented in Figures 2.4 and 2.5. The plots show a relatively sharper wetting front obtained from non-linear relation compared to that of constant diffusivity.

In general, the shape of unsaturated wetting front should show a relatively uniform wetting near the source and a sharp decrease in wetting at the far side. However, plots in Figures 2.4 and 2.5 show a sharper decrease with depth from the beginning. The reason could be attributed to the small value of ratio between diffusivity at initial and final moisture states ($D(\theta_i)/D(\theta_f)$). A similar trend has been found by Gardner and Mayhugh (1958) for a smaller ratio of diffusivity. Besides the program is run for a time period representing a 50 percent wetting only. If it is increased, the earlier slope of the profile might be flatter showing lower suction value for a considerable depth.

The program is run for this alternative form of non-linear diffusivity relation (Equation 2.23) with different index values 'n' = 2.5, 3, 3.5 and 4. The plots of the results for all the analyses show a similar trend of wetting. The shape of the wetting front changes with non-linear diffusion becoming flatter as n increases but once n reaches about 2.5 (the value deduced from comparison with other published relationships) then the effect of further increase is small. Hence, this alternative relation with 'n' equal to 2.5 is adopted for the numerical analysis of this research work.

2.5 Conclusions

In an unsaturated flow, suction potential, instead of moisture content is the driving parameter that results moisture flow. The flow takes place in the direction of higher suction. However, some of the available relations are based on water content, hence where necessary, Gardner's empirical relation between suction potential and wetness, $\psi = A \theta^{-B}$, is used. Meanwhile, when dealing with unsaturated reactive soils as total suction is preferred to matrix suction, this study also considers total suction in describing the moisture flow.

The present work considers a monotonic wetting up or drying out of a uniformly dry or wet subgrade section. It is reasonable to eliminate gravity head as the flow is horizontal and one dimensional. Also it can be assumed that even in a two dimensional case, the elevation head arising from the subgrade thickness is very small compared to the much greater magnitude of suction pressure. At the present stage of research, flow proceeds in either wetting phase or drying phase at a time, and hence the effect of hysteresis is excluded. Also, during the moisture flow in a reactive clay subgrade, the corresponding volume change should affect the volumetric moisture content. The writers found that for a potential volume change

of less than eight percent, the effect on the volumetric water content and corresponding diffusivity value is very small.

With unsaturated flow condition, moisture diffusivity which relates the variation of hydraulic conductivity with the volumetric water content is to be used to define the rate of flow. Since the diffusivity varies with the suction level, a non-linear diffusivity relationship is considered in the study. This alternative non-linear diffusivity relationship is expressed in a power form (Equation 2.7). The magnitude of index 'n' applied in this relation to best describe the wetting front is found to be 2.5. The relation is found to be a convenient form which can be easily applied in the numerical analysis using the readily available computer program, FLAC.

The variation of diffusivity with moisture content obtained by applying this alternative relationship to a typical subgrade moisture condition is compared with that obtained from the relationship (Equation 2.8) presented by Gardner and Mayhugh (1958); and is found to be in reasonable agreement (Section 2.4.3C). Finally, this alternative diffusivity relationship is proposed for the numerical analysis in Chapters VII and VIII.

The suggested alternative non-linear diffusivity relationship is suitable for the analysis of moisture movement in unsaturated reactive clay subgrade that is subjected to the infiltration of rain water through the pervious shoulder and from the sides of a road section. The use of this relationship has not been attempted for other aspects of subgrade moisture movements. Hence it may not be suitable for the solutions of moisture redistribution and drainage problems. However, it will be attempted to apply to the drying phase of relatively wet reactive clay subgrade when the drying will be considered as a monotonic process with water evaporation through the open shoulder or sides of the section. Thus the drying phase will be considered basically as a mirror image of wetting phase.

CHAPTER III

CHARACTERISTICS OF REACTIVE SOILS

3.1 Introduction

Reactive clays are cohesive soils which show significant volume change with change in their moisture content. These clays swell when moisture increases and result ground heave. They shrink when moisture decreases and result ground subsidence. Extent of such volume change depends on the degree of interaction between clay minerals and water under the physico-chemical processes, while overall magnitude of ground movement depends also on external factors such as degree of variation of moisture environment and in situ soil and site conditions.

As this research study is concerned with the mechanical behaviour of reactive clay subgrade subjected to the moisture variation, the detailed consideration of soil mineralogy and surface chemistry are excluded. Rather, the description of the reactive clay characteristics discussed in this chapter, follow mechanistic approach.

3.2 Identification of Reactive Clays

Consistency limits provide a basic background to study the swelling/shrinking potential of reactive soils. A combination of Atterberg limits and colloidal contents help to identify the volume change potential of clays. Colloidal content indicates the amount of active fraction in soil responsible for such behaviour. Plasticity index demonstrates the magnitude of range of moisture content for a soil to remain in plastic condition. Thus higher plasticity index means greater volume of water required to reach the liquid limit which in turn relates higher expansion as water fills the voids and makes film around the clay minerals. Chen (1988) claims that while a high reactive soil manifests higher index property, the converse is not true.

Figure 3.1 illustrates how the expansive clays of Israel lie in the narrow band parallel and just above the 'A' line in the plasticity chart under the Unified Soil Classification System (Kassiff et al. 1969). The plasticity chart shown in Figure 3.1 also includes the location of some other typical reactive clays. Both black earth and grey and brown clays are of high plasticity (CH). The swelling clay soil which was used in the oedometer test to measure the lateral swell pressure parameter for a previous study (Sapkota 1994) also lied in high plasticity inorganic soil zone (CH). Also included in the figure are the typical values of liquid limit and plasticity index which represent different levels of swelling (as defined by Van der Merwe 1975).

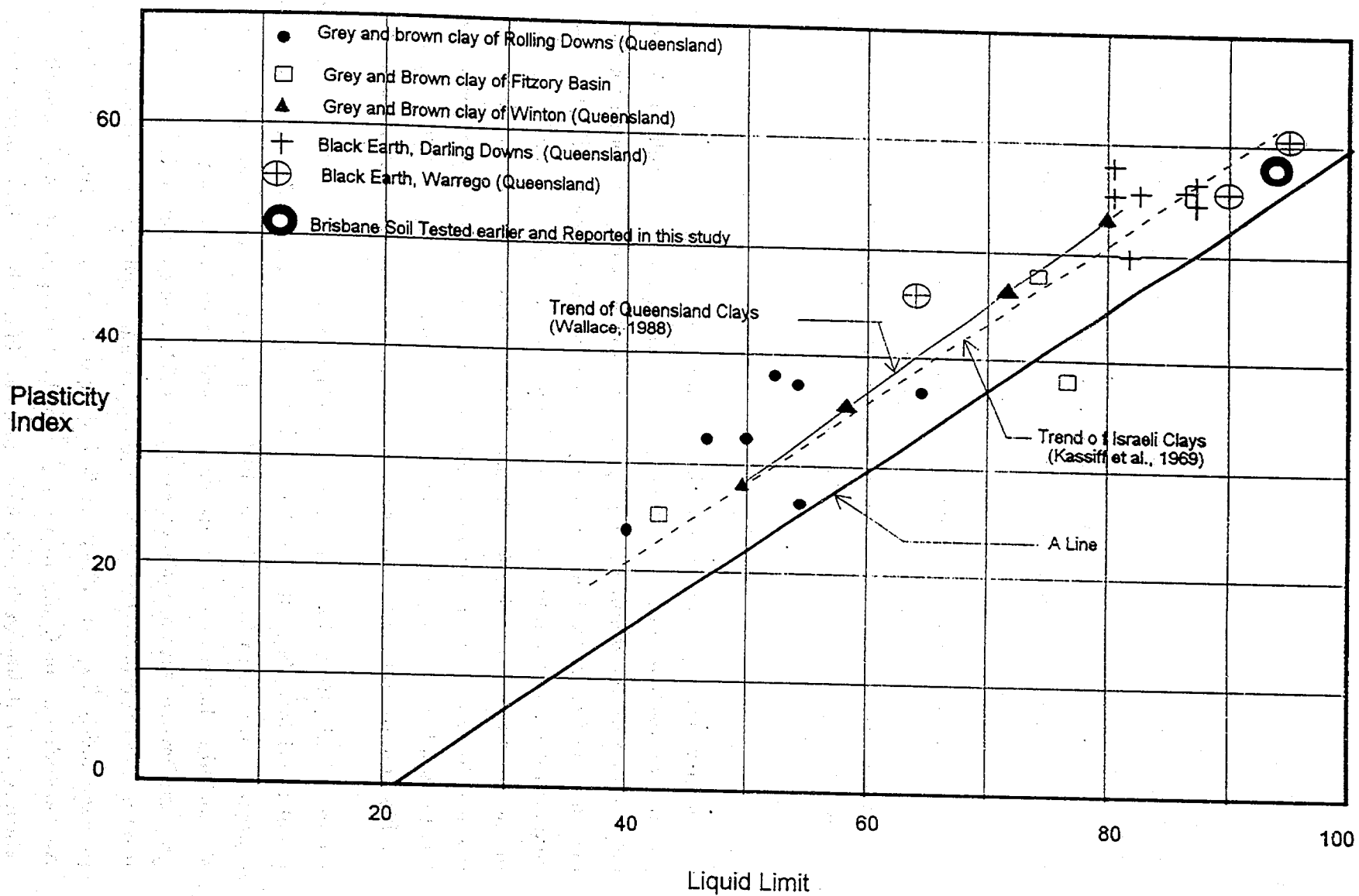


Figure 3.1 Plasticity chart showing typical distribution of reactive clays

3.3 Measurement of Swell/Shrink Characteristics

The laboratory measurement of volume change and swell/shrinkage pressure development using a consolidometer is the most common method used in quantitative analysis. Concept behind the test procedure and the mechanism of swell is well established proving that the test gives direct measurement that is in close compliance to the true field conditions. Pile and McInnes (1984) stated that the current state-of-the-art with respect to natural soil profiles is to conduct either oedometer swell or drying shrinkage tests on "undisturbed" samples. Meanwhile, for a precise analysis of volume increase and associated pressure development it may be important to correct the oedometer test results using Fredlund's (1979) procedure to account for sample disturbance. Regarding mechanistic test on soil shrinkage, a similar oedometer laboratory test for shrinkage under applied pressure have also been reported in the literature (Richards et al., 1983).

Recent advancement and sophistication in field instrumentation technique has made the in situ measurement of swelling/shrinking characteristics more desirable. Numerous reports are available on in situ measurements of ground heave and subsidence. But only a few reports on long term, in situ measurement of swell pressure are published (Richards and Kurzeme, 1973, Brackley and Sanders, 1992, and others). Also, only a few of them includes the in situ evaluation of swell pressure (Ofer, Blight and Komornik, 1984 and Ofer and Blight, 1985). On the other hand, reports on in situ measurements of shrinkage stresses are almost none.

As the proposed mechanistic models of swelling (Chapter IV) and shrinkage (Chapter V) to be applied in this study uses the concept of laterally confined one dimensional volume change, the literature on laboratory oedometer test methods used in the determination of percent swell and swell pressure were reviewed.

3.3.1 Laboratory Measurement of Degree of Swell

Kassiff et al (1973) developed a rational formulation of the relationship between volume, pressure, and suction changes. They made the assumption that the volume change in clay soil is governed by both external (mechanical) forces characterised by the state of stress in soil mass and internal (physico-chemical) forces characterised by the energy state of the soil water, and also stated that at equilibrium these two forces balance each other. They conducted a series of tests in an osmotic oedometer, and applying this concept, they could describe the variation in percent swell with the change in suction and vertical overburden pressure.

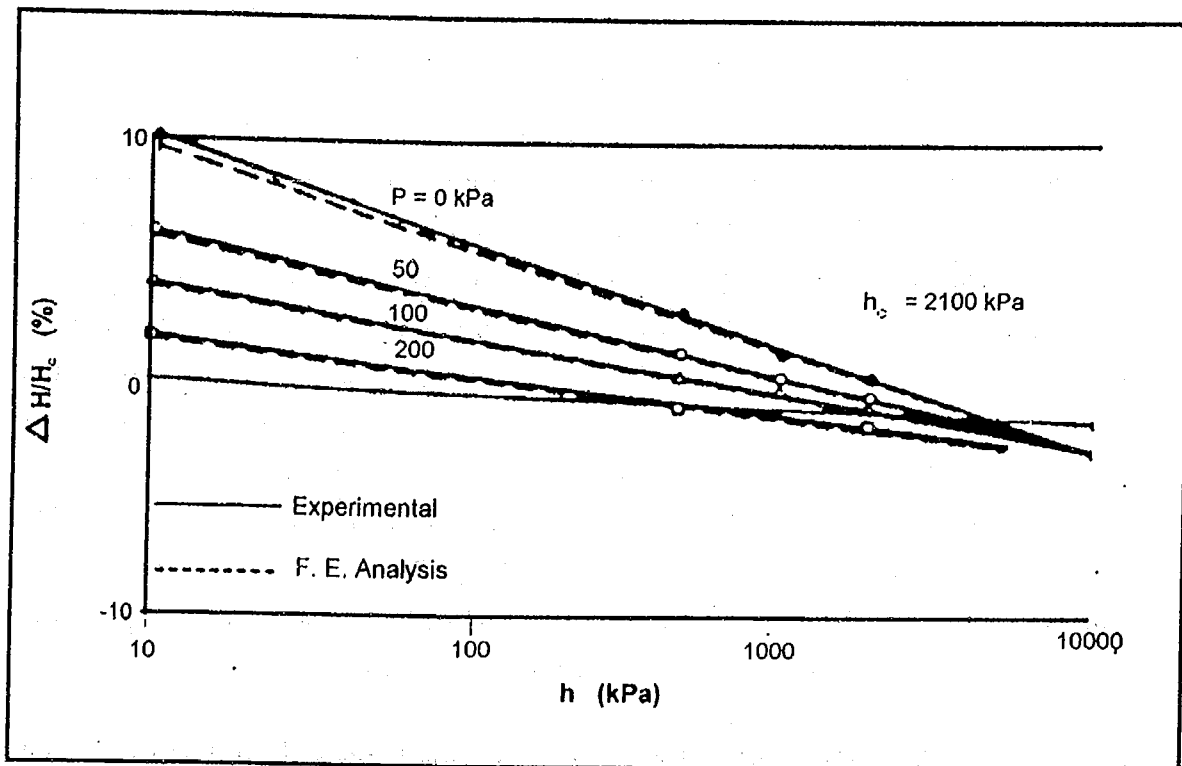


Figure 3.2 Suppression of vertical swell under applied vertical stresses

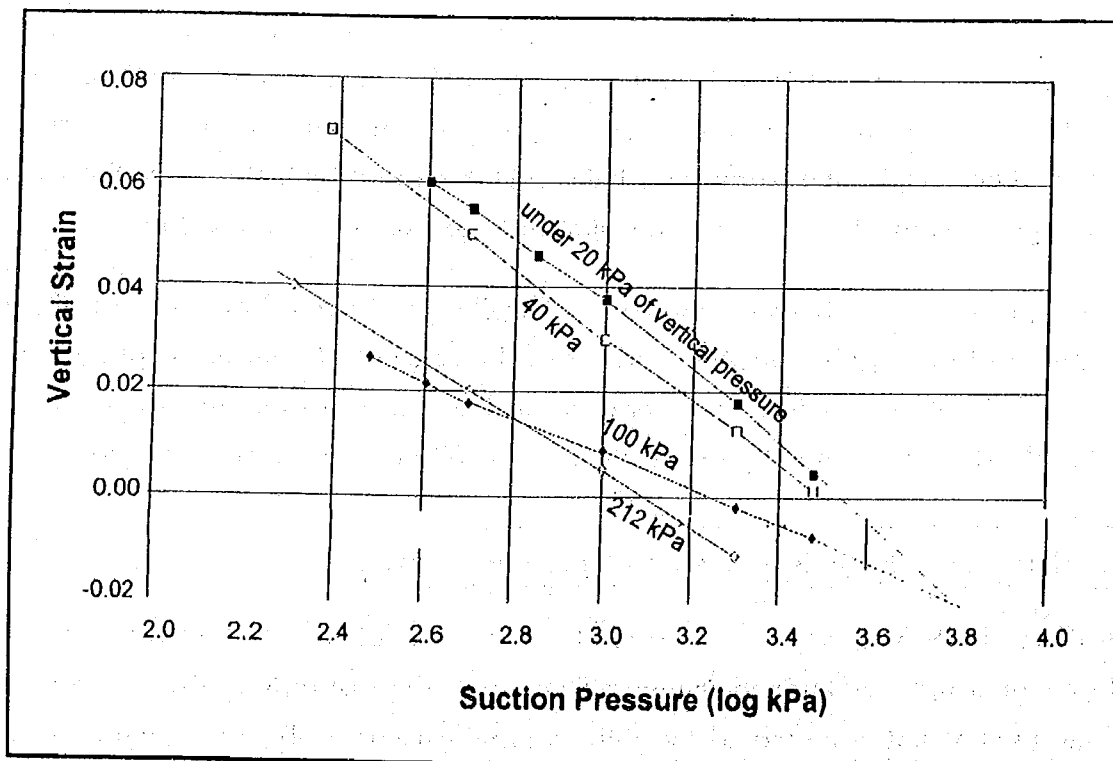


Figure 3.3 Swell suppression in South African swelling clays
(back calculation from Meintjes' test results, 1992)

Using the test data of Kassiff et al. (1973), Richards (1979) described how the percent swell varies linearly with the logarithm of suction pressure and with the

change in vertical applied pressure. Figure 3.2 (after Richards, 1979) illustrates the suppression of heave with increase in vertical pressure. Soil parameters that describe free swell and swell under suppression with an increase in vertical pressure can be evaluated from it. The discussion on this model, its further improvement and application in the present study will be presented in Chapter IV.

Sapkota (1994) conducted analysis on the test data of Meintjes (1992) on South African swelling clays, and found a considerable suppression in vertical swelling with increase in vertical pressure. This is shown in Figure 3.3

3.3.2 Measurement of Shrinkage Potential

Relatively less literature are available in the mechanical analysis and testing of shrinkage behaviour of reactive soils. Particularly, a very few papers discuss the measurement of shrinkage potential in the laboratory or measurement of shrinkage subsidence in the field. A best collection can be found in "Influence of vegetation on the swelling and shrinking of clays" - Fourth Geotechnique Symposium in Print, Geotechnique, 1983.

The laboratory measurements of shrinkage strain with increasing suction followed by Richards et al. (1983), were with two separate methods. One of their two methods results a free shrinkage in which the increase in suction in soil specimen was followed by placing the specimen freely (without any externally applied stress) inside a pressure membrane apparatus. The volume changes together with changes in thickness of the specimen were measured at a regular interval of increasing suction.

Their second method used suction controlled consolidometer in which the soil specimen underwent compression with increase in suction under various applied level of vertical stresses. In over all the samples went through various stress and suction paths.

In this study, the shrinkage measurements from the first method will be useful to obtain free shrinkage parameter, while that from the second method will be applicable to obtain the shrinkage compression parameter. The application of such results in the present simple mechanistic model of shrinkage compression is discussed in Section 5.5.

3.4 Mechanistic Description of Swelling and Shrinking (for Piece wise Linear Elastic Analysis)

The swell and shrink mechanism of a reactive soil is relatively complex. The problem is still often tackled by using theories of elasticity. Most of the governing equations can be developed by applying the basic test conditions like one dimensional confined swelling and shrinkage in an oedometer test or three dimensional free swell and shrinkage condition. The following sections discuss the development of simple stress strain relationship using general theories of one dimensional and three dimensional volume change conditions. The concept will be used in the development of simple material models that will relate swell and shrink strains to swell pressure and shrinkage stress in three dimensions.

A. Volume Change under Three Dimensional Free Condition

For an isotropic soil with Poisson's ratio $\nu_r = \nu_\theta = \nu_z$, and Young's modulus $E_r = E_\theta = E_z$, the relation between elastic stress and strain under three dimensional hydrostatic stress/strain state can be described as:

$$\epsilon_x = \epsilon_y = \epsilon_z \text{ for } \sigma_x = \sigma_y = \sigma_z$$

Then, the strain in one direction, for example vertical 'z' direction,

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$\text{or, } \epsilon_z = \frac{\sigma_z}{E}(1 - 2\nu) \quad (3.1)$$

B. Volume Change under Oedometer

Using the standard boundary condition of a consolidometer with zero radial and circumferential strains ($\epsilon_r = \epsilon_\theta = 0$), radial stress σ_r and vertical strain ϵ_z for an isotropic soil can be determined by the following equations:

$$\epsilon_z = \epsilon_o \left(\frac{1+\nu}{1-\nu} \right) + \frac{\sigma_z(1+\nu)(1-2\nu)}{E(1-\nu)} \quad (3.2)$$

In Equation (3.2), the term $\left(\frac{1+\nu}{1-\nu} \right) \epsilon_o$ represents equivalent laterally confined one dimensional swelling or shrinkage for a three dimensional volume change. The other term $\frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \sigma_z$ represents constrained modulus for swell suppression or shrinkage compression.

Many researchers (Lytton, 1977, Picornell and Lytton, 1984, Wallace and Lytton, 1992b) have described the initial swell strain under oedometer swell/shrink condition as a function of logarithm of change in suction potential. By doing so, the problem of swell and swell suppression can be simplified. This approach is applied in the development of simple mechanistic model of swell and swell suppression which is used in the present study. The writers have extended the approach to shrinkage compression analysis in this study.

Further, if initial strain ϵ_0 is zero in Equation (3.2), then the comparison between Equations (3.2) and (3.1) results a relation between one dimensional constrained strain and three dimensional hydro-static strain as:

$$\epsilon_{z1-D} = \left(\frac{1+\nu}{1-\nu} \right) \left\{ \frac{\sigma_z(1-2\nu)}{E} \right\}$$

results:
$$\epsilon_{z1-D} = \left(\frac{1+\nu}{1-\nu} \right) \epsilon_{z3-D} \quad (3.3)$$

conversely,
$$\epsilon_{z3-D} = \left(\frac{1-\nu}{1+\nu} \right) \epsilon_{z1-D} \quad (3.4)$$

Above two relations (Equations 3.3 and 3.4) are very useful when the one dimensional swelling and shrinkage strains under isotropic condition are to be translated into three dimensional system and vice versa.

3.5 Parameters used in the Description of Volume Change and Associated Stress Development

The volume change behaviour of an unsaturated reactive clay depends primarily on the applied vertical stress and the change in suction pressure. To relate volume change with these two variables: change in suction and applied stress, different parameters are used by different researchers. In the following sections a review on the use of different parameters in the description is presented, and one of them is selected for the present simple mechanistic model.

3.5.1 Swell and Swell Suppression Parameters

Different investigators like Livneh, Shklarsky and Uzan (1973); Richards (1992); Picornell and Lytton (1984); Uzan, Baker and Frydman (1987); and Wallace and Lytton (1992b) have used various parameters in their respective material models to

describe the swell phenomenon. The main parameters involved relate the increase in volume with change in suction potential or moisture content, and with change in volumetric strain under applied external load. The writers consider the first one as free swell parameter and the other as swell suppression parameter.

A. Free Swell Parameter

Lytton (1977) and subsequently by McKeen (1980), Picornell and Lytton (1984), and Wallace and Lytton (1992b) used this parameter as free swell index or suction compression index and defined it as

$$\gamma_h = \frac{\epsilon_v}{\Delta \log_{10} h}$$

where ϵ_v = volumetric strain, and h = suction pressure.

B. Swell Suppression Parameter

Lytton (1977) and as applied by Picornell and Lytton (1984) and Wallace and Lytton (1992b) introduced this parameter as swell suppression index and defined it as

$$\gamma_\sigma = - \frac{\Delta \epsilon_v}{\Delta \log_{10} \sigma}$$

where ϵ_v = volumetric strain, and σ = applied pressure

These two parameters, γ_h and γ_σ , are the constant characteristics of swelling process which are useful in the theoretical and mathematical descriptions of swell behaviour of expansive clays. The application of γ_h and γ_σ in the present research work is covered in Chapter IV.

3.5.2 Shrinkage and Shrinkage Compression Parameters

The free shrinkage and shrinkage compression of drying soil under applied vertical stress have been often looked as a mirror image of swell and swell suppression. Richards et al., (1983) used instability index ' I_{pt} ' as the parameter to describe the shrinkage of soil due to drying. As discussed in Section 3.5.1 A, originally, I_{pt} was by applied by Aitchison and Woodburn (1969), Aitchison (1970), Lytton and Woodburn (1973) and later by Mitchell and Avalue (1984) to describe the free swell parameter as

$$I_{pt} = \frac{\epsilon_{vert}}{\Delta u}$$

where Δu is the change in suction. Further, Australian Standard Code suggests to conduct shrinkage test of soil sample as an alternative to obtain the magnitude of I_{pt} . Code also presents a method of estimating ground heave by using I_{pt} .

The writers attempt to describe the free shrinkage and shrinkage compression under applied stress in a similar manner to free swell and swell suppression. Hence they use free shrinkage parameter λ_h and shrinkage compression parameter λ_σ which are in the similar form as of γ_h and γ_σ .

$$\lambda_h = \frac{\Delta \epsilon_v}{\Delta \log_{10} h} \text{ and, } \lambda_\sigma = \frac{\Delta \epsilon_v}{\Delta \log_{10} \sigma}$$

where ϵ_v is the volumetric strain under one dimensional oedometer shrinkage compression condition, ' $\Delta \log_{10} h$ ' is the change in suction pressure, and ' $\Delta \log_{10} \sigma$ ' is the change in applied stress under one dimensional confined condition like under an oedometer test condition.

3.6 Conclusions

Available literature on the identification of expansive soils based on the physical properties such as consistency limits, moisture/density, and indexes, show that swelling clays normally lie just above the 'A' line in a plasticity chart and manifest higher values of Atterberg limits and index properties (Figure 3.1).

Available reports on laboratory measurements of vertical heave and swell pressure indicate that oedometer swell testing is the most commonly used method in studying the behaviour of swelling clays. In the present study, the concept of swell and swell suppression under an oedometer test condition is applied in the development of a simplified mechanistic model (Section 4.3). Similarly, the concept of shrinkage compression is attempted to develop in an oedometer test condition. Hence, major emphasis is given in the review and evaluation of available oedometer test procedures. The reported results of swell and shrinkage compression tests under different controlled oedometer test conditions are useful to obtain the model parameters which are applied in the study (Sections 4.4 and 4.5).

Various forms of descriptions of swell parameters adopted by different investigators (Section 3.5.2) to estimate swell and swell pressure are available. Particularly, two swell parameters, namely; (1) free swell index ' γ_h ', and (2) swell suppression index ' γ_σ ', and two shrinkage parameters as (1) free shrinkage index λ_h and (2) shrinkage compression index λ_σ are considered as the major parameters to be used in the present work. These are studied extensively in the subsequent chapters.

CHAPTER IV

MECHANISTIC MODEL FOR SUBGRADE SWELL AND SWELL SUPPRESSION

4.1 Introduction

This chapter aims to develop a theoretical descriptions of the wetting and swelling behaviour of reactive clay subgrades. The expressions for constitutive relations of swelling and swell suppression are derived by using a simple mechanistic model of swell and swell suppression. These expressions will be used in the numerical analyses in the later Chapters. The acquisition of typical soil properties required by the model and the outcomes of simple closed form analyses using this model are also covered. Typical oedometer swell tests conducted previously are also discussed, and the results of these tests are used to validate the model.

4.2 Subgrade Moisture Environment

The present study concentrates on a semi-arid environment in which the potential evapo-transpiration exceeds precipitation for at least one or two months in a year. On the basis of detailed literature review conducted by Sapkota (1994) on the variation of suction potential in reactive clays for such climate, the following soil moisture environment is proposed for the parametric study on swelling and shrinkage of reactive clay subgrade presented in this study:

For upper limit of suction, the overall section of subgrade is considered at extreme dry limit with a uniform value of 3.2 to 3.5 log kPa. This is slightly higher than suction at optimum moisture content (for maximum dry density) but lower than suction of 4.0 log kPa at peak dry state indicated by Richards et al (1970).

The lower limit is considered under flooded or fully saturated condition with sufficient water for movement of moisture through the underneath of pervious shoulder. A moisture potential of 1.5 log kPa that accounts for osmotic suction of soil water is considered.

The depth of subgrade that undergoes moisture changes is considered to lie between 600 mm and 1200 mm. For detailed numerical analysis, a depth of 600 mm is considered

A typical seasonal changes of subgrade moisture profile was presented in Section 2.4.1 (Figure 2.1).

4.3 Characterisation of the Swelling Behaviour of Subgrade

In this study, a simplified relation to measure volumetric strains experienced by an elemental volume of soil resulting from a change in water potential is selected to describe the swelling and swell suppression behaviour of reactive clay subgrade. The concept was initially proposed by Lytton, 1977 and subsequently extended and applied by Picornell and Lytton, 1984; Wallace and Lytton, 1992b; and Wallace and Sapkota, 1992. The relation is:

$$\frac{\Delta V}{V} = -\gamma_h \log_{10}\left(\frac{h_f}{h_i}\right) - \gamma_\sigma \log_{10}\left(\frac{\sigma_f}{\sigma_i}\right) \quad (4.1)$$

where $\frac{\Delta V}{V}$ is volumetric strain, γ_h and γ_σ are the free swell and swell suppression indexes, respectively, σ_i is the threshold stress level (refer Section 4.4), and σ_f is the applied stress. The first term determines the free swell condition while the second term accounts for decrease in swell under applied stress condition.

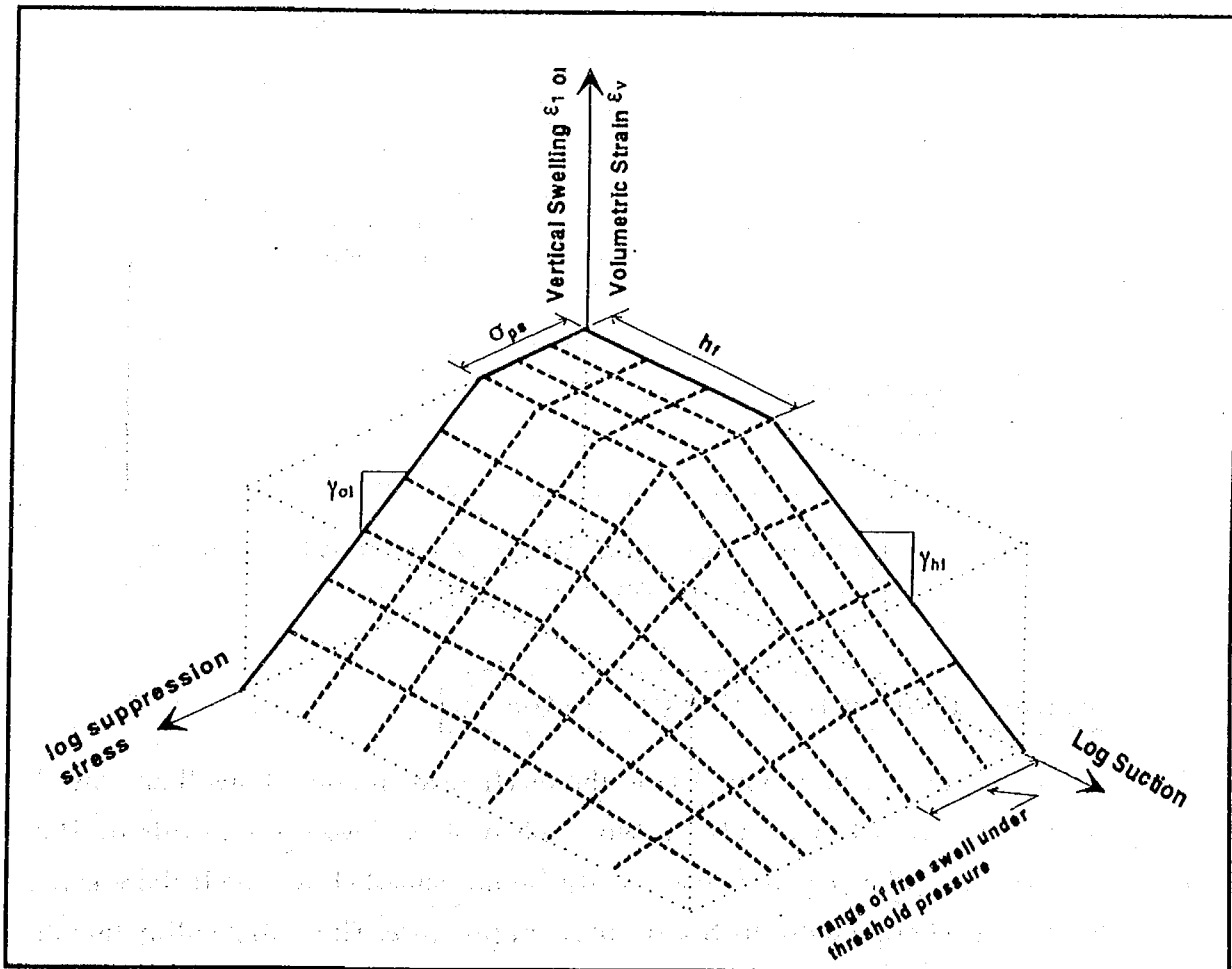


Figure 4.1 Description of swell and swell suppression in three dimensions

The writers present this swell and swell suppression characteristics in a three dimensional plot of Figure 4.1 which is similar to Fredlund's (1979) three dimensional diagram shown in Figure 4.2 representing total stress, matrix suction,

and void ratio. The diagram in Figure 4.1 illustrates how the swell characteristics of a given soil sample vary with applied stress and total suction between extreme wet and dry limits. The volume changes are expressed in terms of vertical strain in the oedometer test such that ϵ_1 is equal to the volumetric strain ϵ_v in an oedometer having zero lateral strain.

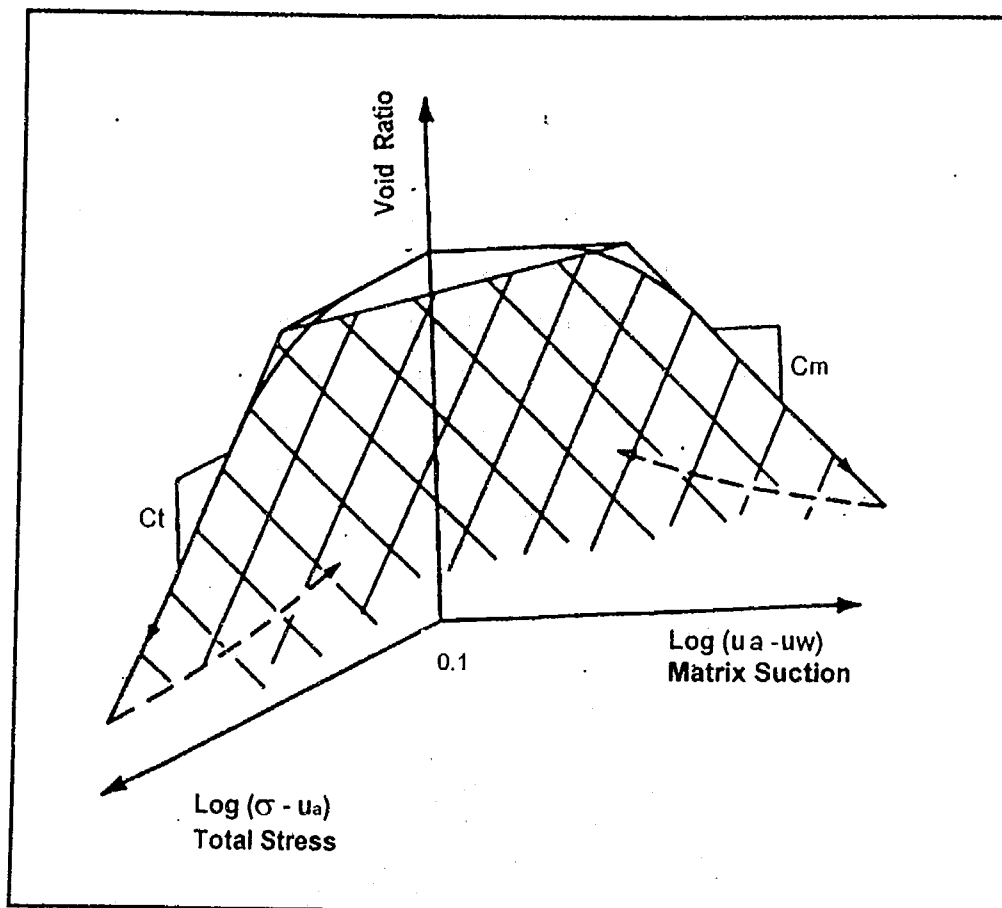


Figure 4.2 Fredlund's three dimensional plot of stress and suction paths for unsaturated soils

4.4 Determination of Material Parameters

To apply the relation of Equation (4.1) in the mechanistic model of swell and swell suppression, the swell test results at two applied stress levels are required. The lower one, σ_{ps} at 7 kPa (1 psi), is used as threshold value below which there is no effect on volume change with further decrease in pressure. The initial estimation of magnitude of σ_{ps} depends on one's judgment, however, a more correct value can be obtained by conducting a few oedometer tests at lower stress levels. The upper value of applied stress level is chosen as 100 kPa (14 psi), which is in the order of magnitude of high swelling pressure encountered in a swelling soil profile. Using the concept of free swell and swell suppression, the swell characteristics under light

overburden pressure without any suppression can be defined by the first term in the right hand side of Equation (4.1) as

$$\Delta \epsilon_1 = \gamma_{h1} \cdot \Delta(\log h) \quad (4.2)$$

where $\log h$ represents logarithm of suction, $\Delta \epsilon_1$ represents the vertical swell strain in the oedometer test, and γ_{h1} represents the free swell index under laterally confined swelling condition. For a practical range of suction varying from 4.5 log kPa to 1.5 log kPa, the free swell index ' γ_h ' is treated as constant.

Similarly, if the soil profile is subjected to a higher overburden pressure which suppresses the potential swell, then the swell suppression at the extreme wet limit can be described by the relationship

$$\Delta \epsilon_1 = \gamma_{\sigma 1} \cdot \Delta(\log \sigma_1) \quad (4.3)$$

where σ_1 is the applied overburden stress, and $\gamma_{\sigma 1}$ is the swell suppression index under confined compression.

The two indexes mentioned above, ' γ_h ', and ' γ_{σ} ', which are employed in the relation of Equations (4.2) and (4.3), and together in Equation (4.1), are the two constant characteristics of soil (Picornell and Lytton, 1984).

Figure 4.3 illustrates how the swell is suppressed under a vertical applied stress of 100 kPa. The underlying assumption in Figure 4.3 is that $\gamma_{\sigma 1}$ varies linearly with the log suction and is zero at the point of zero swell suppression as suggested by the data in Figure 3.2. This zero swell suppression is a hypothetical point of convergence which lies at a suction in excess of dry limit. Intuitively it should occur at shrinkage limit. Physically, this means a further reduction in moisture will not make any change in volume for any suppression stress level. The magnitude of suction pressure at zero swell suppression lies normally within 4 to 4.5 log kPa, which is also found in the plot of Figure 3.2. This convergence value is higher than a value of 3.2 log kPa obtained by Sapkota (1994) in his analysis of Richards' test results on Hindmarsh clay of South Australia (Richards 1977). Likewise, the back analysis illustrated in Figure 3.3 on Meintjes' (1992) test results yields the convergence point at approximately 3.8 log kPa. However, other values of convergence can also be considered for parametric studies.

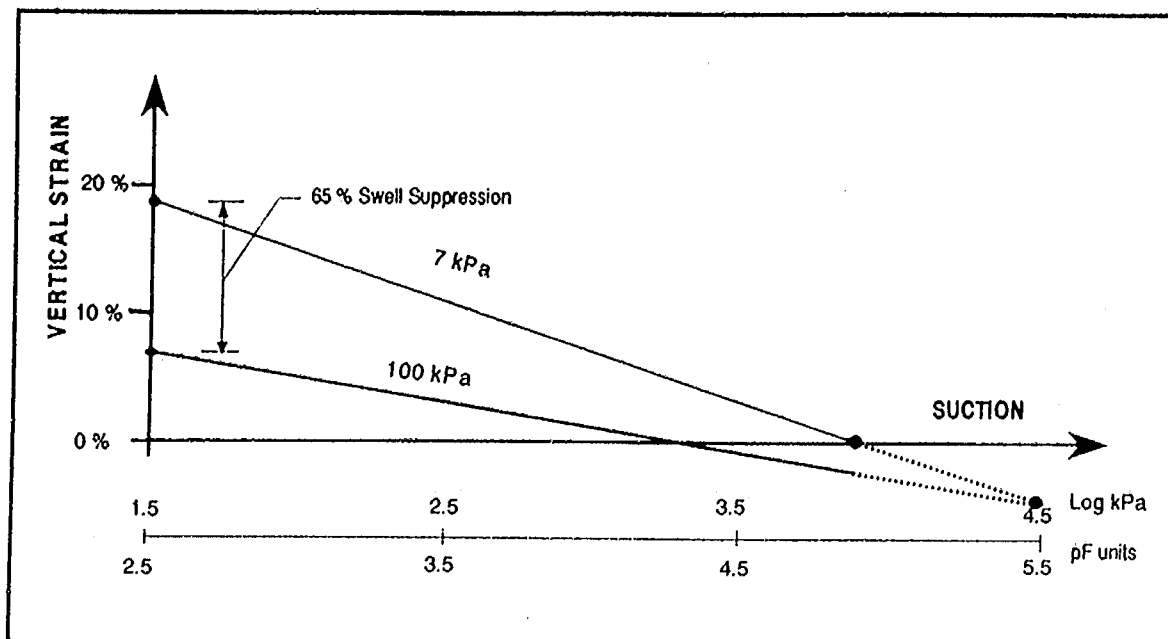


Figure 4.3 Characterisation of swell and swell suppression in an oedometer test condition

For the parametric analysis in Sections 4.5.2 and 4.6, a convergence suction pressure of 4.5 log kPa is taken initially. In later application, a value of 4.0 log kPa is selected for the numerical analysis as will be discussed in Chapter VIII.

4.5 Development of Swell and Swell Suppression Equations Describing Lateral Swell Pressure and Heave

The concept of piece wise linear elastic analysis discussed in Section 3.4 is used to develop the constitutive relations for swelling and swell suppression under applied stress. The general boundary conditions associated with oedometer and triaxial test conditions are applied. Two types of material models, the first model representing anisotropic behaviour of swelling soil, and the second one representing a stress dependent isotropic behaviour are developed. The detailed discussion on these models are presented in Appendix A, here only the second model which describes isotropic stress dependency of swell suppression modulus is discussed briefly as it will be applied later in the numerical analysis in Chapter VII and VIII.

In both models, it is assumed that Poisson's ratio ' ν ' for swell suppression is constant, that is, independent of stress level and direction. It is also assumed that the swell suppression in the principal stress direction is described by E_{s1} (defined by Equation A.2 of Appendix A). Furthermore, the soil is considered to swell from a relatively dry stiff initial state. The rebound deformations that are likely to occur as a result of the removal of in situ stresses in this initial dry state are considered to be much smaller than the potential free swelling strains. Wallace and Lytton (1992b)

justified these conditions as typical of those which could occur in the upper few metres of a reactive soil profile.

4.5.1 Constitutive Relations

Stress Dependent (Isotropic Case)

In this model Wallace and Lytton (1992b) assumed that the modulus of swell/swell suppression (E_s) varies with the 'nth' power of the octahedral normal stress:

$$E_s = E_{s1} = E_{s2} = E_{s3} = B \left(\frac{\sigma_1 + 2\sigma_3}{3} \right)^n \quad (4.4)$$

where 'B' is a parameter that relates swell modulus to the stress level. This model is selected for the numerical analysis in Chapter VIII. The reason for its choice is its ability to describe non-linear suction and stress dependent swell modulus.

In determining the Young's modulus of swell suppression E_s as the function of octahedral normal stress and log suction, the initial value of B parameter can be taken as β described by Equation (A.2) of Appendix A for two quarter level vertical stresses of 30 and 70 kPa. Later with this initial value of B ($= \beta$) lateral stresses are computed. With the new value of octahedral stress, the exact value of B is found iteratively by using Equation (4.4). (More detailed discussion on calculation of B values is available in Sapkota (1994) or Wallace and Sapkota (1992, 1994)). Following this procedure, the sensitivity of B parameter with the index 'n' for swelling of an extremely expansive soil is presented in Figure 4.4 below.

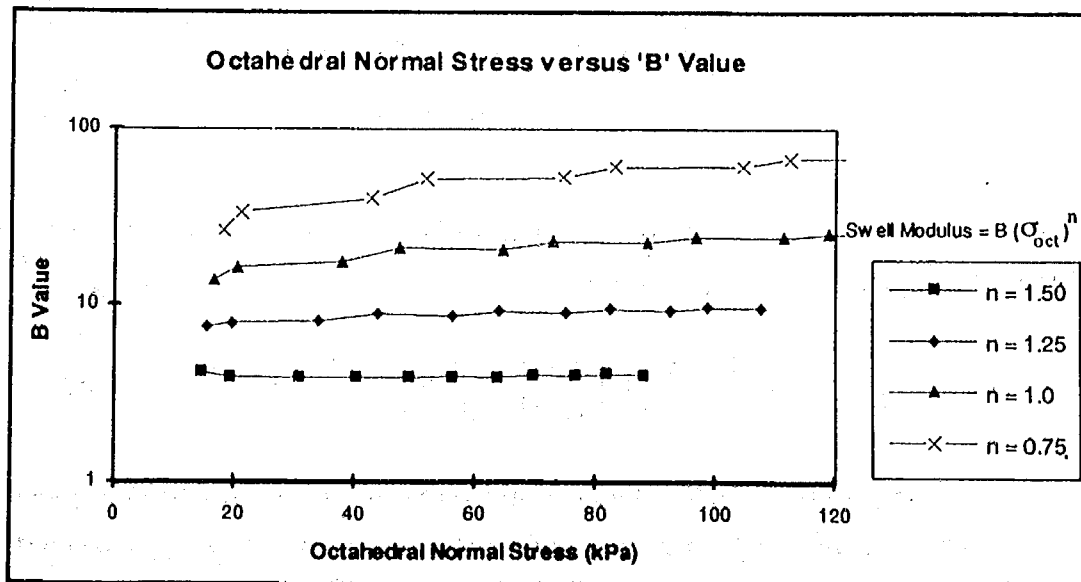


Figure 4.4 Sensitivity of B value with stress level

As shown in Figure 4.4, for values of 'n' ranging between 1.0 to 1.5, B value is more constant over the range of octahedral normal stress ' σ_{oct} '. Therefore, 'n' value of this order gives a more consistent description of swell modulus ' E_s ' for isotropic swelling soils. Although $n = 1.50$ appears to be the closest to give a consistent value of swell modulus over a range of mean stress level, a value of $n = 1.0$ is considered for the present analysis as this value of 'n' leads to an instructive, closed form solution. In addition, it reduces further degree of non-linearity in the computation of modulus besides the already existing non-linear variation of B with suction level.

4.5.2 Equations for Lateral Swell Pressure and Heave

When fissures and shrinkage cracks are present in a dry soil, the development of lateral swell pressure is reduced as a result of a release of initial stresses until such cracks are closed. The exact estimation of this effect on swell pressure measurement is very complex because of the associated difficulty in determining the volume of cracks that are present within a soil profile. However, some adjustments can be made to account for such effect by introducing some coefficients or parameters. Wallace and Lytton (1992b) has indicated that the effect of crack on the swelling pressure can be estimated by introducing a crack closure factor ' ξ ' in the calculation of lateral strain. The final compressive strain of the soil with application of ' ξ ' is presented in equation A.24 (Appendix A).

The derivations of lateral swell pressure and heave is discussed in Appendix A. Here the final forms are presented. The lateral swell pressure coefficient ' K_s ' for a swelling clay profile with ' K_s ' greater than 1 is:

$$K_s = -0.5 + 1.5 \text{ EXP} \left[\frac{\left\{ B \epsilon_{30} (1 - \xi) - (1 - 2\nu) \ln(\sigma_1 / \sigma_{ps}) \right\}}{1.5(1 - \nu)} \right] \quad (4.5)$$

The equation of vertical heave with coefficient of lateral swell pressure ' K_s ' > 1, can be obtained from the following equation

$$\text{Heave} = \epsilon_1 = -\epsilon_{10} + \left(\frac{1 - 2\nu}{B} \right) \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) - \frac{3\nu}{B} \ln \left(\frac{1 + 2K_s}{3} \right) \quad (4.6)$$

This equation has been used by Wallace and Sapkota (1992) to estimate ground heave of a 600 mm thick extremely expansive clay subgrade which is subjected to the wetting and swelling. They found that under an applied vertical stress of 7 kPa equivalent to an overburden pressure of thin asphalt pavement over a granular

base, the vertical heave could reach to about 70 mm. This value of pavement heave falls within the range of 40 to 80 mm as the magnitude commonly observed in the field (Donaldson, 1965, Waters and Kapitske, 1979, and others).

Regarding the lateral swell pressure, Wallace and Sapkota (1992) also estimated lateral swell pressure by using above equation (Equation 4.6) in Hindmarsh clay of South Australia. The field measurement of lateral pressure exerted by this soil against a retaining structure has been reported by Richards (1977). The lateral swell pressure results obtained from the above equation at a depth of 7.5 m was found to be 225 kPa while the measured values reported by Richards are between 300 to 800 kPa. This means the present theoretical model yields a lower bound value. Recognising this, Wallace and Sapkota (1992) conducted a sensitivity analysis of swell pressure to 'B' parameter, and found that for this analysed situation, the variation of lateral swell pressure with an increase in the value of B followed the following pattern described in Table 4.4 below.

Table 4.1 Variation of swell pressure with swell modulus coefficient 'B'

B value	21.6	28.1	32.4	35
Lateral Swell Pressure (kPa)	225	415	605	755

4.6 Typical Values of Soil Properties

A. *Poisson's Ratio, ν*

The value of Poisson's ratio ' ν ' can be calculated by comparing the swell strains in confined compression, and in free three dimensional swell as given by the ratio

$$\frac{\epsilon_v \text{ confined}}{\epsilon_v \text{ 3D free}} = \frac{1}{3} \frac{1 + \nu}{1 - \nu} \quad (4.7)$$

where ϵ_v is volumetric strain.

Pile and McInnes (1984) indicated that this ratio is generally less than 2/3 while other investigators found values down to about 1/2. Livneh et al.(1973) applied the oedometer test results of Komornik and Zeitlen (1965, 1970) on highly expansive clays and obtained a value of ν as 0.25. Uzan, Baker and Frydman (1987) found ν value for swelling marl from 0.1 to 0.3. From these and some other available data, it seems reasonable to assume a range of values of ν to vary from 0.2 to 0.35. For the present analysis, a value of $\nu = 0.3$ is adopted.

B. Free Swell Index, γ_{hl}

Wallace and Sapkota (1992) back calculated the typical values of free swell index for expansive soils of Australia published in the Australian Code for Residential Slabs and Footings (AS 2870). By assuming that the soil at a site is uniform over the depth of seasonal moisture movements, and that Poisson's ratio for swell suppression is equal to 0.3, the following values were obtained

Table 4.2 Back calculated magnitude of free swell index

Site classification	Heave (mm)	Free swell index for confined compression ADELAIDE	Free swell index for confined compression SYDNEY
Highly Reactive	40 - 70	0.031 - 0.053	0.048 - 0.085
Extremely Reactive	70 ->100	0.053 - 0.076	0.085 - 0.121

As depicted in Table 4.2, the value of swell index for the present theoretical parametric study is considered to vary between 0.035 to 0.120. For the numerical analysis, values of 0.035 and 0.07 are considered.

C. Swell Suppression Index, γ_{o1}

As indicated in Figure 4.3, the swell suppression index ' γ_{o1} ' decreases with increase in logarithm of suction pressure. In the analysis, an initial value of γ_{o1} at wet limit for wetting of extreme dry state (4.5 log kPa) to an extreme wet state (1.5 log kPa) will be used. For an intermediate range of suction, γ_{o1} is computed with log linear variation with suction pressure. In the following, typical values of γ_{o1} published in the literature as well as values obtained from parametric analysis of available swell pressure correlations are discussed.

Analysis on Meintjes' (1992) results on South African expansive clays (LL = 89 %, PI = 57 %, and % clay fraction = 62 %) tested under controlled moisture intake indicated that for a suction range between 2.7 log kPa to 3.5 log kPa, γ_{o1} varies from 0.08 to 0.022 with an intermediate value of 0.053 at a suction value of 3 log kPa. Back analysis on the response of Hindmarsh clay reported by Richards' (1977) yields values for γ_{o1} of 0.046 at suction of 1.5 log kPa and 0.033 at 2.3 log kPa.

Some correlations between index properties and swell pressures can be used to define the approximate range of the values of γ_{o1} for an early studies. There are several correlations available that relate swell pressure and heave with index

properties of soil such as liquid limit, plasticity index, initial water content and dry density etc. A detailed discussion on the selection of correlations and application of these correlations in finding the magnitude of swell pressure index $\gamma_{\sigma 1}$ is available on Sapkota (1994) and Wallace and Sapkota (1994).

Picornell and Lytton (1984) back calculated the values of $\gamma_{\sigma 1}$ from their swell pressure test results on swelling marl, and found $\gamma_{\sigma 1}$ to vary from 0.0715 for a specimen having an initial moisture content of about 7 % to 0.115 for moisture content of about 4.7 %.

Test on a typical expansive clay (LL = 90% and PI = 57%) of Brisbane, Australia conducted by Sapkota (1994) resulted a magnitude of free swell index of about 0.054. The swell suppression index of this soil at wet limit (1.5 log kPa) is found to be about 0.06 under 100 kPa as upper range of applied stress and 0.102 under 15 kPa as lower range.

From these available information on the values of $\gamma_{\sigma 1}$, it is concluded that the range of $\gamma_{\sigma 1}$ for expansive clays lies from 0.03 to 0.12. Values of 0.04 and 0.08 at wet limit are selected for use in the numerical analysis. This leads to the selection of $\gamma_{\sigma 1}$ from a wider distribution of expansive soils having about medium swell to extremely high swell potential.

4.7 Validation of Swell and Swell Suppression Model with a Set of Oedometer Test Results on Undisturbed Soil Specimen

A set of laboratory tests on expansive soils of Brisbane were carried out by the first author for a similar study conducted previously at the Queensland University of Technology, Brisbane in Australia. The objective of the tests were to validate the proposed model of swell and swell suppression. The details on the test specimen and the procedure are available in Sapkota (1994), here some of the results are reproduced as a part of validation of the theoretical model.

The tests were carried out through a laboratory oedometer. A modified consolidometer ring capable of measuring lateral swell pressure was employed. It was initially developed by Komornik and Zeitlen (1965) and later applied by Ofer (1980) and Ofer and Blight (1985). Figure 4.5 illustrates the section of the modified consolidation ring used in the test series.

Undisturbed, specimens with known initial suction measured by a thermo-couple psychrometer were used. The suction potential after the final stage of swelling was

also measured using the filter paper technique. During the swelling period, a continuous record of lateral swell pressure and vertical deformations were taken.

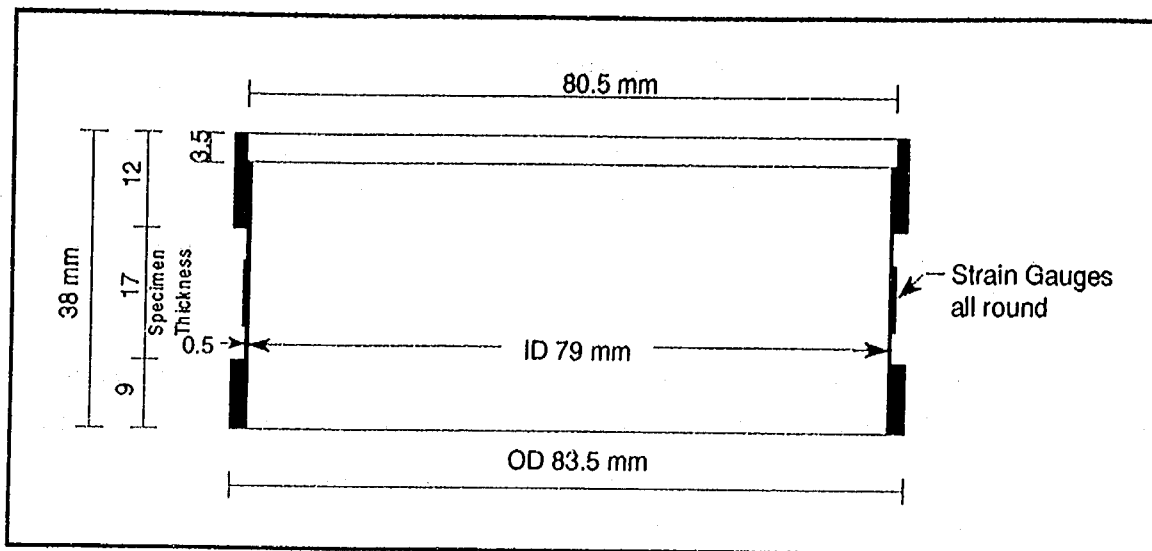


Figure 4.5 Modified oedometer cell to measure lateral swell pressure

Constant vertical pressure of three different magnitude of 7 kPa, 15 kPa, and 100 kPa were applied to the soil specimen just after inundation. The adoption of this loading condition was based on the concept of free swell and swell under suppression applied in the simplified mechanistic model (Section 4.5). The lower values represent typical overburden pressure which is close to the weight of the overlying pavement and self weight of subgrade at a depth of about 600 mm. The higher value of 100 kPa was selected to cover a fairly wider range of pressures.

Lateral swell pressure was measured through the micro strain reading that developed in the strain gauges due to hoop stress in the cell wall as shown in Figure 4.5.

4.8 Parametric Analysis of Test Results to Validate the Swell/Swell Suppression Model

The analysis used the theoretical equations of swell and swell suppression discussed in Section 4.5. Theoretical values of lateral swell pressure and swelling modulus were calculated by using equations. The values of soil parameters such as ' γ_h ', ' γ_o ', range of decrease in suction pressure etc., required by the equations were obtained from the test measurements.

The values of free swell ' γ_h ' and swell suppression ' γ_o ' parameters required in the theoretical model were calculated by using these heave measurements, the change in suction and the applied vertical stresses in the following expressions (Equations 4.2 and 4.3):

$$\gamma_h = \frac{\Delta \varepsilon_v}{\Delta (\log h)}, \text{ and } \gamma_{\sigma_1} = \frac{\Delta \varepsilon_v}{\Delta (\log \sigma_v)}$$

where ε_v is the vertical strain, $\Delta \log h$ is the change in log suction, and σ_v is the applied vertical stress from an oedometer swell test.

A 4.0 log kPa of total suction was considered as the suction above which there is no suppression of swell with increase in stress (Section 4.4). This value of suction pressure was used in the calculation of γ_{σ_1} . The value of γ_{h1} was 0.0538, and remained constant for parametric analyses of all tests. γ_{σ_1} varied with tests. The details on the computation of these parameters and the other parameter 'B' required in the estimation of modulus of swell suppression under isotropic model of swell/swell suppression are available in Sapkota (1994).

Table 4.3 presents a comparative results of measurement and calculated values of lateral swell pressure. As discussed in Section 4.5.2, lateral swell pressure is very sensitive to the value of B parameter. The result of a typical analysis presented earlier in Table 4.1 indicated that a 30 % increase in B value resulted an 85 % increase in predicted lateral swell pressure. Therefore, in a situation of under prediction of lateral swell pressure, the study suggests to apply a coefficient to increase in B value. With an increase in B, the magnitude of swell suppression modulus increases by

$$E = B' \left(\frac{\sigma_1 + 2\sigma_3}{3} \right), \quad \text{where } B' = C \cdot B$$

Table 4.3 Estimation of lateral swell pressure with modified value of B parameter in *Stress Dependent Isotropic Model*

Test No	Applied Vertical Stress	Measured Lateral Stress	Computed Theoretical Lateral Stress							
			B	Lateral Stress	1.25 B	Lateral Stress	1.5 B	Lateral Stress	1.75 B	Lateral Stress
7 kPa as threshold pressure										
3	100	240	39.61	82	45.52	115	59.42	155	69.33	206
4	100	190	36.49	107	45.61	154	54.74	216	63.86	296
6	100	210	34.10	64	42.62	86	51.15	114	59.67	147
7	15	100	16.22	22.7	20.27	27.5	24.33	33	28.38	39.4
15 kPa as threshold pressure										
3	100	240	45.16	110	56.46	145	67.75	187	79.04	239
4	100	190	45.54	154	56.93	214	68.32	291	79.70	392
6	100	210	33.78	79	42.23	98	50.67	121	59.12	147

Note: In total 8 number of tests were carried out. Test No. 1 was under 7 kPa vertical pressure (considered as free swell test) and test data used as a reference for free swell. Test 2 had some technical difficulty. Test 5 was conducted under 15 kPa vertical pressure like Test 7, and used as free swell reference when 15 kPa is considered as threshold pressure. Test 8 was an additional test whose data is not used in the above table.

In Tables 4.3 lateral swell pressure was re-estimated by using the multiplying coefficient C as 1.25, 1.5, and 1.75.

When compared with the measured values of swell pressure, the results in the Table 4.3 show that a coefficient of 1.5 to 1.75 may be suitable to multiply B value for the test condition. The value of such a coefficient can be also estimated for other types of swelling soil if test results, in the form of similar swell tests are available.

4.9 Discussions and Limitations

Wallace and Lytton (1992b) attempted to study the effect of change in the value of Poisson's ratio ν from 0.3 to 0.2 to interpret the oedometer test. They found an increase in lateral swell pressure by 22 percent when ν is decreased from 0.3 to 0.2. In the formulation of the model, it is assumed that Poisson's ratio remain constant which is generally unacceptable. In addition, if it changes during the wetting and swelling process, even by a small amount, the error associated could be significant.

As discussed in Section 4.5.2 and in Table 4.3, the model under predicts the lateral swell pressure. Since swell pressure is primarily dependent on the magnitude of swell suppression modulus ($= B \sigma_{oct}$), it may be suggested to increase B parameter by multiplying it with a coefficient. This correction coefficient can be obtained from the measured swell pressure from oedometer swell tests. For example, in this research study, the value of this correction factor obtained from the swell test (Table 4.3) lies between 1.5 to 1.75.

The variation of swell suppression modulus with moisture condition is very critical. During the analysis in a previous study (Sapkota 1994), it was found that about three quarter of swell pressure develops at only half of the ultimate suction change at shallow depth. This results in a large variation in moduli with very high modulus of unwetted soil to small swell modulus of a wetted one. The ratio between these two moduli was found to be at least 10. Such difference may result in the initial yielding of the wetted zone.

It is still uncertain as to what value of threshold pressure should be considered. To overcome the difficulties of accurately predicting the swell behaviour of a subgrade, swell and swell suppression tests should be conducted precisely over a range of stresses and suction that are close to the expected field conditions. Use of results obtained from such controlled tests is expected to greatly improve the accuracy of model results.

The effects of existing crack volume and crack closure due to debris collection is not well understood at this stage. Furthermore, some thin cracks may close before the middle stage of swelling. Also, the model does not take into account the possible change in crack closure factor during the swelling process.

4.10 Conclusions

Swelling behaviour of an expansive clay subgrade is a very complex phenomenon as it is affected by various factors pertaining to climatic conditions and hydrology of the area, mechanical characteristics of soils, geometry and boundary conditions, as well as the nature and magnitude of the overburden pressure. By using an idealised description of swell and swell suppression characteristics, it is possible to represent the effects of several factors that interact in the development of vertical heave and swell pressure. For this purpose, a simplified mechanistic conceptual model of swell and swell suppression developed by Wallace and Lytton (1992b) is extended to analyse the wetting and swelling behaviour of an expansive clay subgrade. Most of the data on swell and swell suppression had been acquired from one dimensional oedometer tests or from field observations of swelling in expansive clay profiles. These data are analysed to obtain soil properties and parameters necessary for the mechanistic model.

In the mechanistic model of swell and swell suppression, the swelling process is described by two characteristic behaviours: (1) free swell, and (2) swell under suppression in an oedometer condition. When these characteristic swell behaviours are described in three dimension, the plot (Figure 4.1) becomes similar to Fredlund's (1979) diagram describing the constitutive behaviour of unsaturated soils. This plot also illustrates the variation of vertical strain in an oedometer test condition with the log of applied stress and the log of total suction.

The present model uses two indexes, namely; (1) free swell index ' γ_h ', and (2) swell suppression index ' γ_o '. The first index is taken as constant while the second one varies with the suction level (Section 4.4). Also, analysis of available swell test

results showed that the value of suction for zero swell suppression (Figure 4.3) lies around 4 to 4.5 log kPa.

Application of the model produces two constitutive relations (Section 4.5). In this study, the constitutive relation which describes the swell modulus varies with the 'nth' power of the octahedral normal stress (Equation 4.4) is adopted. Poisson's ratio 'v' is assumed as constant during the swell and swell suppression. The parametric study shows that this relation gives a consistent description of swell modulus over a range of octahedral normal stress (Figure 4.4). Also it is in a convenient form to apply in the numerical analysis.

The swell modulus depends largely on the value of B parameter. As such, the accuracy of analysis depends on obtaining an accurate value of B. Having obtained the values of soil properties and swell indexes, an iterative procedure can be applied to obtain a B value under a particular vertical stress and suction level (Section 4.5.3). A more accurate B value can be obtained from the result of precisely conducted oedometer swell tests (Table 4.3).

The magnitudes of the swell parameters γ_h and γ_σ can be estimated by using the swell pressure correlations or from back analysis of published results on similar soil and site conditions. As discussed in Section 4.6, the analyses of published results suggest a typical range of free swell index ' γ_h ' between 0.035 and 0.12, and a typical magnitude of swell suppression ' γ_σ ' between 0.03 and 0.12. These ranges represent soils having low to extremely high swell and swell suppression behaviour.

At the present stage, this proposed simplified mechanistic model has some limitations as described in Section 4.9. Particularly, the model is not robust enough to describe the detailed stress path followed during the process of wetting and swelling. Hence, to improve the model against these limitations, the writers recommend for further refinements of the model in order to precisely describe the swell and swell suppression by rigorously following the stress and suction paths.

CHAPTER V

MECHANISTIC MODEL FOR SUBGRADE SHRINKAGE COMPRESSION

5.1 Introduction

In this chapter a mechanistic model that describes the shrinkage and compression characteristics of reactive soils is discussed. The model is developed by extension of the concept of swell and swell suppression characteristics of reactive soils discussed in Chapter IV. It considers total suction (which comprises both matrix and osmotic suctions) and applied stress as the major variables responsible for volume decrease and development of associated shrinkage stress. Uniaxial stress and suction dependent moduli of shrinkage compression are deduced and used to derive the shrinkage compression equations. These relations are used to study the shrinkage stress and subsidence that develop in a shallow soil profile subjected to a monotonic seasonal drying.

5.2 Characterisation of Shrinkage Behaviour

Figure 5.1 describes how the shrinkage characteristics of a given initially wet soil sample vary with applied stress and log of total suction between the extreme wet and dry limits. It assumes that the shrinkage strain developed due to drying is linearly related with log suction. The model views down the "total stress" analysis under applied stress condition, and assumes that the elastic compression is linearly related with the change in logarithm of applied stress. It follows the shrinkage path in a manner similar to Fredlund's three dimensional diagram of the constitutive surface for an unsaturated soil (Fredlund and Morgenstern, 1976).

Like in the swell and swell suppression model discussed in Section 4.3, this conceptual model of shrinkage compression requires results of shrinkage tests under two different applied stress levels. The lower stress level should be a threshold value below which any changes in applied stress will bring no effect in compression due to drying. It is represented by σ_c in Figure 5.1. For the purpose of present analysis, value of this stress level is selected as 7 kPa (1 psi) as used in the traditional free swell test in an oedometer. The magnitude of upper level σ_d' in the figure, depends on the value of overburden in a soil profile, with a minimum as the threshold pressure. For a parametric analysis, a 100 kPa maybe selected as it represents an upper limit of the order of magnitude of vertical stress normally observed under light structure. However, for an analysis of a field situation, the

value selected should represent vertical pressure in the soil profile. Also similar to swell model, this shrinkage model assumes that due to osmotic effects and/or due to lack of effect of total suction on volume change below a threshold value, a minimum value of total suction of $1.5 \log \text{ kPa}$, oa in the figure, represents the state of soil water potential at the initial soaked condition. The final value of total suction for an analysis of volume change due to drying may be conservatively taken as the total suction at shrinkage limit of a soil such that drying beyond this value would not cause further volume change. Typical value of total suction at shrinkage limit represented by ob' in the figure, may lie around 4 and $4.5 \log \text{ kPa}$.

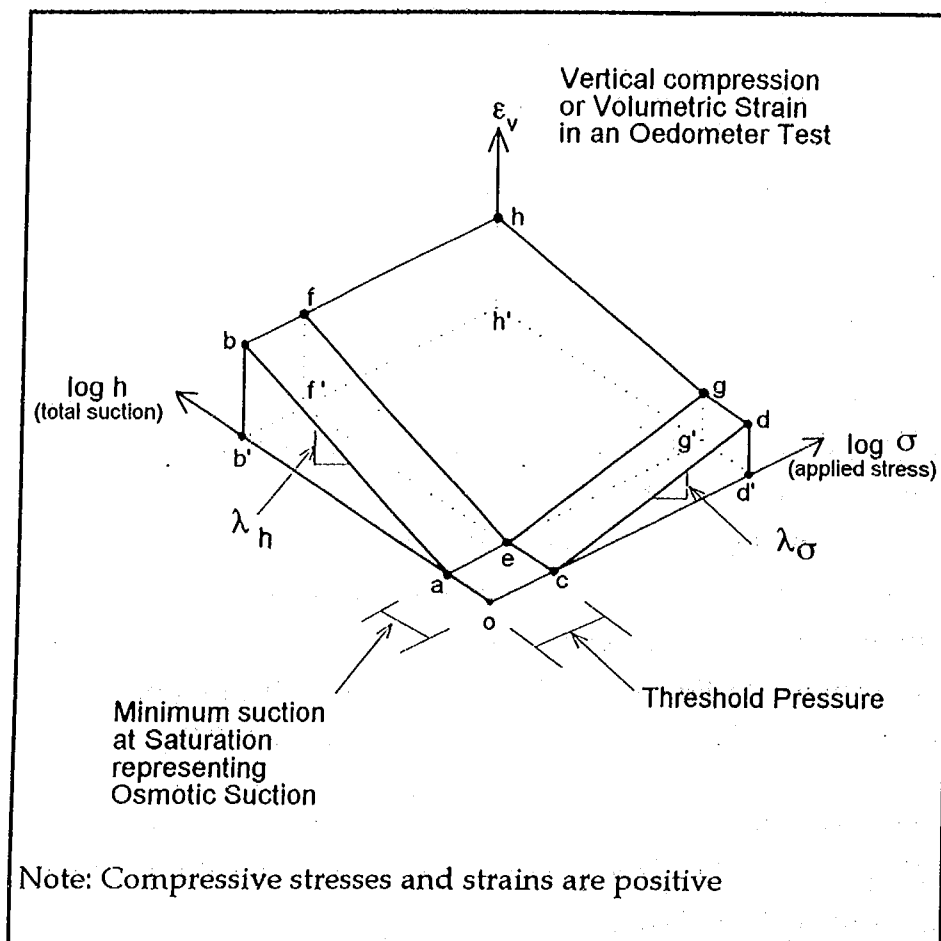


Figure 5.1. Shrinkage strain path in three dimensions

An important underlying assumption made in the model is that there lies a certain value of total suction at which the curves of shrinkage compression under threshold stress and under applied stress levels converge together. This is a hypothetical point of convergence in compression versus $\log$ total suction curve (line hb in Figure 5.1) at suction in excess of extreme dry limit. Intuitively, the value of this total suction should also represent that of shrinkage limit. For the present

analysis purpose, a value of 4.0 log kPa is considered. However, several values can be considered in a parametric study.

By applying concept of linear relationship between volume decrease and increase in log of total suction, volumetric compressive strain under light pressure can be described by

$$\Delta \varepsilon_{vh} = \lambda_h \Delta \log h \quad (5.1)$$

where λ_h is the slope of curve ab or ef in Figure 5.1. It is termed as free shrinkage index for a one dimensional confined shrinkage condition and is defined in a form similar to suction modulus of Fredlund and Morgenstern (1976). $\Delta \varepsilon_{vh}$ is the volumetric shrinkage due to change in log of total suction ($\Delta \log h$), when the soil sample is considered to undergo a one dimensional confined shrinkage.

On the other hand, volumetric compression of a drying soil due to applied stress can be described by

$$\Delta \varepsilon_{v\sigma} = \lambda_\sigma \Delta \log \sigma \quad (5.2)$$

where λ_σ is the slope of curve cd or eg in Figure 5.1. It is termed as shrinkage compression index for a one dimensional confined shrinkage condition. It relates to Young's modulus of shrinkage compression by describing volumetric compression strain to log of change in applied stress.

Combining together, the total volumetric compressive strain of shrinkage due to drying and applied stress becomes:

$$\varepsilon_v = \varepsilon_{vh} + \varepsilon_{v\sigma} = \lambda_h \Delta \log h + \lambda_\sigma \Delta \log \sigma \quad (5.3)$$

Note that λ_h and λ_σ are the constant characteristics of shrinking soils. Also according to the model, λ_σ at the suction representing the point of convergence becomes zero. This means, when the soil is at shrinkage limit, an increase in stress will cause only elastic structural compression. The soil will not have any shrinkage compression.

5.3 Compressibility and Strength Behaviour

In a general one dimensional oedometer compression test of a non-reactive soil, the vertical or volumetric strain ε_v developed can be obtained from the usual relation as:

$$\epsilon_v = \frac{(1 + \nu)(1 - 2\nu)}{(1 - \nu)} \frac{\sigma_z}{E} \quad (5.4)$$

According to the present model of soil shrinkage, the vertical strain due to applied stress in the above test condition can be described by Equation (5.2). If these two relations are compared, the value of Young's modulus ' E_{sh} ' of a shrinking soil under applied stress becomes

$$E_{sh} = \frac{2.3 (1 + \nu)(1 - 2\nu)}{\lambda_o (1 - \nu)} \sigma_z \quad (5.5)$$

$$\text{or } E_{sh} = \beta_c \sigma_z \quad (5.6)$$

$$\text{where } \beta_c = \frac{2.3 (1 + \nu)(1 - 2\nu)}{\lambda_o (1 - \nu)} \quad (5.7)$$

If the concept of λ_o to become zero at the point of convergence is applied, Equation (5.5) results an infinite or excessively high value of modulus of shrinkage compression for suction at or close to shrinkage limit. In that stage the model instructs to change modulus of compression from shrinkage compression modulus to the Young's modulus of load deformation of a dry stiff clay. Theoretically it means when the soil is close to shrinkage limit, there will be no volume change by further reduction of moisture. But there will be elastic strain when a load is applied to this soil, and this strain is elastic structural strain. Therefore, to obtain this strain under applied load condition, Young's modulus of load deformation is applicable.

In the development of the present simplified conceptual model of shrinkage and compression, the volume changes are expressed in terms of vertical strain ϵ_1 in an oedometer test situation, which is equal to volumetric strain ϵ_v for the zero lateral strains. However, it should be noted that although the current explanation is in terms of oedometer test results which are the most common swell/shrink tests conducted in a routine laboratory works on reactive soils, the concept can be applied in a similar manner with other tests or field situation such as under isotropic compression. Equation (5.8) below relates the elastic strain developed in vertical direction under one dimensional oedometer test to that under isotropic three dimensional compression condition.

$$\epsilon_{z1-D} = \frac{1 - \nu}{1 + \nu} \cdot \epsilon_{z1-D} \quad (5.8)$$

Equation (5.1) described shrinkage strain developed due to drying under load free or under threshold pressure in an oedometer test condition. Now to describe the linear free shrinkage strain in a 3-dimensional free condition the relation modifies to

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \lambda_h \cdot \Delta \log h \cdot \frac{1 - \nu}{1 + \nu} \quad (5.9)$$

Also volumetric strain in three dimensions is considered as:

$$\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad (5.10)$$

5.4 Constitutive Relationship for Shrinkage Compression

A constitutive relationship is developed to derive the expressions relating shrinkage strains to shrinkage stresses for three dimensional axi-symmetric compression. The relationship presents a uniaxial stress dependent model and leads to a close form solution.

In the model it is assumed that Poisson's ratio for shrinkage compression remains constant, that is it is assumed to be independent of stress level and direction. Also soil undergoes shrinkage due to monotonic drying from a uniformly wet state to a relatively dry state.

An element of soil as shown in Figure 5.2 in an *axi-symmetric condition* is considered to undergo shrinkage under controlled environment in which vertical principal stress which may be overburden pressure alone or combination with other stresses, does not exceed a limiting value, σ_1 , and the lateral principal stresses $\sigma_2 = \sigma_3$, may be varied to limit the shrinkage strain in the horizontal direction. In order to visualise the field situation where shrinkage stresses develop upon drying, a soil element undergoes shrinkage in four stages as described in Figure 5.2.

For this hypothetical soil element, shrinkage compression in any principal direction is dependent on the magnitude of the principal stress in that direction. Accordingly Young's modulus of deformation in the principal direction can be described by Equation (5.5) with the appropriate principal stress. Principal stress in the illustrated situation is in the vertical direction. Therefore, in Equation (5.5) the shrinkage compression modulus has become function of vertical stress. Also, this implies that, for the present axi-symmetric case, the vertical modulus should generally be different from the horizontal modulus.

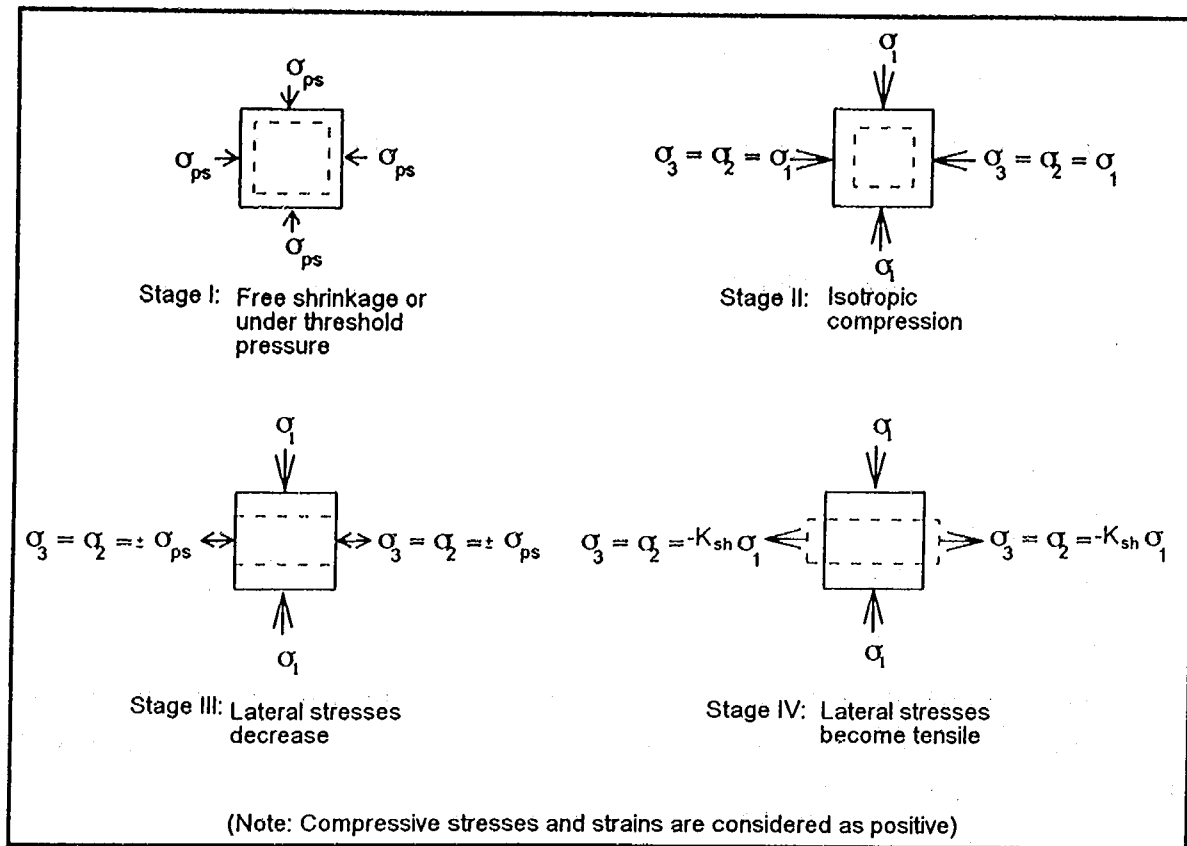


Figure 5.2 Stages of shrinkage due to drying under applied vertical stress

Vertical Compression and Lateral Shrinkage Strain

The detailed derivation of equations for development of principal strains at each stage is presented in Appendix B. Summing up the strains developed at all stages, vertical strain ϵ_1 resulting shrinkage subsidence and horizontal strain ϵ_3 are:

$$\epsilon_1 = \epsilon_{1_0} + \frac{1}{\beta_c} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) + \frac{2\nu}{\beta_c} \ln \left(\frac{K_{sh} \sigma_1}{\sigma_{ps}} \right) \quad (5.11)$$

$$\text{and } \epsilon_3 = \epsilon_{3_0} - \frac{\nu}{\beta_c} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) - \frac{1-\nu}{\beta_c} \ln \left(\frac{K_{sh} \sigma_1}{\sigma_{ps}} \right) \quad (5.12)$$

Lateral Shrinkage Stress

K_{sh} in Equations (5.11) and (5.12) is the ratio of lateral shrinkage stress to applied vertical stress, and determines the magnitude of lateral tensile stress $K_{sh} \sigma_1$ that can develop in a drying soil profile. To determine the value of K_{sh} , consider $\epsilon_3 = 0$. Then the expression for K_{sh} becomes

$$K_{sh} = \left(\frac{\sigma_{ps}}{\sigma_1} \right) \cdot \text{Exp} \left[\frac{\beta_c \epsilon_{3_0} - \nu \ln(\sigma_1 / \sigma_{ps})}{(1 - \nu)} \right] \quad (5.13)$$

With the substitution of β_c and ϵ_{3_0} using Equations (5.7) and (5.9) respectively the relation for K_{sh} becomes

$$K_{sh} = \left(\frac{\sigma_{ps}}{\sigma_1} \right) \cdot \text{Exp} \left[\frac{2.3(1 - 2\nu)(\Delta \log h)(\lambda_h / \lambda_o) - \nu \ln(\sigma_1 / \sigma_{ps})}{(1 - \nu)} \right] \quad (5.14)$$

This expression of K_{sh} indicates that by varying λ_h and λ_o but maintaining their ratio constant, magnitude of K_{sh} remains unchanged.

5.5 Magnitude of Soil Properties and Shrinkage Parameters

A. Poisson's Ratio

As described by Equation (5.8) Poisson's ratio ' ν ' relates shrinkage strains under 1-dimensional confined condition and isotropic 3-dimensional situation. Literature reviewed in Section 4.6, suggested that the order of magnitude of Poisson's ratio for a swelling clay generally should be about 0.2 to 0.35. Meanwhile, a review on magnitude of Poisson's ratio for shrinking soil is presented below.

From the result of their two dimensional restrained shrinkage test on a remoulded Israeli heavy clay, Uzan et al. (1973) found a typical magnitude of Poisson's ratio as 0.20. In a more recent paper on analysis of cracking in drying soils, Morris et al. (1992) used a value of Poisson's ratio as 0.35. From these and other available data it seems reasonable to assume Poisson's ratio for shrinking soil to lie within the range of 0.2 to 0.35. A value of 0.3 is selected for the present study.

Note that the shrinkage process of a drying soil effectively under goes *drained* condition because the shrinkage and drying phenomena occur side by side with the same rate as of movement of moisture. This suggests that the Poisson's ratio should not be 0.5 as in an undrained situation.

B. Shrinkage Indexes

In terms of present approach for a parametric study, the magnitude of shrinkage indexes λ_h and λ_o can be obtained from the published results of shrinkage measurements under laboratory test condition and/or from field observations.

Richards et al. (1983) have presented results of volume decrease of Adelaide clay soils (South Australia) under load free 3-dimensional shrinkage with measurement of matrix suctions, as well as volume decrease under controlled suction oedometer shrinkage compression tests under two different applied stress levels. Analyses are conducted on their results of tests on black earth with the application of Equations (5.8, 5.9 and 5.10) as appropriate. With the assumption that Poisson's ratio of soil is 0.3, it appears that the value of free shrinkage index λ_h ranges between 0.06 to 0.12 for very high to extremely high reactive black earth. While analysis of their other results of similar tests on relatively less reactive red-brown soil suggests that λ_h ranges between 0.02 to 0.04. Similar method when applied to their measurements of shrinkage strain under applied load suggests the shrinkage compression index λ_σ should range between 0.01 for moderately shrinking to 0.04 for extremely shrinking reactive clays. Note that in obtaining the ranges of magnitude of λ_h and λ_σ values of matrix suction are used while the present model requires the total suction as the variable parameter. Therefore the magnitude of these parameters may become slightly different if osmotic part of the total suction also increases in the drying process.

Australian Standard AS 2870, "Residential Slabs and Footings" (1990) provides a consistent basis for prediction of volume change for typical Australian reactive clays. Applying the data of potential subsidence on different types of clays upon increase in suction pressure presented by the Code a set of typical values of free shrinkage index λ_h can be back calculated as shown in Table 5.1. In the computation, soil at the site is considered to be uniform over the depth of seasonal moisture variation and that Poisson's ratio of shrinkage compression is 0.3.

Table 5.1 Magnitude of free shrinkage index for typical reactive soils of Australia

Site classification	Subsidence mm	Free shrinkage for confined compression λ_h	
		Adelaide	Sydney
Highly reactive	40 - 70	0.026 - 0.045	0.04 - 0.068
Extremely reactive	70 - >100	0.045 - 0.064	0.068 - 0.097

These values of λ_h and λ_σ deduced from the published measurements indicates typical order of magnitude of these parameters. Meanwhile the study suggests that for the detailed analysis of a situation, it is necessary to conduct tests on undisturbed soil samples representing the actual field condition. The development of a suitable test procedure in which shrinkage characteristics of a soil can be

measured under a controlled suction environment has become the subject of ongoing research. This will eliminate the uncertainties of presumed values of these shrinkage parameters.

5.6 Analyses of Lateral Shrinkage Stress and Subsidence

A typical extreme situation of monotonic seasonal drying of a reactive soil profile was considered to study the effect of shrinkage parameters on the development of shrinkage stress. In this parametric study, major variables considered were shrinkage indexes λ_h and λ_σ which describe the degree of reactivity of soil with decrease in suction potential.

A two metre depth of an intact soil profile as illustrated in Figure 5.3, was considered. The soil was initially wet with the total suction described in the figure, and was considered to dry out to reach a uniform total suction of 3 to 3.2 log kPa. A 4.0 log kPa of suction was assumed to be the suction at the convergence of free shrinkage and shrinkage under overburden pressure. Poisson's ratio of soil was taken as 0.3 and remained constant during the shrinkage process. The values of λ_h as 0.03 and 0.04, and λ_σ at initial wet state as 0.02 and 0.03 were considered to cover a moderately to highly shrinking reactive soil. In the analysis, the shrinkage compression index λ_σ was considered as maximum at wet limit i.e., at suction of 1.5 log kPa, and its value was decreased linearly with increasing log suction.

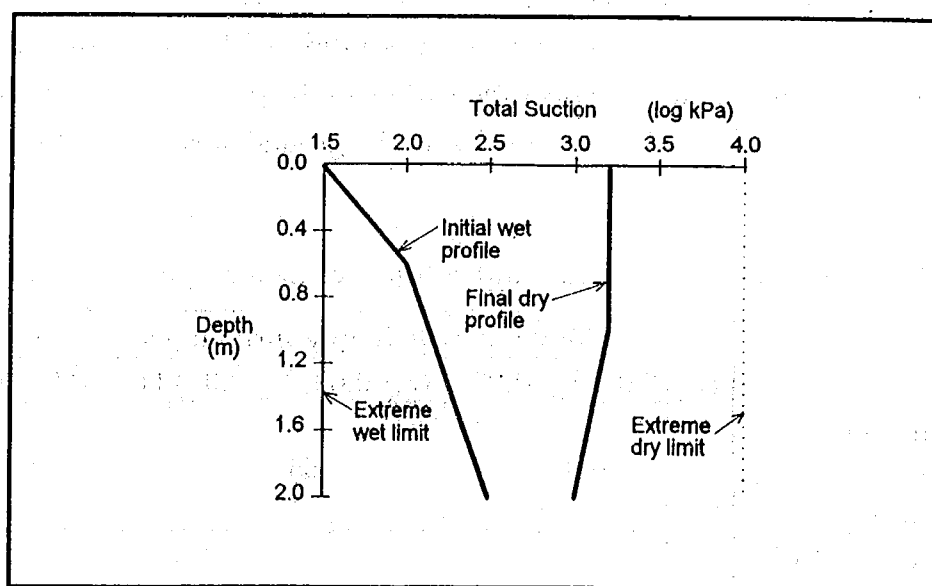


Figure 5.3 Soil moisture profile considered for parametric study

A simple closed form analysis using the simplified conceptual mechanistic model of shrinkage compression developed and described in Section 5.4 of this Chapter was

conducted. The above mentioned magnitudes of soil properties, parameters and site moisture environment as required by the model were used. The succeeding paragraphs discuss the results of this initial analysis.

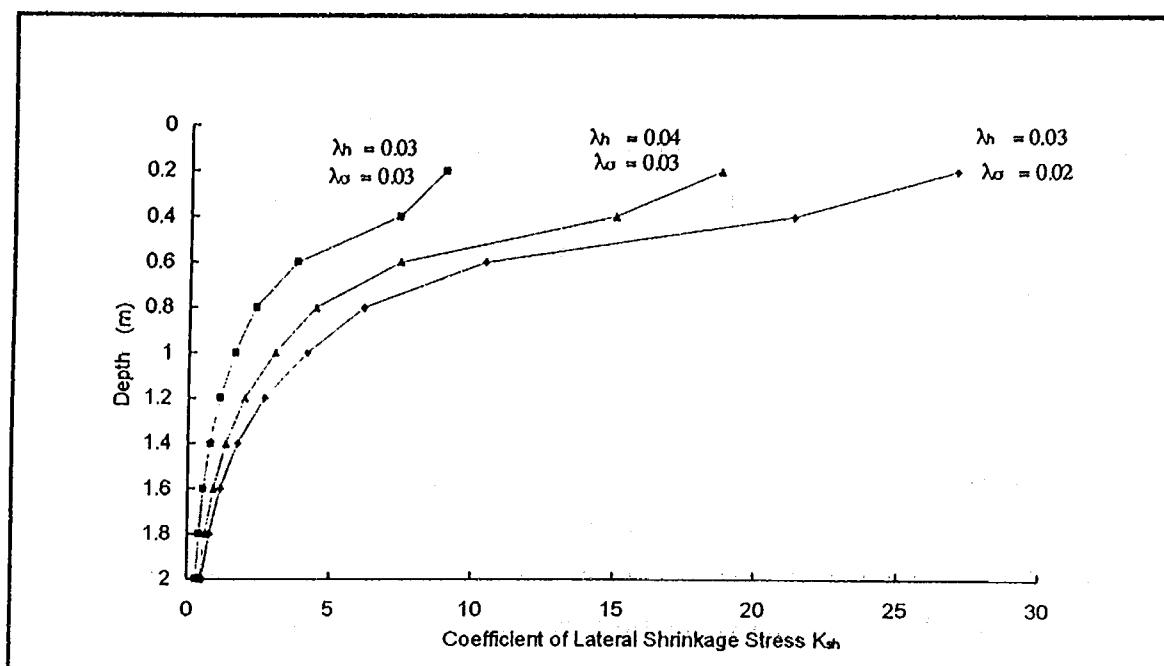


Figure 5.4 Variation of magnitude of lateral shrinkage stress coefficient ' K_{sh} ' with shrinkage indexes

Figure 5.4 above illustrates how the coefficient of lateral shrinkage stress K_{sh} varies with shrinkage indexes for the soil profile and site moisture environment illustrated in Figure 5.3. Table 5.2 below shows the magnitude of lateral shrinkage stress tensile in nature, that can develop near the surface of the profile. Table also includes the magnitude of subsidence that can occur at the ground surface due to drying of top 1 m depth.

Table 5.2 Estimated lateral stresses and subsidence for a range of reactive soils

	Moderately to highly shrinking soils with		
	$(\lambda_h=0.03, \lambda_\sigma=0.02)$	$(\lambda_h=0.03, \lambda_\sigma=0.03)$	$(\lambda_h=0.04, \lambda_\sigma=0.03)$
Maximum lateral shrinkage stress (kPa) at the surface	190	65	130
Subsidence (mm) at ground level for the drying of top 1m	43	45	58

Literature on expansive soils suggest that the tensile strength of reactive clays at their dry state ranges between 60 kPa and 170 kPa (Uzan et al., 1972, Wallace and Lytton, 1992, and others). This implies that the lateral tensile shrinkage stress developed in the present analysis are close to the tensile strength of a soil. However, even though the value may be lower than the tensile strength of a clay at the dry limit, it will be sufficiently larger than the strength of neighbouring wet soil. In that situation, crack will develop on the wet side of the section where there is a relatively steeper gradient of moisture change. This is also confirmed by field observation of Dagg and Russam (1966). They found the development of cracks in the wet side of a reactive clay subgrade in Kenya.

The results of this parametric analysis shows that lateral tensile shrinkage stress is inversely related to shrinkage compression index λ_g and directly related to free shrinkage index λ_h . While as mentioned earlier and also was found in the analysis, this lateral stress becomes constant when values of both the indexes changed but their ratio kept constant. The increase in lateral stress with decrease in λ_g can be easily perceived if looked at Equation (5.5) where the modulus of shrinkage compression increases with decrease in λ_g . As a result with higher modulus, larger stress would be developed than with the lower modulus for an equal strain level. Regarding the effect of λ_h , Equation (5.14) indicates that the coefficient of lateral shrinkage stress ' K_{sh} ' increases with increase in value of λ_h .

Subsidence of ground is estimated by using Equation (5.11). Result in Table 5.2 indicates subsidence of ground surface due to desiccation of top 1 m depth. The magnitude of subsidence is relatively higher because of extreme drying of soils having moderate to high reactivity. Results indicates that subsidence increases with increase in any indexes λ_h or λ_g but higher with increase in λ_h . However because of very small magnitude of overburden pressure, effect of λ_g is not clearly perceived. In their literature review, Richards et al. (1983) reported a differential settlement of 40 mm occurred in a profile of reactive soil in South Australia. Other observations of settlement available in the literature ranges around 50 mm for seasonal drying of about 1 to 2 m depth of soil. This implies that the present estimation yields values close to the reported field observations.

5.7 Conclusions

The mechanics of shrinking soils is notably very complicated than ordinary soils. Therefore it is widely accepted that the development of a mechanistic model that can perfectly characterise the shrinkage compression behaviour of reactive soils is a

significantly difficult task. In such situation, the present work could describe characteristic shrinkage behaviour of such soils in a relatively simple manner. It could explore complex nature of the problem and highlight some of the most important variables in a given situation.

Typical properties and parameters of shrinking soils that are responsible for development of shrinkage cracks and subsidence are identified. The study also presented a quick way of estimating magnitude of these parameters by back analysis of available results. Meanwhile, it suggested to obtain these magnitude directly from the laboratory test on undisturbed soil samples with test conditions representing the actual field situation. The study also highlighted the appropriate tests and test procedure.

At this stage, the model inherits some assumptions and limitations. Further improvement of the model is currently undergoing.

Results of a typical situations suggested that a significant magnitude of lateral tensile stress can develop near the surface of desiccating soil profile. This stress is responsible for the development of shrinkage cracks. The order of magnitude of subsidence estimated appeared to be close to the observed values published in the literature.

CHAPTER VI

DISTORTION AND CRACKING MECHANISMS IN LOW COST ROAD PAVEMENTS ON REACTIVE CLAY SUBGRADES

6.1 Introduction

In this study, the behaviour of a seal coated unbound granular road pavements is studied under the effect of volume change of reactive clay subgrade. Available relations defining the constitutive behaviour of an unbound granular pavement material were briefly reviewed. The selection of suitable criterion to describe the tension failure of granular pavement is discussed to describe the characteristic behaviour of such pavement subjected to a large strain that developed in the subgrade. Finally, a relatively simple tensile failure criterion that can adequately characterise the cracking phenomenon of unbound gravel base in low cost road pavement is proposed for the numerical analysis.

When a reactive clay subgrade wets up due to the seepage of rainwater or dries out due to evaporation of existing moisture through a pervious shoulder or through pre-existing cracks on the surface of pavement, it starts to swell or shrink. In the case of a free or only a partial fixity against lateral displacement on outer edge of subgrade and pavement, subgrade swell results an edge heave and lateral extension, and subgrade shrinkage results edge subsidence and lateral compression. Consequently, these result in the development of vertical as well as lateral stresses/strains in the overlying granular base. The compacted granular material normally has very small tensile strength. Hence, the lateral tensile stresses and edge heave or subsidence developed due to the unequal swelling and shrinking of subgrade between the inner and outer zones are sufficiently large to initiate cracking in the granular layer. Depending on the lateral fixity of the outer edge and the mode of volume change (swell or shrink), such tensile crack starts to initiate either from the pavement surface or from the base course. This type of crack development has been observed to run parallel to the centre line of road and is typically within 0.4 to 1.0 m inside the shoulder (Dagg and Russam, 1966; Wallace, 1988; and Bullen and Wallace, 1992).

As stated above, the unbound granular layer has very small tensile strength which is normally lower than the strength of the overlying bitumen seal coat (at working temperatures) or the underlying subgrade (in normal environment). Therefore, the

strain developed in subgrade and in bitumen (at cracking) are larger than the strain developed at the break of granular layer. In such situation, a relatively simple tensile failure criterion may be adequate to characterise the cracking of granular layer as it normally fails under small strain level for which the stresses/forces developed are quite small for bitumen.

This study does not cover any restraining effect of the thin bituminous seal coat against cracking and crack propagation in the base course. In the analysis the seal coat surface is treated just as an impermeable membrane without any significant strength. The relative contribution of the seal coat will be studied separately in the future.

Most of the researches on crack mechanisms in pavements have been done with fatigue-failure under repeated wheel loads, reflection cracking of asphalt concrete overlays over rigid cement concrete pavement or the propagation of shrinkage cracks developed in the cement treated base. In the present research pavement cracking is considered as a result of stresses induced by volume change of reactive clay subgrade. Since the nature of crack initiation is different, the detailed discussion on cracking mechanisms due to other stress/deformation factors is beyond the scope of the present study. However, available information on basic mechanism of these other forms of cracking is reviewed.

6.2 Constitutive Relationships for Low Cost Road Pavement Materials

As defined earlier, this study is limited to low cost road pavements. The pavement section in consideration consists of a granular layer and a bitumen seal coat with or without thin asphalt concrete layer. The major structural strength is produced by the granular layer only while the top asphalt or bitumen seal coat surface over this granular base normally checks the seepage of rainwater. Although these surfacing may contribute some tensile strength, their effect is not considered in the present analysis. The following sections discuss the behaviour of the granular layer.

6.2.1 Characterisation of Granular Material

It has been widely accepted that granular materials can be successfully used as load distributing layers in the pavement structure. If well compacted, a granular layer with very little or no plastic fines are less susceptible to environmental factors except on the ingress of water. Modified unbound granular materials would

contain small amounts of stabilising agents to improve their performance (eg., by reducing plasticity) without causing significant increase in structural stiffness.

Brown (1978) reported that the characteristic strength properties of pavement materials depend on density and aggregate packing, environmental conditions of temperature and moisture, and repeated loading condition. An unbound granular material develops shear strength as a result of particle interlocking but has no particular tensile strength. This pavement material deforms under shear and densification (under normal stress), and disintegrates through breakdown.

Unbound granular materials in a pavement exhibit anisotropic behaviour. Although it is hard to understand the exact anisotropic behaviour, it has been found that allowing anisotropy produces better fit between calculated and observed pavement deflection under load. However, it is not clear whether this result is due solely to the anisotropic behaviour or stress dependency. It is well recognised that the modulus of an unbound granular material increases with an increase in stress level. This increase depends on the state of stress and type of material. Previous researchers have found the relationship describing modulus as non-linear in nature. A detailed description is covered in the succeeding sections.

6.2.2 Constitutive Relations for Unbound Granular Material

Brown (1978) indicated that the constitutive behaviour of unbound granular materials, unlike bituminous ones, are notably non-linear. The stress/strain curve of these materials show that resilient modulus of elasticity is highly dependent on stress level. The level of dependency of modulus on stress is a function of density, grading and shape, and moisture condition.

Uzan (1985) stated that three different relationships describing the stress/strain characteristics of unbound granular materials are often implemented. These relationships are:

1. the well established $K-\theta$ model, which relates the resilient modulus 'K' to the bulk stress ' θ ', or $K-\sigma_3$ model where resilient modulus is related to the confining pressure ' σ_3 ' (Hicks & Monismith, 1971),

$$E_r = K_1(\theta)^{K_2} \quad (6.1)$$

$$\text{or} \quad E_r = K_3(\sigma_3)^{K_4} \quad (6.2)$$

where θ is the sum of principal stresses, σ_1 , σ_2 and σ_3 , and K_1 to K_4 are constants that depend on material and test conditions. Basically, this relation finds the value of Young's Modulus at relatively small strain level.

2. the hyperbolic law proposed by Kondner (1963) for static loading, refined by Duncan and Chang (1970), and extended by several researchers (Monismith et al. 1977, Barksdale, 1972) as

$$E_1 = \left[1 - \frac{R_f(1 - \sin \phi)(\sigma_1 - \sigma_3)}{2c \cos \phi + 2\sigma_3 \sin \phi} \right]^2 K p_a \left(\frac{\sigma_3}{p_a} \right)^n \quad (6.3)$$

where

$$R_f = \text{the failure ratio, defined as} \quad R_f = \frac{(\sigma_1 - \sigma_3)_{\text{at failure}}}{(\sigma_1 - \sigma_3)_{\text{ultimate}}}$$

K = a modulus number

n = an exponent determining the rate of variation of E_1 with σ_3 and

p_a = atmospheric pressure

K and n are pure numbers which depends on material type. Further, c and ϕ are usual cohesion and friction angle.

3. the fundamental one relating the bulk and shear moduli to octahedral stresses and stress path (Pappin and Brown, 1980, Brown and Pappin, 1981).

All of these relations were reviewed in detail in a previous study by Sapkota (1994). He found that these relationships are applicable under repetitive wheel load of traffic but are not applicable in the present loading condition. Conclusion of his analysis on the applicability of above relationships are discussed below.

6.2.3 Assessment of Available Models to Characterise the Behaviour of Granular Layer Subjected to Stresses due to Subgrade Volume Change

The three different non-linear stress/strain relationships discussed previously are often used to characterise the behaviour of granular soil. However, the suitability or preference of one to the other depends on many factors viz., material type and its properties, loading characteristics and conditions, load/displacement boundary conditions, and degree of accuracy required, to mention a few. Basically, the first and third models, namely; $K - \theta$ model and Brown and Pappin model, are developed for granular material under the concept of repeated wheel load condition. Furthermore, the first model employs stress dependent resilient modulus to describe the constitutive behaviour (Equation 6.1 and 6.2). The hyperbolic model is

developed for static loading on cohesive or granular soil. It has been widely used in the analysis of geotechnical structures. In recent years this model has been used by some researchers (Germann and Lytton (1989), Lytton (1989), and others) to analyse pavement behaviour using back calculations.

The load on the pavement in the present research problem is due to the swelling or shrinkage of the subgrade. The stresses developed are normal (vertical and horizontal), and shear stresses acting from the underneath of the pavement as shown in Figure 6.1. There exists an interface between subgrade and pavement on which lateral shear stresses are developed due to the lateral expansion or contraction of reactive clay subgrade relative to non swelling/shrinking pavement. The most critical stresses are vertical normal stresses whose intensity is increasing towards the edge of the pavement thereby causing bending. Meanwhile, the lateral tensile stress develops basically due to lateral dragging by the horizontal component of swell pressure or shrinkage stress in the subgrade.

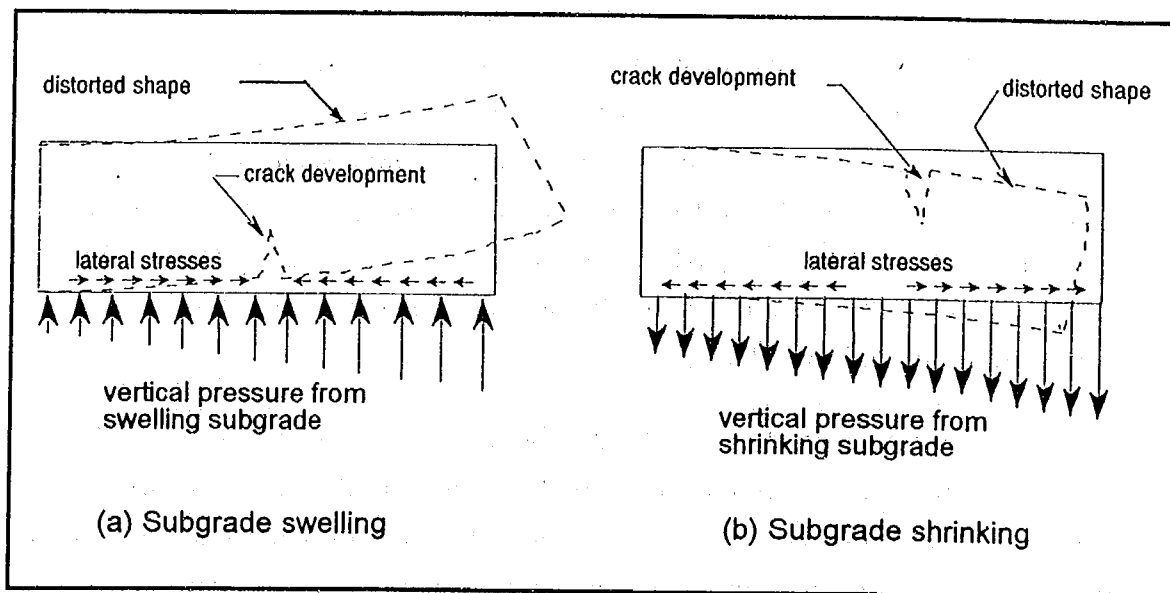


Figure 6.1 Stresses in pavement due to subgrade volume change

The process of overall stress development is due to the volume change of the subgrade which results from an increase in moisture content. Wetting of the subgrade is supposed to start as a result of seepage of rain water from the underneath of unsealed shoulder or side slopes of an embankment. Similarly the shrinkage of subgrade occurs due to the evaporation of moisture from the shoulder and sides. These processes result static load condition rather than a dynamic or repeated loading. Thus the first model, K - θ model, and the third model (Brown and Pappin model) do not seem to be applicable to the present research problem.

The hyperbolic model in Equation (6.3) gives a stress dependent elastic modulus and is suitable for incremental elastic analysis. However, there are some limitations in using this model. A major weakness is its inability to include the effect of dilation particularly when soil is constrained from movement, say fixity in lateral boundary condition. Also, it neglects the effect of intermediate principal stress.

Sapkota (1994) attempted to apply this hyperbolic model in a numerical analysis of a situation similar to the present condition. In applying this model in the computer program, the values of material properties such as cohesion c , friction angle ϕ , and some coefficients such as R_f , K , n , and p_a need to be defined. Based on the published literature on granular pavement material, a value of $\phi = 40^\circ$ and a small value of cohesion of 10 kPa were selected. The value of atmospheric pressure p_a was taken as 101 kPa. R_f was assumed to be 0.80 for the first trial, and values of K and n were taken as 1820 and 0.20 to determine the initial tangent modulus at the first step, and 700 and 0.35 respectively for other steps (Duncan and Chang, 1970).

When the program was attempted to run with these properties and parameters, the program could not run successfully. This is attributed to the development of tensile stress in major/minor principal stress direction which resulted the model mathematically inapplicable. In terms of program, the expression in Equation (6.3) developed a negative variable subjected to a real index. Physically, this should be because of a very small overburden vertical compressive stress due to the self weight alone, while the section is subjected to a higher lateral and shear stresses as a result of subgrade volume change.

Uzan (1985) also criticised the use of hyperbolic model in analysing stress/strain behaviour of granular material in a road pavement. Accordingly, the major deficiency of the model is that it fails to describe the effect of dilation and the accumulation of shear strain under the low minor principal stress compared with the major principal stress as is the case in pavement layers.

6.3 Crack Mechanism and Failure Criteria

A large number of studies on deformation and cracking of flexible as well as rigid pavement have been conducted in the past. Most of the researches concentrate either in reflection cracking in flexible pavement overlay or fatigue cracking of asphalt concrete and cement treated base materials. The fundamental principle of cracking and crack propagation in both studies are based on the theories of fracture mechanics.

Marchand and Goacolou (1982) used mechanistic approaches for the solution of cracking in pavements. Their model is based on Irwin's diagram of fracture mode shown in Figure 6.2.

The application of fracture mechanics principle is essential to understand the crack growth phenomenon. Its elementary theory tells that when an element of solid is loaded in such a way that a crack initiates at any place within the element, the stress intensity increases heavily at the tip of crack as shown in Figure. 6.3, and the material at that zone starts to yield. If the load on the element remains constant, stress increases continuously due to decreasing sectional area, and ultimately the element will break.

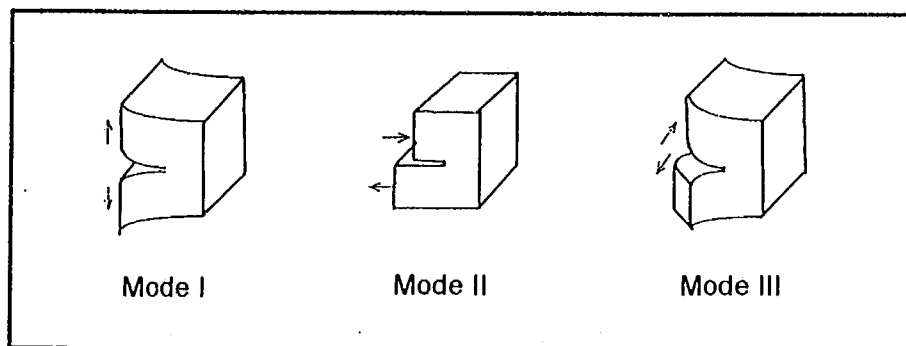


Figure 6.2 Irwin's diagram of fracture modes

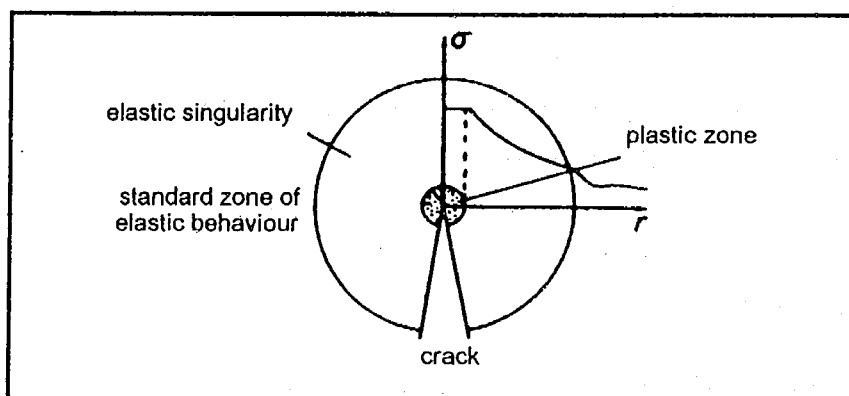


Figure 6.3 Stress configuration at tip of crack

Irwin's criterion of fracture modes, shown in Figure 6.2, can be applied to simulate the mechanism of cracking in a pavement. Typical examples of crack developments in a pavement can be described as: Mode I which can develop due to thermal contraction, bending, and lateral extension of the pavement; Mode II which generally develops when slippage occurs in overlays due to traffic load; Mode III which may occur due to dragging effect of wheel load parallel and near the edge of an existing crack.

This concept of pavement cracking based on fracture modes are applicable when at least a tiny crack has already initiated, and the material is trying to resist further development of this crack. Therefore, the concept is principally useful to determine the life of an asphalt overlay against reflection cracking of pre-existing cracked concrete base under traffic load.

In the present research work, the distortion of a pavement occurs due to the transmission of subgrade swell/shrink effects as shown in Figure 6.1. Consequently, the nature of load condition is purely static. Further, the material which is an unbound gravel layer has very little tensile strength. In such a situation, pavement cracking is basically due to tension failure resulting from lateral extension/contraction transmitted from the lateral swelling or shrinkage of subgrade as well as bending due to the unequal vertical swell and shrink of subgrade zones. In this situation, a suitable simplified failure criterion is necessary to describe such cracking and crack propagation.

6.4 Simplified Elasto-Plastic Model with Limited Tensile Strength to Characterise the Constitutive Behaviour and Crack Mechanisms in Low Cost Pavements against Subgrade Volume Change

Reviewing the limitations associated with all the three available constitutive models and their non applicability to the present research work, a simplified elasto-plastic bi-linear model is proposed for this study. This model is used to describe the constitutive behaviour of granular layer when subjected to forces due to subgrade volume change. It uses Mohr-Coulomb failure criterion with cohesion and friction angle as the strength parameters. Also, as the tensile strength of the granular layer is very small (Section 6.5), the model uses tension cut-off defined by the tensile strength of the material.

Figure 6.4a illustrates a diagrammatic representation of the proposed model, and highlights its closeness to the stress dependent elastic modulus described by a hyperbolic model. The peak strength at shear failure is obtained from the geometry of failure envelop of Mohr's circle as described by Figure 6.4b. Mathematically this relation can be expressed as

$$(\sigma_1 - \sigma_3)_f = 2 c \cos\phi + (\sigma_1 + \sigma_3) \sin\phi \quad (6.4)$$

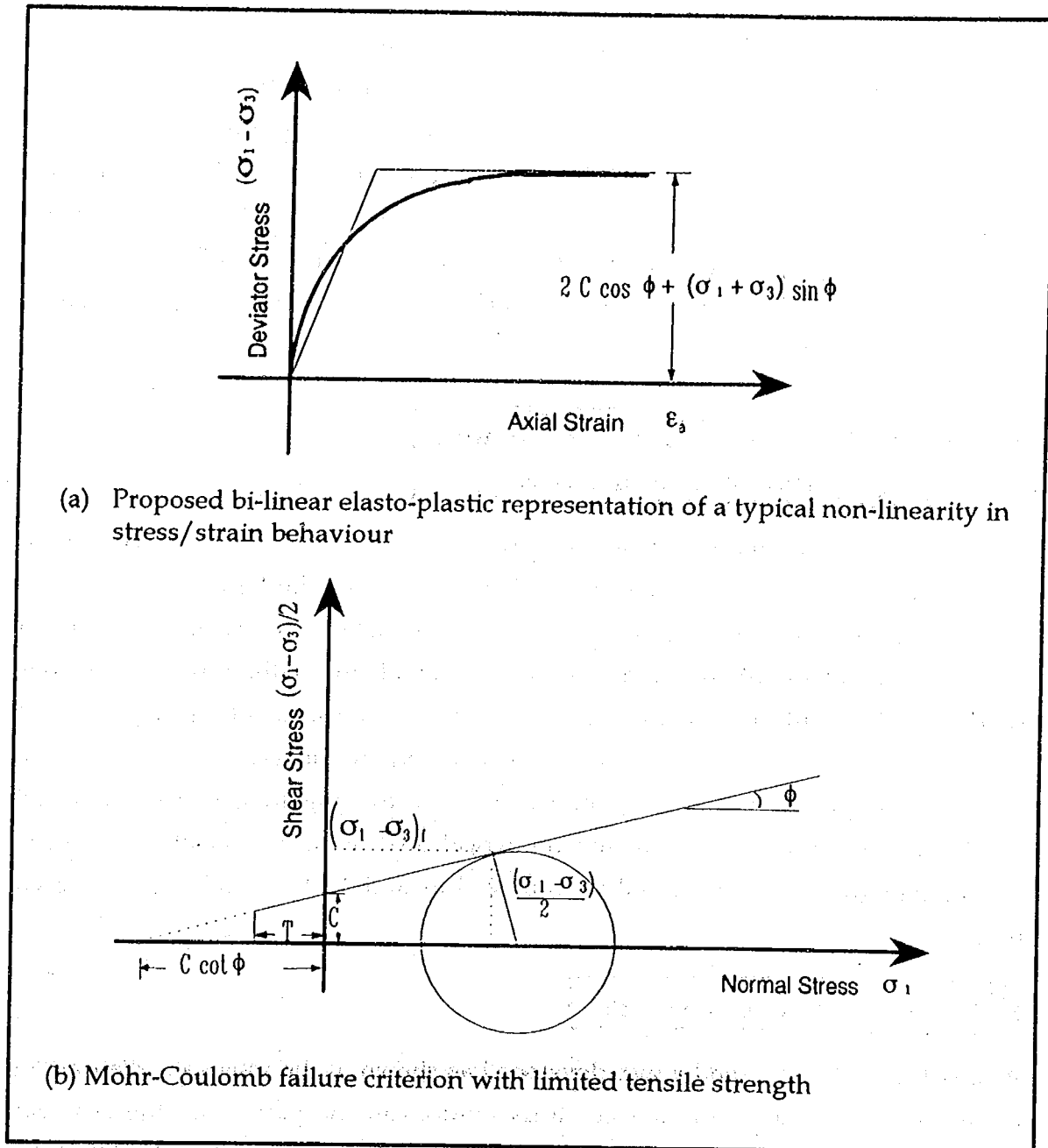


Figure 6.4 Simplified elasto-plastic Mohr-Coulomb model with limited tensile strength

If tensile strength of the material is lower than the plastic apex value, the model will consider failure by tension. This criterion is incorporated in the Mohr-Coulomb material model of the computer program FLAC (Itasca, 1992) which is used in the numerical analysis presented later in Chapters VII, VIII, and IX.

The formulation of plasticity in the analysis is based on Mohr-Coulomb failure criteria where shear yielding follows non-associative flow rule while the volumetric yield follows associative flow rule. In a two dimensional plane strain analysis the Cartesian stress components are resolved into principal stresses in the order of $\sigma_1 <$

$\sigma_2 < \sigma_3$. For shear yield in Mohr-Coulomb criterion, only the two principal stresses, namely; major principal stress σ_1 and minor principal stress σ_3 are used. Tests on tensile failure criterion are made after correcting for plastic flow. The program requires the magnitude of specified tensile strength and its upper limit is controlled by plastic apex limit (cohesion/tan (friction)). During the analysis, the program checks if any of the principal stresses is exceeding the tensile strength. An elaborate discussion on tensile failure/tension cut off criterion adopted in the program is presented in detail in Section 9.2.

6.5 Laboratory Simulation of Pavement Cracking due to Lateral Stress

A prototype laboratory model test on crushed rock gravel base was conducted previously (Sapkota 1994) to study the behaviour of unbound base under the effect of subgrade lateral stressing. The test could also determine the tensile strength characteristics of an unbound granular pavement with and without thin bituminous seal coat. The result of which provided the data for the selection of magnitude of tensile strength for such type of pavement used in the study. Details of the experimental program carried out in each type of tests together with the obtained results and their application in the numerical analysis are available in Sapkota (1994). Only the concept behind the test procedure and the test results are discussed here.

6.5.1 Development of the Model

A "box type" physical model was developed as shown in the schematic diagram in Figure 6.5. To simulate the tensile stress induced in the pavement, the box was constructed in two halves. During the test of the pavement model, each half of the box was joined together and later jacked apart during the loading. Figure 6.5 illustrates the typical test mechanism of the model.

The main features of the model included:

- The pavement specimens consisted of road base and thin bitumen seal coat compacted in the box to represent a total section of a road pavement.
- The location of failure plane and failure mechanism were controlled so that the location and propagation of crack were reproducible for all tests. This was done by "splitting" the model into halves (Figures 6.5) at the longitudinal centre. The

positioning of failure plane at the centre distributes the loading forces uniformly within the cross section.

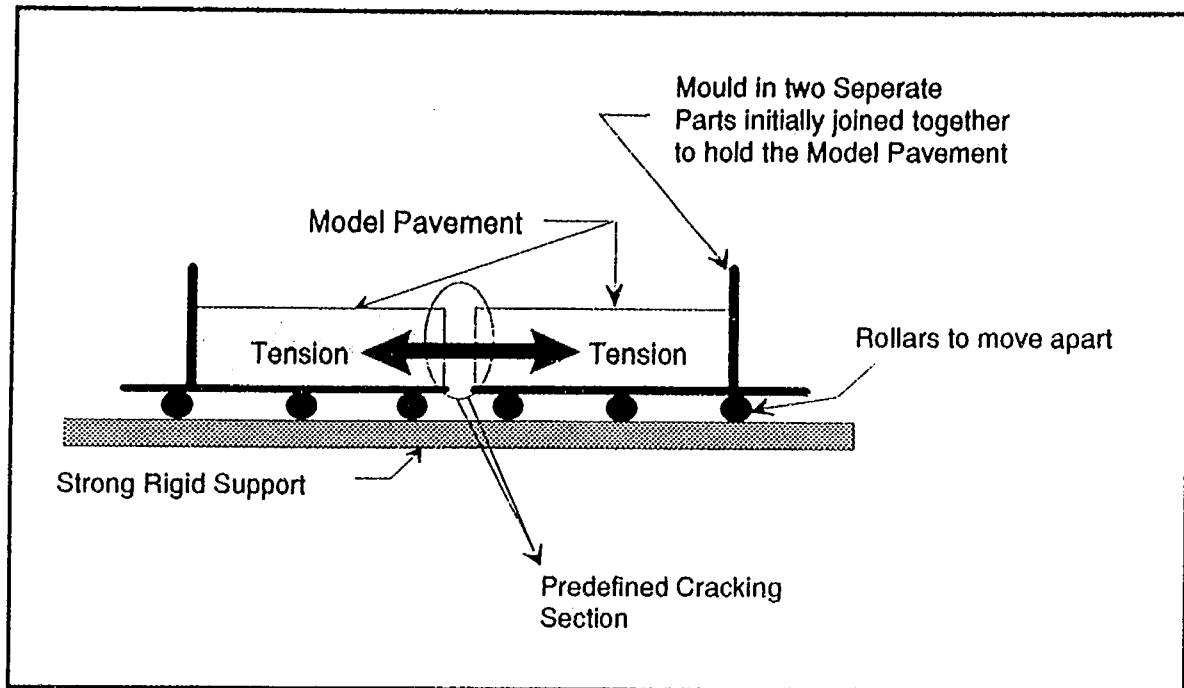


Figure 6.5 The pavement model test

- In the field, stresses are transmitted through the subgrade/pavement interface. To simulate this condition, the tensile load was applied at the base of pavement. To achieve this, the aggregate particles were used to provide roughness at the interface to support the shear loads between sample and the base.
- The lateral boundary conditions were made free to eliminate the frictional resistance between the model pavement and side walls. The effect of end walls was neglected (as it was remote) and the width of pavement section was made at least 3 times the depth. The lateral boundary was made free. Stress/strain condition at failure section under uniform stress field was considered as plane strain with zero lateral strain.
- The box when properly assembled as in Figure 6.6 was sufficiently stiff against any deflection or deformation during compaction. It rested on steel rollers, supported on a rigid base. The configuration gave a smooth surface with minimal friction.

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6.5.2 Construction of Model Box

Figure 6.6 shows a photograph of the test pavement. The box was made up of thick plywood sections on the base and sides. The wood sections were connected by steel brackets and supporting steel sections. The overall inside dimension of box was 670 mm long, 300 mm wide, and 180 mm deep. The thickness of model pavement was normally around 100 mm, while the width and length were the inside dimension of the mould. The friction base was made up of two PVC sheets (3 mm thick). Aggregates of about 9 mm size were adhered to the upper surfaces by means of a Flemoset (a PVC glue resins). One sheet was secured to each half using screws to simplify removal for cleaning. A 40 mm strip at the centre (20 mm at the inner edge of each PVC friction base) was made friction less by not fixing aggregates. This was done so that the force applied to the pavement was direct tension. Also, by this means the failure plane and conditions could be kept constant and reproducible.

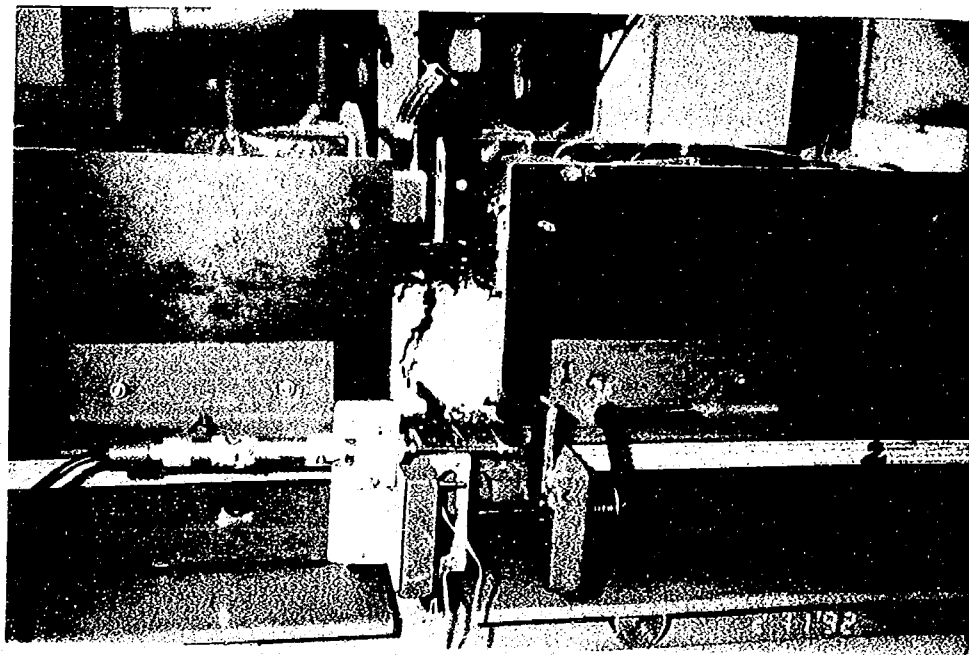


Figure 6.6 Assembly of mould for pavement model test (Photograph was taken right after the test on a seal coated surface. Note the measuring devices and the loading arrangements on one side.)

The loading system required hand turning of fine thread bolts (Figure 6.7) to jack the two halves of the box apart to break the pavement sample. This jacking load was applied on both sides of the box, to provide uniform loading across the section. The system comprised a large bolt threaded through the side plates on each side of the box. The tips of the bolts applied pressure on the load cells on the opposite half of the box and consequently forced the apparatus apart. The loading was made uniform through a winding attachment.

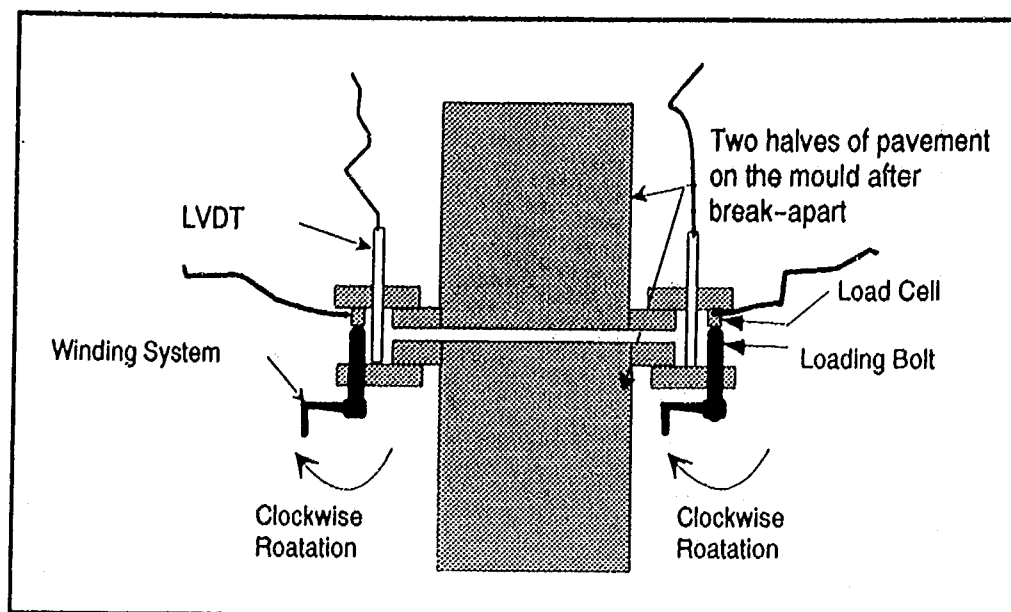


Figure 6.7 A plan view of the test set-up and loading system

6.5.3 Measurement Systems

The load measurement system had to record the jacking load that caused fracture of model pavement. The position for measuring load was at the jacking points on both sides of the box. For this, the end of bolts applied the load through a piece of metal with strain gauges acting as a load cell. The sum of these two load measurements defined the strength of the pavement.

Since the tensile strength of pavement is relatively small, the direct strain that would develop in the load cell under normal stress would be very small. Therefore a more sensitive flexural load cell arrangement shown in Figure 6.8 was developed. The load cell consisted of 10 mm thick aluminium plate with hole drilled to give flexural action as shown in Figure 6.8. Four strain gauges, 5 mm in size were located with two on loading face (giving compressive strain) and two on back face (giving tensile strain).

The load cells were attached to the steel side plates and a small indentation was made at the loading face of the cell to help guide the jacking bolt to the correct loading position. LVDTs (Linear Voltage Displacement Transducers), were used to measure the displacement of each half of the box during the jacking process. The calibration of these measuring systems were carried out properly. A detailed discussion on calibration of measuring devices is presented in Sapkota (1994).

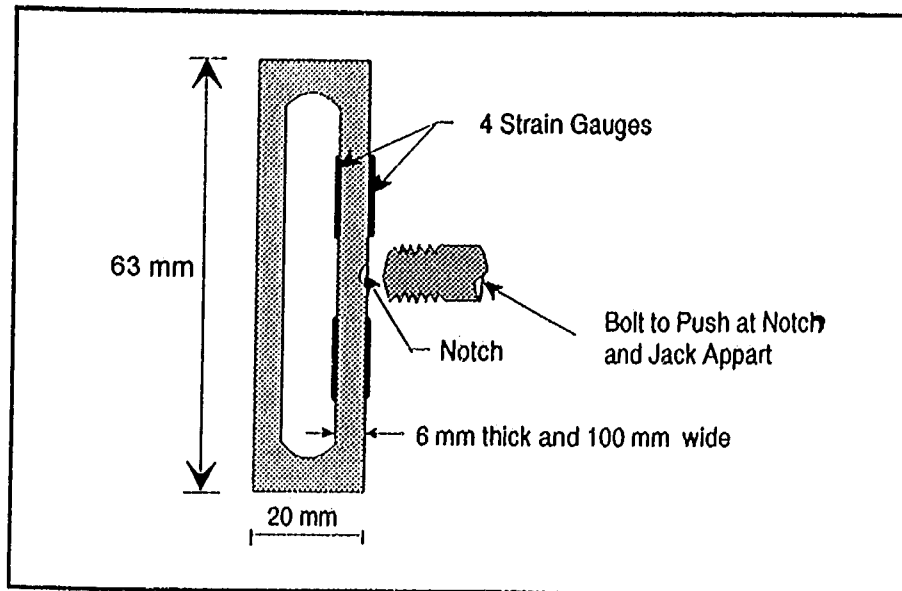


Figure 6.8 Load cell configuration used in the test

6.5.4 Test Material and the Preparation of Specimen

A. Base material

The base material used in this study was obtained from the Readymix quarry at Mt. Petrie, in Brisbane. The supplier's description is a Class C1 Road Base. The tests were conducted with aggregate sizes less than 19 mm. Aggregate particle density was 2.96 ton/m³. The base material was defined as GW, a well graded, very sandy gravel, with low plasticity fines.

B. Bitumen

A very light cut back primer having 50 % bitumen content was prepared by mixing a Class 170 bitumen in the kerosene. This served for first coat priming. For the final seal coating, a commercially available CRS - Cationic Rapid Setting emulsion supplied by EMOLEUM (Australia) Ltd. was used. Their application rate is presented in the succeeding section.

C. Specimen Preparation and Testing

The major parts of the testing included the preparation of aggregates for specimen, compaction, application of primer and emulsion, set-up of measurement devices, and finally, load application. The main steps involved in the test were as follows:

- A thoroughly mixed aggregate sample, at about optimum moisture content (7 % for the test material) was poured into the box. Using a Kango

Compaction Hammer, the sample was compacted evenly over its surface for 2.5 minutes to form the pavement model. Afterwards, the disturbance of the sample and the box was kept as minimum as possible. The sample was left for 36 hours under lamp to dry uniformly at least on the surface and slowly. The surface of the pavement was visually checked for drying.

- A light thin priming coat with 50% Class 170-Bitumen cutback was applied at an application rate of approximately 0.7 litre per square metre. It was applied by brushing over the dried surface and left for 6 hours to penetrate and set.
- A CRS-Emulsion seal coat was then applied over the primer using a chemical sprayer with a wider nozzle. A good fan of spray was obtained and a single coat was applied at a rate of application of about 1.10 litre per square metre. (The application of cutback and emulsion together resulted about 1 mm thick film of bitumen after drying).
- The specimen was left for another 12 hours to set the emulsion. The detachable windows on each side were taken out and the final measurements of plan dimension and thickness of the pavement at the proposed failure section were measured.
- The measurement system consisting of a high speed recorder "Assistant Plus Data Logging System" was set up to record the data from the load cells and LVDTs. The load was applied manually at a constant rate of application of about 1.2 mm per 30 seconds (0.04 mm/sec). Once the test was completed, samples for moisture content determinations were collected from different locations of the failed section.

6.5.5 Test Results and Discussions

The values of optimum moisture content of the test sample under standard compaction, and modified compaction efforts, were found to be about 7 percent and 6 percent, respectively. The control of moisture content and dry density of a test specimen is important when a strength test is to be performed in road materials. In the tests, moisture contents were recorded by taking samples at different depths of the failed section. Values were found to lie within 6 ± 0.7 %. Also the values of dry density were found to lie between 2.10 to 2.30 ton/m³. This indicated that under the applied compactive effort for each test specimen, the moisture content and dry density were fairly reproducible and the effect of variation of moisture content/dry density should be relatively small.

The detailed discussion on the result of measurements of tensile strength of section with and without seal coat are available in Sapkota (1994). In brief, an average of 1.54 kPa was measured as an ultimate tensile strength of the unsealed section. This strength for seal coated section reached to 5.42 kPa.

Sapkota (1994) has commented on the measured tensile strength of seal coated section as to be representative tensile strength of an average for the gross cross-section of 295 mm width and 97 mm thickness. From the displacement relationship it appears that the gravel should have failed well before this peak resistance was achieved. Therefore, neglecting the very small contribution of the gravel, the 5.42 kPa tensile strength corresponds to a peak resistance of about 520 kPa of the 1 mm thick seal coat.

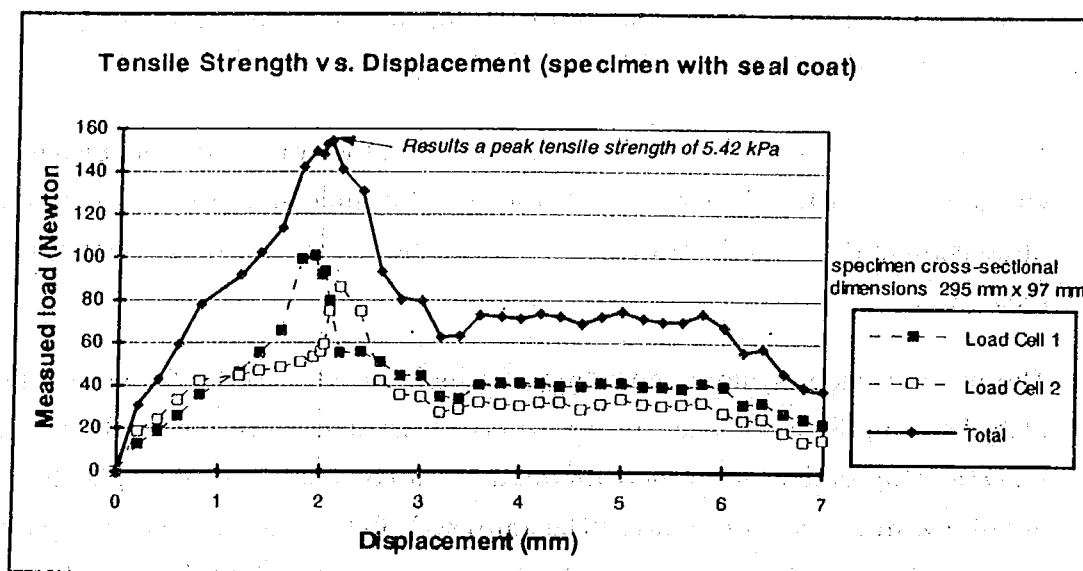


Figure 6.9 Tensile strength characteristics of seal coated road base

The load versus displacement plot of test on section having seal coat is reproduced in Figure 6.9. The plot illustrates how the stress develops with displacement.

The above test method was of relatively new approach which attempted to simulate the actual cracking of pavement being subjected to lateral extension. Sapkota (1994) indicated the following observations during the tests:

- (1) Cracks in pavement, as expected, always started from the bottom face of mid-section
- (2) Once a crack initiated, the strength of the section was gradually decreased. The decrease was observed to be relatively slower in specimen with seal coat.

- (3) It was also observed that during loading, after the specimen had reached peak strength, the seal coat started to extend. During the gradual propagation of crack towards the top surface, the bitumen film became thinner and started to sag. This phenomenon demonstrated visco-plastic nature of bitumen. To understand such effect of seal coat, more tests were required to measure precisely its characteristic strength and consequently, its contribution to the overall tensile strength of the section.
- (3) Finally, the magnitude of the measured tensile strength of a seal coated unbound granular base was about 5.42 kPa.

In the last testing (January 1994) of unbound gravel bases which had been dried back as required for priming and then wetted up under a plastic sheet without applying a seal coat have indicated a gravel tensile strength of about 4 kPa.

6.6 Conclusions

In this study, the pavement section is considered to be of low cost construction in which a relatively thin unbound granular layer is covered with a thin bituminous seal coat. In the analysis, it is considered that only the unbound granular base provides strength to the pavement. The strength of seal coat is not considered.

The study does not cover any repeated wheel loads due to the moving traffic. At present load in pavement is considered due to volume change (both swelling and shrinkage) of underlying subgrade. These stresses/strains developed in the subgrade are ultimately transmitted to the overlying pavement in the form of vertical surface heave or subsidence, lateral extensions or contraction, and bending. Compacted granular material have very little tensile strength, and under such deformation, the granular layer starts to develop cracking. This form of cracking mechanism has been adopted here.

Different available constitutive relationships such as resilient modulus to bulk stress $K-\theta$ or confining stress $K-\sigma_3$ model, hyperbolic model, and model relating bulk and shear moduli to octahedral normal stress were reviewed. The suitability of one over another depended on many factors viz., material types, forms of construction, and loading conditions. Particularly the $K-\theta$ model and the shear and bulk moduli versus octahedral normal stress model are usually applied for the stress/strain analysis under repeated wheel loads. The hyperbolic model although widely applied in the static analysis of general geotechnical engineering problems, has the limitation that it cannot include the effect of dilations and accumulation of

shear strain under low minor principal stress (Uzan, 1985). The writers also experienced similar difficulty in the description of hyperbolic variation of Young's Modulus when applied to the unbound pavement section.

The mechanisms of pavement cracking are normally described by Irwin's criterion with three modes of fracture (Section 6.3). This approach is applied extensively in the analysis of fatigue crack under wheel load by Majidzadeh et al. (1971, 1973, and 1987). However, in the present study pavement cracking appears under different loading mechanism, and hence a different approach is adopted. The cracking of pavement is considered to initiate by tension failure of an elemental pavement, and crack propagation takes place with an increasing number of failed elements. Therefore a simplified constitutive model consisting a bi-linear stress strain relationship to describe the load deformation behaviour of granular base is considered. The model uses Mohr-Coulomb failure criterion with limited tensile strength. This model will be applied in the numerical analysis presented in Chapters VII, VIII, IX and X.

Model pavement tests were conducted to measure the tensile strength of unbound crushed stone base with and without seal coat. The test results showed as would be expected, a very small tensile strength of about 1.5 kPa for unbound base without seal coat. When a thin seal coat is applied to the same section, the tensile strength of the over all section increases to 5.40 kPa.

CHAPTER VII

NUMERICAL MODELLING: PROCEDURE AND PARAMETERS

7.1 Introduction

This chapter develops an integrated numerical analysis procedure of the physical behaviour of moisture flow (Chapter II), and swelling and shrinking of subgrade behaviour (Chapters IV and V), and response of a pavement (Chapter VI) to subgrade volume change. A typical pavement and subgrade geometry and the associated flow, and deformation boundary conditions are presented in Figures 7.1 and Section 7.2.3 respectively. Typical material properties and model parameters essential in the analysis are selected from the earlier chapters. They are applied to develop approximate non-linear relations in a form suitable for numerical analysis (Section 7.3). In this chapter a thermal analogy describing the translations of moisture movement equations to heat diffusion equations with non-linear coefficient is discussed. Also, the stress/strain relations of wetting and swelling as well as drying and shrinking subgrade and of overlying low cost road pavement are presented. The verifications of proposed numerical procedures by comparing typical results obtained from the program with the results obtained from closed form solutions available for simple cases are presented in Appendix E. Finally, this chapter serves as a background information for the numerical analysis of a typical problem in Chapter VIII.

7.2 Numerical Description of Problem

The overall analysis work of the present study can be divided into three major sections that involves the following phenomena, namely

- 1) movement of moisture in unsaturated expansive clay subgrade when moisture infiltrates or dries out through the pervious shoulder and/or sides of a road section,
- 2) stress/strain behaviour of reactive clay subgrade which wets up and swells, or dries out and shrinks, and
- 3) the response of overlying pavement to subgrade volume change,

Initially, these were treated individually and later analysed together such that the overall analysis was interactive with these different phenomena. The theoretical and mathematical descriptions of these phenomena were discussed fully in the

earlier chapters. Here, only the basic components required to develop a numerical solution of the soil/pavement interaction problem are covered.

7.2.1 Finite Difference Formulation

In the present analysis, mathematical relations which included equations related to the stress/strain behaviour and the unsaturated moisture flow, are solved by Finite Difference Method. An available software FLAC (Fast Lagrangian Analysis of Continua, Itasca, 1992) is used for this purpose. The FLAC program uses finite difference coding to solve the sets of partial differential equations. *(The detail on incorporation of mathematical formulations describing flow equations, equations of motion, constitutive relations and boundary conditions, etc., can be found in FLAC Users Manual).*

The selection of the computer program is based on its ability to:

- satisfy geometry, grid generation, and flow and displacement boundary conditions
- allow non-linear potential dependent conductivity
- allow the use of non-linear suction and stress dependent modulus, and suction dependent strength parameters
- provide choice of various built-in mechanical models and interfaces
- provide good input and output scheme and is versatile, and
- provide quick results of analysis at shorter computer time.

With these features, FLAC is found to generally satisfy all the requirements for the present numerical analysis. With this, writers were relieved from the tedium of having to write separate finite element coding to solve a specific task.

In FLAC, each element behaves according to its force and boundary restraints. When stresses are high enough, the material starts to yield and flow. Also, the grid can deform if the large strain mode is selected. The current versions (Version 3.0 and later), have additional capability to run with user defined constitutive relations. This makes the program versatile and effective in the analysis of many typical geotechnical problems. In the present analysis, the writers employed this advantage to describe the stress and suction dependent stress strain behaviour of a swelling and shrinking subgrade. The another major feature of the program is its thermo-mechanical option. The program simulates the transient flow of heat in materials resulting in the development of thermally induced stresses. It has the

advantage that it allows the use of non-linear diffusivity, thus making it possible to solve both non-linear diffusion and elastic stress strain analysis together. Some reformulation applied in the analysis is presented in more detail in Section 7.4.

7.2.2. Geometry of Structure

Figure 7.1 presents a typical cross section of a road embankment considered for the analysis. The analysis considers symmetry along the centre line of the road cross section which reduces computer time. However, the present numerical procedure can also be applied for non symmetrical situation. The problem is considered under plane strain conditions and strain along the length of road is ignored. The section is basically comprised of three kinds of materials: (1) at the bottom, zones 1 and 2, are of natural and/or compacted reactive clay subgrade, (2) at the middle, zone 3, a pervious compacted granular base course, and (3) at the top surface, zone 4, an impermeable bituminous seal coat.

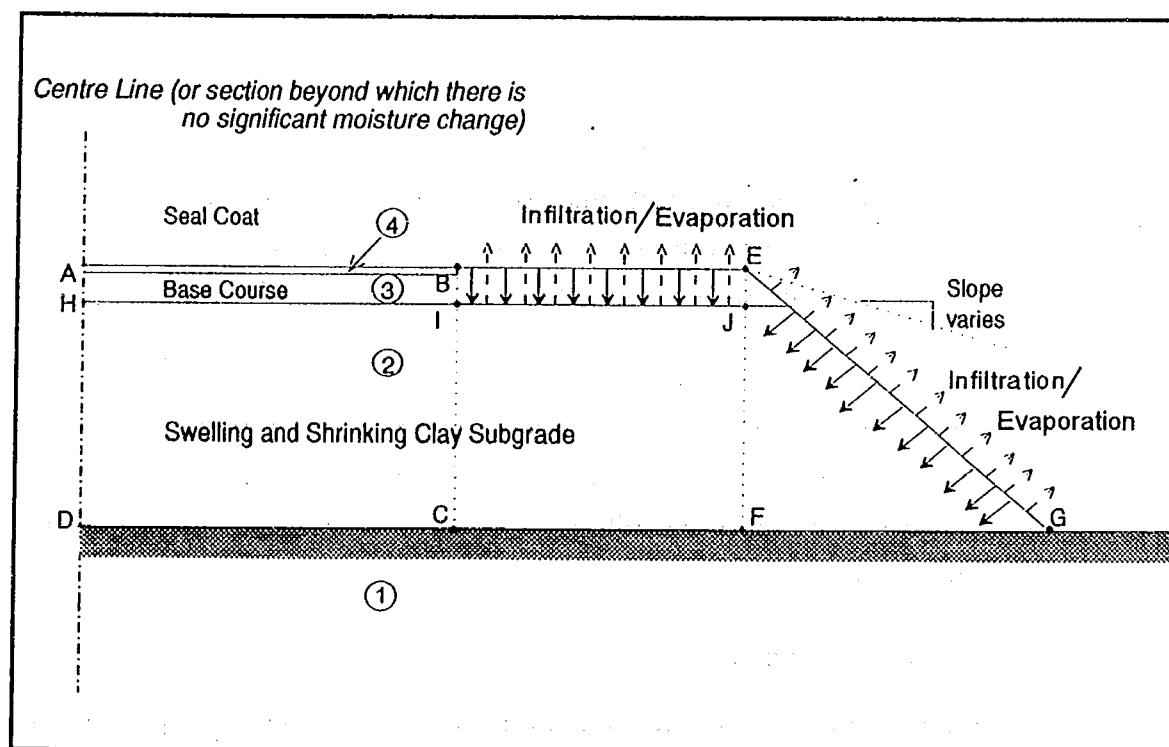


Figure 7.1 A typical cross section of road embankment considered for analysis

Two rectangular sections ABCD and AEFD, are considered. In section ABCD, subsection ABIH represents pavement and HICD represents the reactive clay subgrade. Moisture entry or exit in section ABCD is in a horizontal direction through the vertical face IC. This flow may be considered as seepage entry or exit through the sides of an embankment. In the other section, AEFD with AEJH as

pavement course, the moisture entry or exit is considered to be through the bottom face of the pervious shoulder IJ, where flow takes place in two dimensions: laterally and vertically.

7.2.3 Boundary Conditions

The boundary conditions associated with the overall analysis work were divided into flow and displacement boundary conditions.

A. Flow Boundary Conditions

With respect to the flow boundary conditions, the following are considered (refer Figure 7.1).

- (1) There is no flow condition across the vertical face AD along the centre line because of symmetry along the cross section. In case of a non-symmetric cross section AD may be considered as the section beyond which there is no significant change in the moisture environment.
- (2) The top face of subgrade is also considered as a "no flow boundary" to account for the fact that the compacted granular base has relatively larger pore sizes which is less prone to suck moisture against gravity by virtue of the capillary or any other form of suction, and
- (3) The bottom face of the compacted subgrade is assumed to have no flow boundary condition by considering the maximum effective moisture variation and diffusion to occur only within the depth of compacted subgrade layer.

B. Displacement Boundary Conditions

The displacement boundary conditions for lateral and vertical swelling or shrinking of subgrade and their effect on the overlying pavement were set to closely simulate field conditions as follows (refer Figure 7.1):

- (1) Lower face DC or DF is fixed against vertical displacement.
- (2) Top surface of pavement as well as subgrade is considered to be free for both horizontal and vertical displacements.

- (3) The left vertical face AD is assumed to be fixed against lateral displacement which is justified if symmetry is considered along the centre line of a road cross section. For a non-symmetric cross section or when AD is not a centre line of a symmetric section, lateral fixity of AD is still valid when there is no significant moisture variation beyond this face. Vertical displacement is allowed on this face.
- (4) Vertical faces BC and EF are considered free for vertical displacement. For lateral displacement, three extreme cases are considered;
- (a) free lateral displacement throughout the depth of both pavement and subgrade . Such free lateral and vertical displacement boundary conditions represent situations having:
- (i) a road section with narrow shoulder and steep verge such that subgrade and pavement sections considered for analysis have very small side support as illustrated in Figure 7.3 a(i).
 - (ii) a deep and relatively wide existing crack extending from the pavement surface to the bottom of compacted subgrade. Such crack illustrated in Figure 7.3a(ii) results to a free lateral displacement in a swelling/shrinking of subgrade and until the crack closes in case of swelling. It also allows the overlying pavement to extend or contract laterally as a result of subgrade lateral movement.
- (b) fixed against lateral displacement throughout the depth for both pavement and subgrade, . This type of lateral fixity represents typical field situations having:
- (i) outer edge of both pavement and subgrade sections supported by a retaining structure throughout the depth of section as illustrated in Figure 7.3b(i). *(This situation provides lateral fixity for swelling subgrade only).*
 - (ii) a relatively wide overall section of road in a shallow embankment such that the outer edge of the section analysed is subjected to a significant lateral support from the remaining outer width. In this situation the pavement layer is

considered to be continuous and extends across the width of shoulder. It is illustrated in Figure 7.3b(ii). (This situation provides lateral fixity for both swelling and shrinking subgrade).

- (c) fixed against lateral displacement for the pavement section and free for remaining subgrade . This type of lateral fixity represents situations having a wide and deep vertical pre-existing crack inside and near the outer edge of subgrade section. Such crack may develop due to shrinkage of subgrade soil. On the other hand, the pavement section is free from cracks and extends across the shoulder. It is illustrated in Figure 7.3c.

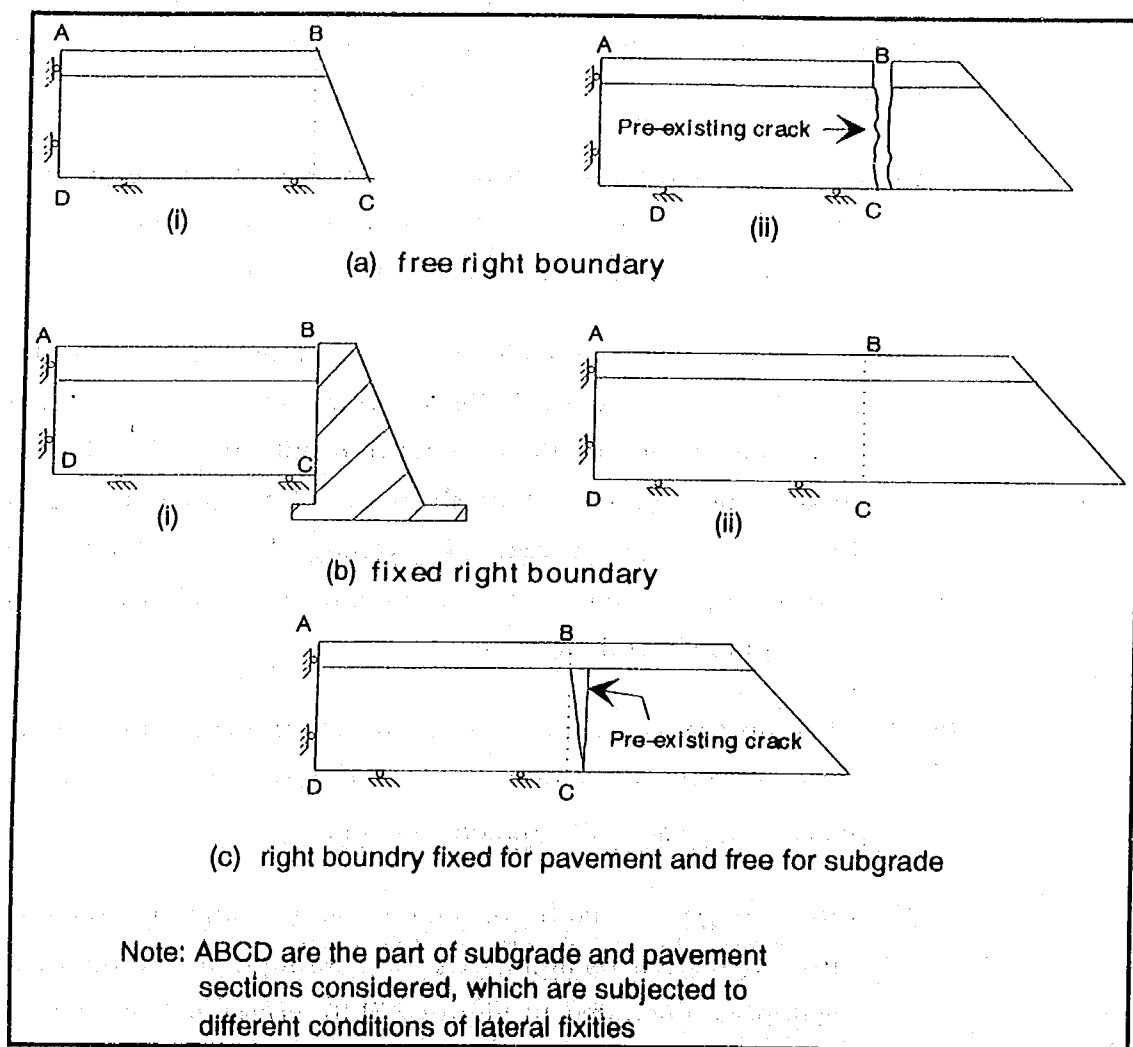


Figure 7.2 Different extreme situations of lateral boundary conditions at the outer edge of pavement

The above mentioned conditions of lateral fixities represent extreme conditions. In most of the actual field situations, the outer edge of both pavement and subgrade

sections experience partial lateral support. The degree of lateral support is significantly affected by the presence of discontinuity, loose materials, etc. For example, a well compacted outer section of subgrade (ie., section outside the right edge as in Case (b)(ii)), will provide a relatively higher lateral support than for the same outer section consisting of loose material or minor shrinkage cracks and fissures. Similarly, the lateral support of pavement section is also affected if its outer section consists of some minor cracks developed during construction or due to traffic loads, and any other form of structural defect and discontinuity.

In overall, it is extremely complex to understand the exact nature of partial support particularly to estimate the degree to which it supports, and to understand its behaviour during the wetting up or drying of reactive clay subgrade. Neither it is easy to characterise the effect of partial support in the numerical analysis. To overcome these complexities, the study considered appropriate to conduct numerical analysis under extreme situations of lateral fixities. And to understand the effect of such extreme lateral fixities, the study conducted a parametric analysis described in Chapters IX and X.

7.2.4 Interfaces

Interfaces exist between the top surface of subgrade and the bottom face of granular base, and the top surface of granular base and bottom face of seal coat. The behaviour of the first interface is significant in the analysis of transmission of forces and displacements generated by the volume change of subgrade to the overlying pavement. The degree of transmission depends on the shear stiffness of the subgrade/pavement interface. Due to the difficulty in the assessment of interface strength and stiffness parameters, this interface is not included in the present analyses. However, some vertical interfaces within the pavement section are considered to simulate the crack opening (refer Section 8.6 for detail). The effect of the interface between seal coat and granular base is not considered. As mentioned in Section 5.1, the seal coat is assumed not to contribute any strength.

7.3 Material Behaviour and Parameters

Three kinds of materials considered in the analysis are; a reactive clay subgrade, an unbound granular base, and a thin bituminous seal coat. In modelling of moisture flow, only the reactive clay subgrade will be analysed as this is the layer which subjects to moisture variation and volume changes. The pavement course is a non-swelling/non-shrinking material and is therefore not considered for moisture flow.

It is assumed that there is no significant variation in moisture in this layer for an increase or decrease of moisture content in the underlying subgrade, and as such, there is no change in stress/strain behaviour of the pavement material during the subgrade wetting or drying.

In dealing with stress/strain and strength behaviour of two different layers of pavement (as explained in Chapter VI) it is considered that the seal coat does not provide any separate strength against the distortion and cracking of pavement. It serves only as an impermeable layer against the infiltration of water through the pavement surface. The granular base course provides major strength to the pavement and is considered to behave as an elasto plastic material with limited tensile strength, and which follows the general Mohr-Coulomb failure criterion discussed in Section 6.4.

The strength and stress strain behaviour of a reactive clay subgrade are considered to be dependent on moisture potential. And the moisture movement equations are reformulated as heat diffusion equation (with non-linear coefficient) such that the existing thermo-mechanical programs of FLAC can be used. Thus, in terms of program, the strength and stress strain behaviour of clay is temperature dependent. The analysis uses Mohr-Coulomb plasticity material model with the development of thermal stresses and strains. The modulus of elasticity is varied non-linearly with temperature (ie., suction). An incremental linear elastic analysis is employed to modify the stiffness with stress level and suction. The modelling of constitutive behaviour with varying material properties is presented in the later parts of this section and in Chapter VIII.

7.3.1 Swelling and Swell Pressure Development

As discussed in Section 4.5.1, the constitutive behaviour of a swelling clay is dependent on the state of stress developed due to wetting and swelling. In the analysis, the stress/strain behaviour of such clay is characterised by using the concept of stress dependency in isotropic material in which swelling and swell suppression in any direction is dependent on the magnitude of octahedral normal stress. The choice of the model is based on its consistent description of Young's Modulus of swell suppression ' E_s ' over a range of mean normal stress, and its simplified form able to be used in an incremental analysis (Section 4.5.1). The B parameter involved in the relation of swell suppression modulus ' E_s ' reproduced below is a function of swell indices and the range of suction pressure

$$E_s = B \left(\frac{\sigma_1 + 2\sigma_3}{3} \right)^n$$

where the index 'n' is taken as unity for the present analysis (Section 4.5.1).

A theoretical analysis is carried out to determine the magnitude and variation of B with suction level. For this analysis, the relationships of swell and swell suppression modulus described in Chapter IV and discussed in detail in Appendix A are adopted. Initially, the coefficient β in the relation $E_{s1} = \beta \sigma_1$, used for the uniaxial model, is calculated as

$$\beta = \frac{\sigma_1}{0.435 \gamma_{\sigma_1}} \frac{(1 + \nu)(1 - 2\nu)}{(1 - \nu)}$$

where Poisson's ratio $\nu = 0.30$, and swell suppression index $\gamma_{\sigma_1} = 0.04$ and 0.08 .

This value of β is used as an initial estimation of B in stress dependent model where $E_s = B \left(\frac{\sigma_1 + 2\sigma_3}{3} \right)$. Using this value of E_s and the vertical stress σ_1 as 30 kPa and 75 kPa representing roughly two quarter points between 0 and 100 kPa, the B value is computed iteratively at each different suction level. The value of σ_3 is obtained by using $K_s \sigma_1$ where K_s is the coefficient of lateral swell pressure (refer Equation A.26, Appendix A). The above analysis is performed for an intact soil mass such that the crack closure factor ξ is considered as zero.

One of the objectives of this research study is to carry out a parametric study covering a wider range of reactive soils. In the numerical analysis a typical range of values of swell parameters are applied. As suggested in Sections 4.6, three sets of values of swell indices: $\gamma_h = 0.035$, $\gamma_{\sigma_1} = 0.04$ representing high swelling clays with high swell suppression; $\gamma_h = 0.07$, $\gamma_{\sigma_1} = 0.04$ representing extremely high swelling clays with high swell suppression; and $\gamma_h = 0.07$, $\gamma_{\sigma_1} = 0.08$ representing extremely high swelling clays with very high swell suppression are considered. The analytical determination of B value was conducted for these three sets of swell indices.

The result of analysis summarised in Figure 7.3 illustrates that the magnitude of B decreases non-linearly with increase in wetness. Approximate relations were also considered to describe closely such non-linear behaviour. These relations presented in Table 8.1 in terms of both suction and thermal parameter, are useful as they can be easily applied in the numerical program to modify stiffness properties with the stresses and suction levels.

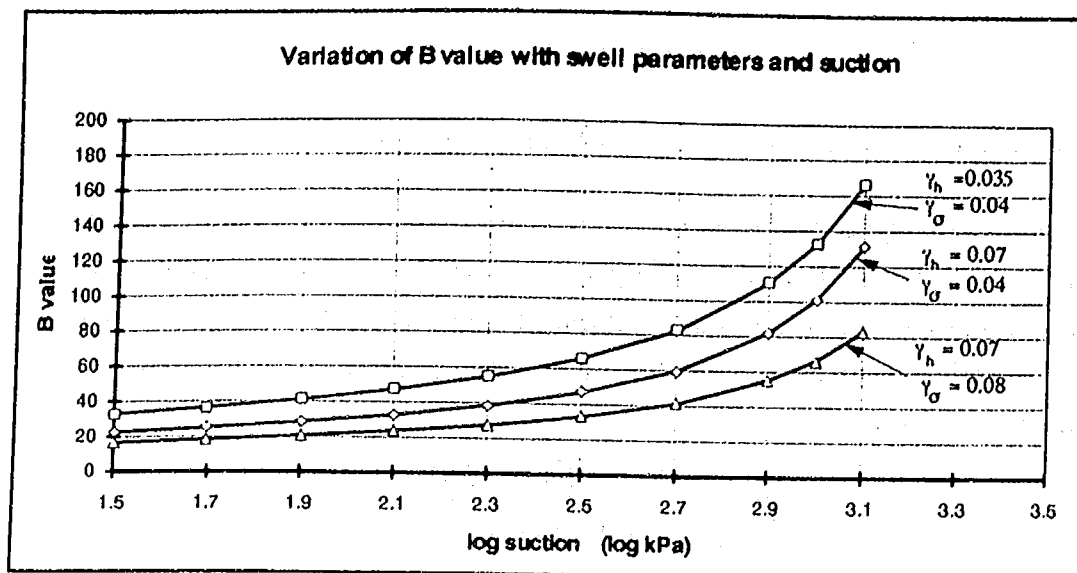


Figure 7.3 Variation of B value with log suction

While the analytical determination of swell/swell suppression modulus discussed above is thought to be adequate at the very low overburden pressure encountered in the pavement, its general use must be seriously questioned as it is based on swell suppression characteristics at the wet limit which ignores the complex suction and stress path as the soil wets up. Richards (1977 and 1980) derived some constitutive relationships for one or two particular soil types which take suction/stress path into account. The advantage of the present approach (and any further refinements) is that it is able to relate to known swell characteristics over a wide range of expansive soils. Further improvement of this approach will form part in future research.

7.3.2 Shrinkage and Shrinkage Stress Development

The shrinkage behaviour of subgrade upon drying is also characterised with non linear suction and stress dependent stiffness modulus. However, instead of octahedral normal stress, in this case the normal vertical stress is considered representing an oedometer type one-dimensional shrinkage compression. As discussed in Section 5.3, the shrinkage compression modulus is described by Equation (5.5) rewritten here as:

$$E_{sh} = \frac{2.3 (1 + \nu)(1 - 2\nu)}{\lambda_\sigma (1 - \nu)} \sigma_z$$

where λ_σ is the shrinkage compression index, ν is Poisson's ratio and σ_z is the vertical stress.

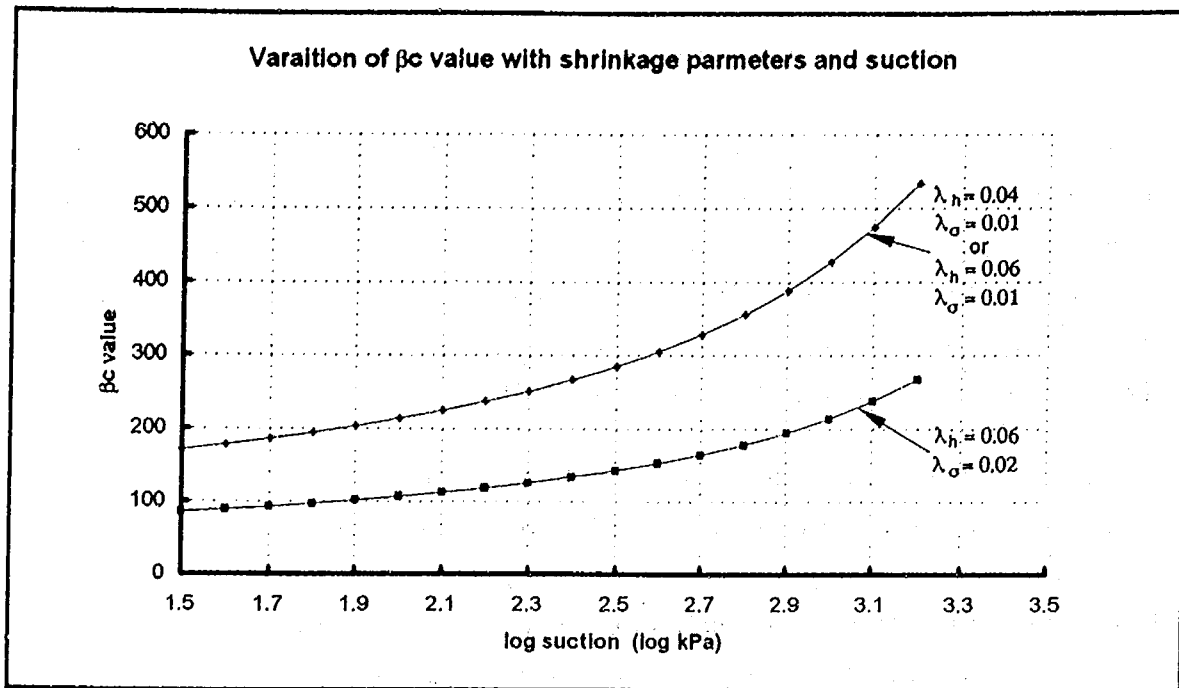


Figure 7.4 Variation of coefficient ' β_c ' of shrinkage compression modulus with log suction

The procedure of analysis for subgrade shrinkage follows a similar approach as swelling. To cover a wide range of reactive soils, the study considers three sets of free shrinkage index λ_h and the shrinkage compression index λ_σ . Figure 7.4 presents the variation of coefficient ' β_c ' of shrinkage compression modulus with log suction for a range of soils from medium to very high shrinkage.

7.3.3 Shear Strength Parameters of Reactive Clay Subgrade

The values of shear strength parameters, cohesion ' c ' and friction angle ' ϕ ', are needed to be defined in the Mohr-Coulomb model. The role of cohesion and friction in the proposed model will be presented in Chapter VIII where discussion on the selection of suitable failure criterion is covered. Here, the variation in cohesion and tensile strength with suction is discussed.

Note that the moisture flow and stress/strain phenomena investigated occurs under drained condition. This is attributable to the very slow process of moisture diffusion in unsaturated clays. Analyses are performed under applied stress condition and is based on total stress analysis.

The angle of internal friction is assumed to remain constant with a typical value in the range of 15° to 20° . The other parameters, cohesion and tensile strength, are considered to vary linearly with the logarithm of suction pressure as

$$c = A + \frac{\Delta c}{\Delta \log h} \log h$$

Considering the boundary values of cohesion at maximum wet and dry conditions, the constant A involved in the above equation can be determined. The tensile strength of reactive clay is assumed to be twice the cohesion. This value will be significantly lower than the plastic apex value obtained from cohesion/tangent ϕ for smaller friction angles. Selection of typical values of these parameters used in the analysis is discussed further in Chapter VIII.

Richards (1977, 1992) considered suction change as a variable to define variation of undrained cohesion and friction angle of wetting and swelling on Hindmarsh clay of South Australia. Using the plots of suction pressure vs. deviator stress obtained from the experimental results, he found the yield criterion for this clay as cohesion $c = 0$ and friction angle for a change in soil suction as $\phi_h = 8.3^\circ$.

7.3.4 Stress/Strain Behaviour of Granular Base

The stress strain behaviour of granular base is discussed in Section 6.4. For the present numerical analysis, it is assumed that the material behaves in an elasto-plastic manner with limited tensile strength. As explained in Figure 6.4a, the non linear stress strain behaviour of granular material is idealised with a bi-linear elasto-plastic model. The value of yield stress is computed from the geometry of Mohr's envelop shown in Figure 6.4b. Tension failure of the granular section is examined by checking the tensile stress generated in the section whether it is greater or lesser than the tensile strength of the material. Also, as the section has relatively small tensile strength, a tension cut off defined by tensile strength of the material is considered in place of the plastic apex (cohesion/tangent ϕ).

The pavement reaction to the forces applied as a result of subgrade volume change is of the major concern. As discussed in Chapter VI, the unbound granular layer used in a seal coated road pavement is a major strength layer in compression but it is weak in tensile strength. Hence, failure in pavement is investigated by considering the magnitude of tensile strain developed in the base. This is because the tensile strain able to be sustained by the overlying seal coat or underlying swelling subgrade is sufficiently large enough to result in cracking in the granular unbound base.

7.4 Procedure of Numerical Analysis

The numerical analysis proposed for this study uses a simplified description of the swelling and shrinking characteristics of a reactive clay subgrade in terms of thermal analogy of the moisture diffusion and volume change. As mentioned in Section 7.2.1, the present study employs FLAC's thermo-mechanical analysis program.

The constitutive properties of subgrade which vary with changes in suction potential and stresses, are adjusted at each step according to the new values of suction and octahedral normal stress level. To modify the strength and stiffness with suction and stress level, the writer has written a separate routine which is called by the source program after every incremental decrease in suction potential and change in stresses.

A. *Numerical Translation: Thermal Analogy*

A major feature in the analysis is the reformulation of some thermal relations to represent wetting and swelling or drying and shrinkage phenomena. The basic substitution involved in the mechanical analysis is that free swell and shrink is analogous to isotropic thermal expansion and contraction of a material. Therefore, the coefficient of thermal expansion and contraction are set equal to linear free swell and shrinkage strain per unit change in logarithm of suction pressure (ie., in log kPa). The other thermal parameters involved are specific heat, temperature dependent thermal conductivity in transient flow, and mass density.

The thermal properties which are essential for the thermo-mechanical analysis of FLAC are discussed here. These properties are extended and translated to describe unsaturated flow of water in soil. A comparative chart in Table 7.1 summarises the dimensional analyses for numerical works and discusses the translation of material parameters.

Using the above translation of unsaturated flow properties into heat flow properties, the thermal analogy of unsaturated flow to heat diffusivity is accomplished. Typical values of these properties used in the numerical analysis are discussed in the succeeding sections.

Table 7.1 Thermal analogy

	Unsaturated Flow of Water	Heat Diffusion in Media
1. Conductivity	Generally known as permeability 'K' and describes rate of water flow through a unit area under a unit hydraulic gradient resulting from suction potential $K = \frac{\text{m}^3}{\text{sec}} \cdot \frac{1}{\text{m}^2} \cdot \frac{\text{m}}{\text{m}} = \text{m/sec}$	Heat conductivity 'K' describes flow of heat through a unit area under a unit temperature gradient $K = W/\text{m}^2 = \frac{\text{J}}{\text{sec}} \cdot \frac{1}{\text{m}^2} \cdot \frac{\text{m}}{^\circ\text{C}} = \frac{\text{J}}{\text{sec m } ^\circ\text{C}}$
2. Specific water/heat capacity	Specific water capacity $C(\theta)$ determines the water volume retained in unit total volume of soil under a unit suction potential ψ $C(\theta) = \frac{d\theta}{d\psi} = \frac{\text{m}^3}{\text{m}^3} \cdot \frac{1}{\log \text{kPa}} = 1/\log \text{kPa}$	Specific heat capacity is the energy required to change a unit mass to a unit degree rise in temperature ($^\circ\text{C}$) $C_p = \frac{\text{J}}{\text{ton } ^\circ\text{C}}$
3. Mass density	Total mass contained in a unit volume of soil-water, with the unit selected as ton/m^3	Total mass contained in a unit volume of heat flow media, selected unit as ton/m^3
4. Diffusivity	Hydraulic diffusivity in an unsaturated soil media relates variation of hydraulic conductivity with change in water capacity $D(\theta) = \frac{K(\theta)}{C(\theta)} \text{ m}^2/\text{day}$	Thermal diffusivity is a function of heat conductivity, specific heat and mass density as $D = \frac{K}{\rho C_p} = \frac{\text{J}}{\text{sec m } ^\circ\text{C}} \cdot \frac{\text{m}^3}{\text{ton}} \cdot \frac{\text{ton } ^\circ\text{C}}{\text{J}} = \text{m}^2/\text{sec}$
5. Coefficient of expansion or contraction	For reactive clays, the coefficient of expansion or contraction is defined as volume change per unit decrease or increase in suction potential with the unit as $1/\log \text{kPa}$	In thermal condition, the coefficient of expansion or contraction is considered as volume change per unit increase or decrease in temperature with unit as $1/^\circ\text{C}$

B. Transformation Variable

Moisture potential is considered in terms of $\log \text{kPa}$ and, in the thermo-mechanical analogy, changes in moisture potential in $\log$ suction is the same as in thermal potential in degrees Centigrade. Using $\log$ suction rather than suction as the independent variable is valid for wetting of the homogenous subgrade because it is assumed, as is inferred by Equation (4.2) and Figure 4.1, that the volumetric moisture content is linearly related to $\log$ suction. $\log$ suction is therefore a better conditioned and simpler variable than suction because swell and swell suppression

or shrinkage and shrinkage compression are more linearly related to log suction than to suction. (Further consideration is required before applying this simplification to heterogeneous soils with different suction and moisture characteristics but it may still be valid where the difference in the volumetric moisture content of soils is secondary to the effect of suction changes.)

When studying results, it is nice to have the water flowing in the same direction as the flow of heat in the analogy so, to give the same sense of moisture movement as heat flow and to simplify the swelling characteristics, the present analysis uses a transformation variable as:

$$T = C - \log(h)$$

where C is the minimum value of suction potential above which there is zero swell suppression or zero shrinkage compression under the applied normal stress. The value of C for a particular soil can be determined from the results of oedometer swell/shrink tests. For the case studied here, suction at zero swell suppression or shrinkage compression is assumed to lie between 4.0 and 4.5 log kPa. For example, if value of C is taken as 4 log kPa, then the relation becomes $T = 4 - \log(h)$.

This implies, for example a soil wetting up from 3.2 log kPa to 1.5 log kPa is equivalent to a metal heating from 0.8 °C to 2.5 °C. Similarly drying of a soil from 1.5 log kPa to 3.2 log kPa is equivalent to the metal cooling from 2.5 °C to 0.8 °C.

C. Thermal Parameters

As depicted in Table 7.1, two major thermal parameters are required to be defined to apply thermal analogy. These parameters are specific heat and diffusivity. The detailed discussion on the descriptions of these parameters and their role in unsaturated moisture flow are presented in Chapter II, while the method of obtaining their typical values is presented Appendix C. For example, if the volumetric moisture content is increased from 15 percent to 45 percent under a suction change from 3.5 log kPa to 1.5 log kPa, the resulting specific heat capacity in terms of specific water capacity is $C_h = 576\,000$ kJ/ton °C. This value of specific heat is used to obtain non-linear suction dependent moisture conductivity relationship in Section 8.4.1.

7.5 Conclusions

A numerical analysis procedure was developed to analyse the physical and mathematical descriptions of the wetting and swelling as well as drying and shrinkage processes of expansive clay subgrades and the stress/strain behaviour of the overlying pavement. For this purpose, the finite difference solution technique was applied using an available computer program, FLAC (Itasca, 1992).

Basically, three major parts were involved in the overall analysis procedure. These were moisture movement in unsaturated reactive clay subgrade, stress/strain development in wetting and swelling as well as drying and shrinking subgrade, and the response of pavement to subgrade volume increase. The analysis used a two dimensional plane strain environment in which the strain along the length of the roadway was considered as zero. FLAC was used in the analysis because of its ability to satisfy all the requirements in regard to geometry, grid, and flow and displacement boundary conditions. It allowed the application of non-linear suction and stress level dependent modulus and strength parameters. FLAC also has various built in models of mechanical behaviour including interfaces. Thus the tedium of developing a separate finite element/finite difference coding was eliminated.

Typical grid generation with rectangular elements was selected (Section 7.2.3). Three sets of possible fixity against lateral displacement of individual subgrade and pavement sections were considered. As discussed in Section 7.2.3B, these lateral fixity conditions represent extreme situations. In most of the actual field situations there would develop a partial lateral support from the outer sections. However it is more complex to apply the actual field situations of lateral fixity in the numerical analysis. Hence lateral fixities representing extreme situations were applied for the numerical analysis in Chapter VIII as well as drying and shrinkage, and their effect on the response of subgrade/pavement system is analysed in Chapter IX and X.

The material parameters required in the numerical description of wetting and swelling as well as drying and shrinkage of subgrade were identified. Their magnitudes were estimated based from the information covered in the theory chapters (Chapters II, III, IV, V and VI). Using these as initial values, the numerical descriptions of non-linear suction and stress dependent stiffness and strength parameters were developed (Section 7.3) and will be used in numerical analysis in Chapter VIII.

An important feature in this analysis procedure was the translation of moisture flow and swelling/shrinking process into a heat diffusion and thermal stress/strain development. The readily available thermo-mechanical option of FLAC was utilised. The parameters, namely; moisture diffusivity, specific water capacity, and hydraulic conductivity, which are involved in describing the unsaturated flow were re-described in terms of thermal parameters. Also, a free swell parameter was identified as equal to the coefficient of thermal expansion. Similarly a free shrinkage parameter was identified as the coefficient of thermal contraction. A transformation variable was applied which translates log of suction potential into an equivalent temperature in degree Celsius.

For the analysis of swelling and swell pressure development in the subgrade, the stress dependent isotropic mechanistic model was selected where the modulus of swell suppression varied with the stress and suction levels. The swell suppression modulus was taken as $E_s = B\sigma_{oct}$. The B value is analytically determined (Figure 7.3) which was derived from a wide range of combinations of magnitudes of swell ' γ_h ' and swell suppression ' γ_σ '. The range of values of γ_h and γ_σ selected represented clays of high swelling to extremely high swelling behaviour, covering a wider distribution of expansive soils.

For subgrade shrinkage analysis, a non-linear uniaxial stress and suction dependent modulus of shrinkage compression was employed. The shrinkage compression modulus was taken as $E_{sh} = \beta_c \sigma_1$, the value of β_c was obtained directly from the relationship (Equation 5.5) described in Chapter V.

Finally, the numerical procedure developed here will be utilised in Chapter VIII where it will be applied in the analysis of a typical situation of subgrade/pavement interaction.

CHAPTER VIII

NUMERICAL MODELLING: APPLICATION

8.1 Scope of Numerical Analysis

The objective of this chapter is to apply the numerical analysis procedure developed in Chapter VII to study the interaction between a swelling and shrinking clay subgrade and a seal coated unbound granular pavement. The earlier part describes a typical situation of a road section considered for the numerical analysis, and, also covers the magnitude of properties and parameters required to characterise the behaviour of subgrade and pavement. The later part presents a review of the preliminary results on moisture flow obtained from the analysis. The detailed parametric study on the results mechanical response of subgrade and pavement obtained from the analyses of this problem is left to Chapters IX and X.

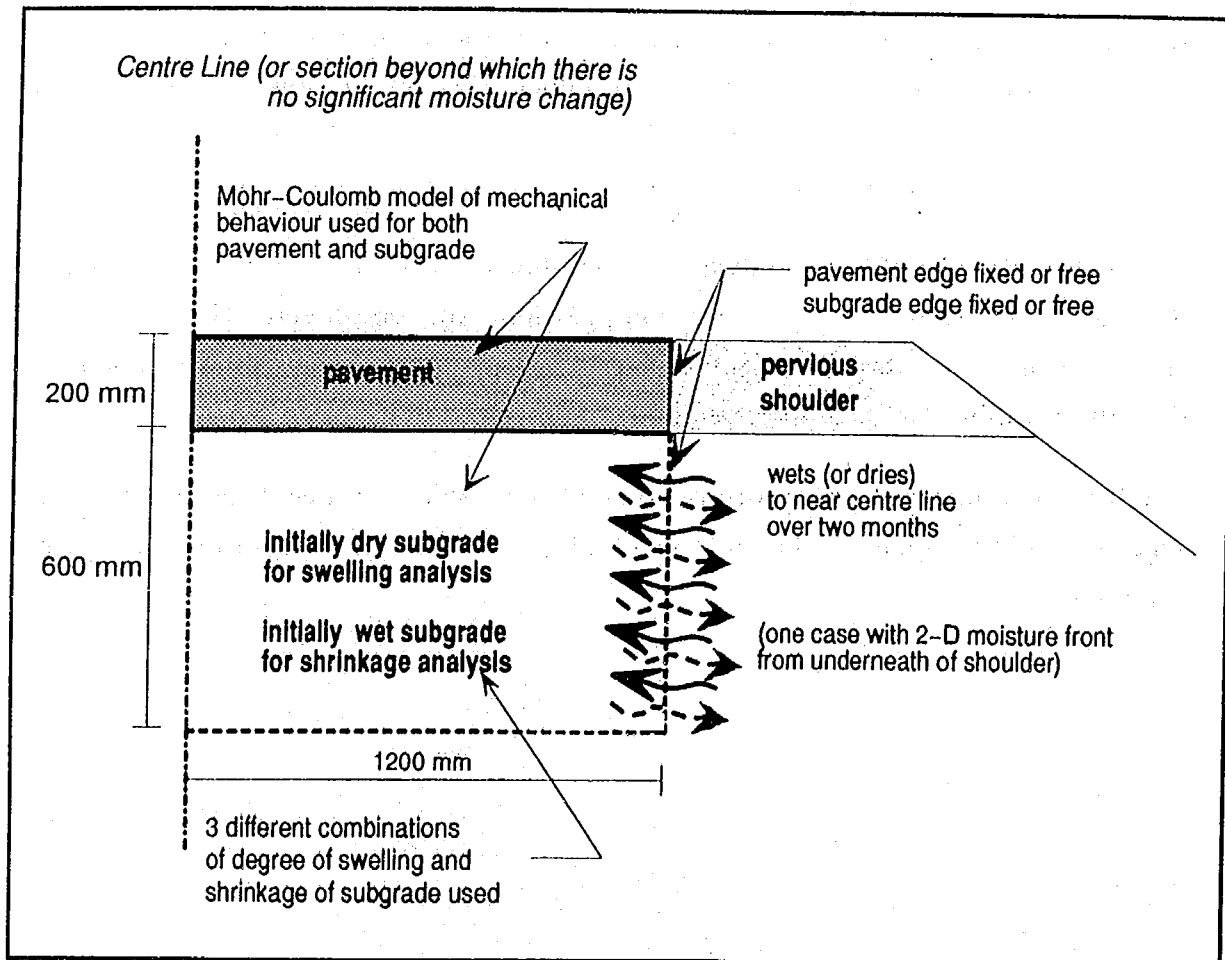


Figure 8.1 Features of the physical problems considered in the numerical analysis

Figure 8.1 depicts a typical pavement situation where the subgrade soil is subjected to a large variation in moisture environment. The movement of moisture was

considered under a monotonic wetting from a uniformly dry stage to a relatively wet stage for swelling analysis. And monotonic drying from the end of wet stage of wetting process obtained from swell analysis to a relatively dry stage for shrinkage analysis. Both wetting and drying phenomena start from the right edge of subgrade section and propagates inwards for wetting or outwards for drying. In one situation, a two dimensional moisture flow starts through the underneath of pervious shoulder. The flow boundary condition shown in Figure 8.2 was considered for all cases of analyses in 1-D flow condition. The boundary condition for 2-D flow is illustrated in Figure 8.6.

The section of subgrade analysed is considered as an intact mass without any major shrinkage cracks or fissures. Such cracks and fissures would tend to relieve swell pressure during initial crack closure. The effect of minor cracks and fissures would be included in the parameters measured in swell and shrinkage tests on undisturbed specimens. The three sets of values of swell and shrinkage parameters discussed in Sections 7.3.1 and 7.3.2 are applied to cover a wide range of reactive soils. As discussed in Section 8.2.2 later, the subroutines required in the program to modify the stiffness using stress and suction dependent material model were written separately for each set of subgrade swell and shrinkage parameters.

Three sets of lateral deformation boundary conditions representing extreme field situations were considered. Details of these are covered in Section 7.2.3 (Figure 7.2). The typical cases analysed are discussed later in this chapter. The analysis uses a plane strain condition with strain along the road length taken as zero.

The pavement and subgrade materials were characterised using Mohr-Coulomb model of mechanical behaviour. The inclusion of a set of vertical interfaces at pre-located possible crack development zones were incorporated in one case. Opening of these interfaces simulated crack initiation and propagation.

As mentioned in Chapter VII, the numerical procedure employed an existing, versatile micro-computer program FLAC (Itasca, 1993). An example of the application of FLAC is found in the work of Wallace and Lytton (1992a) where the thermo-mechanical option of the program was applied in their study of the characteristic behaviour of a small block of swelling clay. Another example of the application of the thermal analogy is the work of Uzan et al. (1987) who applied NASTRAN program to analyse a deep excavation in swelling marl. Both these examples analysed the situation of wetting and swelling of soil, in this study the writers have attempted to extend this concept of analysis into drying and shrinkage.

8.2 Description of Typical Problem

An idealised sketch of a typical road section consisting of an unbound granular pavement layer surfaced with a seal coat overlying a moisture and volume changing reactive clay subgrade is illustrated in Figure 8.1. The analyses were conducted separately for wetting and drying phases.

8.2.1 Geometry of Structure, Grid Distribution, and Boundary Conditions

Figure 8.2 illustrates a typical geometry of subgrade/pavement composite section under the different boundary conditions discussed in Chapter VIII. The section consists of a 1200 mm wide by 600 mm deep subgrade surfaced with a 200 mm thick pavement.

The section was divided into rectangular finite difference zones. In the vertical direction, the pavement layer was divided into 4 layers of closely spaced grids each 50 mm thick. The subgrade depth was divided into 10 layers each of 60 mm depth. Along the lateral direction, both pavement and subgrade were divided into 23 columns having variable width from 100 mm to 30 mm. Dense grids (width of 30 mm and 50 mm) were provided near the outer edge where the moisture enters or exits the subgrade section. It is because the region near the outer edge subjects a sharp gradient of suction potential at the initial stage of both wetting and drying, and thus needs to have a finer mesh for better distribution of suction potential and associated stresses.

In some cases analysed, the section included both horizontal and vertical interfaces. The horizontal interface was considered to represent a separation between subgrade and overlying pavement. The vertical interfaces within the pavement layer were applied to represent possible crack development in the pavement layer. The location and alignment of vertical interfaces were selected by reviewing the results of analyses of cases having no interfaces and noting the location of area of pavement having maximum tensile stress developed after certain period of subgrade wetting and volume change. Such an interface behaved as a longitudinal crack in the pavement. Further details on these interface characteristics are presented in Section 8.6.

The lateral deformation at the left edge was considered as fixed throughout the depth. As mentioned in Section 7.2.3, the assumption of lateral fixity of left edge is possible for both symmetric and non symmetric road sections, as long as there is no

significant seasonal variation of moisture change takes place at that section (Uzan et al. 1972). Thus this edge may not necessarily be the centre line of a road.

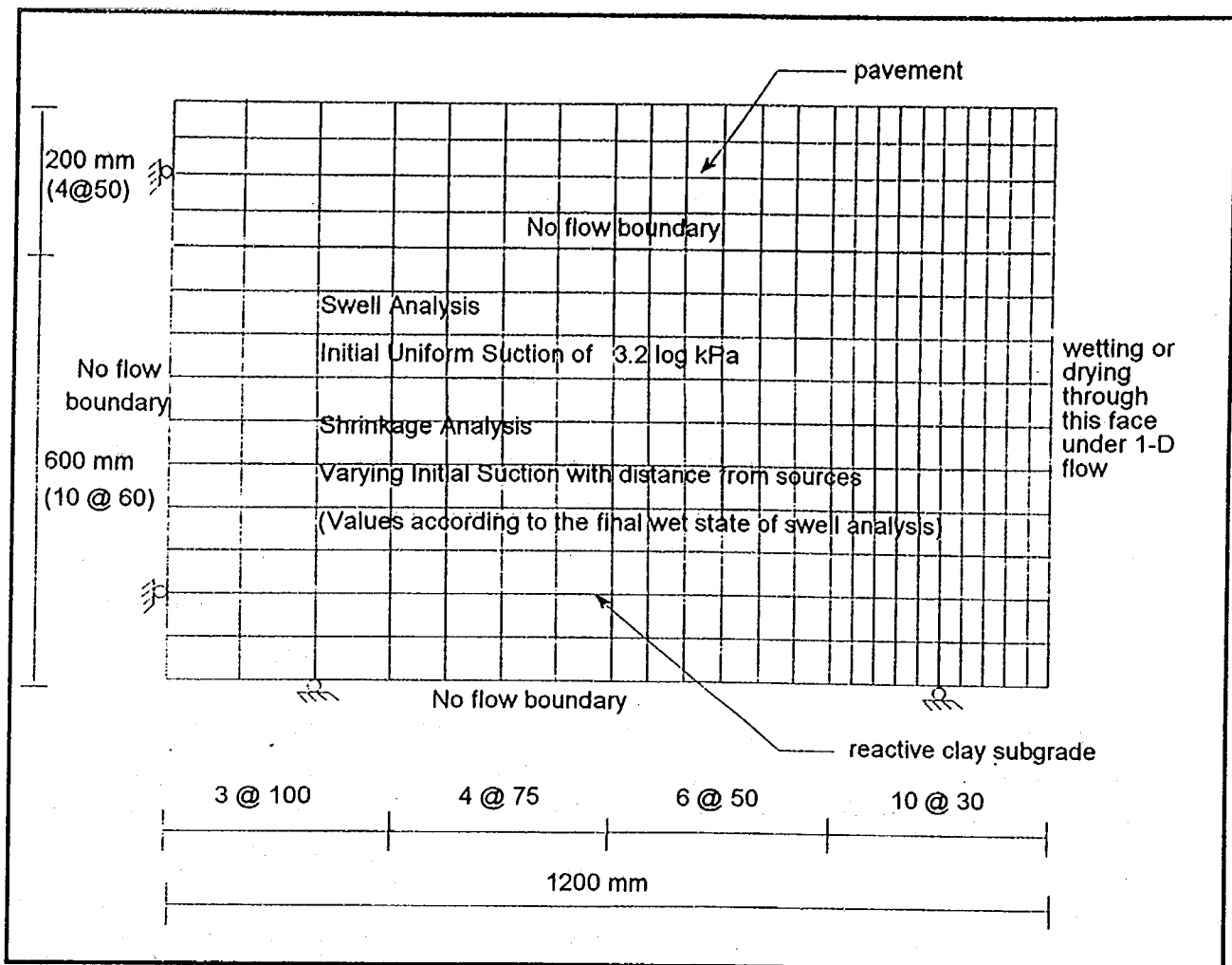


Figure 8.2 Typical geometry, grid distribution and boundary conditions used in the analysis (for one dimensional horizontal flow condition)

The right edge lateral deformation was studied, as discussed in Section 7.2.3 and illustrated in Figure 7.2, under three different cases simulating three different extreme situations. The first case assumed that the right edge had a free lateral deformation boundary through out the depth . The second case assumed the right edge to be fixed against lateral deformation through out the depth of section . The third case considered a free right boundary for lateral movement for the subgrade and fixed for the pavement .

Only the base of the subgrade was considered as fixed against vertical movement while all other edges were free to move vertically. The rationale in the selection of these boundary conditions is discussed fully in Section 7.2.3.

8.2.2 Subgrade Soil Characteristics

Basically, the numerical analysis dealt with two kinds of materials, namely; a reactive clay subgrade section, and a granular layer acting as an overlying pavement structure. The top surface, which was a bituminous seal coat, was not considered in the structural analysis. However, its imperviousness against infiltration of rain water was taken into consideration in dealing with flow boundary conditions and as source and location of moisture entry or exit.

A. Wetting up and Swelling of Subgrade

The strength and stiffness of subgrade are dependent on suction potential and on octahedral normal stress level. The variation of Young's Modulus of swell suppression with suction potential and stress level is presented in Figure 7.3 of Section 7.3.1. In Table 8.1, such variation of modular properties are expressed in terms of shear and bulk moduli with temperature using thermal analogy discussed in Section 7.4. The analysis was based on the assumption that Poisson's ratio remains constant at all stages of subgrade wetting and swelling processes.

Table 8.1 Variation of subgrade stiffness with temperature for swell analysis

Swell Indexes		In Terms of Suction Potential log kPa		In Terms of Temperature °C		Remarks
γ_h	γ_σ	B value	Young's Modulus E kPa	Shear Modulus G kPa	Bulk Modulus K kPa	Poisson's Ratio, ν
0.035	0.04	$\frac{65.409}{(3.493 - \log h)}$	$B \sigma_{oct}$	$\frac{25.157 \cdot \sigma_{oct}}{T - 0.507}$	$2.17 \cdot G$	$\nu = 0.3$, constant
0.07	0.04	$\frac{43.553}{(3.432 - \log h)}$	$B \sigma_{oct}$	$\frac{16.751 \cdot \sigma_{oct}}{T - 0.568}$	$2.17 \cdot G$	$\nu = 0.3$, constant
0.07	0.08	$\frac{32.713}{(3.493 - \log h)}$	$B \sigma_{oct}$	$\frac{12.582 \cdot \sigma_{oct}}{T - 0.507}$	$2.17 \cdot G$	$\nu = 0.3$, constant

B. Drying out and Shrinkage of Subgrade

In the shrinkage analysis also the strength and stiffness of subgrade were dependent on suction potential. But the stress dependency of the stiffness was based on vertical stress level. The variation of Young's Modulus of shrinkage

compression with suction potential and stress level is presented in Figure 7.4 of Section 7.3.2. In Table 8.2 below such variation of the modular properties are expressed in terms of shear and bulk moduli with temperature. Here also Poisson's ratio was assumed to remain constant at all stages of drying and shrinkage.

Table 8.2 Variation of subgrade stiffness with temperature for shrinkage analysis

Shrinkage Indexes		In Terms of Suction Potential log kPa		In Terms of Temperature °C		Remarks
λ_h	λ_σ	β_c value	Young's Modulus E_c kPa	Shear Modulus G kPa	Bulk Modulus K kPa	Poisson's Ratio, ν
0.04	0.01	$\frac{427.15}{(4 - \log h)}$	$\beta_c \sigma_v$	$\frac{164.288}{T} \sigma_z$	$2.17 \cdot G$	$\nu = 0.3$, constant
0.06	0.01	$\frac{427.15}{(4 - \log h)}$	$\beta_c \sigma_v$	$\frac{164.288}{T} \sigma_z$	$2.17 \cdot G$	$\nu = 0.3$, constant
0.06	0.02	$\frac{213.58}{(4 - \log h)}$	$\beta_c \sigma_v$	$\frac{82.144}{T} \sigma_z$	$2.17 \cdot G$	$\nu = 0.3$, constant

C. Both Swell and Shrinkage Analyses

The relation for variation of Mohr-Coulomb shear strength parameters with temperature was considered as same for both swelling and shrinkage processes. The following was considered in the analysis.

For all types of soils; cohesion = $85.88 - 32.35 T$ (Section 8.4.2), tension = 2 cohesion, and friction angle $\phi = 35^\circ$.

In the above tables (Tables 8.1 and 8.2) and the expression for cohesion, the value of T in °C is defined as $(4.0 - \log h)$, where $\log h$ is the logarithm of total suction potential defined in terms of log kPa. The factor 4.0 represents the value of suction potential above which there is no swell suppression or shrinkage compression under the applied stress.

8.2.3 Range of Change in Suction Potential

A. Wetting up and Swelling of Subgrade

The swell analysis begins at the end of a long dry period where the subgrade is considered to have an equilibrium total suction potential of about 3.2 log kPa. It

was presumed that the site analysed suddenly reaches an extreme wet condition as a result of intense rainfall, and that this wetness remains for a considerable period (at least 60 days), providing a continuous source of moisture available to propagate inwards from the outer edge of the section.

In the analysis, initially an instantaneous wetting of the outer edge was carried out. Afterwards, an alternating analysis of wetting and swelling of subgrade was continued. While conducting mechanical analysis, the analysis also accounted for the interaction between subgrade and pavement by analysing the suppression of swell due to the weight of pavement layer and the stress/strain response of pavement to subgrade volume increase. The scope of the analysis does not cover the swell suppression effect of traffic loads being transmitted to the swelling subgrade.

An appropriate value of soil diffusivity was selected by choosing a "reasonable period" of subgrade wetting. With the selected value of diffusivity, the estimation of wetting period for the alternate analysis of heat flow and mechanical equilibrium, and the examination of result of moisture flow are presented in Sections 8.5 and 8.7 respectively.

B. Drying out and Shrinking of Subgrade

Shrinkage analysis begins at the end of a long wet period. The subgrade section is considered to have an initial total suction representing the wet condition reached by a subgrade due to monotonic wetting for a period of two months. This is obtained by taking suction values at the end of swell analysis conducted for a two months wetting. Thus the values of initial total suction varies with the distance from the source with a minimum value of about 1.5 log kPa near the outer edge, and increasing value in the inward direction.

Also, it was assumed that the site starts to dry continuously for at least two months (64 days) such that a continuous unsaturated moisture flow takes place from inside section to the outer edge as the dried outer edge has the constant higher suction potential of about 3.2 log kPa. The instantaneous drying of the outer edge and the later alternate analysis of drying and shrinkage of subgrade were carried out in a similar manner as were for wetting up and swelling of subgrade.

The selection of diffusivity value and the period of drying were taken as the same of wetting process assuming that the drying is a mirror image of wetting.

8.3 Selection of Material Model

Mohr-Coulomb model was used for both subgrade and pavement to take into account the possible plastic behaviour with shear yielding and tensile failure. Limited tensile strength criterion was applied to interpret the crack initiation and propagation in the pavement layer.

In an earlier study, the first writer (Sapkota, 1994) he used a varieties of combination of mechanical models to represent subgrade and pavement. In that study he used Ubiquitous joint model (a built in model available in FLAC program) to represent pavement in one case. The specific feature was that with this model the pavement material could be described as a laminated structure such that the model should account the plastic failure as well as slip of the joint representing cracks in the pavement. However, because of the lack of data on joint properties, shear and strength properties of joints were considered the same as the shear and strength properties of continuum. By doing so, the results of analysis with joint model could not differentiate from the results obtained by using general Mohr-Coulomb model. Hence in this study, the use of ubiquitous joint model is not adopted.

8.4 Typical Material Properties

In analysing the subgrade section, two sets of soil properties were used to characterise its volume change behaviour. The first was a set of properties that described the moisture movement during the wetting and drying processes, and the second was a set of properties required for the mechanical analysis of the swelling and shrinkage processes.

In characterising the behaviour of pavement, only the strength and stiffness properties were required to describe stress strain behaviour and failure criterion. Since the pavement layer was not considered to play any role during the moisture movement phenomenon within the clay subgrade, the flow properties of granular layer were not needed in the analysis.

8.4.1 Unsaturated Moisture Flow Properties

As mentioned earlier, FLAC's thermo-mechanical analysis program was employed to model the moisture flow and volume change of subgrade. The thermo-mechanical option of FLAC allowed the use of non-linear diffusivity. Application of

such non-linear suction dependent diffusivity yielded a sharper wetting front which is in close approximation to the observed wetting fronts.

To obtain typical diffusivity values, the analysis assumed a total wetting period of about 100 days which represented about 90 percent wetting under constant diffusivity whose magnitude is considered to be at a saturated state. A diffusivity value of $0.0122 \text{ m}^2/\text{day}$, which gave a 90 percent wetting of a 1200 mm wide soil section in 100 days was selected. Note that the program units are in seconds but treated as though they are in days to represent the field situation. As defined, and derived, in Appendix C, the relation of temperature dependent conductivity obtained by using the above diffusivity value is

$$\text{Conductivity, } K = 1096 + 696 T^{2.5}$$

where T is the temperature in $^{\circ}\text{C}$ ($=4.00 - \log \text{ kPa}$), and unit of K is in $\frac{\text{kJ}}{\text{m sec } ^{\circ}\text{C}}$.

The other thermal properties required in the description of the wetting process are specific heat and mass density. Magnitudes of these properties selected for the analysis were taken as:

$$\text{Specific heat} = 576000 \frac{\text{kJ}}{\text{ton } ^{\circ}\text{C}}, \text{ and mass density} = 1.7 \text{ ton/m}^3.$$

In the study, thermal parameters thus defined in wetting up and swelling processes of subgrade were also used for the drying out and shrinkage. The decision was made on the basis that both wetting and drying processes were considered as monotonic, starting from a uniform equilibrium moisture potential and reaching to a relatively wet (for swelling) or dry (for shrinkage) state. Therefore the magnitude of these parameters were considered to be applicable for both wetting and drying.

8.4.2 Swell/Shrink Characteristics and Mechanical Properties of Subgrade

A. Swell/Shrink Characteristics of Subgrade

To simulate the swelling and shrinking behaviour, a coefficient of isotropic linear thermal expansion/contraction must be set. This was considered to be equal to the linear free swell and shrink strain per unit change in logarithm of suction in kPa unit. The free swell index ' γ_h ' discussed in Chapter IV and applied to describe the swell behaviour in the present research model was based on one dimensional oedometer condition. As such the typical γ_h mentioned in Table 8.1 (Section 8.2.2 A)

corresponded to $\left(\frac{1+\nu}{1-\nu} = 1.85; \text{ where } \nu=0.3\right)$ times the free swell in three dimensional swell condition. Hence in the analysis the values of coefficient of thermal expansion ' α ' was computed by dividing the magnitude of γ_h by 1.85 .

Similar to the case of swell, free shrinkage index λ_h in the proposed shrinkage model belongs to the one dimensional confined shrinkage measurement. Hence while obtaining the coefficient of thermal contraction ' α ' for shrinkage analysis, the magnitude of λ_h presented in Table 8.2 was divided by 1.85.

B. Mechanical Properties of Subgrade

Swelling Subgrade: The typical mechanical properties used for swelling clay subgrade at its initial unwetted state representing a stiff clay under drained condition were:

Young's Modulus $E = 5000 \text{ kPa}$,	Poisson's ratio $\nu = 0.3$
mass density $= 1.7 \text{ ton/m}^3$	friction angle $\phi = 20^\circ$
dilation $\psi = 0^\circ$	cohesion $c = 60 \text{ kPa}$, and
tensile strength $= 2 \times \text{cohesion} = 120 \text{ kPa}$	

Shrinking Subgrade: The magnitude of initial stiffness of subgrade elements depended on the value of log total suction of these elements. The suction values were obtained from the two months wetting carried out in swell analysis. With these suction values, the program computed initial stiffness of these subgrade elements by applying the appropriate relation presented in Table 8.2 of Section 8.2.2 B. The values of other properties such as mass density, Poisson's ratio, friction and dilation angles were the same as in swell analysis and remained constant. The values of cohesion and tensile strength at the initial wet state were computed by the program in a similar way as stiffness by applying relations given in Section 8.2.2 C .

As noted in Section 7.3.3, for both swelling and shrinking processes the analyses considered drained condition and were based on total stress analysis under applied stress condition.

As will be discussed in Section 8.5, the mechanical behaviour of subgrade was analysed assuming that the response of both wetting up and swelling, as well as drying out and shrinking phenomena could be modelled as an incremental linear elastic or elasto-plastic model with the strength and stiffness parameters adjusted according to the magnitude of stresses and suction potential. The adjustment of properties was accomplished using the relations discussed in Tables 8.1 and 8.2.

The variation of cohesion with the log of suction potential was considered to be linear with a magnitude of 60 kPa at the dry stage (ie., at suction of 3.20 log kPa) and 5 kPa at wet stage (at suction of 1.5 log kPa). In an equation form, the variation in cohesion is equal to $c = -43.52 + 32.35 (\log h)$. At all stages, the tensile strength was considered as twice the cohesion.

8.4.3 Mechanical Properties of Pavement

The mechanical analysis of pavement layer considered a section that was composed of an unbound granular material having very low cohesion and tensile strength. The elastic stiffness properties of the granular layer were the same as the stiffness of clays at the dry state with a Young's Modulus $E = 5 \text{ MPa}$, Poisson's ratio $\nu = 0.3$, but with a higher mass density $= 2.2 \text{ ton/m}^3$. This pavement modulus seems low when compared with the resilient moduli for pavement design. But it is applied in a different loading condition in which tensile forces due to subgrade lateral strains are causing the development of tensile stresses in pavement. In a pavement having very low tensile strength, these tensile stresses are critical in resulting cracks.

Other typical properties used for Mohr-Coulomb Model are

- cohesion $= 10 \text{ kPa}$
- tensile strength $= 10 \text{ kPa}$
- friction angle $\phi = 40^\circ$, and
- dilation angle $\psi = 0^\circ$

The tensile strength of 10 kPa applied in the above models was an assumed value. It is higher than the measured value of 5.4 kPa presented in Section 6.5.5. However the difference is small compared to the value of tensile stress that is found to develop due to the transmission of lateral swelling stresses of a subgrade. Thus it is expected that the use of this small tensile strength would not affect the stress/strain data from the numerical analysis.

8.4.4 Units used in the Analysis

The primary units used in the analysis for linear dimension, mechanical and flow properties, and parameters are m, kPa, tonne, and days. FLAC uses elastic properties in terms of shear modulus 'G' and bulk modulus 'K' instead of Young's Modulus 'E' and Poisson's ratio 'ν'. These are determined by using common relations as

$$G = \frac{E}{2(1 + \nu)}, \text{ and } K = \frac{E}{3(1 - 2\nu)}$$

8.5 Solution Procedure

The general procedure adopted in the analysis followed the sequence of events simulating as closely as possible the actual field situations. With this logic, the overall solution procedure was divided into three major steps of analysis. The primary step involved setting up of initial equilibrium stress/strain condition under general body forces due to gravity. Then the instantaneous wetting of moisture entry boundary in swell analysis, and instantaneous drying of moisture exit boundary in shrinkage were applied. Finally the alternate analysis of the simulated moisture flow and mechanical equilibrium were carried out. The following sections discuss in detail the solution steps followed in the analysis.

8.5.1 Initial Stress Equilibrium

A. *Swell Analysis*

The aim was to obtain an equilibrium initial stress/strain conditions under gravity forces. The analysis was carried out under a plane strain environment with out-of-plane strain (ie., along the length of road) considered as zero. All the three components of stresses: σ_{xx} , σ_{yy} , and σ_{zz} where σ_{zz} were considered by the program as out of plane stress, are to be initialised.

At first, the vertical stress σ_{yy} was initialised with gravity stress using the unit weight of pavement and subgrade materials. The lateral out of plane stress σ_{zz} was calculated by taking the product of Poisson's ratio and the summation of the other two normal stresses, such as $\nu(\sigma_{xx} + \sigma_{yy})$. For the initial stage, the lateral stress σ_{xx} was assumed to be zero. This represented the situation where any residual lateral stress along the x-direction developed during construction, and the effect of lateral earth pressure due to a vertical gravity stress are released because of a free boundary condition of the sides. This assumption may not be true in all cases, eg., in cases with fixity for lateral displacement. However it was anticipated that the error involved would be consistent and very small in comparison with the very high magnitude of lateral stress developed due to swelling.

The initial magnitude of stresses σ_{yy} and σ_{zz} for all pavement and subgrade grids were estimated using their mass densities of 2.20 ton/m³ and 1.70 ton/m³, respectively. The values of σ_{yy} and σ_{zz} thus calculated were presented in a file of FLAC input command, in Appendix D. Also included in Appendix D is a function subroutine written by the writers which enabled external modification of subgrade

strength and stiffness with respect to the values of the normal stresses and suction potential.

The initial properties of subgrade were selected assuming high shear and tensile strengths to avoid localised yielding during the initial gravity stress distribution. The initial values were taken as; cohesion = 200 kPa, friction angle $\phi = 40^\circ$ and tensile strength = 400 kPa. These properties were updated with their "true values" mentioned in section 8.4.2 B once equilibrium under the initial gravity stress was achieved. Afterwards, the program was re run to allow for any deformation.

Under the externally initialised gravity stresses, the internal acceleration due to gravity was set to zero. The program was then run either to a defined number of steps or by setting of an unbalance force that is less than a pre-set value, such that the program ran iteratively until the unbalance force reaches the convergence value. In the analysis, this converged unbalance force was set to 0.3 % of the initial unbalance force. The program continues running until an equilibrium was attained under the specified initial stresses for each of the cases having prescribed displacement boundary conditions. The response of the structure against the initial stresses was checked by examining the state of yield condition for each grid. The state of yield condition indicated if any yielding or plastic flow within any grid occurred under the specified initial stress conditions.

Once the yield states were checked and found to be under stable condition, the program was again run with the true strength and stiffness properties of subgrade and pavement material and inclusion of gravity as 9.81 m/sec^2 to simulate the actual situation under gravity stresses. At this stage, the magnitude of different stress components σ_{xx} , σ_{yy} , and σ_{zz} were printed and checked with the initialised values. The magnitude of these stresses should be the same as when initialised at the beginning, however, σ_{xx} values which were initially set to zero, were usually found negligibly higher than zero. This could be the result of inclusion of 0.3 percent error applied in setting the unbalance force as discussed before. Again checks were made with yield states and velocities and found to be in elastic state. Finally, once the structure was in a stable condition, it was considered to be at equilibrium under initial stresses.

B. Shrinkage Analysis

The procedure of obtaining the initial stress/strain equilibrium condition for an initially wet subgrade was exactly similar to that followed for swell analysis mentioned above in Section A. The only difference was that the magnitude of initial

strength and stiffness parameters were belonging to the wet state. The magnitude of these parameters were computed by the program. The stress components σ_{xx} , σ_{yy} and σ_{zz} with their values obtained in a similar way mentioned in swell analysis, were initialised. During the stages of initialisation with externally applied stresses and application of gravity later, checks were made in a similar manner and results obtained were found to be consistent as before in swell analysis.

8.5.2 Moisture Flow Simulation

A. Wetting Process

The analysis was based on suction change as the driving parameter to cause moisture flow. It was achieved by translation into thermal diffusion described in Sections 7.4 and 8.4.1. The moisture movement process was analysed in two stages: sudden wetting of the source boundary, and successive wetting of the interior zones. In both stages, the period of wetting for each step was estimated with the following consideration:

To simulate the sudden wetting of the outer boundary from where moisture enters and the rapid initial swelling at that boundary, suction potential was decreased from 3.20 log kPa to 1.5 log kPa (ie., with temperature increased from 0.8 °C to 2.5 °C) in successive eleven steps of 0.1 days resulting in a total period of 1.1 days.

For a 1200 mm length of moisture flow in one dimension, as shown in Figure 8.2, it was observed during trial runs that with flow properties of subgrade described in Section 8.4.1, moisture reaches to the section just beyond 600 mm inside from the outer edge in 20 days and 900 mm inside in 40 days. Also it was found that moisture reached more than 1200 mm at 100 days and almost saturated at about 200 days. Considering these wetting periods, a rough estimate could be made in terms of the square of the ratio of lengths of wetting front as illustrated in the example below.

If wetting front reaches 900 mm inside the outer edge in 40 days, then the period required for a wetting front to reach 1200 mm inside was determined as

$$40 \times \left(\frac{1200}{900} \right)^2 \approx 70 \text{ days}$$

With a trial run for 64 days, the decrease in log suction was reached by fifty percent at mid width, ie., at 600 mm distance from the source boundary (temperature increased from 0.8 °C to 1.65 °C corresponding to decrease in suction from 3.2 log

kPa to 2.35 log kPa). Thus for this analysis, a total of 64 days is considered as a fifty percent wetting period and is divided into twenty steps with proportionate to square root of time.

Expressing this in equation form, therefore, the period of wetting in days at any step can be estimated as $n = \left(n \times \frac{\sqrt{t}}{20} \right)^2$.

Applying this formula for t as 64 days, the ratio is determined as $R = \frac{\sqrt{t}}{20} = \frac{8}{20} = 0.4$

For example the period required for 6th step = $(6 \times 0.4)^2 = 5.76$ days. Thus the step wise periods in days for 20 steps are:

Step No.	Wetting Period (days)	Step No.	Wetting Period (days)
1	0.16	11	19.36
2	0.64	12	23.04
3	1.44	13	27.04
4	2.56	14	31.36
5	4.00	15	36.00
6	5.76	16	40.96
7	7.84	17	46.24
8	10.24	18	51.84
9	12.96	19	57.76
10	16.00	20	64.00

In the above stepping periods, instantaneous wetting discussed previously, covers up to 1.1 days in eleven steps. Hence the continued stepping time covers remaining 18 steps in rounded number of days as 1.5, 2.5, 4, 6, 8, 10, 13, 16, 19, 23, 27, 31, 36, 41, 46, 52, 58 and 64 days.

B. Drying Process

In this study as mentioned earlier, the drying process was followed in an exactly similar way as wetting process. Here also, two stages were considered: instantaneous drying of the outer edge as an exit boundary and later the successive drying of inner zones through time. The periods for both instantaneous and

successive drying were the same as instantaneous and successive wetting, and thus the same stepwise period for 18 steps consisting rounded number of days was followed.

8.5.3 Alternate Analysis of Moisture Flow and Mechanical Equilibrium

As mentioned earlier, to simulate as closely as possible the true field situation, the analysis was stepped through time in shorter periods to the transient flow for wetting or drying of subgrade. Then, for the moisture conditions at the end of this short period, the analysis incorporated swelling or shrinkage strain as a result of increase or decrease in moisture content in terms of temperature over the period and corresponding mechanical analysis of induced thermal forces and displacements. Thus, in this way the solution steps through time as series of alternating analysis of thermal transients and stresses and strains. At each interval between mechanical analysis, the soil stiffness and strength parameters were adjusted according to the new values of soil suction potential and normal stress level. This was accomplished with the help of a subroutine called by the main program after each transient moisture flow. No attempt was made to carry out a series of iteration of mechanical analysis with modified stiffness and strength properties and moisture flow as a truly semi-coupled approach. It is assumed that the cumulative errors involved without this iteration were consistent and kept small by carefully choosing the length of the time increments as the analysis proceeded.

When solving the wetting and swelling phenomena together using a coupled approach with moisture flow and stress/strain analysis at the same time, it was found to take several hours of computer time to solve a simple example problem. This made the approach not suitable for further use. It was decided that the solution steps through time as a series of alternating analysis of thermal transients and elastic parameters. While conducting an alternate analysis, the length of flow path is assumed to be constant during the flow interval. However, since the total volume change is considered as less than 8 percent, it is expected that the error associated with the assumption of constant volume during the moisture diffusion is very small, of the order of less than 5 percent. The basis of this assumption has been discussed in Section 2.3.

For mechanical analysis, the behaviour of subgrade was analysed assuming that the soil response could be characterised by an incremental linear elastic or elasto-plastic model with plastic failure criterion satisfied by the shear failure of Mohr-Coulomb

shear strength. The plastic strain rates in Mohr-Coulomb model were described by non-associative flow rule.

8.6 Inclusion of Interfaces in Pavement in Swell Analysis

In actual field condition, there exists an interface at the contact between pavement and subgrade layer. The strength and stiffness of this interface will be different than both the overlying pavement and underlying subgrade. Usually the interface shear strength should be lower than the shear strength of overlying and underlying layers. Because of the difference in strength and stiffness of the interfaces, it plays significant role in the transmission of forces and displacement from swelling or shrinking subgrade to the overlying pavement. With this reason, it is necessary to include the interface elements in the subgrade/pavement connection surface. On the other hand, it is equally important to apply accurate magnitude of interface strength and stiffness properties in order to obtain a reliable result on the effect of interface. But it is difficult to assess an accurate magnitude of this interface property and hence in most of the analyses, except in one case of swell analysis, the interface is not included. Figure 8.3 below shows the location of interfaces with a new geometry of grid applied in that case.

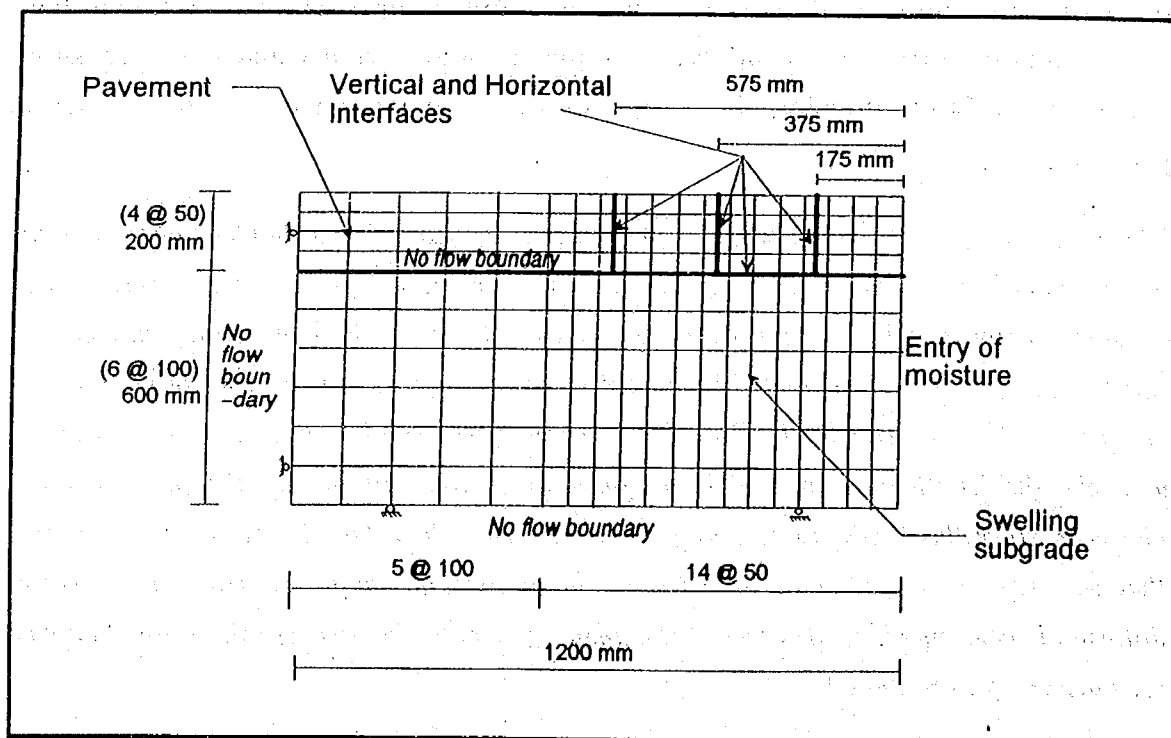


Figure 8.3 Location of interfaces to simulate possible crack opening in pavement in the case of subgrade swelling

Obviously, the swelling and shrinking of underlying subgrade were higher near the outer edge of pavement. This resulted more distress in overlying pavement near the outer edge relative to inner part. Consequently, development of pavement cracks were more likely to occur in the outer half ahead of inner one. Hence, three vertical interfaces in the pavement were located in a regular intervals on the outer half of the section and made through out the depth of pavement. The purpose of providing these interfaces was to simulate possible crack opening in the distressed and distorted pavement as a result of subgrade swell pressure and volume change.

Shear strength and stiffness properties of these vertical interfaces were taken the same as of pavement elements. The magnitudes of interface properties considered are: cohesion = 10 kPa, tensile strength = 10 kPa, and friction angle $\phi = 40^\circ$.

Horizontal interface was provided in between pavement and subgrade to account for the change in material properties and stiffness in subgrade due to the change in suction potential and stress level. But for the initial analysis, the effect of the horizontal interface between pavement and subgrade was eliminated by assuming a relatively high strength as:

cohesion = 200 kPa, tensile strength = 400 kPa, and friction angle $\phi = 30^\circ$.

According to the program, for these type of interfaces which have no effect on the stress/strain behaviour of the section, an exact value of interface stiffness was not very important. A good rule of thumb is to set the normal stiffness ' K_n ' and shear stiffness ' K_s ' not less than ten times the equivalent stiffness of the neighbouring elements. As such these stiffness values selected for the present case were $K_n = K_s = 5.6 \times 10^6$ kPa.

8.7 Review on Results of Subgrade Wetting/Drying

The analysis mostly used one dimensional moisture front which entered or exited from the right vertical edge of the subgrade as depicted in Figure 8.4. The subgrade section had no flow boundary condition on all other edges including the interface between the subgrade and pavement. After wetting or drying for instantaneous and for different periods, the contours of suction potential in the one dimensional horizontal flow condition, as would be expected, were just vertical lines. Right after the instantaneous wetting, moisture had not reached to a considerable length of section inside the outer edge. Similar was with the drying, the decrease in moisture of inside regions was almost negligible. Variation in moisture undertook in inner zones only after a considerable period of moisture flow.

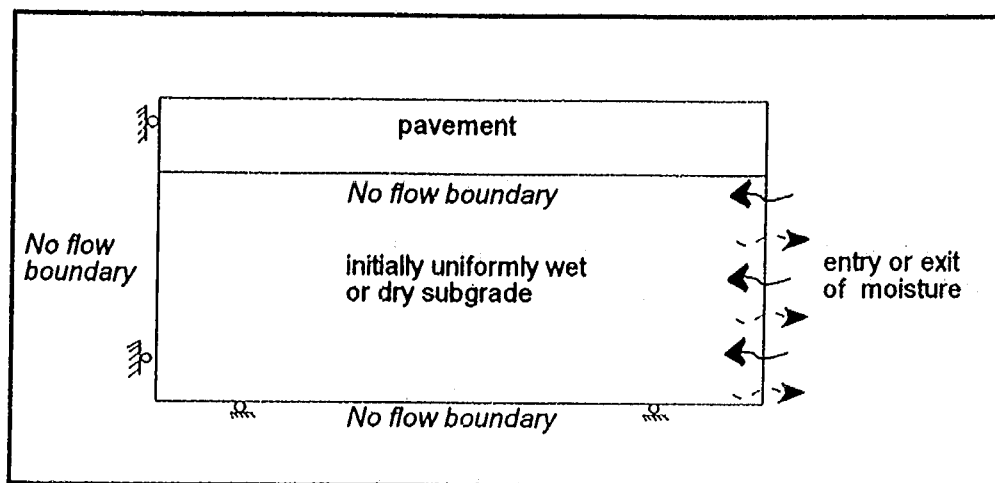


Figure 8.4 Typical section for 1-D flow profile

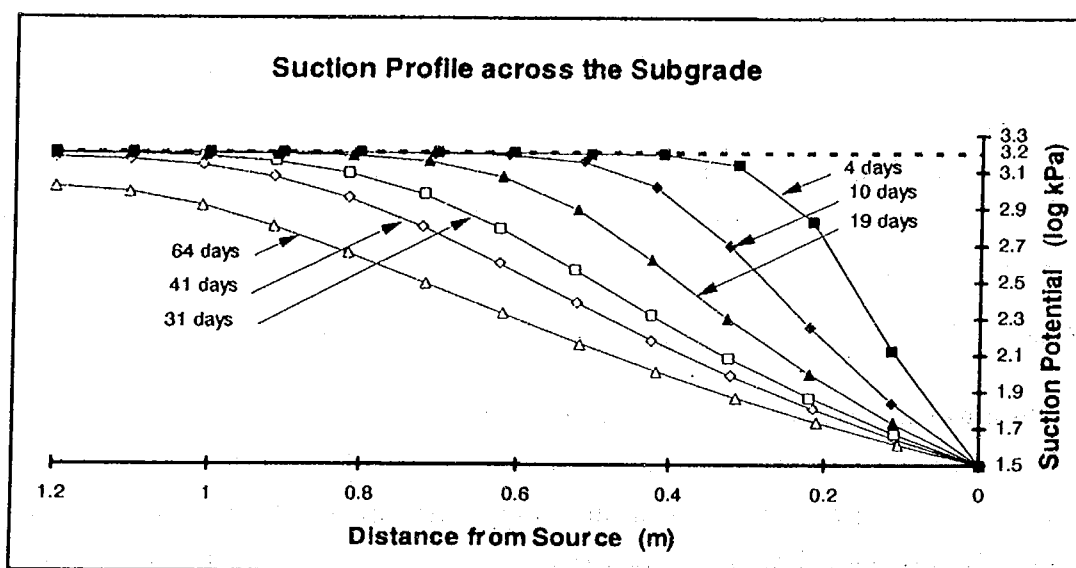


Figure 8.5 Suction profile after a continuous and different periods of wetting under one dimensional flow through the vertical outer edge of the subgrade

Figure 8.5 shows a typical wetting profile across the width of subgrade that was obtained after a continuous wetting for different periods ranging from 4 to 64 days. The wetting period of 64 days was considered to represent a 50 percent wetting of the section (Section 8.5.2). Similar was the drying profile which is not included here.

A typical case with two dimensional flow in wetting, where flow starts from the base of pervious shoulder (Figure 8.6a) was also studied. Figure 8.6b shows the contours of suction potential after 64 days of continuous wetting and swelling of subgrade. It illustrates how suction varies throughout the half section of overall subgrade. As would be expected, there existed a more highly wetted zone adjacent to the pervious shoulder.

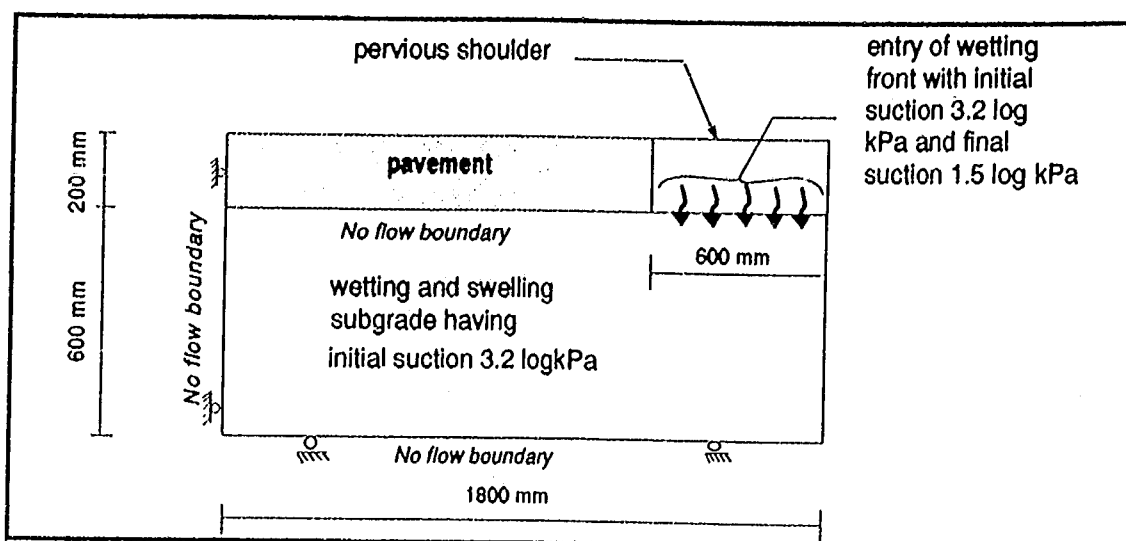


Figure 8.6 (a) A typical wetting situation with 2-D flow starting from underneath of the pervious shoulder

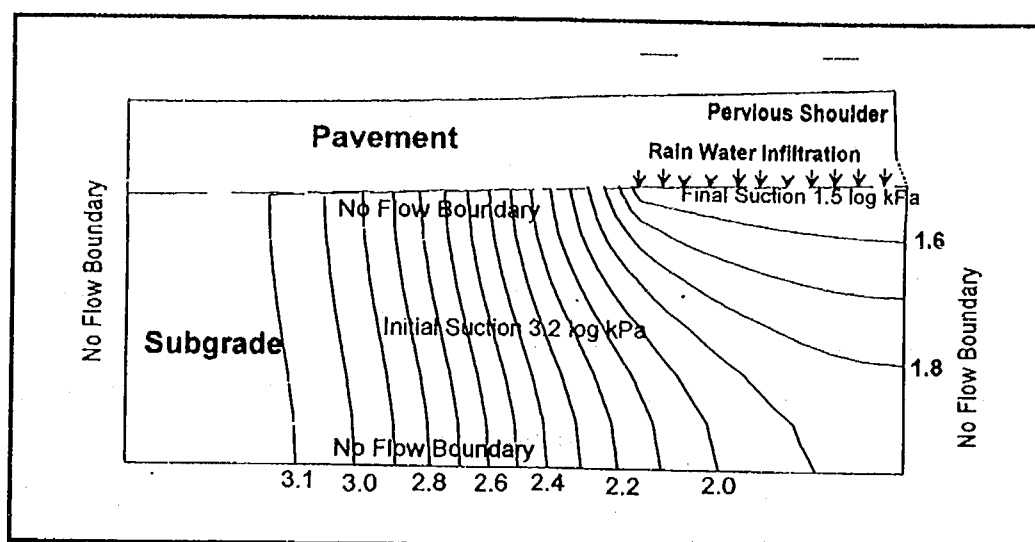


Figure 8-6 (b) Contours of suction potential after a continuous wetting for about 2 months

In both cases of wetting under 1-D as well under 2-D flow, even after continuous wetting of 64 days, a substantial part of the interior of subgrade was still dry and showed a suction potential of higher than 2.3 log kPa (a 50 % decrease value). Similar was the situation with drying: total suction of interior of subgrade was still close to the initial value even after drying of outer edge for 64 days.

By allowing swelling or shrinking of subgrade during the transient flow of moisture, as would be expected from field situation, the length of flow path should change, with an increase in path in swelling case and decrease in shrinking case. This should result to some differences in the value of suction potential (ie., temperature in the program) for the same node under different swell/shrink strain conditions.

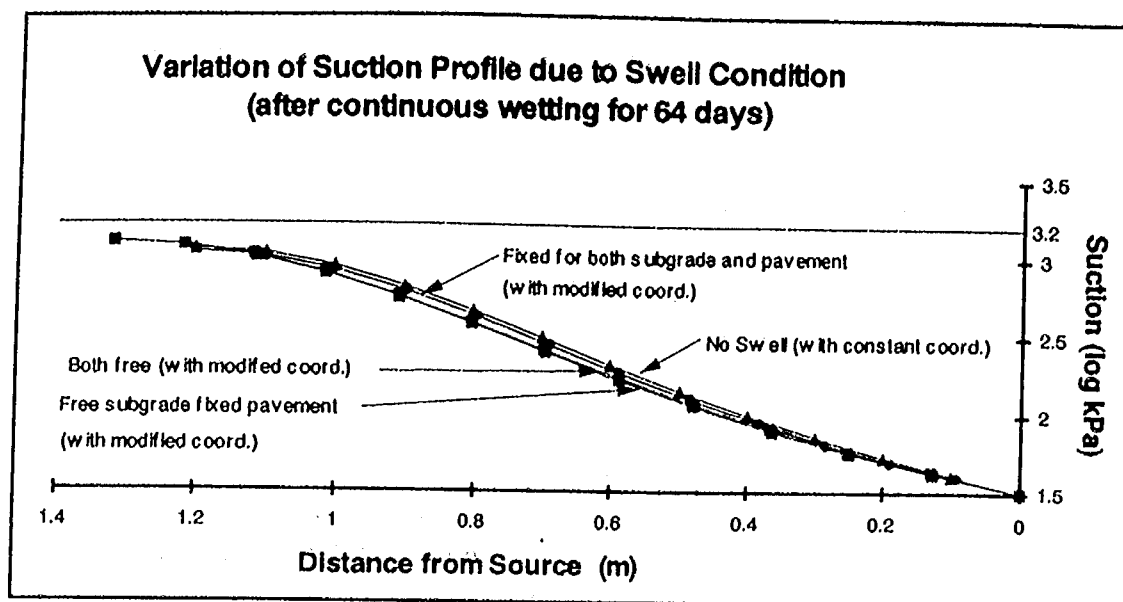


Figure 8.7 Suction profile under different swelling conditions

Figure 8.7 summarises the results obtained from the wetting up and swelling analysis conducted on subgrade having a very high swelling properties, ie., Case 2-

a $\begin{matrix} \gamma_w = 0.07 \\ \gamma_o = 0.04 \end{matrix}$, under one dimensional horizontal flow. Figure also reveals that, even with the modified coordinates of the section under large strain mode, very close suction profiles were observed with swelling under different lateral fixity. The maximum difference of suction at the mid section of a 1200 mm width of subgrade was found to be about 5 % higher in the case of no nodal coordinate changed condition than in nodal coordinate changed condition representing a free swell condition. This difference is small in sections close to source and remote boundary. It indicated that the error associated even with constant nodal coordinates would not make erroneous result. This would justify a similar argument made in Section 2.3, where in the derivation of flow equation a total volume was considered to normalise the volume of water. Similar was the situation with drying where the effect of length flow path was minimal in the overall drying process.

8.8 Summary of Analysis

The numerical analysis procedure discussed in Chapter VIII was applied to analyse a typical field situation idealised in Figure 8.1.

In the analysis, the wetting and drying of subgrade were studied under one dimensional horizontal moisture flow with a two dimensional moisture flow for one case of both wetting up and drying of subgrade. The coefficient of linear

thermal expansion ' α ' was obtained by multiplying with a factor $\left(\frac{1+\nu}{1-\nu} = 1.85\right)$ to the free swell parameter ' γ_h ' for swell analysis and to the free shrinkage parameter ' λ_h ' for shrinkage analysis.

Three different relations (Table 8.1) were used to describe Young's Modulus of swell suppression E_s (or G , K) during wetting and swelling. These relations were based on different sets of magnitudes of swell and swell suppression indexes. Similarly, another three different relations (Table 8.2) were used to describe Young's Modulus of shrinkage compression E_{sh} (or G , K) during drying and shrinking. These relations were based on different sets of magnitudes of free shrinkage and shrinkage compression indexes. Thus the study covered a wide range from medium to extremely high reactive clays.

In the analysis of wetting up and swelling of subgrade, the strength and stiffness at the initial state represented unwetted stiff clay under drained condition. Particularly, the magnitude of initial stiffness which is the load deformation modulus of unwetted soil is differentiated from the swell suppression modulus, where the former is at least ten times higher than the later. In the case of analysis of drying out and shrinking clay subgrade, the initial strength and stiffness belong to the wet state which were calculated by applying the relations of Table 8.2, and during the drying stages they were subsequently modified.

By applying gravity stresses before wetting or drying of the subgrade, the initial state of stresses representing field conditions was achieved (Section 8.5.1). In every mechanical calculation, the analysis was run until the converged unbalance force was less than 0.3 percent of initial unbalanced force.

To simulate the instantaneous wetting at the moisture entry boundary (right edge) of subgrade and the rapid initial swelling of nearby soils, the log suction at this boundary was decreased from 3.2 log kPa to 1.5 log kPa (T increased from 0.8 to 2.5) in 11 very short time steps. This covered an initial wetting period of about 1.1 days. Afterwards, an alternating analysis of wetting and swelling was carried out such that the solution stepped through time as a series of alternate analysis of thermal transients and stresses/strains. It was assumed that the error involved with alternate analysis would be consistent and kept small by carefully selecting the length of time increments. Similar procedure was followed for drying case with instantaneous drying of the outer edge to 3.2 log kPa in 11 very short time steps resulting 1.1 days, and later alternating analyses of drying and shrinkage.

The results at the end of two months wetting under one dimensional horizontal flow and swelling (Figure 8.7) showed that by allowing free displacement, and by changing the coordinates of nodal points in grids, the effect of increase in flow path in wetting pattern was small (maximum difference in the mid section was about five percent). The profile at the end of different periods (Figure 8.5) showed that even after a continuous wetting for two months, a substantial part in the subgrade interior was still relatively dry and had a suction potential greater than 2.3 log kPa. During the drying process also even after 64 days of continuous drying, inner zones of subgrade were still significantly wet.

The primary objective of this chapter was to apply the analysis procedure to a typical situation and study the initial result of moisture flow in order to ascertain the applicability of procedure and ability to achieve the correct order of magnitude of results. Review of results of analysis confirmed that the program satisfied the required flow conditions. The detailed parametric study on the results of numerical analysis will be presented in Chapter IX and X.

CHAPTER IX

NUMERICAL MODELLING: PARAMETRIC STUDIES ON THE EFFECTS OF SUBGRADE SWELLING

9.1 Introduction

In Chapter VIII, a typical section of road consisting of a thin seal coated granular pavement that is resting over a reactive clay subgrade (see Figure 8.1) was considered in the numerical analysis. The different combinations of swell parameters were selected in the analysis, and the different displacement boundary conditions were applied. The results of numerical analyses indicated that the development of heave and lateral swell pressure upon wetting of a reactive clay subgrade, and consequent response of overlying pavement depended on magnitudes of several soil parameters and boundary conditions. In this chapter, the effects of variations of these soil parameters and boundary conditions are discussed.

9.2 Characteristics of Program Output and Cases Analysed

Before presenting a detailed parametric study of the result of analyses, a few points that need to be understood are discussed in this section. At first, notations followed by the program and how tension failure/cut-off indicated in program output are discussed. Later a table describing the details of initial conditions of different cases analysed and an illustration figure describing the location of elements and zones are also presented. These serves as a guidance to study the results presented in the remaining sections in this Chapter and also in Chapter X.

9.2.1 FLAC's Notations and Concept of Tension Failure

While interpreting results obtained from the computer program 'FLAC' for a parametric study, it is necessary to understand a few minor details on the analysis rule followed by the program. FLAC considers the following axial notations, and sign conventions:

FLAC handles the tension failure of a Mohr-Coulomb material in the following way:

If the major principal normal stress (σ_1) exceeds the tensile strength 'T', the latter is cut to zero, and all the three principal stresses (in tension) are set to

zero. In this case, the zone was considered to have failed in tension, and hence unable to sustain further tensile stresses.

If the intermediate principal stress (σ_2) and/or the minor principal stress (σ_3) exceed/s the tensile strength 'T', then the tensile strength is cut to zero and the corresponding stress set to zero.

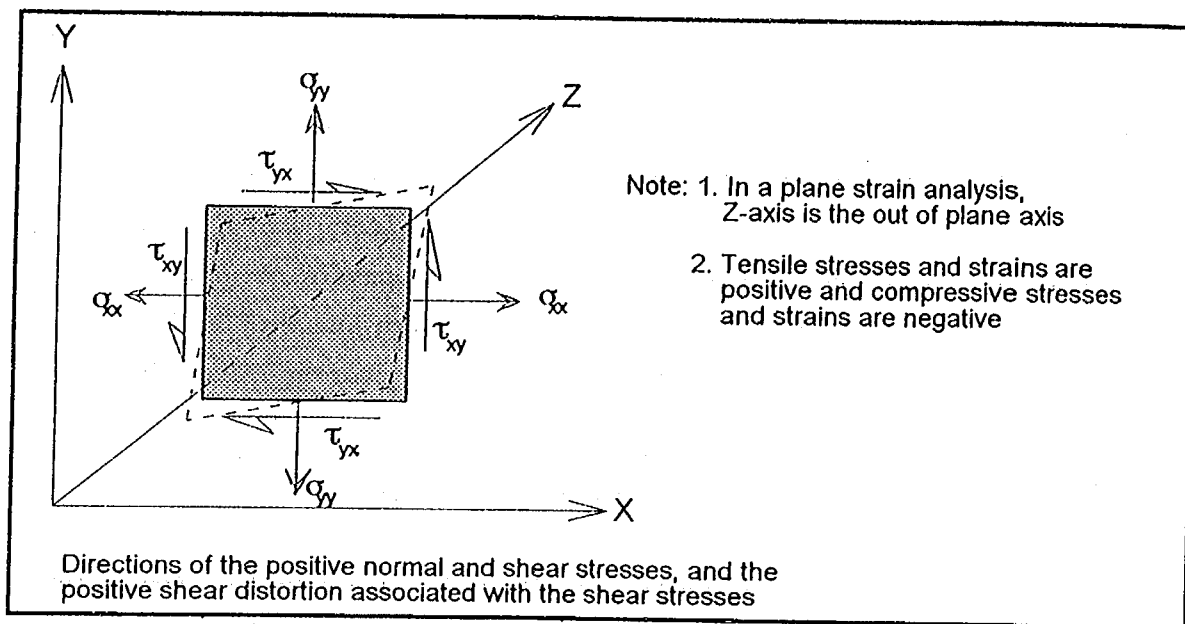


Figure 9.1 FLAC's axial notations, and sign conventions for stresses and strains

This concept of tension failure/cut-off was applied in the analysis when the pavement layer is modelled with Mohr-Coulomb material having a limited tensile strength 'T' of 10 kPa. This magnitude of 10 kPa as tensile strength was lower than the plastic apex limit of the model (which was $C \cot \phi$, where cohesion 'C' = 10 kPa, and friction angle ' ϕ ' = 40°).

When the major principal stress (σ_1) exceeds the tensile strength (10 kPa), the section failed in either general tension or uniaxial tension depending upon the nature of the minor principal stress. If only the minor principal stress (σ_3) exceeded the tensile strength (10 kPa), then that stress was set to zero and the tensile strength of the element became zero. However, the element could still remain in an elastic state and the program redistributed the stresses in the surrounding elements. This often resulted with the element becoming back under compression in minor principal stress when the pavement section was subjected to the new values of stresses coming from the swelling subgrade. The tension failure and yielding of any element in the section were examined by checking the plastic state of the system.

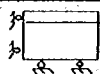
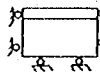
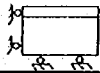
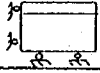
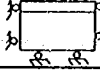
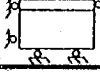
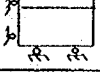
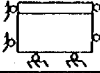
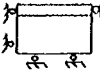
Tension cut-off was examined by checking the tensile strength at any particular stage, using the program.

Meanwhile, it is important to note that FLAC's output of stress analysis in general comes in terms of axial stress values in Cartesian co-ordinates such as σ_{xx} as lateral normal stress, σ_{yy} as vertical normal stress and σ_{zz} as out of plane normal stress. Thus obtained output values of σ_{xx} , σ_{yy} and σ_{zz} may not belong to the major or minor principal stresses. Therefore an element may undergo tension failure/cut off even though the value of any or all of these σ_{xx} , σ_{yy} and σ_{zz} are only close to tensile strength of the section.

9.2.2 Cases Analysed

Table 9.1 presents a guide to the different cases of analyses with respect to lateral fixity, magnitude of swell and swell suppression parameters. These references are applied to all cases of analysis of material models used for pavement and subgrade, as well as for all types of flow conditions.

Table 9.1 References for different cases of analysis

Case	Lateral fixity	Magnitude of swell parameters	Remarks
1-A		$\gamma_h = 0.035, \gamma_\sigma = 0.04$	High swelling
1-B		$\gamma_h = 0.035, \gamma_\sigma = 0.04$	High swelling
1-C		$\gamma_h = 0.035, \gamma_\sigma = 0.04$	High swelling
2-A		$\gamma_h = 0.07, \gamma_\sigma = 0.04$	Very high swelling
2-B		$\gamma_h = 0.07, \gamma_\sigma = 0.04$	Very high swelling
2-C		$\gamma_h = 0.07, \gamma_\sigma = 0.04$	Very high swelling
3-A		$\gamma_h = 0.07, \gamma_\sigma = 0.08$	Very high swelling and swell suppression
3-B		$\gamma_h = 0.07, \gamma_\sigma = 0.08$	Very high swelling and swell suppression
3-C		$\gamma_h = 0.07, \gamma_\sigma = 0.08$	Very high swelling and swell suppression

Similarly, Figure 9.1 presents a reference diagram for the locations of elements and zones in the section that are referenced frequently in the following sections in this Chapter as well as in Chapter X.

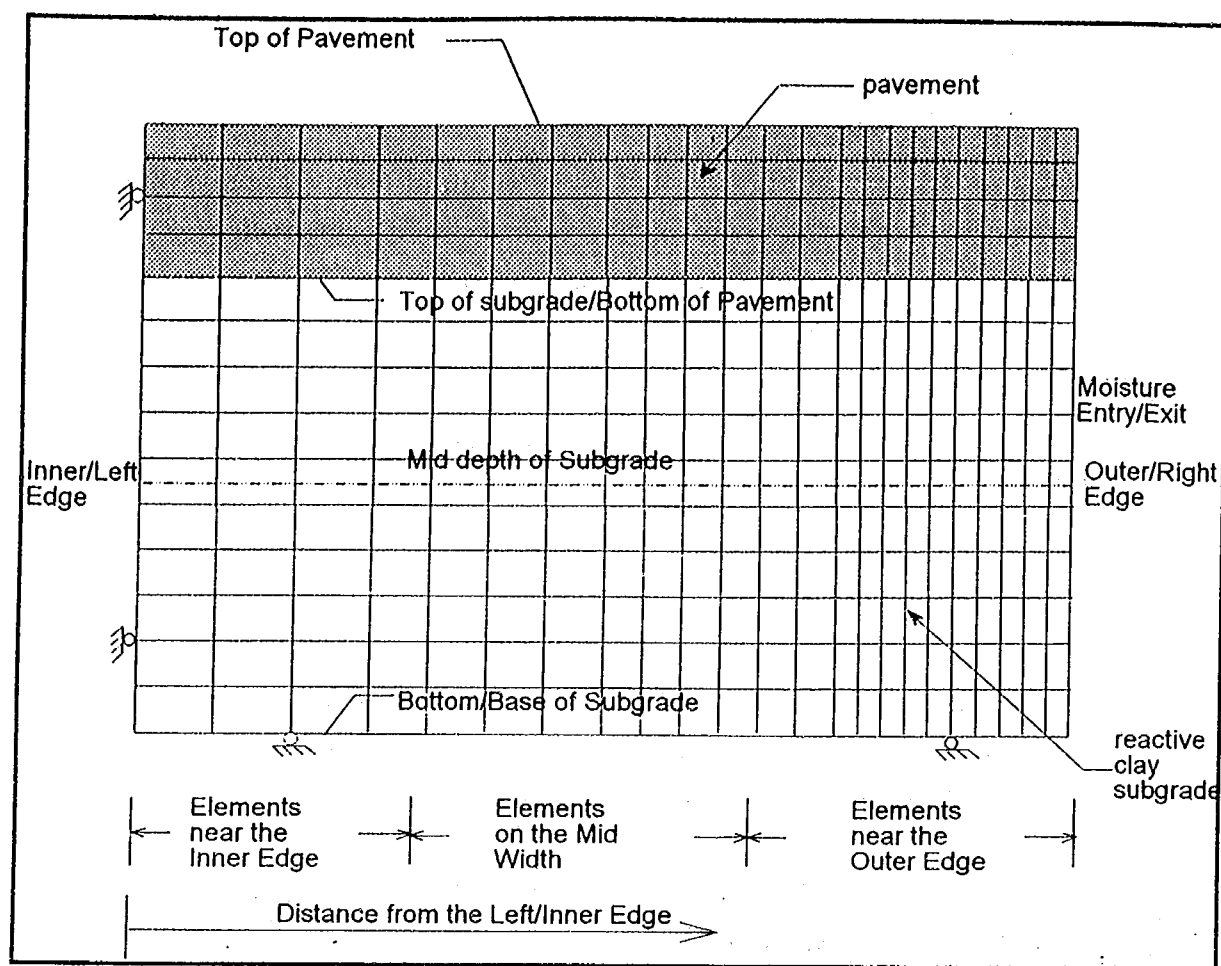


Figure 9.2 Reference diagram for location of elements and zones

9.3 Parametric Study

The comprehensive results of numerical analysis on subgrade wetting up and swelling undertaken in Chapter VIII are applied in the present parametric study. A review of results with particular emphasis on the development of stresses and displacements in both pavement and subgrade sections revealed that the stress/strain response of subgrade and overlying pavement is sensitive to the following parameters.

1. Period of wetting
2. Restraint against lateral displacement
3. Magnitude of swell and swell suppression parameters
4. Wetting pattern in subgrade

9.3.1 Wetting Period

The wetting of subgrade was modelled under horizontal one dimensional flow. As shown in Figure 8.4, moisture enters through the right edge of the subgrade section.

Figure 8.5 illustrates the typical suction profile obtained from the analysis indicating the progressive wetting of the subgrade section. The following analysis is based on this wetting profile.

A. Subgrade Response

The strength and stiffness of subgrade soil depended largely on the degree of wetting. With increasing wetness, the decrease in modulus of swell suppression followed the pattern described by Figure 7.3. As discussed in Section 8.2.2 decrease in Mohr-Coulomb strength parameters followed linearly with the increasing wetness.

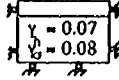
To study the effect of wetting period, results of analysis of 'Case 3-B'  are considered. Case 3-B physically represents a situation where an extremely expansive clay subgrade undergoes a monotonic wetting up and swelling in a laterally confined boundary conditions.

Figure 9.3 shows the progressive development of lateral normal stress σ_{xx} along the X direction at the mid-depth of the subgrade section after different periods of wetting: after instantaneous wetting of boundary, and after continued wetting for a period of 4, 10, 19, 31, 41 and 64 days. Because of the lateral fixity on both edges, the lateral stress σ_{xx} that developed during wetting, and experienced by all elements in the section, were compressive. The stress development was progressive from the outer edge where moisture entered and the subgrade started to swell.

As seen in Figure 9.3, even after an instantaneous wetting of the outer edge, significant magnitude (about 16 kPa) of σ_{xx} was developed on the outer edge elements and transmitted to unwetted inner zones. The constant magnitude of σ_{xx} (about 11 kPa) in the inner zone was the response of the elements still possessing initial stiffness. The constant magnitude of σ_{xx} developed in the inner elements were obtained until about 10 days of modelled wetting when these inner elements were still unwetted. After about 31 days (considered as one month here in after), all the elements were wetted to certain degree. The maximum value σ_{xx} then became about 75 kPa at the outer most element and slowly decreasing inwards to a value of around 60 kPa on middle of the inner half. The slight decrease in σ_{xx} on the middle elements were apparently because of less wetting but significant decrease in stiffness. With further wetting to 41 days and 64 days the development of σ_{xx} were relatively consistent. At 64 days (considered as two months here in after) of wetting, an almost constant magnitude of σ_{xx} (about 68 kPa) was developed in all the elements.

Note that some of the subgrade elements near the source of moisture started yielding at an early stage of wetting. This yielding phenomenon resulted from the reduced strength and stiffness experienced by the wetted subgrade elements. These elements were at the same time subjected to greater swelling pressure because the inner elements next to them were relatively dry and stiff. However, such elements previously at yield, came back to an elastic state when the wetting front advanced further and wet up inner zones such that the inner ones also had lower strength and stiffness. The effect of such initial yielding was found to be insignificant when subgrade mechanical responses were considered.

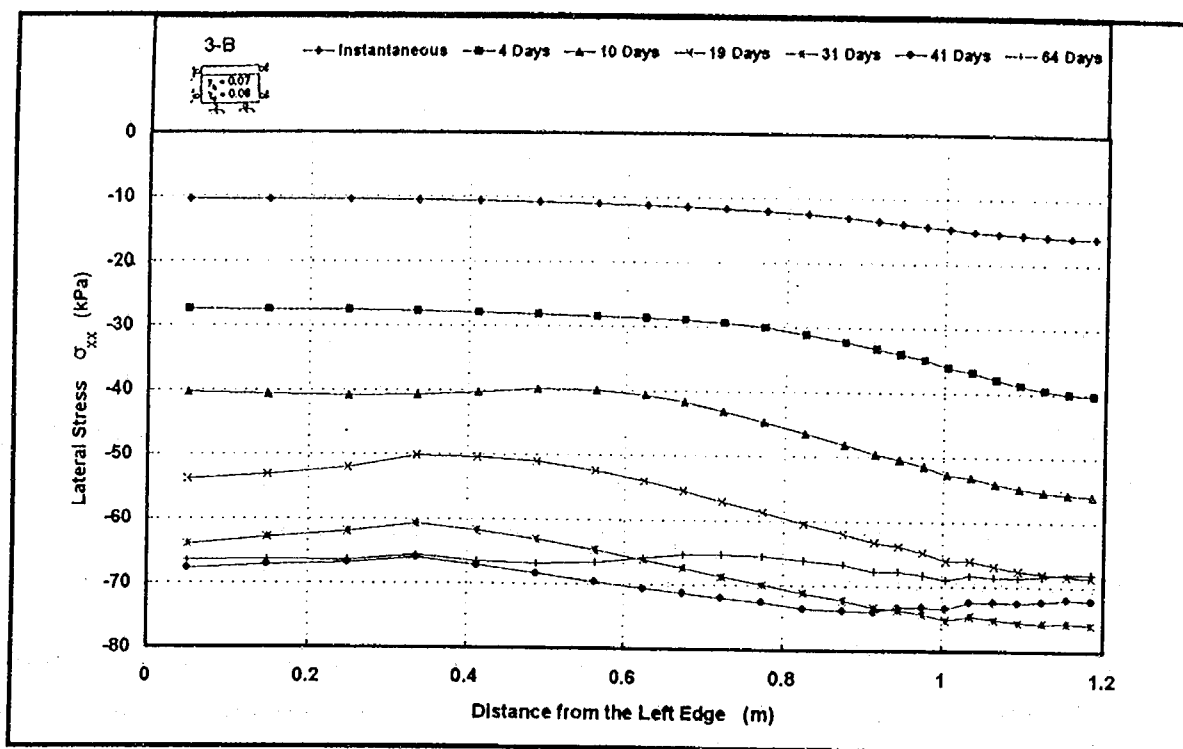


Figure 9.3 Development of lateral stress σ_{xx} at the mid depth of subgrade section.

Results in Figure 9.3 also indicate that in the wetted zones of the outer half, the magnitude of the lateral compressive stress developed decreased with the increase in wetting period after one month. This was attributed to the reduced stiffness of these elements. For an example Young's modulus of an outermost element at the mid depth of subgrade decreased from an initial 5000 kPa at the unwetted state to 895 kPa after one month, 880 kPa after 41 days and further dropped to about 828 kPa after two months of wetting.

Figure 9.4. shows the development of vertical stress σ_{yy} , along the mid-depth of the subgrade. As the wetting front advanced, the location of change in nature of stress from compression to tension and back into compression, moved inwards. The peak

magnitude of both compressive and tensile stresses decreased with an increase in the wetting period. The significantly higher change in magnitude of these stresses in earlier stages of wetting was basically due to the higher differences in stiffness between the wetted and unwetted zones. The effect was pronounced at the wetting periods less than one month. After a month when almost all elements were wetted to some degree, development of σ_{yy} was consistent in the outer zones. Similar to lateral stress, magnitude of σ_{yy} in the wetted zones decreased with an increase in wetting period. Further, having a free vertical displacement boundary at the top surface of the pavement, the vertical stress get released as the surface heave develops. As such, the vertical stress in all elements after more than two months wetting would reach the initial overburden pressure (8.45 kPa in compression).

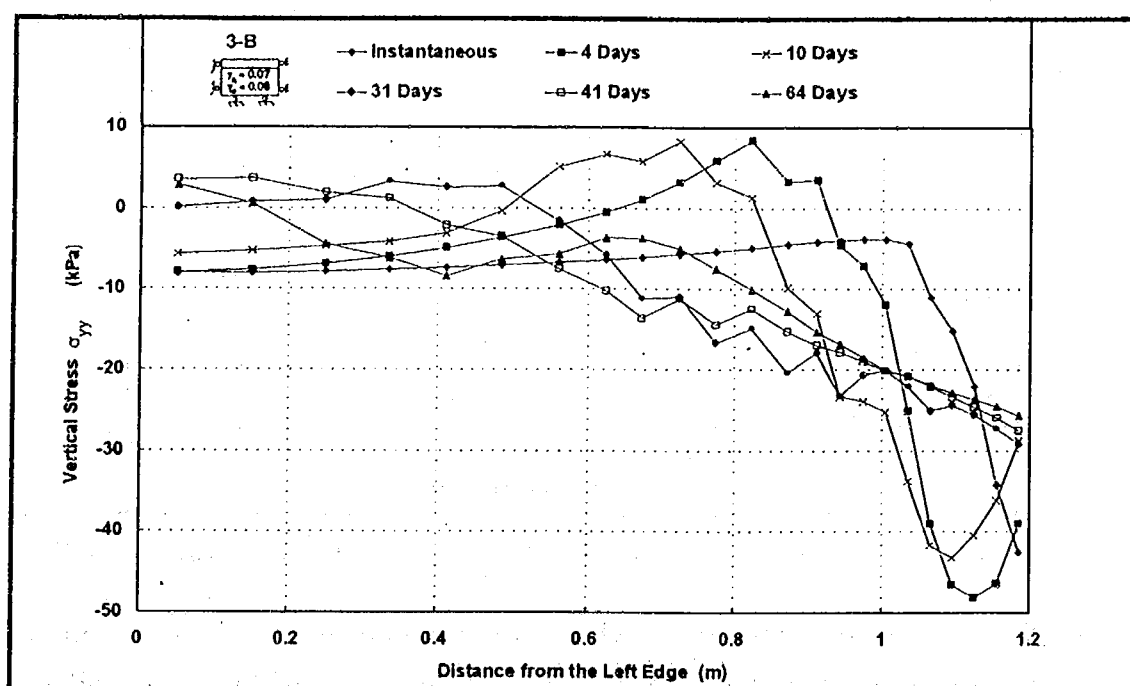


Figure 9.4 Development of vertical stress σ_{yy} at the mid depth of subgrade section.

The development of tensile vertical stress in Figure 9.4 in the inner elements was due to the lifting effect of the distorted pavement as shown in Figure 9.8b. This is because in the analysis the pavement and subgrade were considered as a composite section without any interface elements. The higher value of σ_{yy} at early stage of wetting (up to about 10 kPa observed after 4 days of wetting) was the result of higher stiffness of inner elements as discussed above.

B. Response of Pavement

Figure 9.5 shows the progressive development of heave on the overlying pavement surface as a result of subgrade swelling. As expected, surface heave increased consistently with the period of wetting, because more and more inner elements

wetted up to higher degrees and heave progressively advanced inwards. For the considered situation (Case 3-B $\gamma_w = 0.07$, $\gamma_s = 0.08$) after two months of wetting, the surface heave reached to about 85 mm over the 100 mm width near the outer edge. The extent and magnitude of heave, however, largely depended on the lateral fixity and the values of swell parameters which will be discussed in the latter sections.

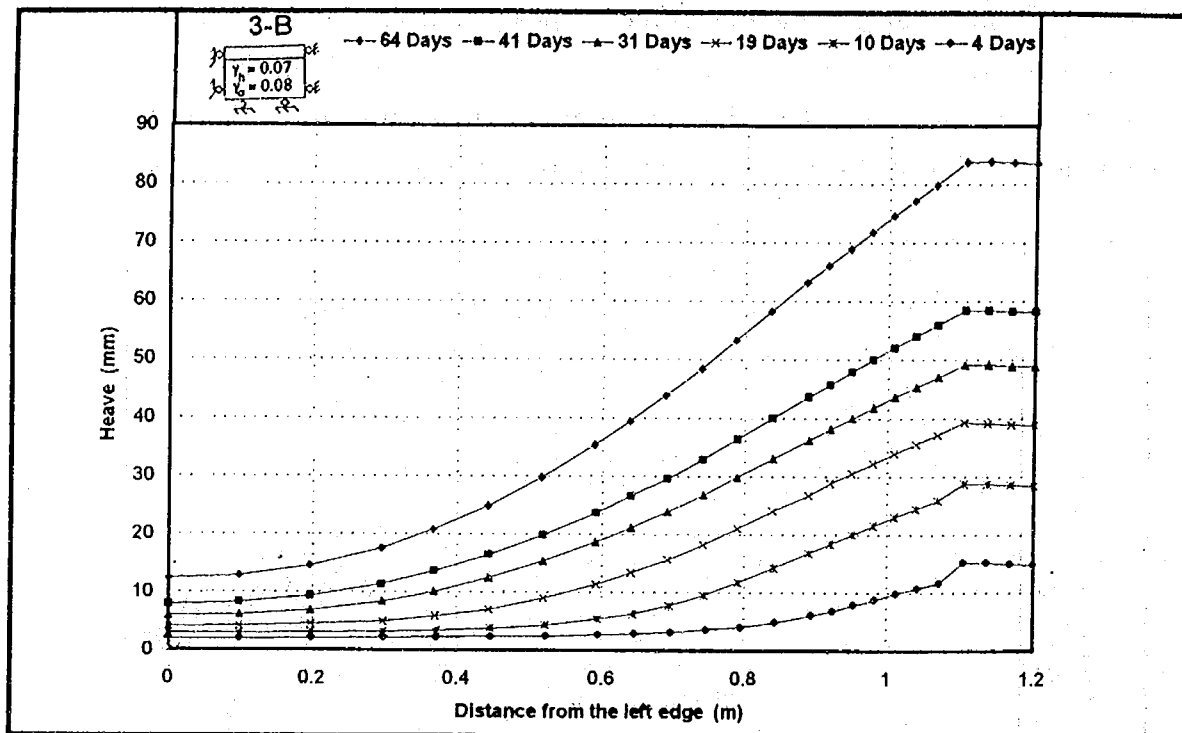


Figure 9.5 Development of pavement surface heave with the wetting period

Figures 9.6 a and b illustrate the development of lateral stress σ_{xx} on the top and bottom faces of pavement. Since the mechanical behaviour of the pavement was modelled with the Mohr-Coulomb material having limited tensile strength ($T=10$ kPa), elements subjected to tensile stress over 10 kPa underwent uniaxial or general tension failure/tension cut-off. Thus the value of σ_{xx} when it exceeded 10 kPa in tension dropped to zero, and therefore the resulting values of σ_{xx} in tension in these figures are always less than 10 kPa.

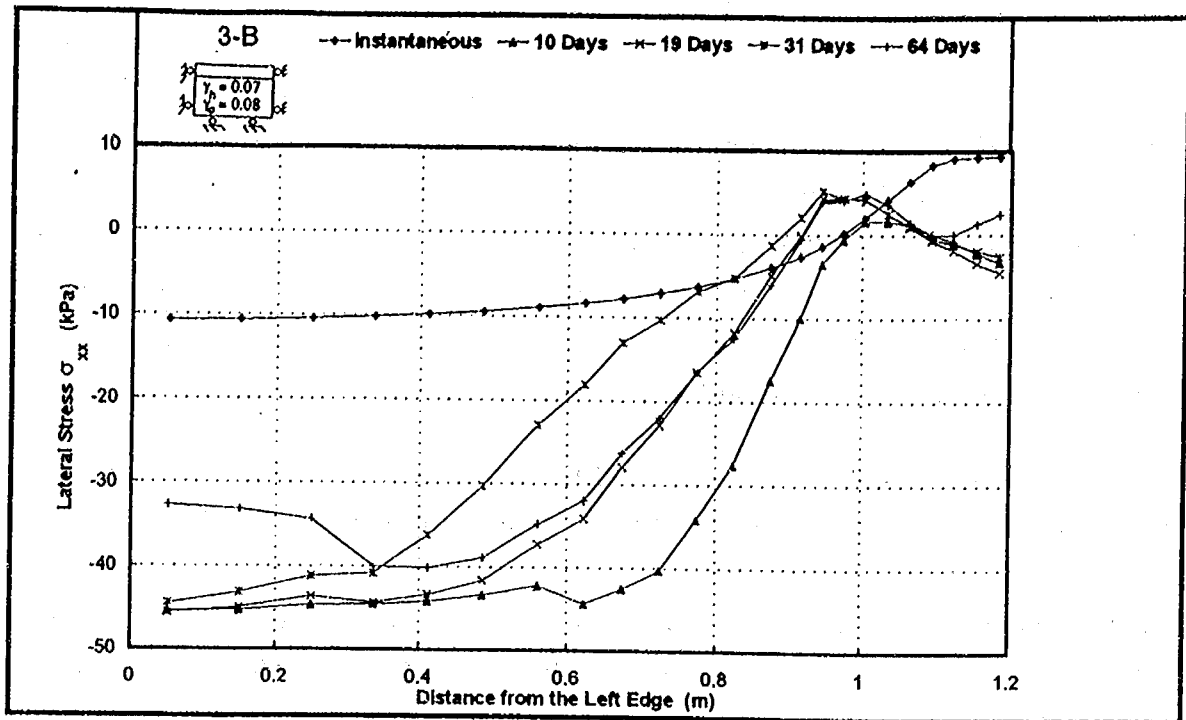
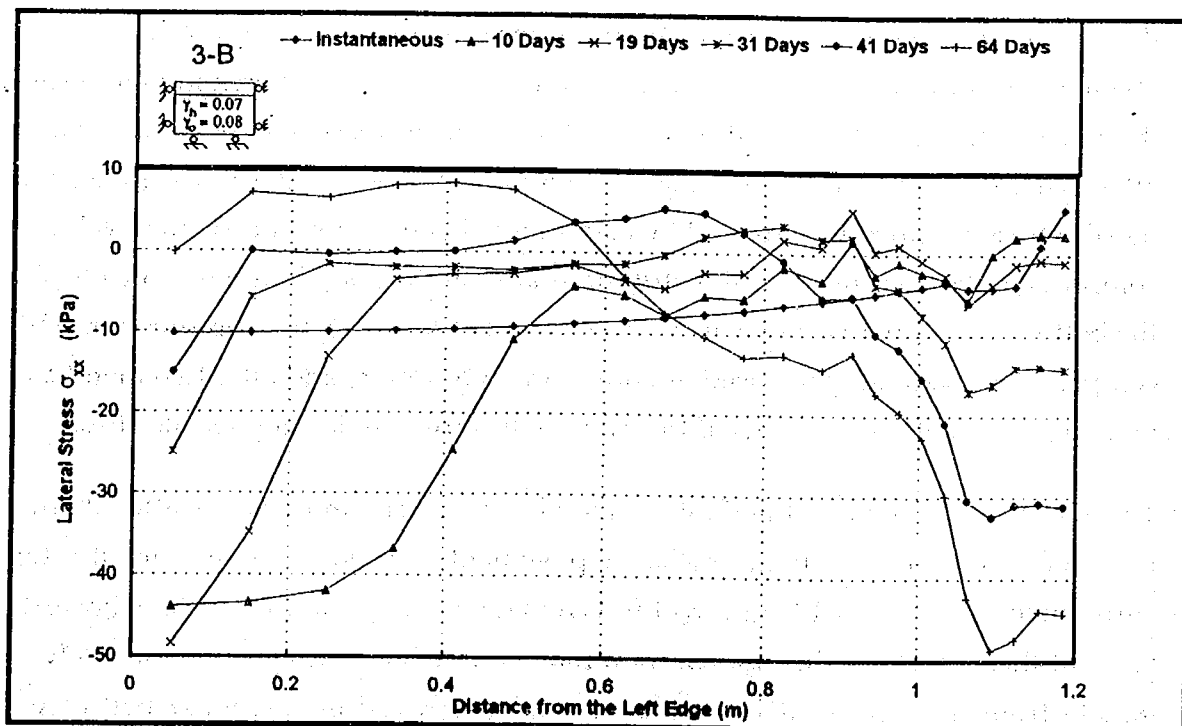

 (a) Lateral stress σ_{xx} on the top face of pavement

 (b) Lateral stress σ_{xx} on the bottom face of pavement

 Figure 9.6 Lateral stress σ_{xx} on the pavement at the different stages of subgrade wetting

An important point which needs to be considered in analysing the development of lateral stress in the pavement section, is the role of the interface between pavement

and subgrade. The degree of transmission of lateral stresses developed in the swelling subgrade, to the overlying pavement, depends largely on the shear strength and stiffness of the interface elements. In the illustrated case (as well as in most of the cases of analysis), this interface was not treated separately. Hence in the absence of an interface the overall section behaved as a composite section and the transmission of the effect from subgrade to pavement layer depended on the relative stiffness of the pavement and subgrade, and the lateral fixity. Further discussion on the role of this stiffness is presented in the succeeding section.

On both top and bottom faces, the nature of progressive development of lateral stress σ_{xx} was consistent with the advancement of wetting front. Initial compressive stress in the inner zones gradually decreased, and the development of tensile stress advanced inwards. Particularly at the top surface, tensile failure/cut-off moved inwards as a result of lateral fixity. Due to this fixity, the distorted shape of the pavement section was a combination of concave and convex curvature as shown in Figure 9.8b in Section 9.3.2. With such distortion, most of the inner elements on top face were under compression at all stages of wetting, while elements at and near the surface on the outer 100 mm width underwent tension failure/cut-off from about 4 days of wetting (which is not shown in the figure). As shown in Figure 9.6 a, the tensile strength close to 10 kPa was developed in these element immediately after instantaneous wetting of the source boundary. After 10 days of wetting, this tensile stress dropped to zero, and instead these elements developed small value of compression because of stress redistribution among the neighbouring elements. On the bottom face, inner elements initially were subjected to higher compression. This compression gradually decreased as shown in Figure 9.6 b, and after two months of wetting, these bottom elements of the inner half started to develop tensile stress.

The above results indicate that in this modelled situation, immediately after about a week of continuous wetting, cracks in pavement start to develop from the top surface within the outer 100 mm width. When the wetting front progresses inwards, some of these cracks in the outer section may close while new crack would likely to initiate from the pavement base and propagate upward on the inner half of the section.

9.3.2 Effect of Lateral Restraint

In this study, for all cases of analyses, it was assumed that left vertical face belongs to either the centre line of the road or the section beyond which there is no significant change in total suction. This indicates that the left boundary would

remain fixed against lateral displacement. The right vertical face was considered to behave in three different manners representing extreme conditions of lateral fixity as described in Section 7.2.3 B.

- | | | |
|----------|---|---|
| Case A - |  | free lateral displacement on the right edge for both pavement and subgrade sections, |
| Case B - |  | restrained against lateral displacement on both the edges for both pavement and subgrade, and |
| Case C - |  | restrained against lateral displacement on both the edges for pavement layer but free for subgrade at the right edge. |

These are the extreme conditions of lateral fixity as described in Section 7.2.3 B and illustrated in Figure 7.2. In general field situation, the remaining section outside the right edge of analysed block usually provide partial lateral support. However, as discussed in Section 7.2.3 B, it is extremely complicated to understand and estimate the degree of such partial lateral fixity. Further difficulty arises in applying this complicated partial fixity into numerical analysis. Therefore analyses had been carried out with these extreme conditions. A parametric study is conducted in this section to understand the effect of such lateral fixities.

Obviously the development of heave and lateral swell pressure depends largely on the displacement boundary conditions of the sides. To study the extent of effect due to different degree of lateral fixities, a typical situation with subgrade soil having magnitude of swell parameters $\gamma_h = 0.07$ and $\gamma_\sigma = 0.08$ representing an extremely high swelling clay is considered.

Before going into further detail on the development of lateral stress and strain in the pavement due to the transmission of force and displacement due to wetting and swelling of the subgrade, it would be necessary to understand the role of the interface on the transmission phenomenon.

A. Treatment of Interface between Subgrade and Pavement

As discussed briefly in Section 9.3.1, the transmission of stresses and strains in swelling subgrade to the overlying pavement depends largely on the role of the interface between them. However in the following case illustrations, this interface is not treated separately, i.e. no interface elements were included to represent it. In the absence of interface elements the transmission of stresses and strains from subgrade elements to the connecting pavement elements depends primarily on the

difference in stiffness of the pavement and the subgrade. The degree of lateral fixity in both subgrade and pavement sections is also important. Under three different cases of lateral fixity: Cases A, B, and C as explained previously, the following situations of transmission of lateral stresses and strains can occur in the elastic state.

- (i) For a free lateral displacement boundary condition on the right edge of both

subgrade and pavement section, as in Case 'As' , the transmission of force and displacement from the swelling subgrade to the overlying pavement is likely to occur in a manner described in the schematic diagram (a) of Figure 9.7. When the stiffness of pavement is significantly higher than the stiffness of the connecting subgrade, the nodes that are shared between the subgrade and pavement will have lower lateral displacement than the nodes just below which are within the subgrade section. On the other hand, the nodes just above within the pavement will have lower lateral displacement than the shared nodes. Moreover, when the subgrade under free lateral boundary conditions extends laterally, it drags the connecting pavement together. This results in the development of higher lateral tensile and shear stresses in the pavement which has higher stiffness. Conversely, the connecting subgrade elements develop lateral stresses in compression as a result of the restraining effect of pavement when the pavement elements undergo smaller displacements because of their higher stiffness.

- (ii) When both pavement and subgrade sections are fixed against lateral

displacement as in Case 'Bs' , the swelling elements of subgrade that are connected to the overlying pavement will develop an inward displacement as shown in diagram B of Figure 9.7. In this case the nodes shared between pavement and subgrade also move inwards. This results in a dragging effect in the connecting pavement elements and leads to the development of tensile or compressive stresses depending on the location of element across the width of the section (near the outer edge or near the inner edge).

On the other hand the pavement elements which possess higher stiffness will develop lower magnitude of tensile displacement as compared to the subgrade elements. Such lower displacement leads to the development of compressive stress in the connecting subgrade elements.

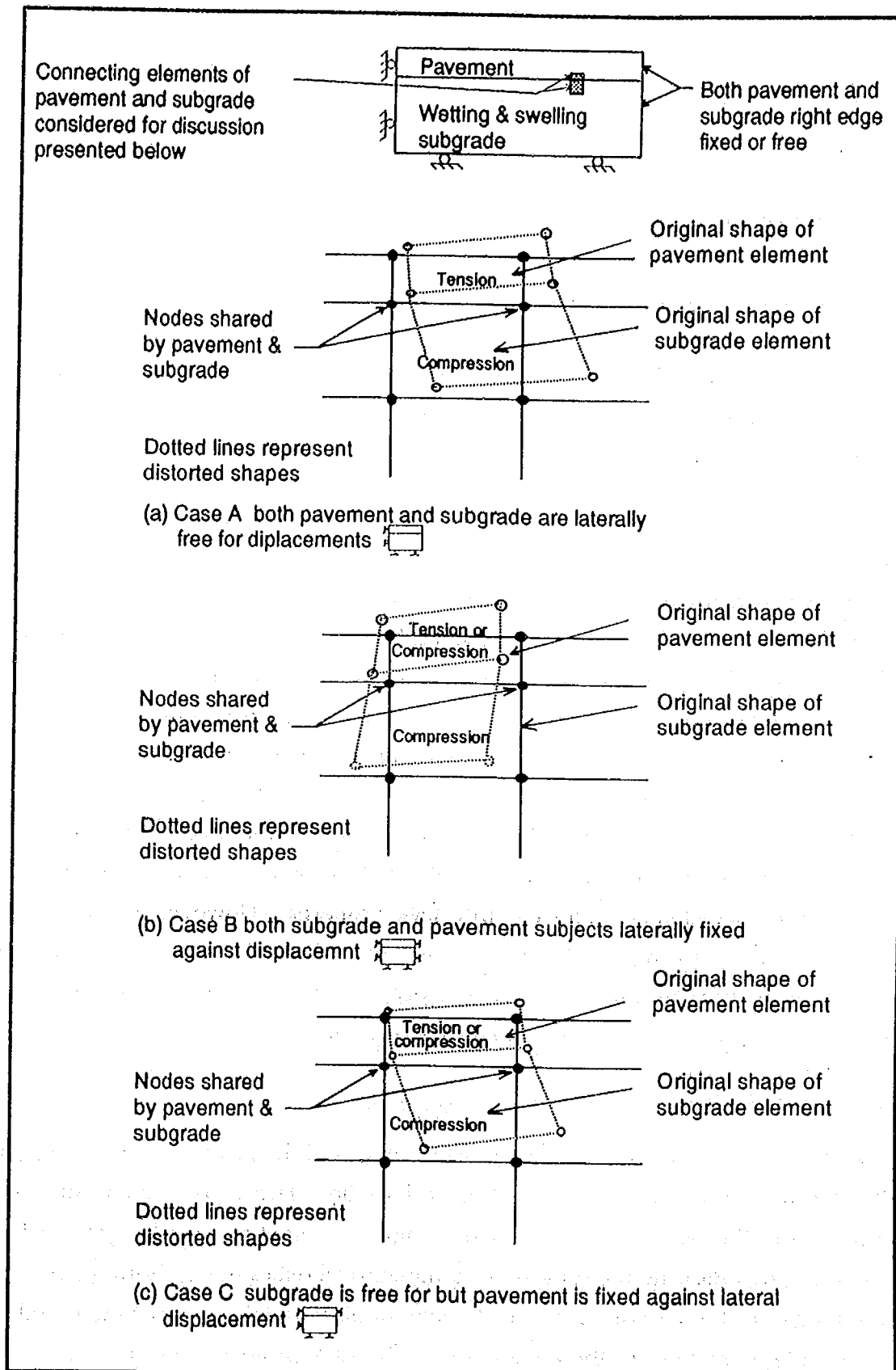


Figure 9.7 Transmission of stresses/strains between connecting elements of subgrade and pavement sections with no interface elements and interface properties

- (iii) When the pavement section is fixed while the subgrade section is free for lateral displacement on the right edge as in Case 'Cs' , the swelling subgrade elements will develop outward displacement as shown in diagram C of Figure 9.7. This outward displacement of the subgrade results in dragging of the connecting overlying pavement elements. But since the pavement section has lateral fixity on both the inner and outer edges, compression occurs on the outer elements and tension on the inner elements which are connecting to the subgrade. Consequently, as discussed in the earlier cases, there develops lateral stress with higher compression on the outer elements and relatively lower compression on the inner elements of subgrade as a result of the restraining effect of pavement.

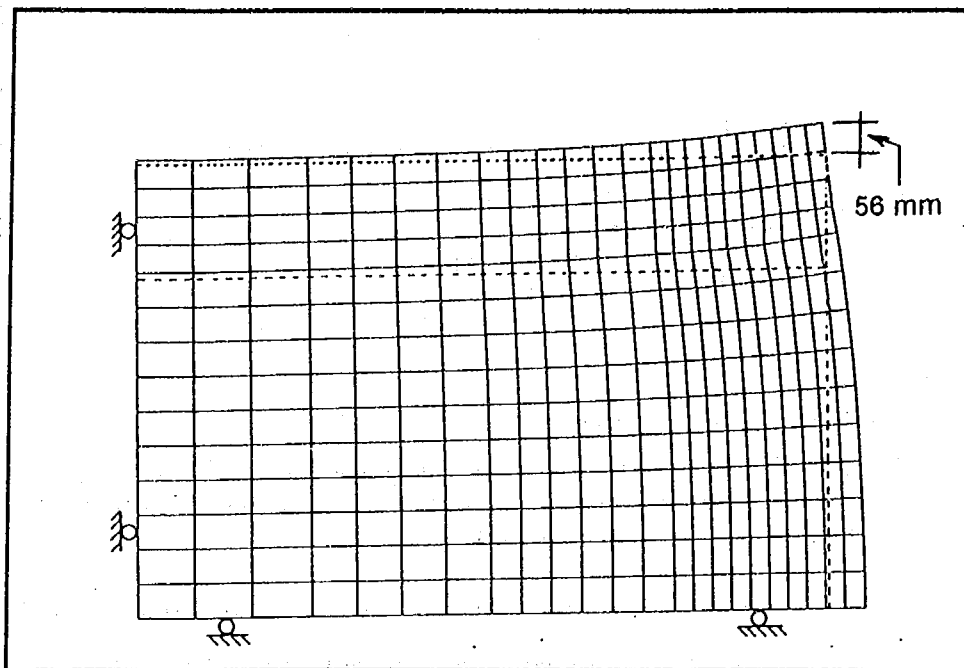
The above discussion is based on the lateral swelling stress developed in the subgrade elements, and its transmission to the overlying connecting pavement element. Meanwhile the swelling subgrade also develops vertical displacement in the form of heave. This is ultimately transmitted to the overlying pavement, and leads to the bending of the pavement as a result of differential heave between the outer and inner elements of subgrade. The nature of development of flexural stresses whether compression or tension depends on the mode of bending which is the function of lateral fixity. Thus, when combined together the overall transmission of stresses and strains of subgrade to the overlying pavement becomes more complex. In such situation it would be necessary to apply properly described interface elements and use interface properties. However, in this study, the analyses are performed without inclusion of interface because of difficulty in assessing proper magnitude of interface properties. It is suggested for future research to conduct analysis with inclusion of interface.

B. Distorted Section and Pavement Surface Heave

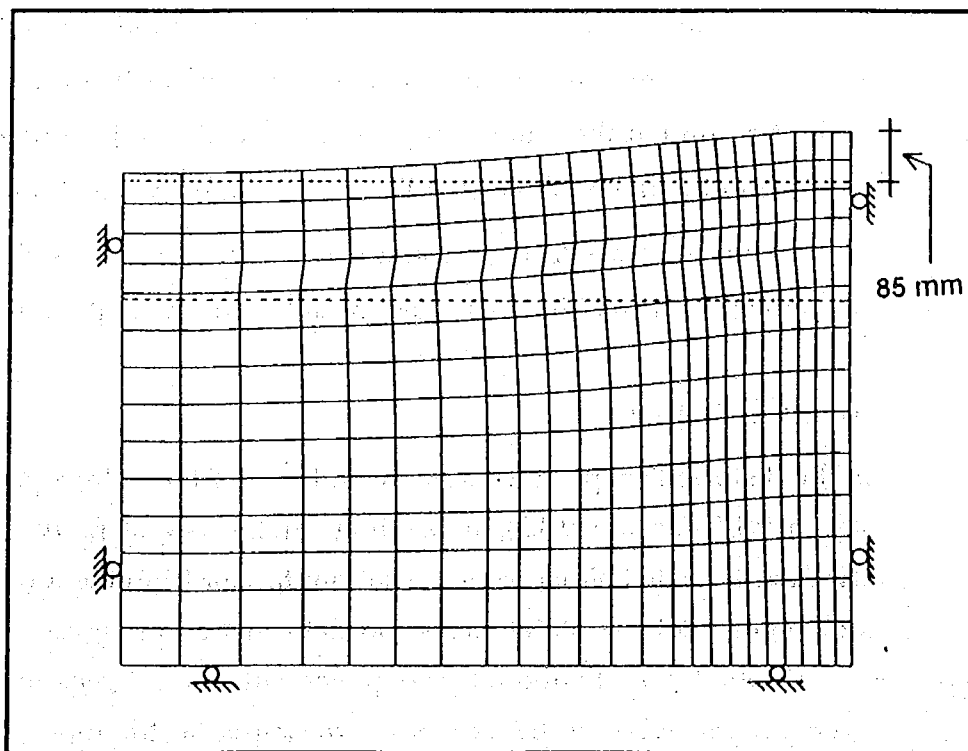
Figures 9.8 a, b and c illustrates the distorted shapes of the pavement and subgrade sections under different situations of lateral fixities. These distorted figures are obtained from the result of analysis with subgrade wetted to a total period of two months. Figures indicate the typical bending characteristics of overlying pavement with changes in the compressive and tensile faces. Particularly, faces subjected to tension initiate cracking in the pavement. Such crack progressively propagates to the inside of the pavement depth.

In Figures 9.8 a, b and c also included is the magnitude of vertical heave developed in the pavement surface after two months of subgrade under different situations

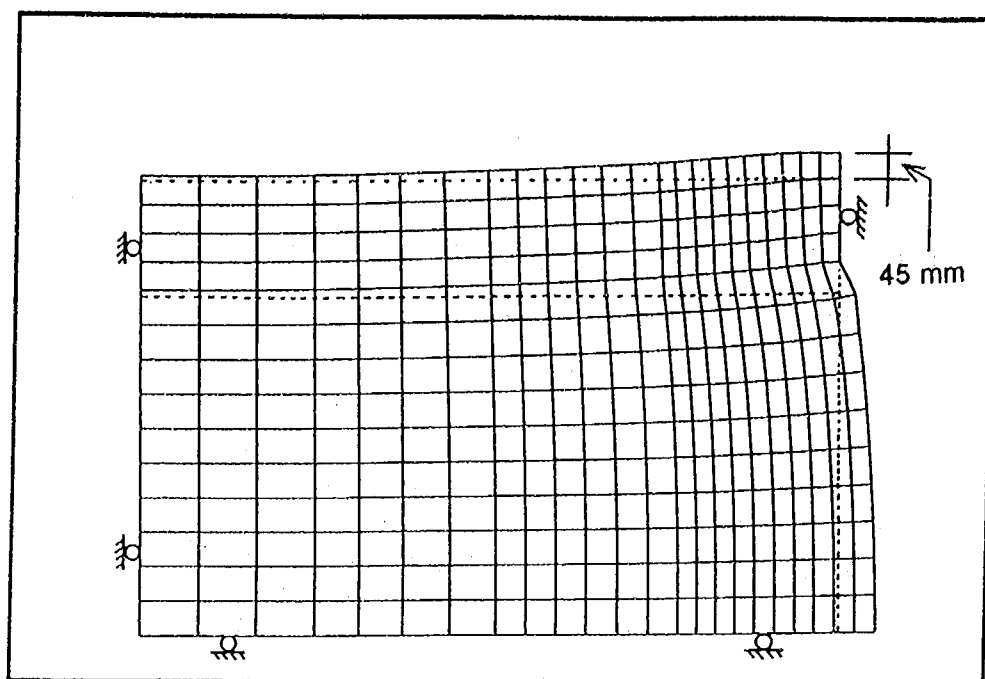
lateral fixity. During the analyses it was observed that the Case 'Bs'  always developed higher value of surface heave than the other two cases, Case 'As'  and Case 'Cs' . Such remarkable difference was because in Case 'Bs' the volume increase in subgrade took place in vertical one dimension.



(a) When right edge is free for both subgrade and pavement



(b) when right edge is fixed against lateral displacement for both subgrade and pavement



(c) when the right edge is fixed for pavement and free for subgrade.

Figure 9.8 Distorted grids when extremely high swelling subgrade ($\gamma_h = 0.07$, and $\gamma_o = 0.08$) wets up and swells for a period of two months (Broken lines indicate the original boundary)

In summary, the results of present analyses showed that when a road section having an extremely high expansive clay subgrade ($\gamma_h = 0.07$, and $\gamma_o = 0.08$) undergoes wetting for a period of two months, the heaving of pavement surface under confined lateral condition i.e., Case 'B' can reach to a magnitude of 85 mm over the 100 mm width near the outer edge. The magnitude of such surface heave reached to a value of 56 mm at the outermost edge in Case 'A', and about 45 mm over the 120 mm width near the outer edge in Case 'C'. Relatively small difference in heave developed between Case 'A' () and Case 'C' () was because the lateral fixity of pavement alone had little effect on the subgrade vertical swell.

C. Lateral Stresses in Pavement

As illustrated in the distorted shape of the sections in Figure 9.8 and the opening of interfaces in Figure 9.17 (Section 9.3.5), depending on the lateral fixity, tension cracks in the pavement initiated either from the bottom face or from the top surface of the pavement. Hence, plots of lateral stress and deformation in Figures 9.9 and 9.10 are presented for the top and bottom face of pavement, respectively. Similarly, heaving (ie., vertical deformation of the section) is measured at the top surface of the pavement (Figure 9.11) as it includes accumulation of over all vertical displacements of the profile.

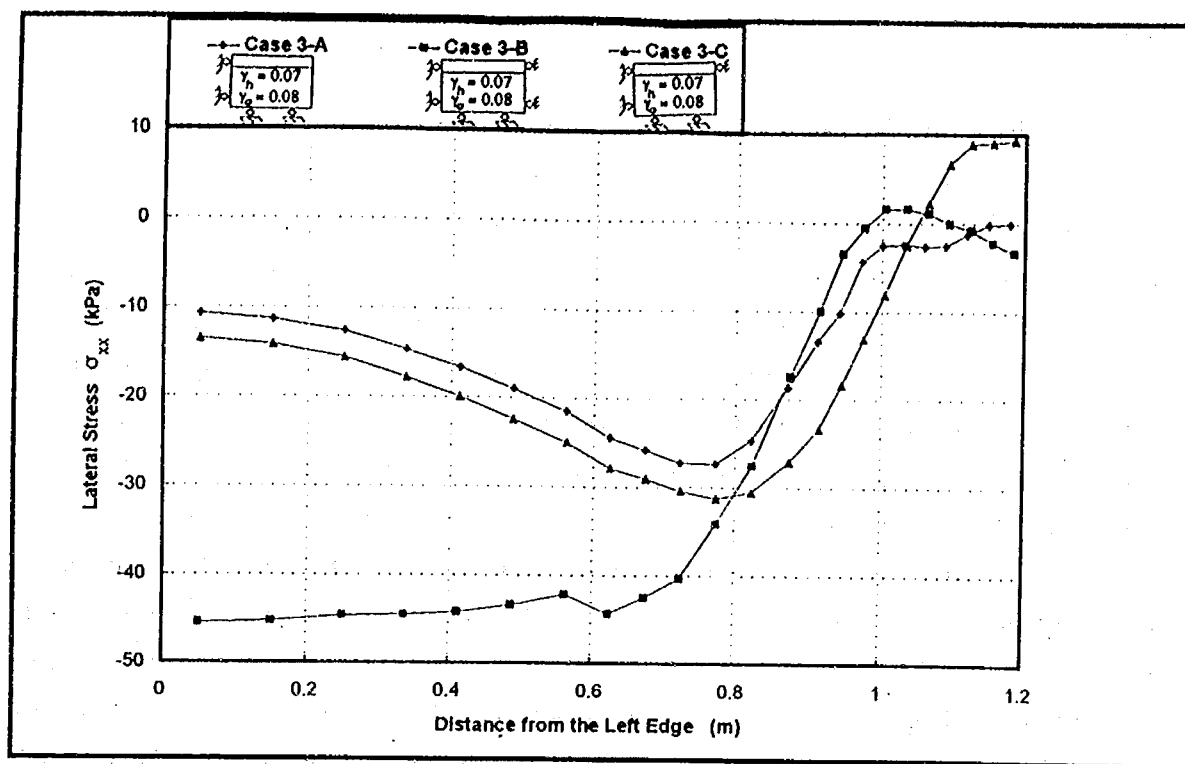
(i) *At the top surface of pavement*

Figures 9.9 a, b and c illustrate the development of lateral stress σ_{xx} on the top surface of the pavement at three different stages of wetting: 10 days, one month, and two months.

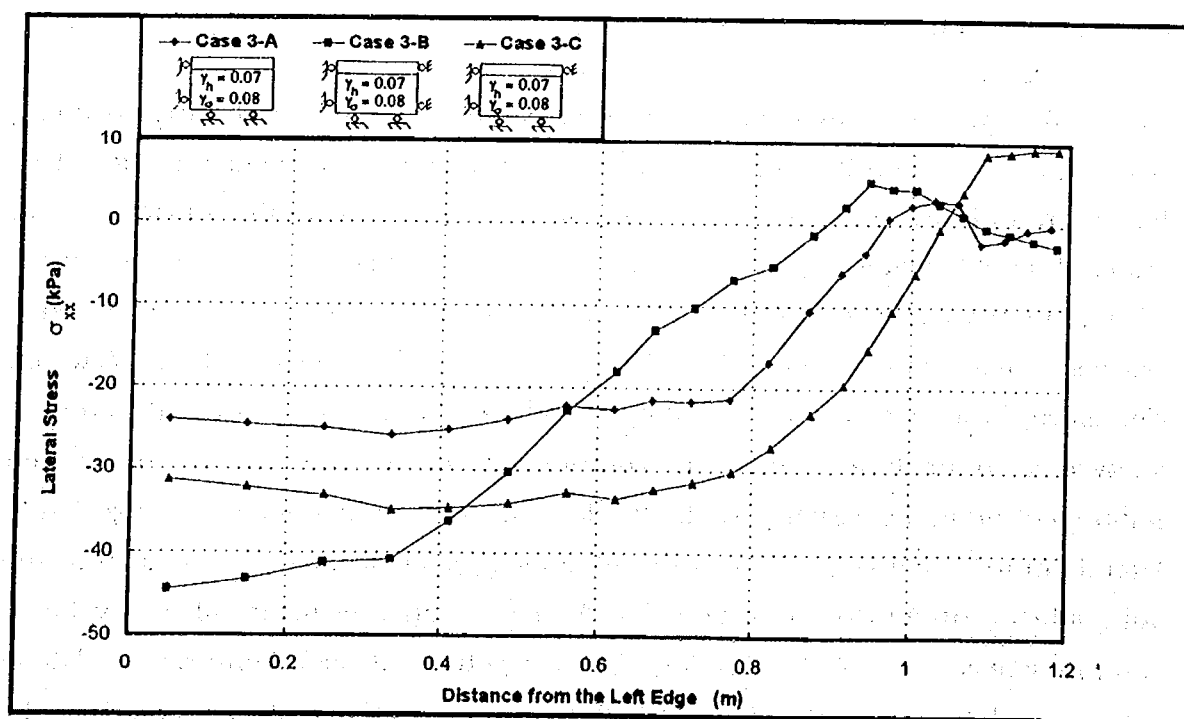
Case 3-A  Initially compressive stress developed in most of the inner elements while little tensile stress developed in the outer ones. The compressive stress was attributed to the bending of the pavement section with compression on the upper face. The development of tensile stress in the outer elements were due to the lateral extension of the pavement because of the lateral displacement of the subgrade. After 10 days of wetting, compressive stress was peak, about 28 kPa, at about mid width and gradually decreasing to 10 kPa in the inner elements. With increasing wetting period the peak value of compressive stress slowly decreased to 18 kPa at the end of two months. Also after a month of wetting, the magnitude of compressive stress were almost uniform for all elements in the inner half, and latter it slowly decreased outwardly. This decrease was due to the release in stress with the development of heave and lateral extension. Figures of one month and two months of wetting ie., Figure 9.9 b and c reveal that some elements about 100 mm inside the outer edge developed some tensile stress of about 6 kPa. This tensile stress on the outer edge surface elements is attributed to the lateral extension of the pavement section.

Case 3-B  As shown in Figure 9.8b, because of the lateral fixity, bending resulted in tension on the upper face of the outer half of the pavement section. This tensile stress in the outer elements were further increased with the transmission of lateral drag of the subgrade describe in Section 9.3.2 A. As discussed in Section 9.3.1, after about a week of subgrade wetting a number of elements near the outer edge reached tensile stress more than 10 kPa and resulted tension failure/cut-off. Therefore these elements in the latter periods, in the Figures 9.9 (a), (b) and (c), showed zero or little stresses in compression or tension because of tension failure/cut-off in the earlier period. The elements on the inner half were subjected to lateral compressive stress as a result of transmission of lateral stresses from the subgrade as discussed in section 9.3.2A. These inner elements also developed compressive stress due to bending. The magnitude of such compressive lateral stress in all the elements of inner half increased continuously and reached the maximum value of about 45 kPa at 10 days (Figure 9.9a). Afterward, the magnitude of this compressive stress decreased gradually and outwardly to a value of about 35

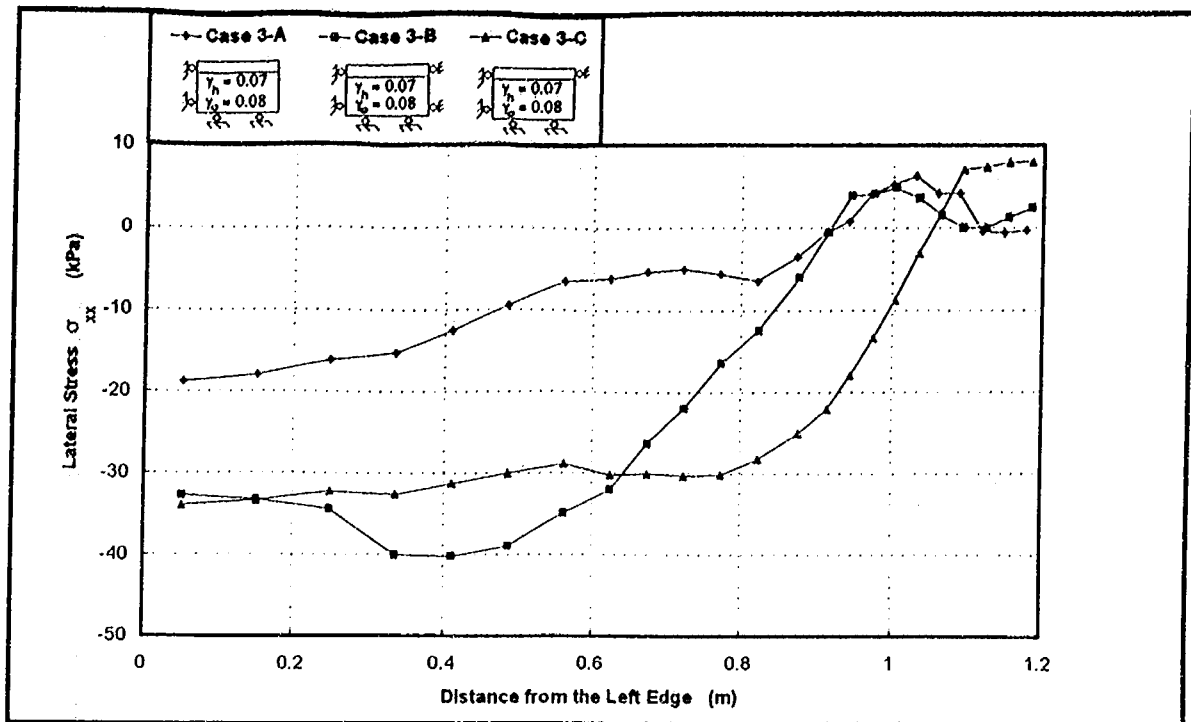
to 40 kPa at the end of two months. Such decrease could be attributed to the gradual reduction in lateral swelling stress in subgrade with increasing wetness described in Figure 9.3.



(a) after 10 days of wetting



(b) after 31 days of wetting



(c) after 64 days of wetting

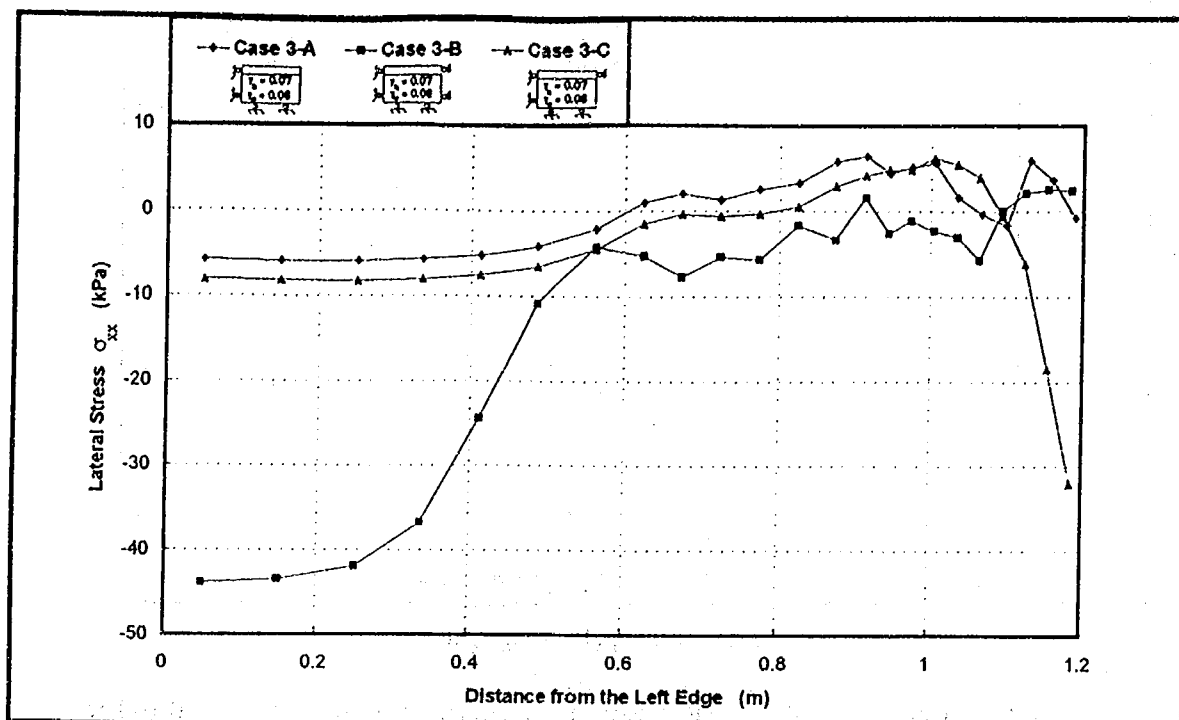
Figure 9.9 Lateral stress σ_{xx} developed on the pavement surface at the different stages of subgrade wetting under three different conditions of lateral fixity

Case 3-C  It reveals a combination of situations presented by Case 3-A  and Case 3-B . As depicted from the distorted shape shown in Figure 9.8c the surface elements of the pavement underwent compression on mid width elements and tension on elements close to outer edge. This could be because, even though the subgrade lateral stress should result compression on elements near the outer edge, the bending stresses adversely effected the nature of stress on the outer elements. When combined the overall results, it resulted tension in the outer edge elements as shown in Figure 9.9a, b and c (for all stages of wetting). Regarding inner elements, at all stages of wetting until two months, they showed compression with a peak value of up to 35 kPa at the end of one month.

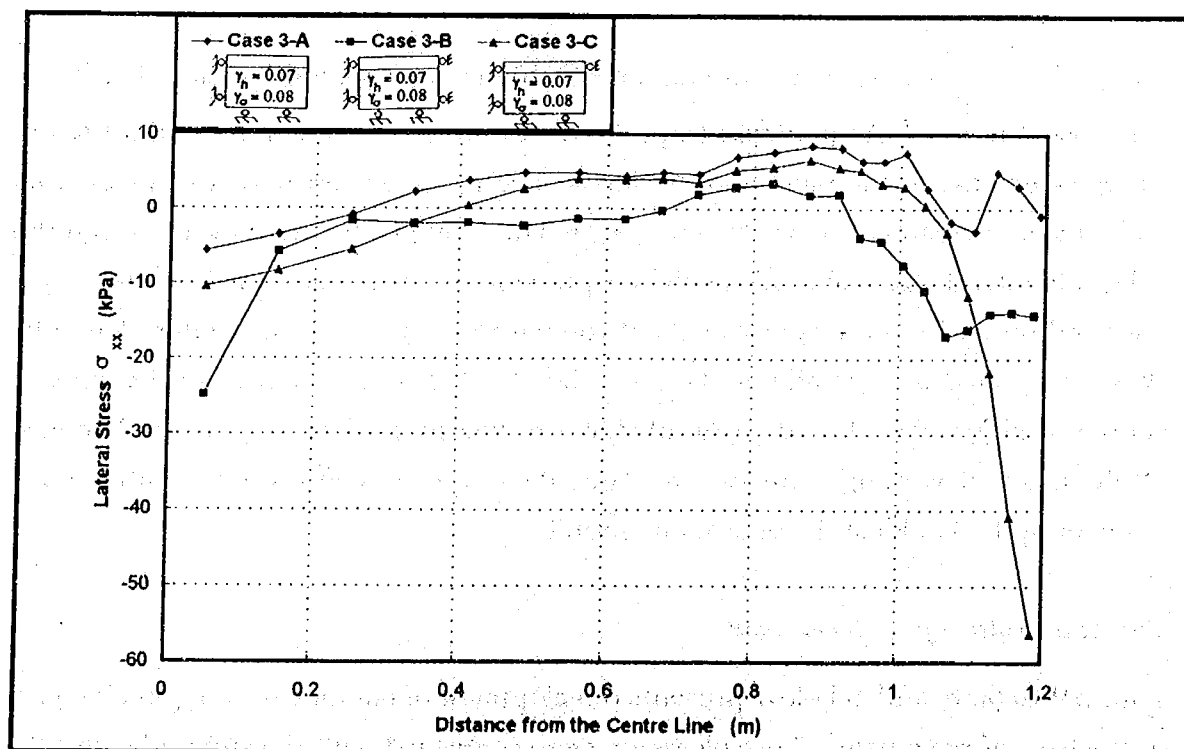
(ii) At the bottom face of pavement

Figure 9.10 (a, b, and c) below presents development of lateral stress σ_{xx} in elements at the base of pavement. These elements were connected with the subgrade, and the transmission of lateral stress of swelling subgrade to these pavement elements were discussed in detail in Section 9.3.2A. As mentioned above, in some situations of lateral fixity, lateral stresses developed due to bending were of opposite nature to

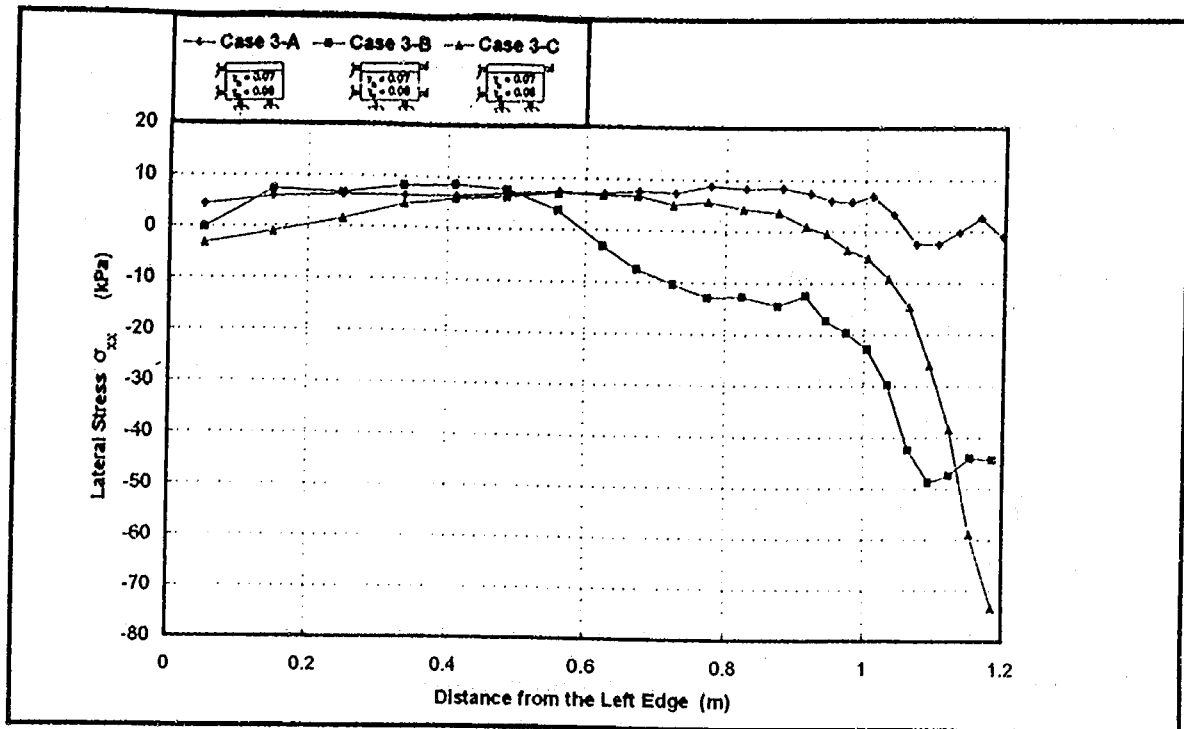
the lateral stresses resulted due to the subgrade extension. In such situation, the response of pavement elements was complicated.



(a) after 10 days of wetting



(b) after 31 days of wetting



(c) after 64 days of wetting

 Figure 9.10 Lateral stress σ_{xx} developed on the base of pavement at different stages of subgrade wetting under three different conditions of lateral fixity

Case 3-A  At the earlier stage (at 10 days) of wetting, inner elements were subjected to small compressive stress of about 6 kPa. This compressive stress gradually decreased with increasing wetting period. As revealed from Figure 9.10b, after one month of wetting, almost all elements except three elements near the outer edge developed tensile stress because of combined effect of both lateral extension and bending. At the end of two months wetting all elements were in tension. Some of the elements had reached their tension limit in the earlier stage of wetting and lateral stress was cut down to zero or came under little compression (as shown by elements within 100 mm inside the outer edge) after the stress redistribution.

Case 3-B  At the earlier stage of wetting higher compressive stress was developed in the inner elements. The peak value reached to about 42 kPa on the innermost element at the end of 10 days. This could be attributed to the transmission of very high subgrade compression as revealed in Figure 9.2. As the wetting period increased the higher compression in the innermost zone gradually decreased and by two months of wetting, inner elements also started to develop tension as a result of bending stress shown in Figure 9.8b. Elements of the outer half underwent compression after a month of wetting due to the transmission of lateral stress and due to the compressive bending stress.

Case 3-C  The nature of lateral stress in almost all elements at all stages of wetting was similar to Case 3-A, except an element near the outer edge (as shown in Figure 9.10 a, b and c) continued to undergo higher compression. It could be because of the artefact of the mesh size or numerical imperfection in the analysis due to the changes in lateral fixity and material stiffness between subgrade and pavement. When the subgrade was wetted to two months, such excessive compression got released when a few elements between 100 to 400 mm inside reached tension cut-off.

D. Tensile Cracking in Pavement

Case 3-A - 

In Case '3-A', after about 10 days of wetting, a column of three elements starting from the base of the pavement near the outer edge (150 mm inside) developed a tensile stress greater than 10 kPa such that they underwent uniaxial tension failure demonstrating the initiation of cracking from the base and propagating upward. This is compatible with the nature of distortion of the pavement section suggested by Figure 9.8a. Later with new stresses transmitted from further wetting and swelling of subgrade, these elements came under small compression.

After two months of wetting, another crack started to initiate from the base as illustrated in Figure 9.8a. Further the results of analysis showed that a few other elements also reached tension cut-off. As the program redistributed stresses within the surrounding elements, these elements afterwards came under compression.

Case 3-B - 

As shown in Figure 9.8b, with this case of lateral fixity, the upper surface of pavement near the outer edge was subjected to tension. From the initial stage of subgrade wetting, it developed higher lateral tensile stress σ_{xx} in the outer elements of pavements. After 4 days of wetting period, a column of elements of the pavement about 100 mm inside the outer edge underwent uniaxial tensile failure due to the major principal stress (σ_1) reaching more than 10 kPa in tension. It was because the top outer elements subjected higher tensile stress as a result of development of higher bending stress near the outer edge. Once the top element failed in tension, the tensile stress was redistributed to the adjoining elements. This resulted higher tensile stress to element beneath which also subsequently underwent uniaxial tension failure. In this way the crack initiated and propagated downward. Such initiation of cracking on the top surface, and propagation

downward could be attributed to the nature of bending of pavement as revealed in Figure 9.8b or opening of interface in Figure 9.17b.

After two months of subgrade wetting some elements of the bottom face in the inner half developed higher shear stresses, and as a result they reached the yield state. Later, as depicted from Figure 9.10c they came under a tensile stress of about 8 to 9 kPa.

Case 3-C 

In Case '3-C', from the initial stage of subgrade wetting (after 10 days), pavement elements at the surface within 150 mm inside the outer edge started to develop lateral tensile stress σ_{xx} of above 9 kPa. But the results did not show any tensile failure of these elements nor any other elements within the pavement section through out the period of wetting from instantaneous wetting of source boundary to the two months of continued subgrade wetting. With increasing wetting period the magnitude of the developed tensile stress in these outer edge elements slowly decreased from 9.4 kPa after 10 days to 9.2 kPa after one months and 8.0 kPa after two months of wetting.

E. Behaviour of Subgrade

As explained in Section 4.5.1, the subgrade swell modulus is the function of octahedral normal stress and suction potential of subgrade soil. Because of its lateral confinement, Case 'B'  developed higher level of octahedral normal stress and hence at the same stage of wetting, subgrade elements in this case, had relatively higher stiffness than in Cases 'A'  or 'C' . The situation of increase in stiffness for Case 'B' was discussed in Section 9.3.1A, where the magnitude of stresses developed became the function of degree of wetting and the lateral fixity. During the analyses, vertical stress σ_{yy} at the mid depth of subgrade showed a higher magnitude of both compressive and tensile stresses with Case 3-B than with Cases 3-A and 3-C. Further, Case 3-A and 3-C behaved with a relatively similar nature of stress development. The reason was obviously because of similar form of lateral displacement boundary of the subgrade section.

The development of lateral stress σ_{xx} depended on the lateral fixity. With the free lateral boundary in the right edge in Cases 3-A and 3-C, there was no lateral stress developed in Case 3-A, where as in Case 3-C, because of lateral fixity of pavement section, there was little stress developed in the upper elements. This was attributed to the restraining effect of the adjoining pavement at the upper zones of subgrade.

9.3.3 Effect of Magnitude of Swell and Swell Suppression Parameters

Section 7.3.1 discussed how the two swell parameters: free swell index ' γ_h ' and swell suppression index ' γ_σ ' are involved in the description of modulus of wetting up and swelling of clay subgrade. Application of the simplified description of non-linearly varying swell modulus with wetness in the numerical analysis was discussed in Section 8.2.2. As seen in Table 8.1, the equation describing variation of swell modulus with suction is different for different sets of values of swell and swell suppression indexes. For a value of suction of 2.0 log kPa, the first set Case-1 ($\gamma_h = 0.035$, $\gamma_\sigma = 0.04$) gives the magnitude of B parameter as 43.81, second set Case-2 ($\gamma_h = 0.07$, $\gamma_\sigma = 0.04$) gives B as 30.41, and the third set, Case-3 ($\gamma_h = 0.07$, $\gamma_\sigma = 0.08$) gives B as 21.91. As such, at the same stage of wetting, stress and deformation behaviour of the subgrade section for a similar lateral fixity would be different for different sets of magnitude of swell and swell suppression indexes.

A typical situation having free lateral displacement boundary for both pavement and subgrade is considered. With this situation, the effect of variation of these swell indices in the response of pavement and subgrade is studied at different stages of wetting.

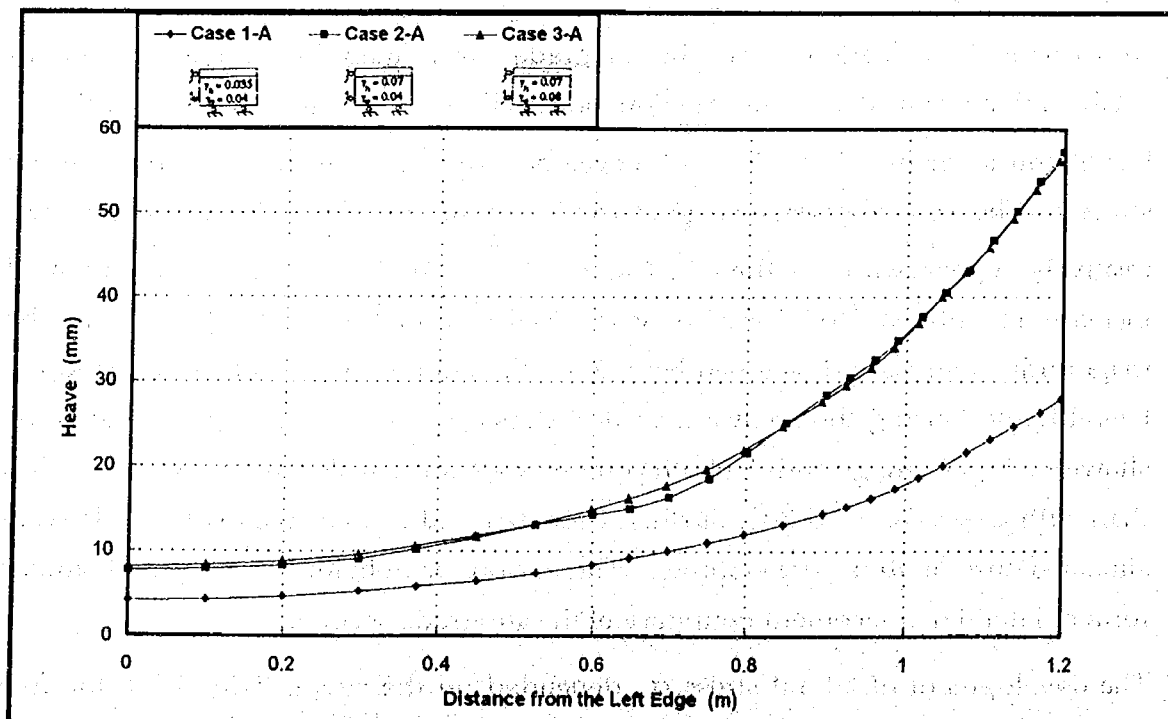


Figure 9.11 Heaving of pavement surface after two months of wetting with different sets of subgrade swell indices

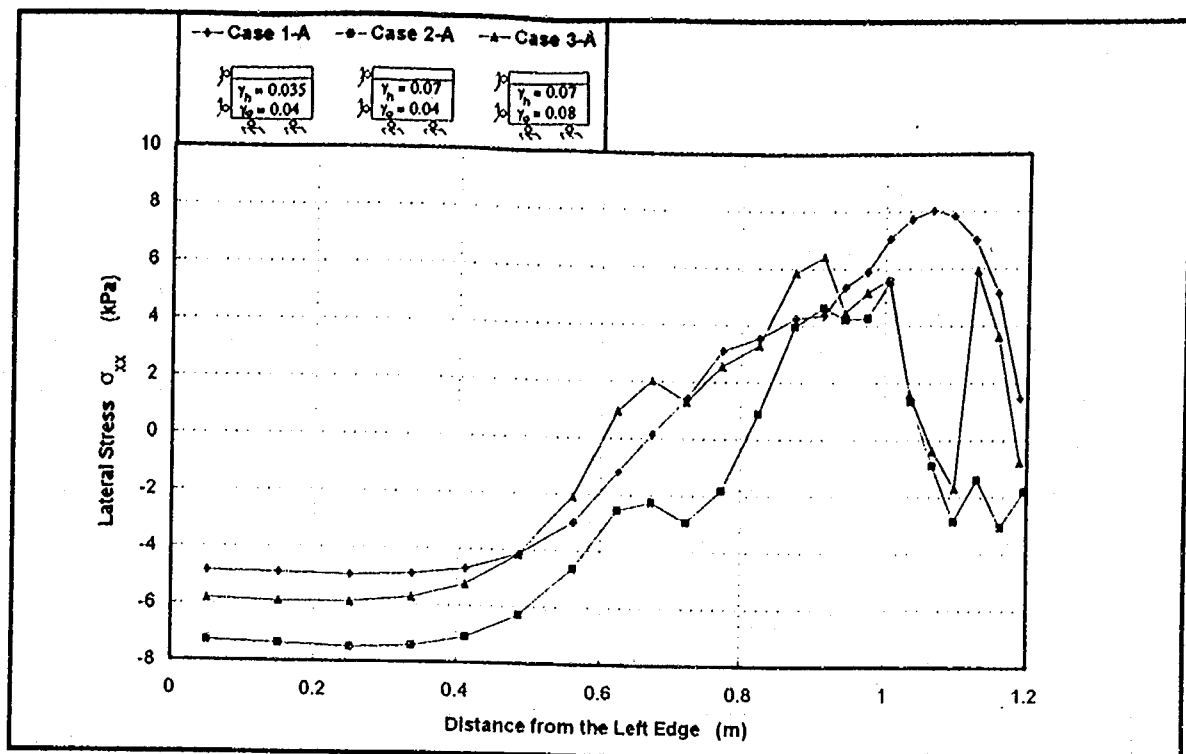


Figure 9.12 Lateral stresses in pavement base after 10 days of wetting for different sets of swell indices in subgrade

A. Pavement Surface Heave

At all stages of wetting, as expected heaving of pavement surface was always higher with Case 2 and Case 3 ($\gamma_h = 0.07$, $\gamma_\sigma = 0.04$; and $\gamma_h = 0.07$, $\gamma_\sigma = 0.08$ respectively) than with Case 1 ($\gamma_h = 0.035$, $\gamma_\sigma = 0.04$). Figure 9.11 shows heaving of pavement surface at the end of two months of wetting. It was observed that without altering the swell index γ_h , there was no significant reduction in heave with an increase of swell suppression index γ_σ from 0.04 to 0.08. The reason was accounted to the very low overburden pressure (only the weight of overlying pavement) to which the subgrade was subjected.

The results presented in Figure 9.11 also indicate that with an increase in swell index γ_h from 0.035 to 0.07, after two months of wetting, the maximum edge heave increased from 28 mm to about 57 mm.

B. Lateral Stressing in Pavement

Figure 9.12 illustrates the effect of swell indices in the development of lateral stress at the bottom face of pavement at the end of 10 days of subgrade wetting. (At the later stages (after one month or more) of wetting, many pavement elements with all sets of swell indices surpassed tension cut-off and hence the plot could not illustrate clearly the effect of swell indices. Therefore the plots for latter periods are not included). The results illustrated in the figure show that the development of

compressive stresses on the bottom face of pavement in the inner half was relatively lower with Case 1-A than with the other two cases Case 2-A and Case 3-A. Regarding tensile stress which develops on the bottom face elements, Cases 2-A and 3-A developed higher values than Case 1-A. The magnitude of the tensile stress developed in the outer half of the pavement with these two cases was higher than the tensile strength of pavement (which is 10 kPa). Therefore a column of elements near the outer edge in each of these Cases 2-A and 3-A went tension failure before 10 days of wetting. Later these elements came under little compression after stress redistribution. This indicate that pavement in Cases 2-A and 3-A ($\gamma_h = 0.07$, $\gamma_o = 0.04$, and $\gamma_h = 0.07$, $\gamma_o = 0.08$ respectively) undergoes pavement cracking ahead of Case 1-A ($\gamma_h = 0.035$, $\gamma_o = 0.04$).

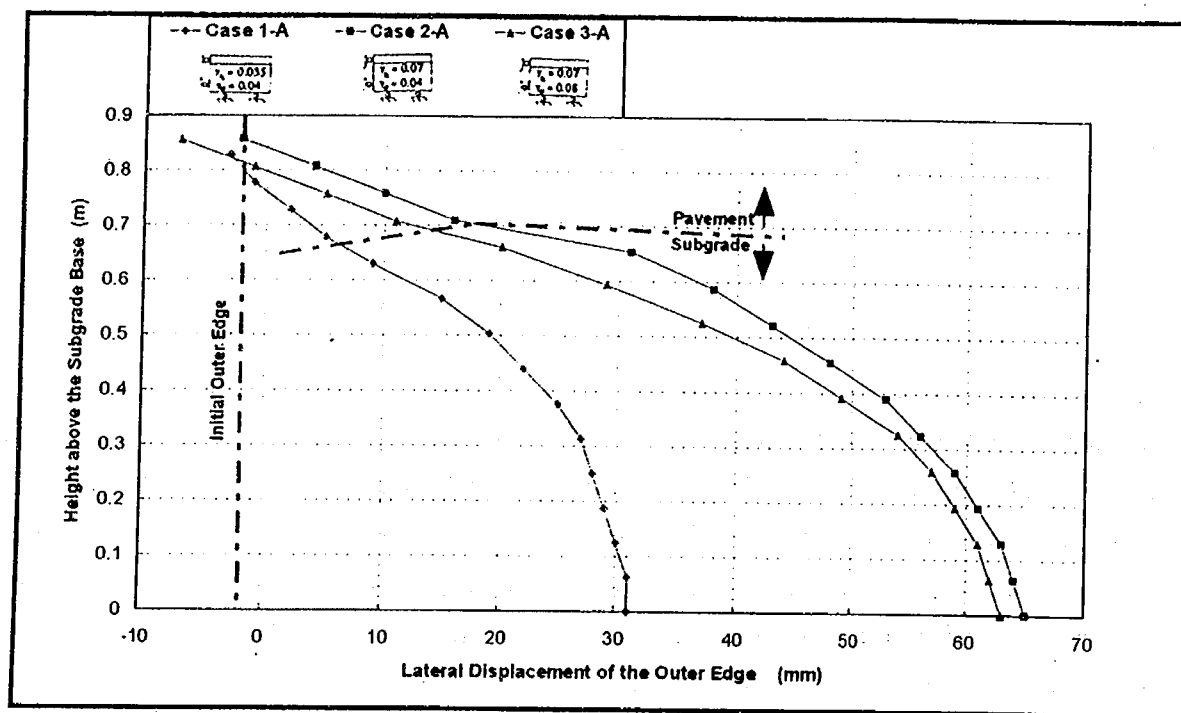


Figure 9.13 Lateral displacement of subgrade after two months wetting under different sets of swell indices

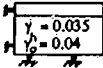
C. Lateral Extension of Subgrade Section

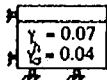
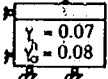
With free right edge, lateral displacement at all stages of wetting was significantly higher with subgrade having higher swell index. As shown in Figure 9.13 with two months of wetting, without altering swell suppression index γ_o , Case 2-A having swell index $\gamma_h = 0.07$ developed higher lateral displacement than Case 1-A (with $\gamma_h = 0.035$). With the constant value of swell index $\gamma_h = 0.07$, an increase in swell suppression index γ_o from 0.04 to 0.08 resulted only a small decrease in lateral displacement as can be seen by comparing Cases 2-A and 3-A in Figure 9.13.

9.3.4 Subgrade Wetting Pattern

A typical situation having free lateral displacement on the right edge  was considered under a two dimensional wetting pattern. The width of the section was increased to 1.8 m to accommodate a 600 mm wide pervious shoulder. The height of the section was still kept 800 mm total with a 600 mm deep subgrade and a 200 mm thick overlying pavement. Moisture in this situation entered from the underneath of pervious shoulder as shown in Figure 8.6 (Section 8.7). Under this pattern of subgrade wetting, the stress/strain behaviour of subgrade and response of pavement are discussed below.

A. Subgrade Stress/Strain Development

With the changing direction of advancing wetting front, the nature and magnitude of stresses developed in the subgrade section varied considerably with the wetting period. Analyses showed that at the earlier stage of subgrade wetting (up to 10 days), elements on the upper one third part of the subgrade section were observed to develop compression due to the restraining effect of pavement. Elements on the lower two third underwent tensile stress in the lateral direction (σ_{xx}) as a result of free lateral swelling. When the wetting period increased to two months, depth of compression zone increased such that almost half of the upper section subjected compression and the lower half subjected to tension. The difference in the magnitude of lateral stress (σ_{xx}) development under different sets of swell indices was found to be relatively small. However, as expected, subgrade having lower value of swell indices as in Case 1-A  showed relatively lower level of stress

development than the one having higher swell indices, Case 2-A  or Case 3-A .

Regarding the vertical stress σ_{yy} , before the wetting of subgrade, all the elements in the section had an initial overburden pressure equal to the gravity stress (eg., 13.7 kPa at the centre of the elements at the base). When the section was wetted to a period of two months, the initial compression in each cases increased in most of the elements due to swelling and due to confinement provided by the overburden pressure of overlying pavement. However because of relatively small overburden pressure, the increase in vertical stress was small. Meanwhile, due to the differential heave between inner part and outer part of pavement surface, the vertical stress was decreased in some inner elements of subgrade because of the

lifting effect of overlying pavement. Also, because of relatively lower overburden pressure, the effect of increase in swell suppression index γ_s was not observed.

B. Response of Pavement

Figure 9.14 shows a comparative development of heave in pavement when the subgrade having similar swell condition wetted up to a period of 2 months under one dimensional wetting front discussed in earlier sections and under two dimensional wetting process. The results presented in the figure indicated that the magnitude of maximum heave under two dimensional wetting is almost equal to the corresponding magnitude under one dimensional flow. For example, the maximum heave of 30 mm, 54 mm and 59 mm were observed in Cases 1-A, 2-A and 3-A under two dimensional wetting where as under one dimensional wetting the corresponding values of heave were 28 mm, 57 mm and 56 mm.

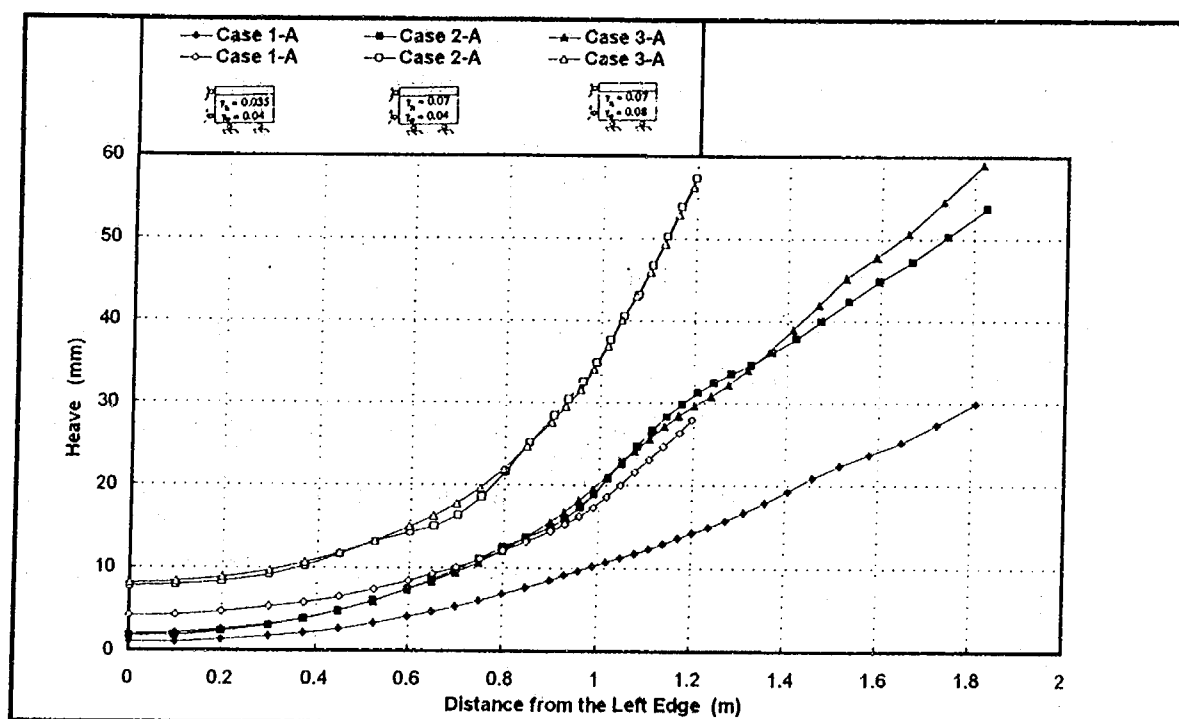
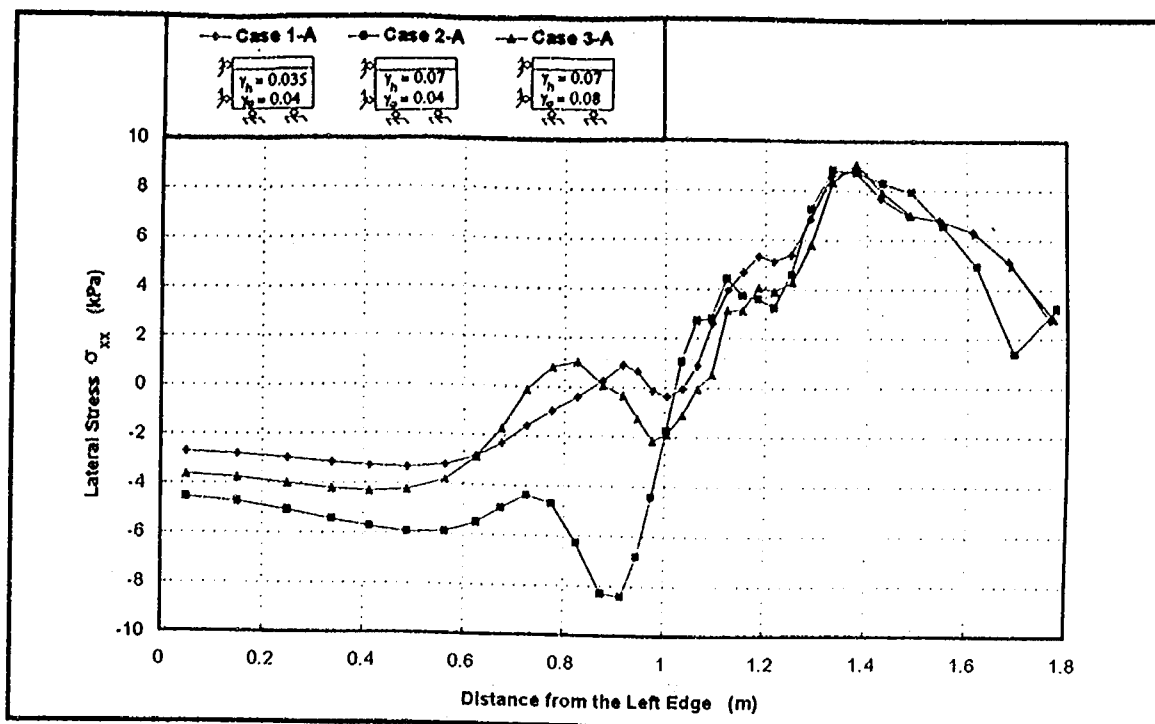
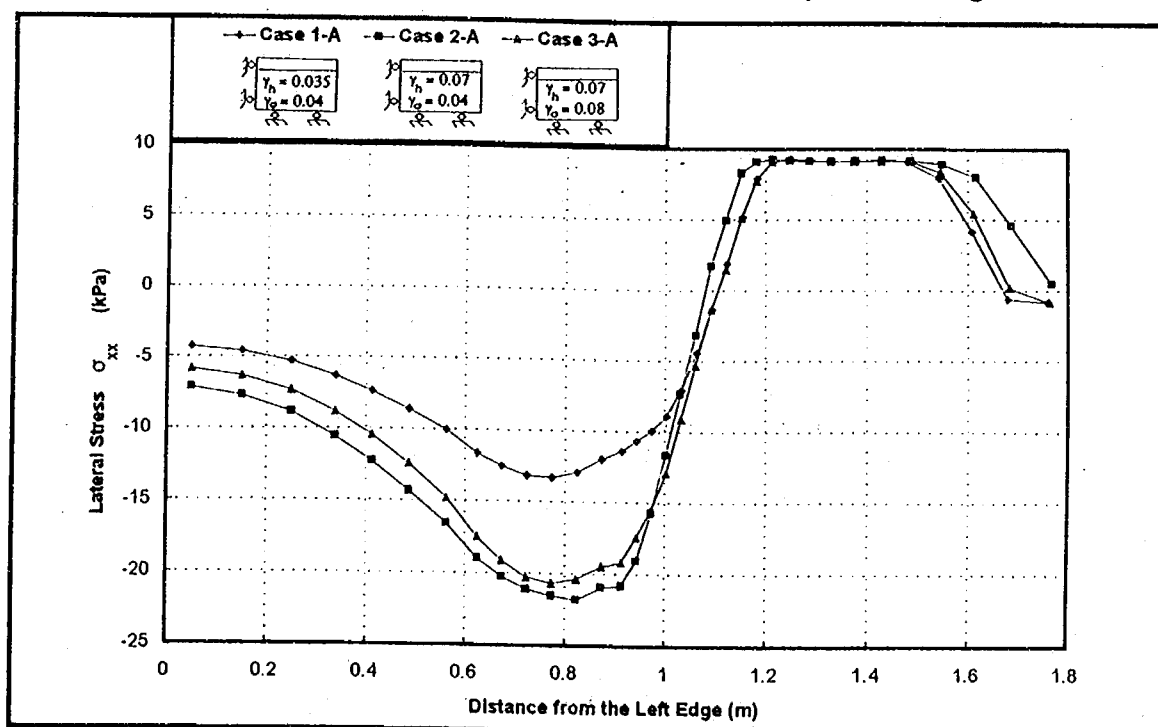


Figure 9.14 Comparative development of heave on pavement surface under one and two dimensional wetting front, when subgrade wets up to two months

Figures 9.15 a and b illustrate the lateral stress ' σ_{xx} ' development in the pavement lower face and upper surface when the underlying subgrade undergoes two dimensional wetting for 10 days. Elements of pavement around the connection between the pavement and shoulder, and from base to the top surface started to reach yield state even at the earlier stage of subgrade wetting. When subgrade was wetted to a period of two months, many pavement elements reached yield state.



(a) At the base of pavement after 10 days of wetting



(b) At the top of pavement after 10 days of wetting

 Figure 9.15 Lateral stress σ_{xx} on the pavement under 2-D flow

As depicted from the Figures 9.15 a and b, the result of analyses showed that after a wetting period of 10 days, in all sets of values of subgrade swell parameters, the elements in the inner half were under small compressive stress on the lower face and relatively higher compression on the upper face. But even at this stage of

wetting, some of the elements around the pavement/shoulder connection reached yield state. This resulted a uniform tensile stress of about 9 kPa developed in all cases at the top surface elements of the section. As a result of yielding, the pavement elements on the bottom face showed inconsistent pattern of lateral stress development. Such yielding of elements near the inner edge of shoulder in each cases were attributed to the excessive change in suction gradient as shown in Figure 8.6b. This also signified the possible initiation of pavement cracking as a result of tensile stress development.

C. Lateral Displacement of the Section

Figure 9.16 shows a comparative plot of lateral displacements of the right edge when the subgrade was wetted to two months under one and two dimensional wetting fronts. The shape of deformation varied considerably with the wetting pattern. With two dimensional wetting, the whole section of subgrade extended relatively uniformly. The reason could be the sharper gradient of decreasing wetting with depth at the subgrade section underneath the shoulder which resulted to decrease in lateral swelling with depth, while an increasing overburden pressure resulted higher lateral displacement with depth. When these two phenomena occurred together, one balanced the other and made the over all displacement more or less uniform through out the depth.

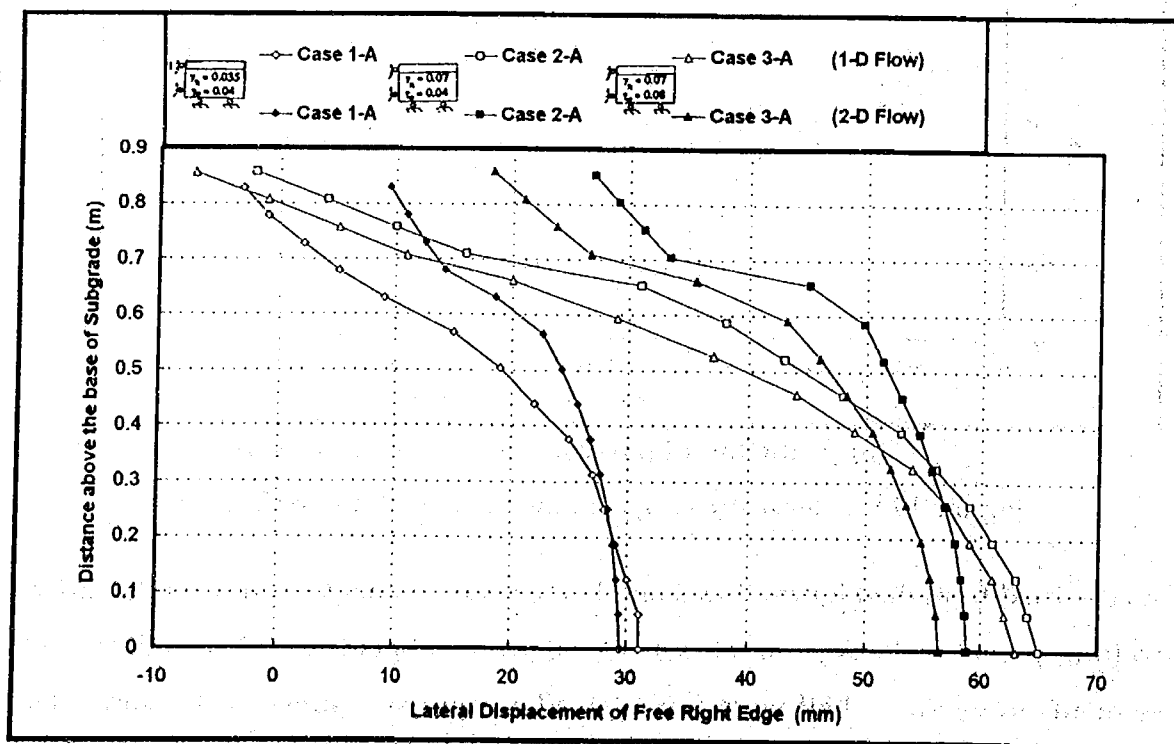


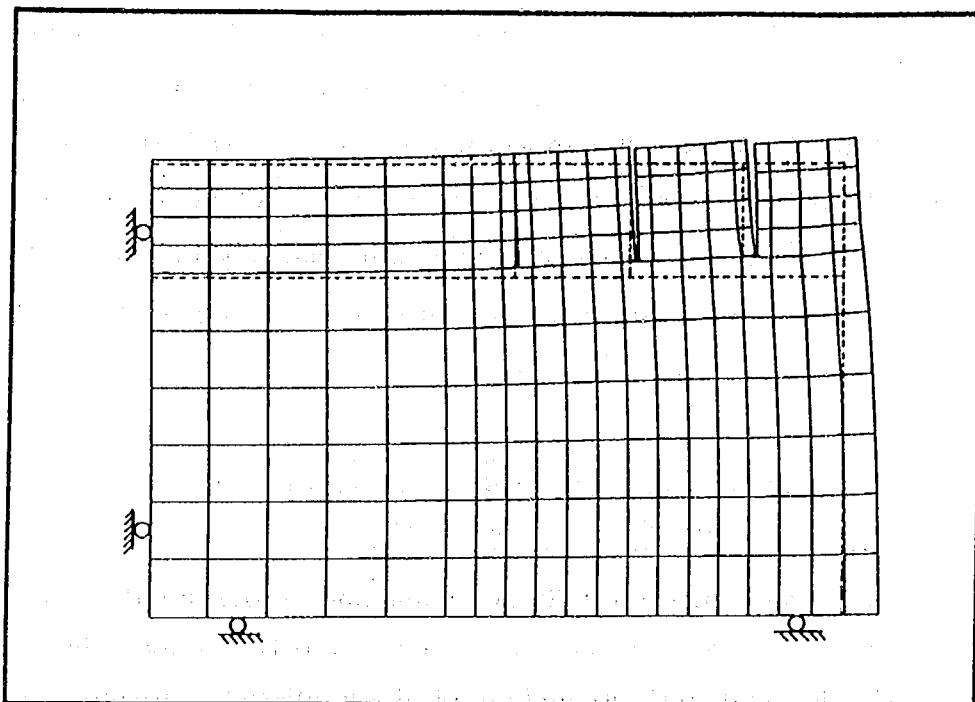
Figure 9.16 Comparative results on lateral displacements under 1-D and 2-D wetting fronts

Regarding the displacement with respect to varying swell indices, as would be expected, the magnitude of lateral swelling was higher with the subgrade having higher free swell index ' γ_h '. Further, as would be expected without changing ' γ_h ' subgrade having higher swell suppression index ' γ_σ ' resulted lower lateral extension under both pattern of wetting front. The nature of lateral displacement in pavement was similar under both situations, but relatively higher displacement under two dimensional wetting. Also, the pavement over higher lateral swelling subgrade showed higher lateral displacement.

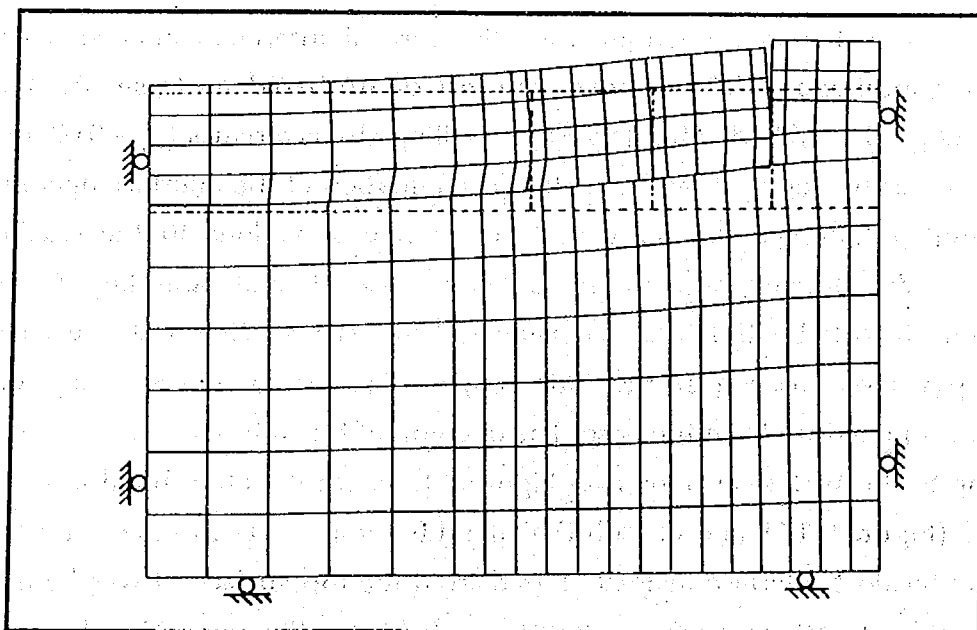
9.3.5 Inclusion of Vertical Interfaces in Pavement for the Simulation of Pavement Cracking

A set of analysis was carried out with horizontal and vertical interfaces within the pavement section and at the plane of connection between the pavement and subgrade. The location and orientation of these interfaces together with the estimation of their normal and shear stiffness were discussed in Section 8.6 (Figure 8.3). In this section, the responses of these interfaces are analysed. Particularly, opening up of these vertical interfaces are of prime interest as they are expected to represent the possible crack initiation and propagation in the pavement.

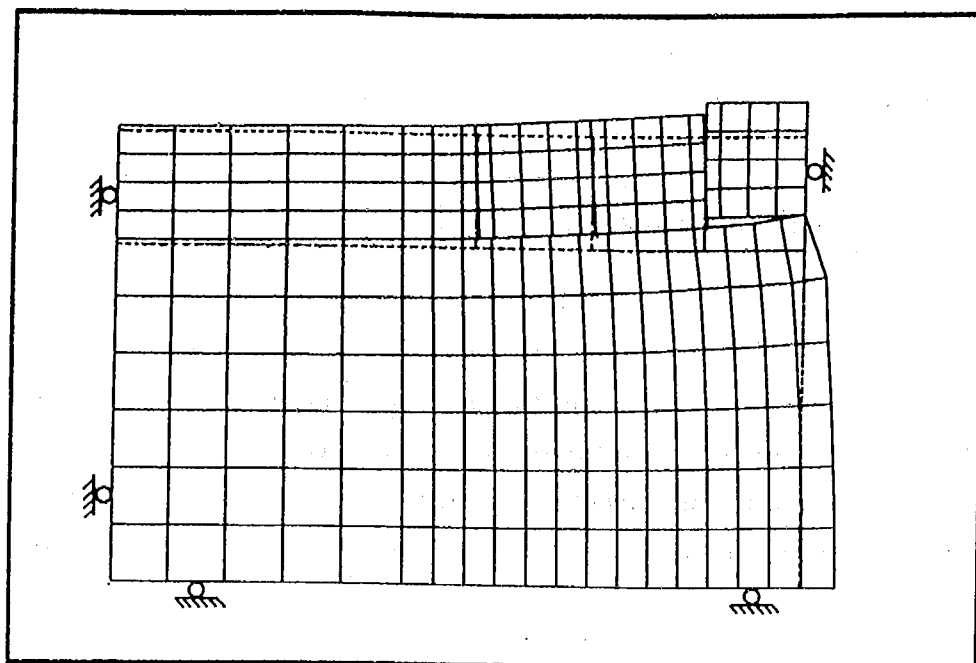
Figures 9.17 a, b and c illustrate how the vertical interfaces opened up under different conditions of lateral fixity. The results are based on Case-3A, B and C where subgrade soil had extremely high swelling characteristics ($\gamma_h = 0.07$ and $\gamma_\sigma = 0.08$) and was wetted up to two months. The initiation of the interface opening and their propagation was similar to the common observations in the cracking of pavement due to subgrade swelling. With a free lateral boundary Case 3-A, pavement distorted with lateral extension and concave bending of the surface. As a result, pavement base near the outer edge subjected to excessive tension and interfaces opened up from the base. The opening of this interface extended upward reaching to the surface as shown in Figure 9.17 a. On the other hand in Cases 3-B and 3-C (Figure 9.17 b and c) with the lateral fixity against pavement, the opening of the outermost interface instead started from the top surface. It was because the pavement bent with convexity on the top surface near the outer edge. Figure 9.17 c also suggests that in the case of Case 3-C, there existed an unequal vertical displacement at the top of the outer most interface. The opening of other inner interfaces were relatively small.



a) When right edge of both pavement and subgrade are free against lateral displacement



(b) When right edge of both pavement and subgrade are fixed against lateral displacement



(c) When right edge of pavement is fixed and subgrade is free in lateral displacement

Figure 9.17 Opening of interfaces applied in a pavement to simulate possible cracking in pavement.

In general, the results showed that the nature of the opening of these interfaces was similar to the cracking of pavement. In all cases, the outermost interface opened up ahead of the inner ones and also had higher width of opening. Particularly in Case 3-C, a vertical shearing was observed in the outer interface. This might be because of difference in lateral fixity between pavement and subgrade which resulted a high shear stress due to the unequal bending.

Since the subgrade section was modelled with no interfaces within, the response of this section should be similar to other analyses done with this model. In an initial analysis some difficulty was observed with the program when dealing with the moisture distribution in subgrade elements around the interface. The program resulted non-uniform wetting of the subgrade section due to the continuous flow of moisture (that is, heat under the thermal analogy) through the interface between subgrade and pavement. This resulted an inconsistency in the subgrade stiffness modification which in turn reduced the accuracy in the magnitude of swell and swell pressure development. This difficulty was successfully overcome by defining temperature of the grid points externally and thus adjusting the value of moisture dependent stiffness. With this, the program could successfully undertake the thermo-mechanical analysis. Meanwhile, as revealed in Figure 9.17c, in Case-3C, there developed an undesirable separation of interface between the pavement and

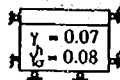
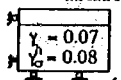
subgrade near the outer edge. This is partially due to the inaccuracy in defining interface properties as input to the program and partly due to the limitation of the program.

9.4 Review of Parametric Analysis

The discussions presented above on the analysis of results obtained at different wetting period for different cases of lateral fixities, different combinations of magnitude of swell parameters, and under two different wetting pattern showed that some of the parameters considered have significant effect on the response of subgrade and the overlying pavement when the subgrade wets up and swells. In this section, the sensitivity of these parameters are evaluated and discussed.

A. Wetting Period

As discussed in Section 9.3.1 for all situations under all types of lateral fixities, magnitude of swell indices, and boundary conditions, the extent of swell pressure and heave in subgrade and consequent response of pavement depended largely on

the wetting period. For a typical situation, Case 3-B  shown in Figure 9.5 (a, b and c), the heave on the outer edge of pavement increased from about 30 mm after 10 days of wetting to 50 mm after a month. Similarly, for Case 3-A , the lateral displacement of the subgrade on the free right edge increased from 50 mm after one month wetting to 63 mm at the end of two months.

The magnitude of lateral stresses σ_{xx} , and the vertical stress σ_{yy} developed in the subgrade were significantly higher at the initial wetting stage and decreased gradually with increasing wetting period. As shown in Figure 9.3 for subgrade mid depth, with an increase in wetting period until the wetting front reaches to the innermost element, for example one months in Figure 9.3, the lateral stress σ_{xx} in compression increased gradually with the increase in wetting period. Its magnitude reached from 15 kPa after instantaneous wetting of the boundary to 75 kPa after a month of wetting. Later σ_{xx} slowly decreased with wetting period such that its value become 68 kPa at the end of two months of wetting. Note that as depicted from Figure 9.3, the magnitude of σ_{xx} was almost constant for all elements from the outer edge to the inner edge.

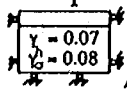
At the earlier stage of wetting up to 10 days, with increasing wetting period the elements at the mid depth of subgrade showed increasing vertical stress σ_{yy} in compression near the outer edge and increasing σ_{yy} in tension around the mid

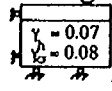
width. Afterwards, σ_{yy} in both compression and tension gradually decreased with wetting period in the latter stage.

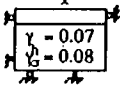
Regarding the lateral stresses in pavement, as shown in Figures 9.6 a and b, the pavement response with the wetting period was relatively complex. The behaviour of lateral stresses in the top surface elements and bottom face elements were controlled more by degree of lateral fixity.

B. Lateral Fixity

With the situations of extreme lateral fixities, their effects in subgrade and pavement response were very significant. The distorted shape of the analysed road section shown in Figures 9.8 a, b and c, indicated that when both the pavement and

subgrade were fixed against lateral displacement as in Case 3-B , heave on the pavement surface at the end of two months wetting reached to about 84 mm

compared to 56 mm in free right edge of Case 3-A , and 45 mm in Case 3-C

 where only the pavement layer was restrained. Regarding lateral stress σ_{xx} in pavement, Figures 9.9 a, b and c show that all the elements on the top face of pavement in Case 3-A subjected to compression while in other two cases: Case 3-B and Case 3-C, a number of elements near the outer edge of top face subjected to tension and reached tension failure or tension cut-off. On the other hand case 3-A developed tension on bottom face.

Regarding the stresses in subgrade, under all stages of wetting, Case 3-B having lateral fixity showed significantly higher magnitude of vertical stress σ_{yy} in both compression and tension compared to other two cases. And obviously Case 3-B had higher lateral stress (up to 68 kPa in compression at mid depth after two months wetting) which was apparently nil or almost nil in other two cases.

C. Effect of Swell Indices

Evidently, higher magnitude of free swell coefficient ' γ_h ' resulted higher swell pressure and heave in the subgrade, and consequently higher surface heave reflected to the pavement. As illustrated in Figure 9.11, after wetting to a period of two months, heave increased by about 100 percent (from 28 mm to 57 mm) when γ_h was increased from 0.035 to 0.07. Similarly, the results in Figure 9.13 indicates that after two months of wetting the lateral displacement of free edge of subgrade

with $\gamma_h = 0.07$ could reach to about 65 mm compared to 30 mm with subgrade having $\gamma_h = 0.035$.

Without altering the magnitude of free swell coefficient γ_h , an increase in γ_o from 0.04 to 0.08, did not have any significant effect in the subgrade swell behaviour. This might be due to very small overburden pressure (just the weight of pavement) which caused insignificant swell suppression even under higher value of γ_o .

At the earlier stage of wetting, lateral stress development in pavement was less affected with increase in the magnitude of swell indices. The effect should more pronounce in the later stage of wetting. But, because of many elements reaching to yield state or tension failure/cut-off at the later stages of wetting, the distinct effect of increase in subgrade swell indices was not very clear from the analysis results. However, higher stress should have developed with higher magnitude of free swell coefficient as it resulted higher surface heave.

D. Wetting Pattern

The nature of vertical and lateral stresses developed in the subgrade due to wetting under two dimension where moisture entered through the base of pervious shoulder was relatively different than that under one dimensional horizontal wetting front. However, after a relatively longer period of wetting (say two months), results of response of an over all structure in terms of maximum heave at the pavement outer edge (Figure 9.14) was almost the same order of magnitude under both types of flow. With the free outer edge, the shape of distortion of pavement excluding the shoulder section, was also similar to one dimensional flow situation. The nature of lateral stresses in pavement bottom face under two dimensional flow condition (Figure 9.15a) was similar to one dimensional flow (Figure 9.12).

Regarding the lateral displacement of free right edge as shown in Figure 9.16, the maximum displacement at the base of subgrade was about of the same magnitude in both cases of flow condition. But the lateral extension of the pavement edge under 1-D flow or shoulder outer edge under 2-D flow were different. With 2-D flow, this lateral extension was higher. It was because under two dimensional flow a number of pavement elements underwent yielding and resulted excessive lateral displacement.

9.5 Conclusions

Edge heave and cracking of pavement due to lateral tensile stress are the major concerns in the analysis. With the right edge free for lateral displacement, the magnitude of maximum edge heave resulting from a continuous wetting of an extremely high swelling subgrade for two months was found to be of the order of 60 mm. When both the pavement and subgrade were fixed against lateral displacement, the section under the same situation resulted a higher value of heave up to 85 mm. The distorted shape which is the nature of pavement bending near its outer edge and the location and propagation of tensile cracks also depended on lateral fixity. After wetting for two months, many elements at the outer half of pavement reached tensile stresses higher than the tensile strength (10 kPa), and hence either failed in tension or reached the tension cut-off. At the initial stages of wetting, lateral stresses in the subgrade when it was fixed against lateral swelling, reached very high compression (up to a maximum of 75 kPa near the outer edge of an extremely high swelling subgrade, Figure 9.3) and vertical stress in tension reached up to a maximum of 8 kPa near the mid width (Figure 9.4). But with increasing wetness, the magnitude of these stresses gradually decreased. It was also observed that the nature and magnitude of these stresses and displacements in both the pavement and subgrade were sensitive to many parameters which have been well discussed above. The following are the critical parameters identified.

1. **Wetting period** is considered the major parameter that directly controls the magnitude of swelling and swell pressure development of the subgrade section. The nature and magnitude of stresses and displacements in the pavement are very sensitive to this parameter.
2. **Lateral fixity against displacement** affects the mechanical response of both subgrade and pavement sections. Distorted shape of pavement on its outer edge including the location of potential cracking and crack propagation is different under different types of lateral fixities on its outer edge. From the extreme situations analysed, a fixed right edge in both pavement and subgrade produced a higher magnitude of heave for all sets of values of swell indexes. When lateral fixity in pavement layer only existed, relatively small effect in the development of both stresses and displacements in pavement were observed.

The above conclusion is based on extreme situations of lateral fixity (Section 7.2.3B) applied in the analysis. However in most of the field situations, the

right edge usually subjects to a partial lateral support. In such situation, the degree and nature of surface heave and cracking would be different, and the road section may behave as a transition between free and fixed condition.

3. **Free swell coefficient** is another major parameter in terms of development of edge heave and lateral displacement of both pavement and subgrade. An increase of about 100 percent was observed in vertical heave when γ_h was increased from 0.035 to 0.07. However, the effect of increase in the magnitude of swell suppression index, γ_o in the stress/strain response of both subgrade was found to be small because of low overburden pressure.
4. **Change in wetting pattern** from one dimensional horizontal wetting front to two dimensional where moisture entered through the shoulder had relatively less effect when the magnitudes of maximum edge heave of pavement and lateral displacement of subgrade are concerned. On the other hand, some effects on lateral extension of pavement were observe. Also the nature of stresses developed during wetting of subgrade was different with two dimensional case. Under two dimensional wetting front, relatively more number of elements at the mid width of pavement as well as in subgrade reached yield states.
5. With the **vertical interfaces included** in the Mohr-Coulomb model of pavement, opening of interfaces represented the possible nature and location of pavement cracking.

CHAPTER X

NUMERICAL MODELLING: PARAMETRIC STUDIES ON THE EFFECTS OF SUBGRADE SHRINKAGE

10.1 Introduction

In this chapter a parametric study is conducted on the results of analyses of response of pavement subjected to the drying and shrinkage of underlying subgrade. The typical section of road consisting of a thin seal coated pavement overlying a shrinking clay subgrade described in Figure 8.1, was considered in the numerical analysis. The different sets of combination of shrinkage indexes were selected, and the different displacement boundary conditions were applied. Like in the swelling subgrade situation, the results of numerical analyses indicated that the development of shrinkage stresses and subsidence upon drying of a reactive clay subgrade, and the response of overlying pavement depended on several subgrade soil parameters and boundary conditions. In this chapter, the effects of variations of these soil parameters, and boundary conditions are discussed.

10.2 Parametric Study

Like in the analysis of swelling subgrade, the results of shrinkage analyses indicated that the behaviour of subgrade and the response of pavement largely depended on the following parameters

1. Period of drying
2. Restraint against lateral displacement
3. Magnitude of free shrinkage and shrinkage compression indexes
4. Drying pattern in subgrade

Before going into the detailed parametric studies, a few observed limitations associated with the program and the analysis procedure for the drying and shrinking of subgrade applied in the present study are indicated here.

A. Limitation Associated with the Conceptual Model

According to the conceptual model of shrinkage compression described in Chapter V the modulus of shrinkage compression ' E_c ' is the function of vertical stress σ_{yy}

and the log of total suction 'log h'. This ignores the effect of lateral stresses σ_{xx} and σ_{zz} which are tensile in nature, for a soil shrinkage under a restrained boundary condition.

B. Modification of Mohr-Coulomb Strength Parameters with Suction

During the analyses, the subgrade strength parameters: cohesion 'C' and tensile strength 'T' were adjusted with respect to the value of the total suction of the elemental soil after every time of alternating analyses of moisture flow and induced stresses and strains. According to FLAC's rule of tensile failure criterion discussed in Section 9.2, once any of the major, intermediate and minor principal stresses exceeds the tensile strength of a Mohr-Coulomb material, the tensile strength will be cut to zero, and the corresponding stress will be set to zero. This means when the tensile stress in any element of subgrade becomes greater than its tensile strength at any stage of drying, the value of tensile strength of that element will be cut to zero. But when program calls the function subroutine for strength and stiffness modification, the same element possessing zero tensile strength due to tension cut-off/failure will update its tensile strength (higher than zero) with respect to the new value of total suction.

When considering the first limitation, it is apparent that the behaviour of shrinking soil in a three dimensional environment would vary significantly with the mean stress level rather than the vertical stress alone. Thus the concept of vertical stress dependency representing one dimensional shrinkage under a typical oedometer test situation inherits serious limitations. The writers acknowledged this limitation, and suggested for future research development to refine and extend the conceptual model to include the effects of lateral stresses also. However, for a parametric study the writers used the existing model to carry out the analyses.

Regarding the second limitation, results obtained at several steps of alternate analysis of moisture flow and mechanical equilibrium indicated that the subgrade elements for the analysed cases, did not reach the tensile stress larger than the strength in any element at all stages of drying. This implies that in the present analyses, the second limitation has not seriously affected the results.

10.2.1 Drying Period

The drying of subgrade was modelled under horizontal one dimensional flow. As shown in Figure 8.4, moisture exits from the right edge of the subgrade section. As indicated in Section 8.5.2 the drying process is based on the reverse of wetting

process. Thus the method of analysis using thermal analogy and the interpretation of results obtained are similar to that of the wetting process.

A. Subgrade Response

As indicated in Figure 7.3.2 the strength and stiffness of subgrade soil depended largely on the degree of dryness. With increasing dryness, the increase in modulus of shrinkage compression followed the pattern described by Figure 7.4. Also, the Increase in Mohr-Coulomb strength parameters followed linearly with the increasing dryness (Section 8.4.2 B).

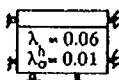
To study the effect of drying period, the results of analysis of 'Case 2-B'  are considered. Case 2-B physically represents a situation where an extremely shrinking clay subgrade undergoes a monotonic drying out and shrinking under a laterally restrained boundary conditions.

Figure 10.1 illustrates the progressive development of lateral normal stress σ_{xx} along the X direction at the mid-depth of the subgrade section after different periods of continued drying: 4, 10, 19, 31 and 64 days. (Note that the period of 31 days is considered as one month, and 64 days as two months). Because of the lateral fixity on both edges representing a fully restrained shrinkage situation, the lateral stress σ_{xx} that developed during drying, and experienced by all elements in the section were tensile. The stress development was increasing towards the outer edge where moisture exited and the subgrade started to shrink.

As seen in the Figure 10.1, even after 4 days of drying, significant magnitude (about 22 kPa) of σ_{xx} was developed on the outer edge elements and transmitted to the relatively wet inner zones. The magnitude of σ_{xx} gradually decreased in the inner zone. This was because the elements in the inner zone were still wet and possess the initial lower stiffness. The magnitude of lateral stress σ_{xx} increased with the period of drying. The increase was sharper in earlier stages (before one month) and gradual afterwards when all the elements were dried to certain degree. The maximum value of σ_{xx} after two months of drying reached to about 65 kPa near the outer edge where moisture exited. This magnitude belongs to the element which has dried from initial total suction of 1.7 log kPa to the final total suction of 2.97 log kPa.

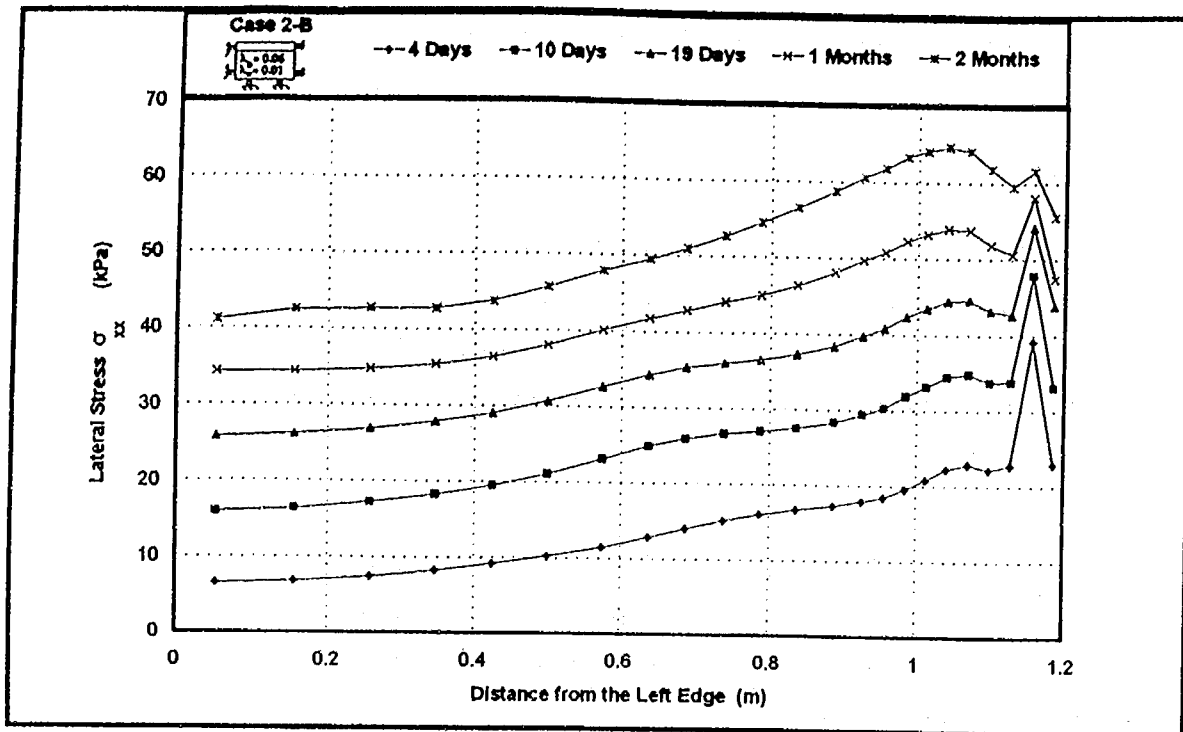


Figure 10.1 Development of lateral stress σ_{xx} at the mid depth of subgrade section.

Note that as seen in Figure 10.1, some subgrade elements near the outer edge where moisture exited started to yield at an early stage of drying. This yielding phenomenon was attributed to the excessive stressing of the outermost element as a result of the instantaneous drying of the exit boundary. This excessive tensile stress developed was transmitted to the adjoining inner element which was still relatively wet and had small stiffness, and therefore it reached yield state. However, elements previously at yield, came back to an elastic state when the drying advanced further, and the inner elements also started to have increased stiffness. This initial yielding near the outer edge was found to have a little effect on the subgrade mechanical response only at the earlier stage.

Figure 10.2 shows the development of vertical stress σ_{yy} along the mid-depth of the subgrade. The magnitude of both compression in the inner elements and tension in the outer elements except the outermost element which is in compression, increased consistently with the period of drying. The plot of two months showed a decrease in magnitude of peak tension.

The development of tensile stress in the outer half of the subgrade section except the outermost element was attributed to the effect of varying lateral shrinkage between subgrade's inner and outer regions. The tensile stress was also developed due to the lifting effect of the pavement as a result of its unequal subsidence.

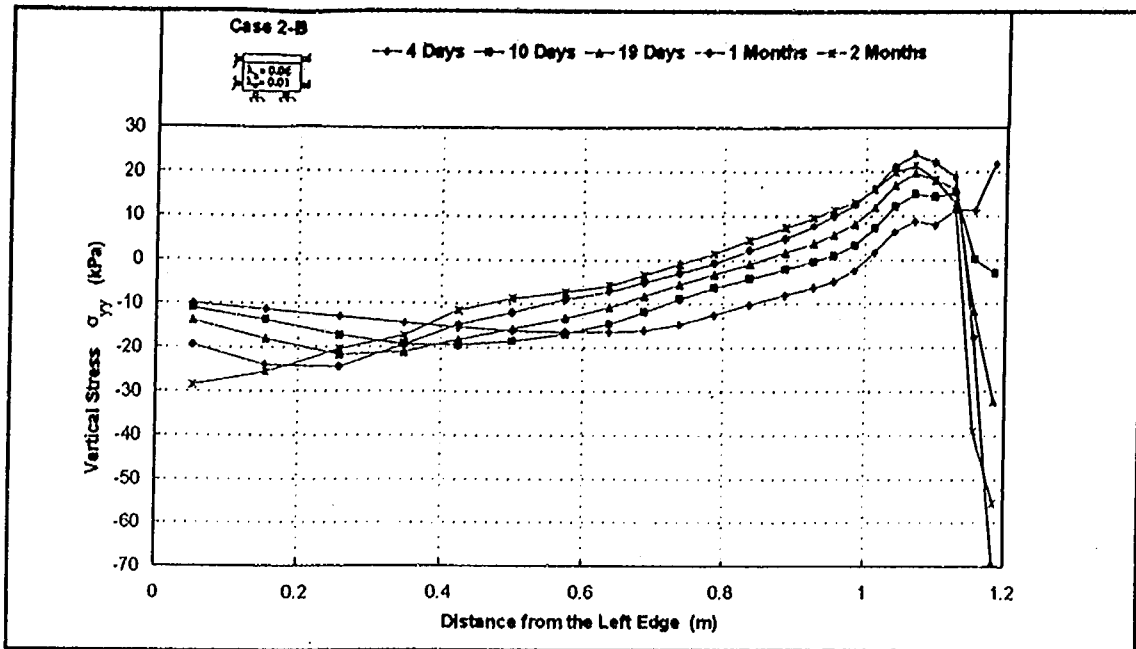


Figure 10.2 Development of vertical stress σ_{yy} at the mid depth of subgrade section.

B. Response of Pavement

Pavement Surface Subsidence

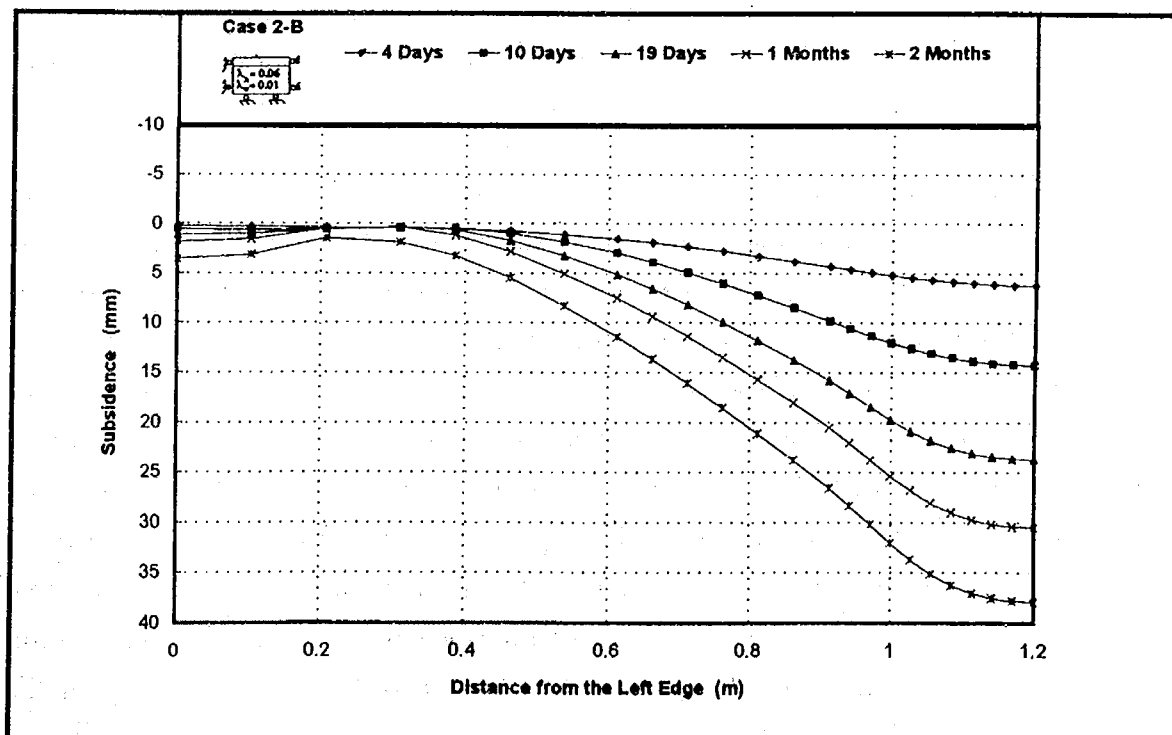


Figure 10.3 Progressive subsidence of pavement surface with the drying period

Figure 10.3 shows the progressive development of subsidence on the overlying pavement surface as a result of subgrade shrinkage. As expected, the surface subsidence increased consistently with the period of drying, because more and

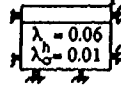
more inner elements dried out to higher degree and subsidence progressively advanced inwards. For the considered situation ('Case 2-B' ) after two months of drying, the surface subsidence reached to about 38 mm over the 100 mm width near the outer edge. The extent and magnitude of subsidence, however, largely depended on the lateral fixity and the values of shrinkage parameters which will be discussed in the latter sections.

Figure 10.3 also indicates that the surface of pavement settled a little (about 4 mm) in the innermost region (close to the left edge) even though the subgrade soil has not dried in this region. The settlement is attributed to the initial elastic compression of soft wet subgrade under the overburden pressure.

Lateral Stresses in Pavement

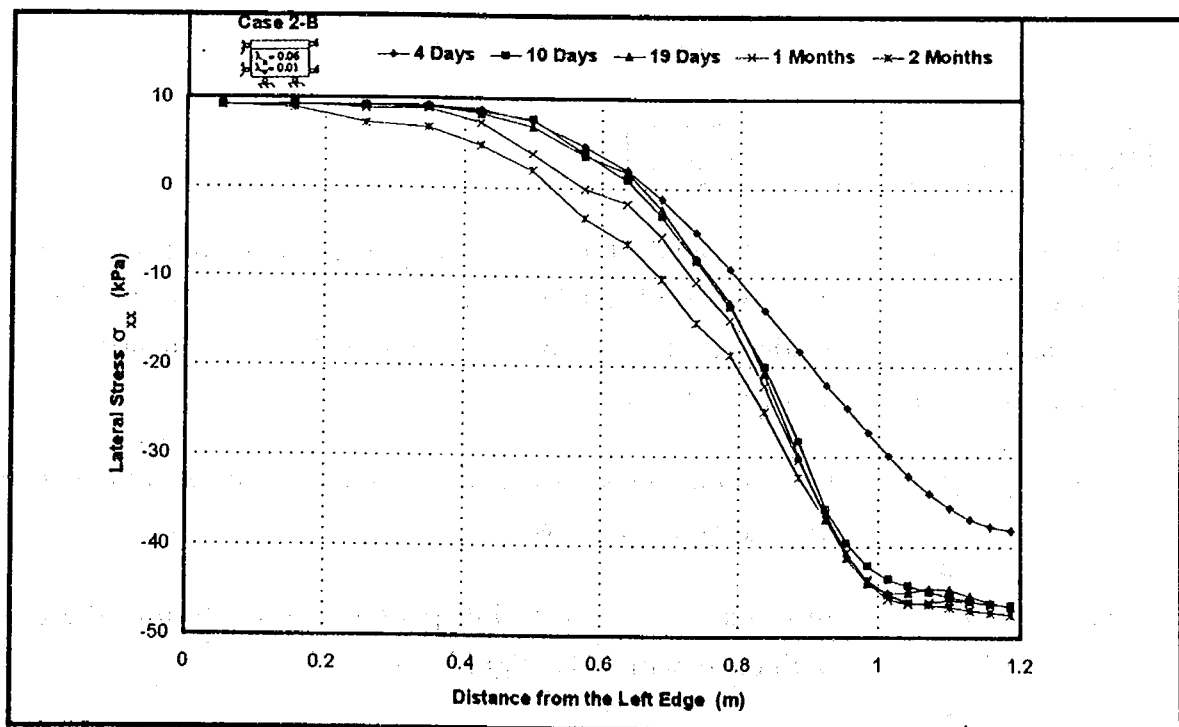
Figures 10.4 a and b illustrate the development of lateral stress σ_{xx} at the top and bottom faces of pavement. Since the mechanical behaviour of the pavement was modelled with the Mohr-Coulomb material having limited tensile strength ($T=10$ kPa), elements subjected to tensile stress over 10 kPa underwent uniaxial or general tension failure or tension cut-off. Thus the value of σ_{xx} in pavement when it exceeded 10 kPa in tension dropped to zero, and therefore the resulting values of σ_{xx} in tension in these figures are always less than 10 kPa.

An important point which needs to be considered in analysing the development of lateral stress in the pavement section, is the role of the interface between the pavement and the subgrade. The degree of transmission of lateral stresses developed in the shrinking subgrade to the overlying pavement depends largely on the shear strength and stiffness of the interface elements. In the illustrated case (as well as in most of the cases of analysis), this interface was treated in a manner described in Section 10.2.2.A. The bond strength, cohesion and friction angle of the interface were 120 kPa, 60 kPa and 30° respectively. Hence with the limited bond strength between the two layers, when the component of shear stress or normal tension exceeds the strength of interface, the interface opens up resulting separation of pavement from the underlying subgrade. This results some deficiency in the transmission of effects of subgrade to the overlying pavement.

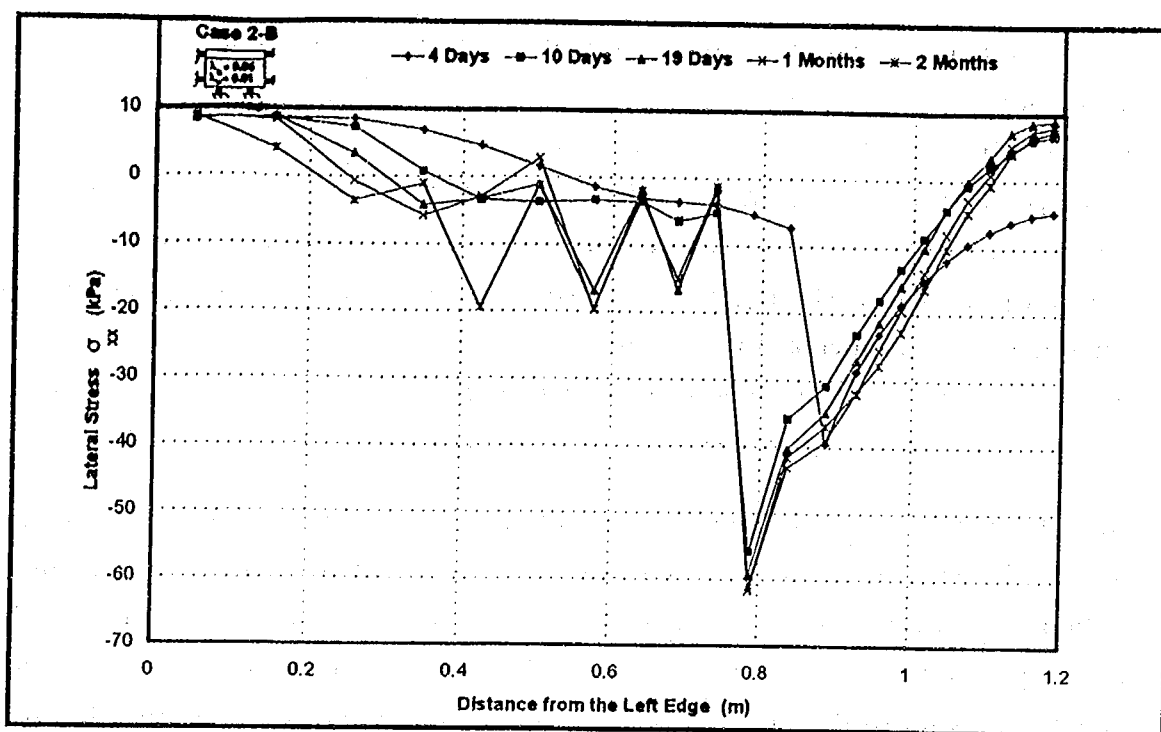
At the top face, as depicted from Figure 10.4 a the nature of progressive development of lateral stress σ_{xx} was consistent with the advancement of drying

process. Initial compressive stress gradually increased in the outer half. Over the width of 250 mm near the outer edge, the compressive stress development was already significantly high (about 35 kPa) after 4 days of wetting. The development of compressive stress remained constant (about 45 kPa) in the elements within this width after 10 days. This was because interface has opened up and the separation between pavement and subgrade occurred. The tensile stress development in the inner half and its increasing magnitude near the inner edge was the result of bending of pavement as illustrated in Figure 10.5 b. The bending of pavement results tension on the upper face near the inner edges.

The lateral stress σ_{xx} developed in the bottom face of pavement was compressive around the mid width and tensile near the inner and outer edge of pavement section. The development of tensile stress in these areas were basically due to the bending of pavement as a result of unequal shrinkage compression of underlying subgrade. As illustrated in Figure 10.4 b the development of σ_{xx} around the mid width of the section was inconsistent as a result of opening of interface and other difficulties associated with the program.



(a) Lateral stress σ_{xx} on the top face of pavement



(b) Lateral stress σ_{xx} on the bottom face of pavement

Figure 10.4 Lateral stressing of pavement due to subgrade shrinkage at the different period of drying

10.2.2 Effect of Lateral Restraint

The concept of lateral fixity against the horizontal displacement presented in Section 9.3.2 in the swelling subgrade analysis was also applied here. The following three situations are considered. The explanation and limitations on the application of these lateral fixities were discussed in Section 7.2.3.

Case A - 

free lateral displacement on the right edge for both pavement and subgrade sections,

Case B - 

restrained against lateral displacement on both the edges for both pavement and subgrade, and

Case C - 

restrained against lateral displacement on both the edges for pavement layer but free for subgrade at the right edge.

Obviously the development of subsidence and lateral shrinkage stress depends largely on the displacement boundary conditions of the sides. To study the extent of effect due to the different degrees of lateral fixity, a typical situation with

subgrade soil having magnitude of shrinkage indexes as $\lambda_h = 0.06$ and $\lambda_o = 0.01$ representing a high shrinking clay is considered.

A. Treatment of Interface between Subgrade and Pavement

Initially the analysis was conducted without providing interface between subgrade and pavement. The overall section of road was considered as a composite section consisting two different materials with different stiffness and strength. The results of analysis showed that with the increase in shrinkage, subgrade elements started to pull down the adjoining pavement elements. This resulted vertical extension of pavement elements. The extension in the height of pavement elements was attributed to the low magnitude of their stiffness (5 MPa). This resulted an undesirable increase in the thickness of pavement.

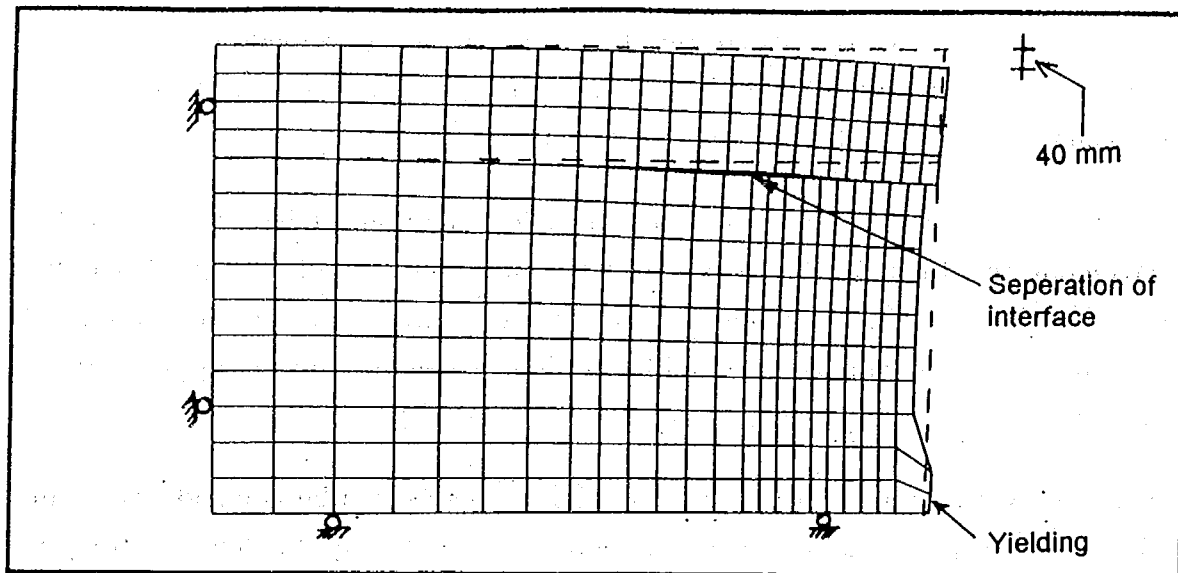
By increasing the stiffness of pavement resulted increasing lateral stresses in the pavement and thus caused early development of tensile failure/cut-off. On the other hand, it was not possible to increase the magnitude of tensile strength of pavement as the existing value of 10 kPa already belonged to the upper limiting value for a seal coated granular pavement (Section 6.5.5).

Finally it was decided to apply interface element between subgrade and pavement. The properties of interface was selected by considering the tensile strength, cohesion and friction angle as equal to the tensile strength, cohesion and friction angle of dried stiff reactive clay which are 120 kPa, 60 kPa and 60° respectively. The assumption was made with respect to the concept of inter layer bond. The selection of magnitude of shear and normal stiffness of the interface is described in Section 8.6.

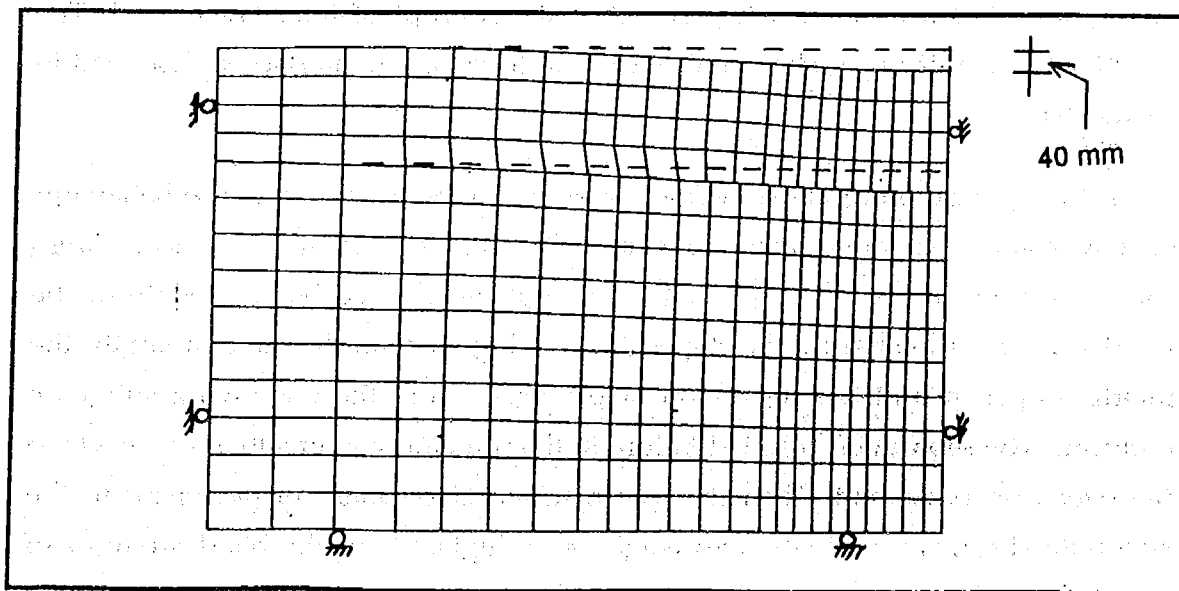
With the application of above defined interface elements, the response behaviour of pavement was more accurately modelled till the tensile or shear stress component did not exceed the magnitude of above defined bond strength of the interface. Once the stresses developed became higher than the bond strength, the interface opened and the separation of pavement from the underlying subgrade occurred. The separation resulted failure in the transmission of subgrade effects to the overlying pavement. Then the true mechanical response of pavement to the subgrade shrinkage with the shrinkage stress higher than the bond strength of interface could not be achieved.

B. Distorted Section and Pavement Surface Subsidence

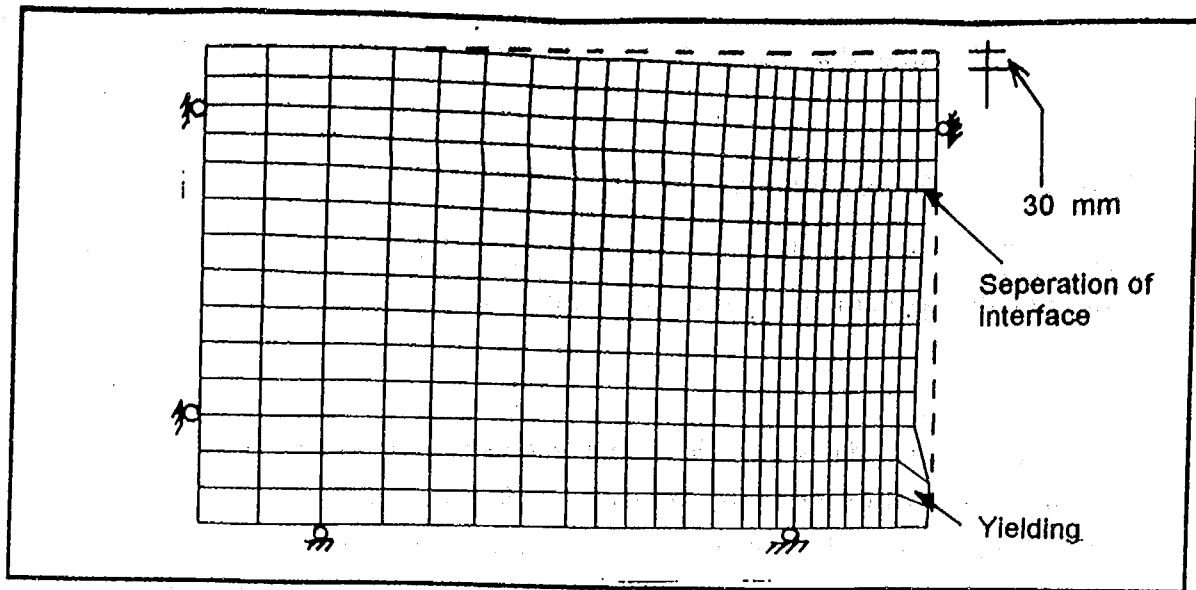
Figures 10.5 a, b and c illustrates the distorted shapes of the pavement and subgrade sections under different situations of lateral fixities. These distorted figures are obtained from the result of analyses with subgrade dried to a total period of two months. Figures indicate the typical bending characteristics of overlying pavement with the changing faces of compression and tension. Particularly, the faces subjected to the tension initiate cracking in the pavement. Such cracks progressively propagate to the inside of the pavement thickness.



(a) When right edge is free for both subgrade and pavement



(b) when right edge is fixed against lateral displacement for both subgrade and pavement



(c) when right edge is fixed for pavement and free for subgrade.

Figure 10.5 Distorted grid of pavement and subgrade sections when high shrinking subgrade ($\gamma_h = 0.06$, and $\gamma_\sigma = 0.01$) dries out and shrinks for a period of two months. (Broken lines indicate the original boundary of the section)

In Figures 10.5 a, b and c also included is the magnitude of vertical subsidence developed in the pavement after two months of subgrade drying under the different situations of lateral fixity. During the analyses it was observed that the Case 'As'  and Case 'Bs'  developed higher values of surface subsidence than the Case 'Cs' .

In summary, the results of present analyses showed that when a road section having an high shrinking clay subgrade ($\lambda_h = 0.06$, and $\lambda_\sigma = 0.01$) undergoes drying for a period of two months, the subsidence of the pavement surface under confined lateral condition i.e., Cases '2-A' and '2-B' can reach to a magnitude of about 40 mm at the outer edge. The magnitude of such surface subsidence reached only to a value of 30 mm at the outermost edge in Case '2-C'.

C. Lateral Stresses in Pavement

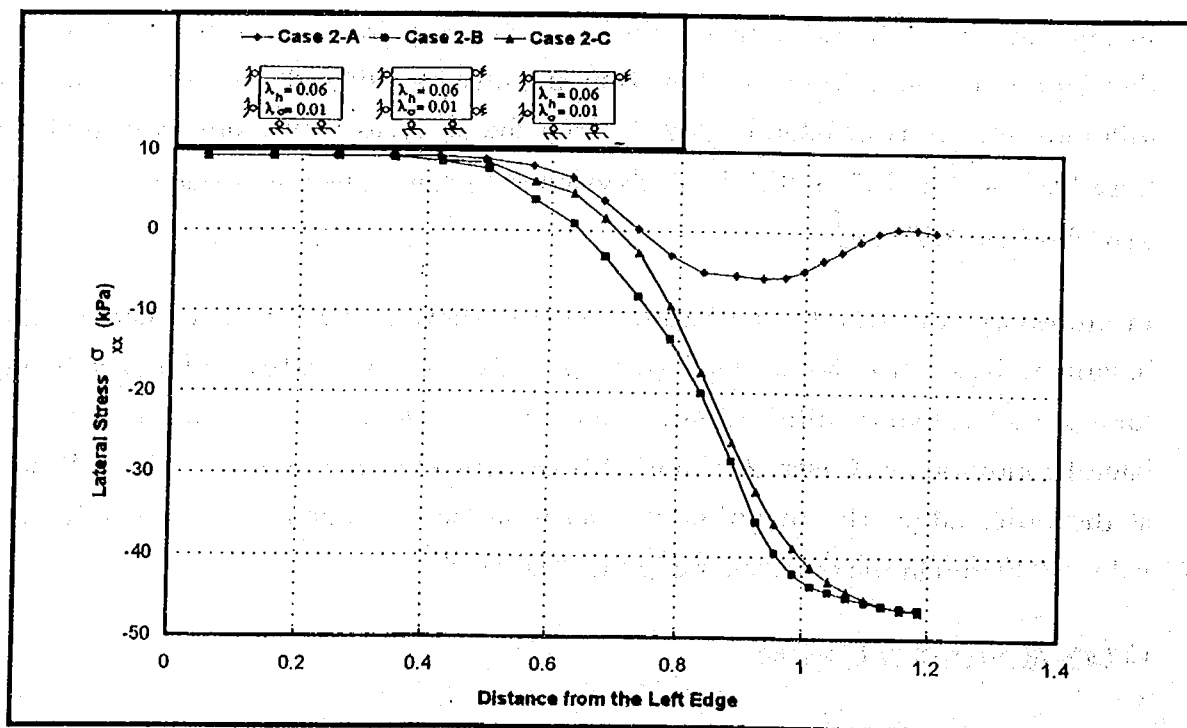
As illustrated in the distorted shape of the sections in Figure 10.5 depending on the lateral fixity, tension cracks in the pavement initiated either from the bottom face or from the top surface of the pavement. Hence, plots of lateral stress and deformation

in Figures 10.6 and 10.7 are presented for the top and bottom face of pavement, respectively.

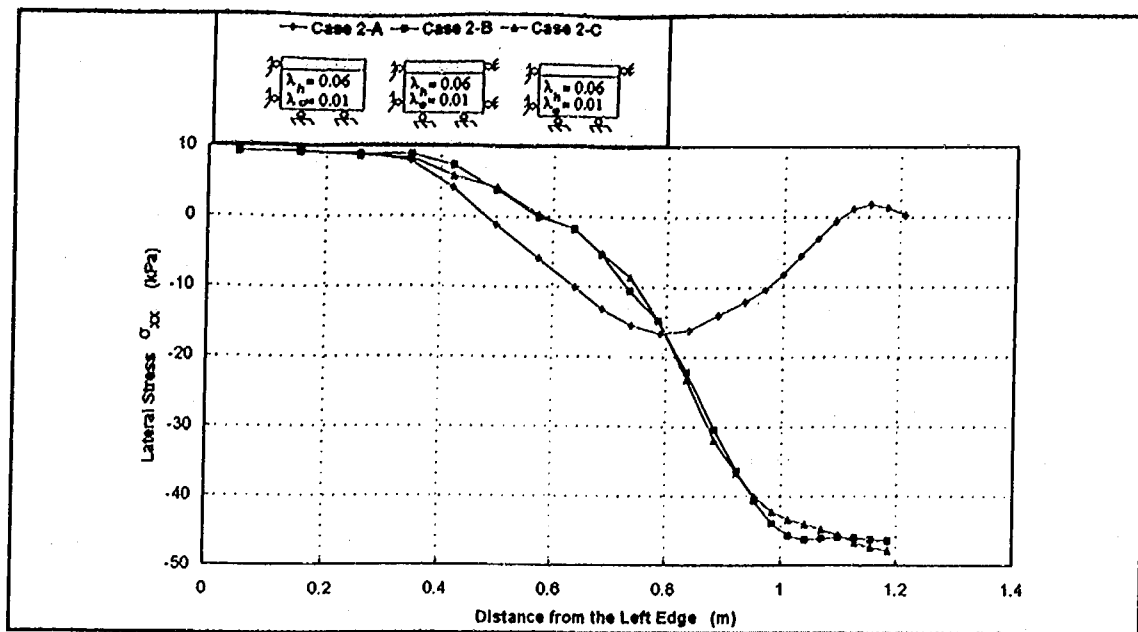
(i) At the top surface of pavement

Figures 10.6 a, b and c illustrate the development of lateral stress σ_{xx} on the top surface of the pavement at three different stages of drying: 10 days, one month, and two months.

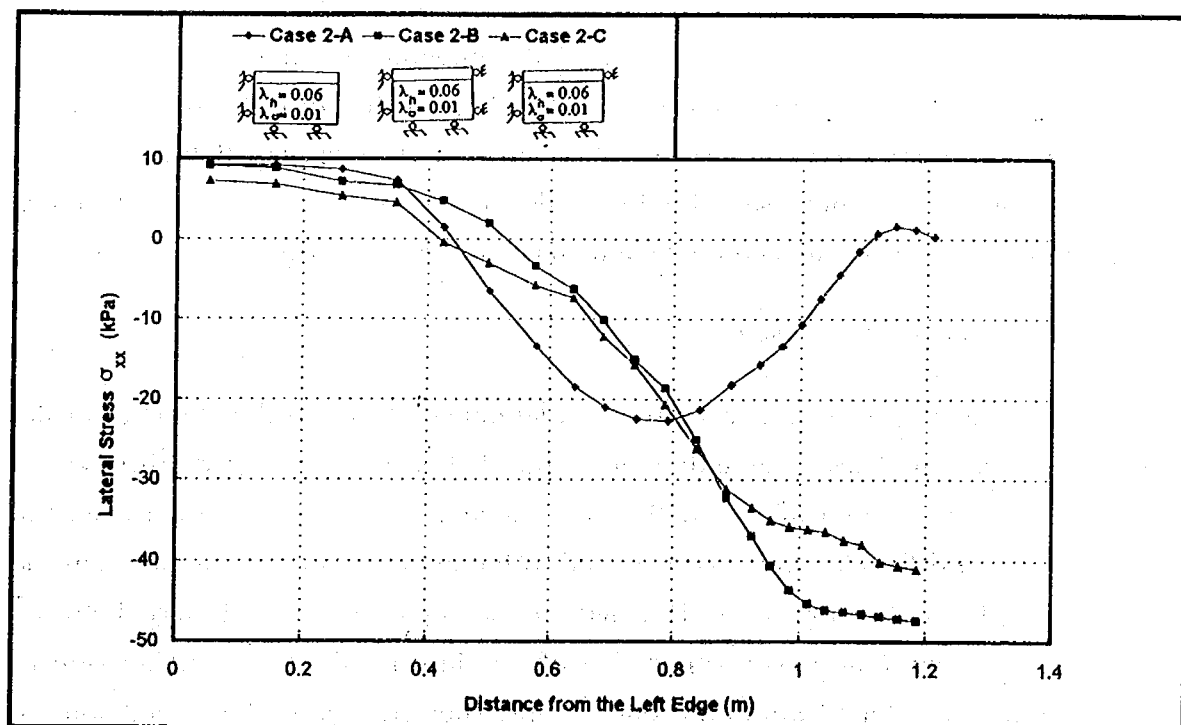
Case 2-A  Tensile stress close to the tensile strength of pavement (10 kPa) developed in most of the inner elements while little compressive stress developed in the elements near the outer edge. The compressive stress was attributed to the transmission of the shrinkage stress that developed on the underlying drying subgrade. The development of tensile stress in the inner elements were due to the bending of the pavement with tension on the upper face. With the increasing drying period the peak value of compressive stress gradually increased to 23 kPa near the mid width after two months of drying. The number elements near the inner edge which were initially under tension gradually decreased with drying period when the compressive stress on the mid width elements increased.



(a) after 10 days of drying



(b) after 31 days of drying



(c) after 64 days of drying

Figure 10. 6 Lateral stress σ_{xx} developed on the pavement surface at different stages of subgrade drying under three different conditions of lateral fixity

Case 2-B  Because of the lateral fixity, bending resulted in tension on the upper face of the inner half of the pavement section. This tensile stress in the inner elements were further increased with the transmission of lateral drag of the subgrade which is tensile in nature. As shown in Figure 10.6 b, from the early period of subgrade drying (about a week) a number of elements near the inner edge reached and remain under tensile stress of 9.2 kPa. This resulted yielding of these inner elements. When the period of drying increased, the compressive stress in the outer elements gradually increased, and some of the elements on the left of the mid width that were in yield under tension slowly fell under compression.

The elements on the outer half as depicted in Figures 10.6 a, b and c, developed compressive stress from the early stage of drying (from 10 days). Its magnitude reached to about 45 kPa and remained constant for the whole drying period of two months.

Case 2-C  It should reveal a combination of situations presented by Case 2-A  and Case 2-B . But because of the opening of interface between the pavement and subgrade occurred at the later stage of drying (after one month of wetting), the lateral stress development as illustrated in Figure 10.6 a, b and c, became similar to that of Case 2-B with tensile stress on the elements near the inner edge, and compressive stress on the elements near the outer edge.

(ii) At the bottom face of pavement

Figure 10.7 below presents development of lateral stress σ_{xx} in the elements at the base of pavement. These elements were connected with the subgrade through an interface. As mentioned above at the later stage of drying (more than a week), lateral shear and tensile stresses developed in the interface became higher than the bond strength of the interface. This led to the separation of interface between pavement and subgrade. The separation of interface resulted no transmission of subsequently developed subgrade effects, and showed inconsistent behaviour of pavement base elements as depicted in Figure 10.7. Due to this difficulty presented by interface separation after 10 days of drying, analysis of results is limited to 10 days.

Case 2-A  At the earlier stage (at 10 days) of drying, inner elements were subjected to a tensile stress of about 9 kPa and these elements reached yield state. In the later stage of drying, some of these elements came back under elastic state when compression developed in the mid width elements gradually increased with increasing drying period. The compressive stress developed after 10 days of drying was 25 kPa near the mid width.

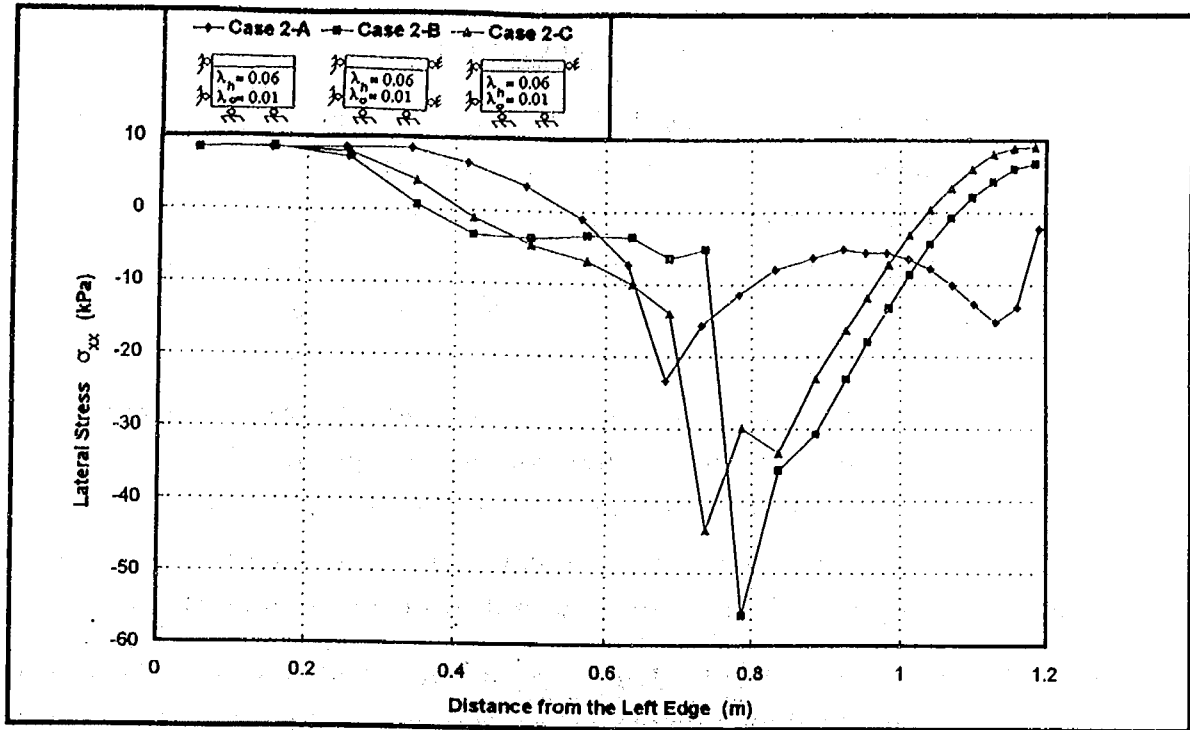


Figure 10.7 Lateral stress σ_{xx} developed on the base of pavement after 10 days of subgrade drying under three different conditions of lateral fixity

Case 2-B  At 10 days of subgrade drying higher compressive stress was developed in the elements of middle section. The peak value reached to about 55 kPa on the element near 800 mm away from the inner edge, and gradually decreased further outward. The elements near the inner edge developed tensile stress. The development of compressive stress around the mid width and tensile stress on the inner edge is attributed to the dragging effect of subgrade under varying degree of lateral shrinkage. The shrinkage of subgrade increases outwards and results dragging of adjoining pavement elements towards the mid width. The magnitude of lateral stress σ_{xx} in tension near the inner edge is about 9 kPa, and some elements underwent shear yielding. Also a few elements near the outer edge developed tensile stress due to the transmission of lateral tensile stress developed by the shrinking subgrade under laterally fixed boundary condition.

Case 2-C  The nature of lateral stress in almost all elements until 10 days drying was similar to Case 2-B as shown in Figure 10.7, with tensile stress near the inner edge and near the outer edge, while compressive stress on the middle section. The magnitude of tensile stress near the outer edge is relatively higher than in Case 2-B because in this case the underlying subgrade is laterally free for shrinkage.

10.2.3 Effect of Magnitude of Shrinkage Indexes

As seen in Table 8.2 (Section 8.2.2), the equation describing variation of shrinkage modulus with suction is different for different sets of values of shrinkage compression index. For an increase in suction from 1.5 log kPa to 2.0 log kPa, the first set of shrinkage compression index as $\lambda_{\sigma} = 0.01$ in Case-1 ($\lambda_h = 0.04$, $\lambda_{\sigma} = 0.01$) and Case-2 ($\lambda_h = 0.06$, $\lambda_{\sigma} = 0.01$) gave β_c as 213.58, while the second set as $\lambda_{\sigma} = 0.02$ in Case-3 ($\lambda_h = 0.06$, $\lambda_{\sigma} = 0.02$) gave β_c as 106.79. The different values of β_c results different magnitude of shrinkage compression modulus. Meanwhile, the free shrinkage index λ_h directly controls the volume change in subgrade with the change in suction level. Thus, at the same stage of drying, stress and deformation behaviour of the subgrade section for a similar lateral fixity would be different for different sets of magnitude of free shrinkage and shrinkage compression indexes.

To study the effect of variation of these shrinkage indexes in the response of pavement and subgrade at the different stages of drying, a typical situation having free lateral displacement boundary for both pavement and subgrade () is considered where both the pavement and subgrade sections were characterised with Mohr-Coulomb material. The following subsections discuss the effect of indexes on the magnitude of subsidence and the lateral stressing in the pavement.

A. Pavement Surface Subsidence

Free shrinkage index λ_h is the major parameter that controls the degree of subsidence. At all stages of drying, as expected subsidence of pavement surface was always higher with Case 2 and Case 3 ($\lambda_h = 0.06$, $\lambda_{\sigma} = 0.01$; and $\lambda_h = 0.06$, $\lambda_{\sigma} = 0.02$ respectively) than with Case 1 ($\lambda_h = 0.04$, $\lambda_{\sigma} = 0.01$). Figure 10.8 shows the pavement surface subsidence at the end of two months of drying. It was observed that without altering the shrinkage index λ_h , there was no significant increase in subsidence with an increase in shrinkage compression index λ_{σ} from 0.01 to 0.02. The reason was accounted to the very low overburden pressure (only the weight of overlying pavement) to which the subgrade was subjected.

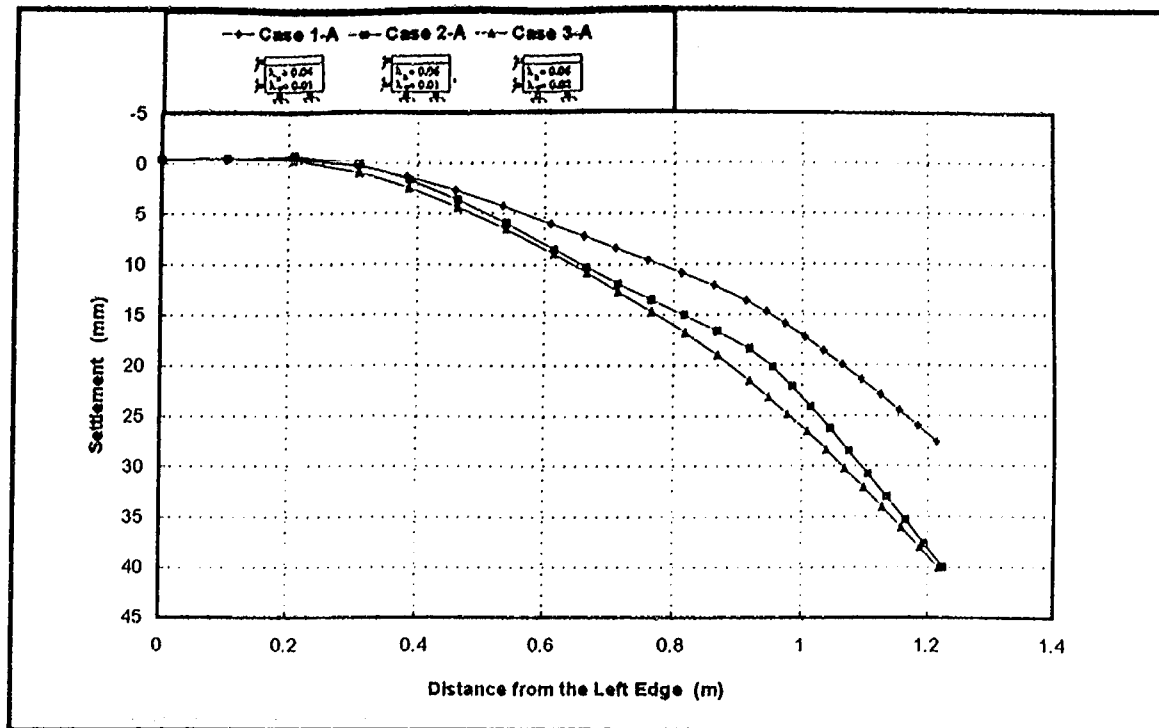
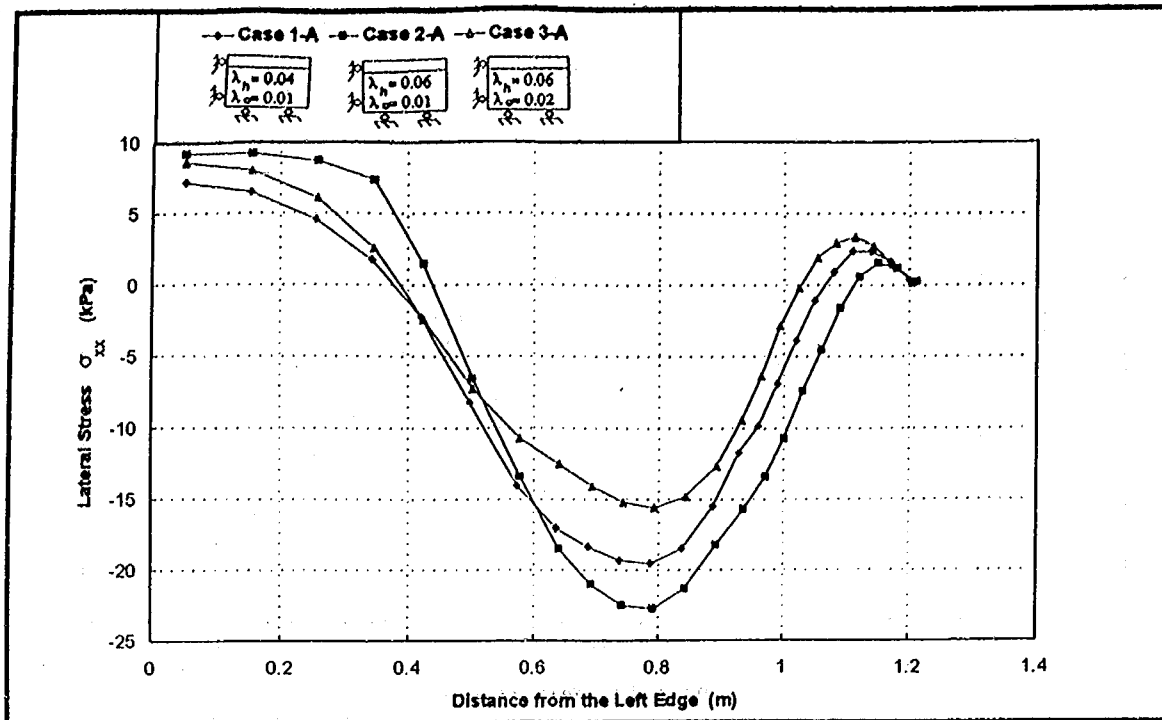


Figure 10.8 Subsidence of pavement surface after two months of drying with different sets of magnitude of subgrade shrinkage indexes

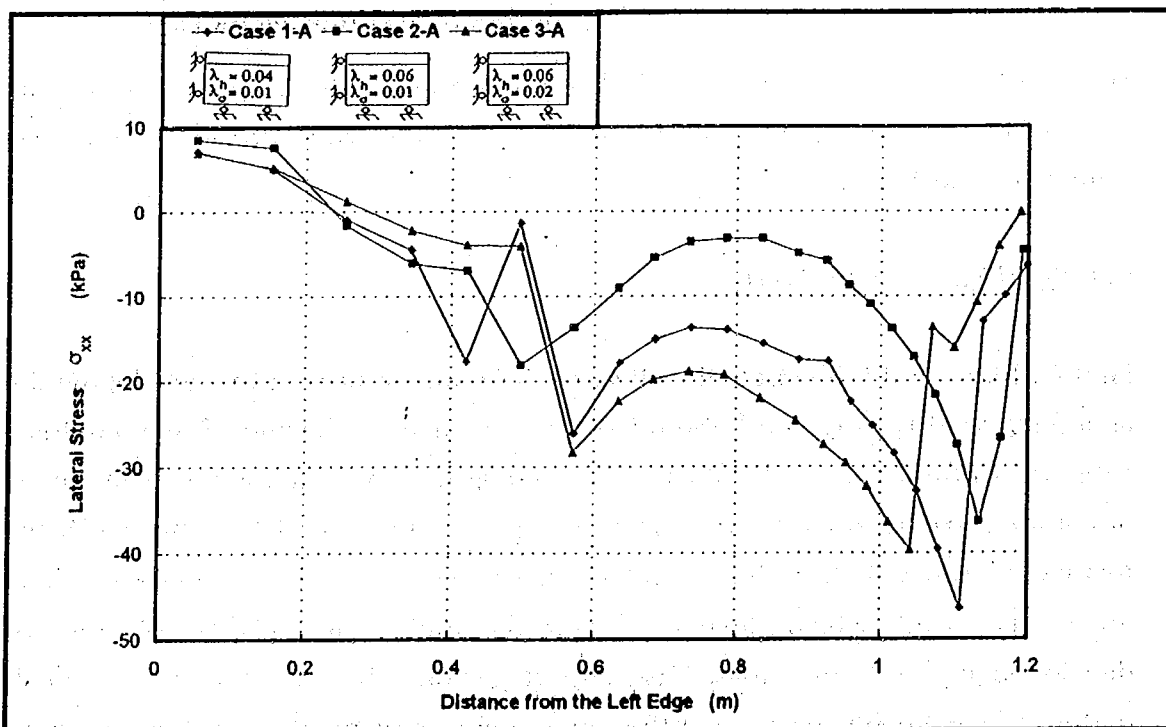
The results presented in Figure 10.8 also indicate that with an increase in l_h from 0.04 to 0.06, after two months of drying, the maximum edge subsidence increased from 28 mm to 40 mm.

B. Lateral Stressing in Pavement

Figures 10.9 a and b illustrate the effect of shrinkage indices in the development of lateral stress at the top and bottom faces of pavement at the end of two months of subgrade drying. The results illustrated in the figures showed that the development of compressive stresses on the top face of pavement around the mid width was relatively lower with Case 3-A than the other two cases Case 1-A and Case 2-A. As for tensile stress which developed near the inner edge, Cases 2-A and 3-A developed higher values than Case 1-A. Regarding the lateral stress s_{xx} on the bottom face elements, because of the separation of pavements from the underlying subgrade occurred as a result of interface opening at the later stage of drying (more than 10 days), the result obtained as illustrated in Figure 10.9 b was inconsistent.



(a) Lateral stress σ_{xx} at the top surface elements of the pavement



(b) Lateral stress σ_{xx} on the elements at the base of pavement

Figure 10.9 Lateral stresses in pavement after two months of subgrade drying for different sets of shrinkage indices

C. Lateral Extension of Subgrade Section

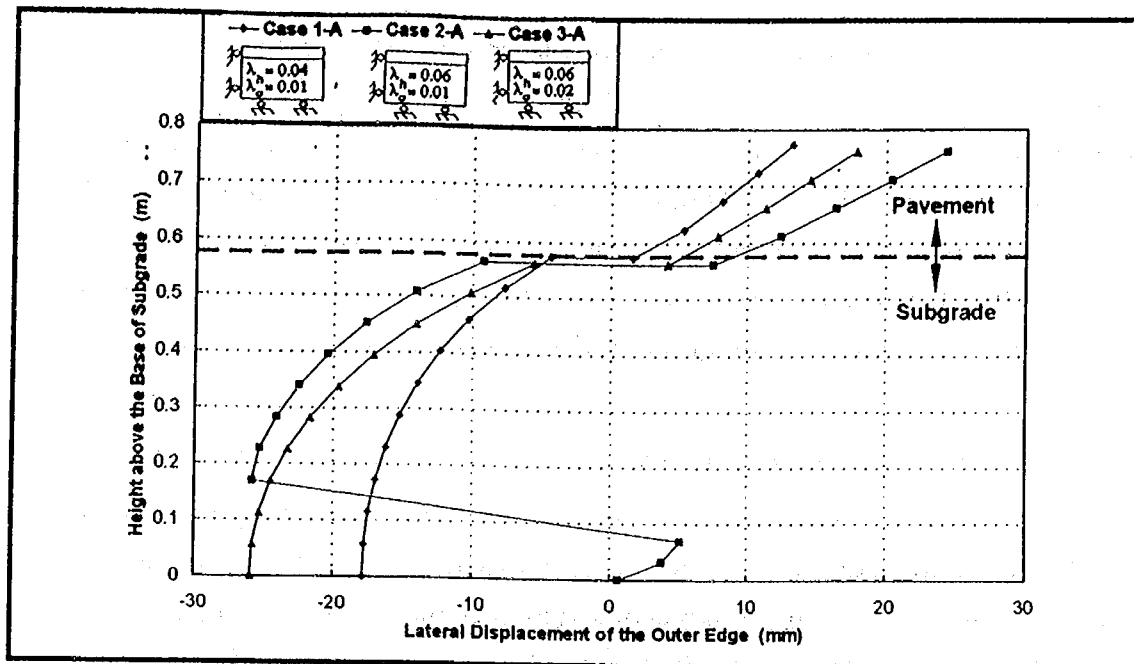
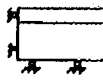


Figure 10.10 Lateral displacement of subgrade after two months drying under different sets of shrinkage indices

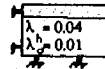
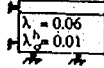
It was observed in the analysis that with the free right edge, at all stages of drying the inward lateral displacement due to the shrinkage was significantly higher with subgrade having higher free shrinkage index λ_h . As shown in Figure 10.10 after two months of drying, without altering shrinkage compression index λ_v , Case 2-A having shrinkage index $\lambda_h = 0.06$ developed higher lateral displacement than Case 1-A (with $\lambda_h = 0.04$). With the constant value of shrinkage index λ_h as 0.06, an increase in shrinkage compression index λ_v from 0.01 to 0.02 resulted only a small difference in lateral displacement as can be seen by comparing Cases 2-A and 3-A in Figure 10.10. Note that because of high vertical stressing under the laterally free condition of the right edge, three elements on the column at the outer edge in Case 2-A underwent yielding and resulted excessive straining outward.

Regarding the lateral displacement of pavement, because of shrinkage subsidence of subgrade on the outer half, the top corner of the outer edge of pavement got bent and extended outward by a magnitude of about 20 mm.

10.2.4 Subgrade Drying Pattern

A typical situation having free lateral displacement on the right edge  was considered under a two dimensional drying pattern. The width of the section was increased to 1.8 m to accommodate a 600 mm wide pervious shoulder. The height of the section was still kept 800 mm total with a 600 mm deep subgrade and a 200 mm thick overlying pavement. Moisture in this situation exited from the underneath of pervious shoulder. Under this pattern of subgrade drying, the stress/strain behaviour of subgrade and the response of pavement are discussed below.

A. Subgrade Stress/Strain Development

With the changing direction of moisture exit, the nature and magnitude of stresses developed in the subgrade section varied considerably with the drying period. Analyses showed that at all stage of drying, elements on the upper half of the subgrade section were observed to develop tension in the lateral direction (σ_{xx}) due to the restraining effect of pavement on the shrinkage of soil. Elements on the lower half underwent some minor compression. With the increasing drying period, depth of tension zone also gradually increased. The difference in the magnitude of lateral stress (σ_{xx}) development under different sets of shrinkage indexes was found to be relatively small. However, as expected, subgrade having lower value of shrinkage indexes as in Case 1-A  showed relatively lower level of stress development than the one having higher shrinkage indexes, Case 2-A .

Regarding the vertical stress σ_{yy} , before the drying of subgrade, all the section had an initial overburden pressure equal to the gravity stress (eg., 13.7 kPa at the centre of the elements on the base). When the section was dried to a period of two months, the initial compression in each cases decreased in most of the elements in the middle width of the section. But the initial compression was gradually increased in the elements near the inner and outer edges. On the other hand, because of relatively lower overburden pressure, the effect of increase in shrinkage compression index λ_v was not observed.

B. Response of Pavement

Figure 10.11 shows a comparative development of subsidence in the pavement surface when the subgrade having similar shrinkage condition dried to a period of 2 months under one dimensional drying process discussed in earlier sections and under two dimensional drying process. The results presented in the figure indicate

that the magnitude of maximum subsidence under two dimensional drying is almost equal to the corresponding magnitude under one dimensional flow in Cases 1-A and 3-A. For example, the maximum subsidence of 27 mm, 30 mm and 38 mm were observed in Cases 1-A, 2-A and 3-A under two dimensional drying where as under one dimensional drying the corresponding values of subsidence were 28 mm, 40 mm and 40 mm. The difference in subsidence under one dimensional and two dimensional drying for Case 2-A is not clear.

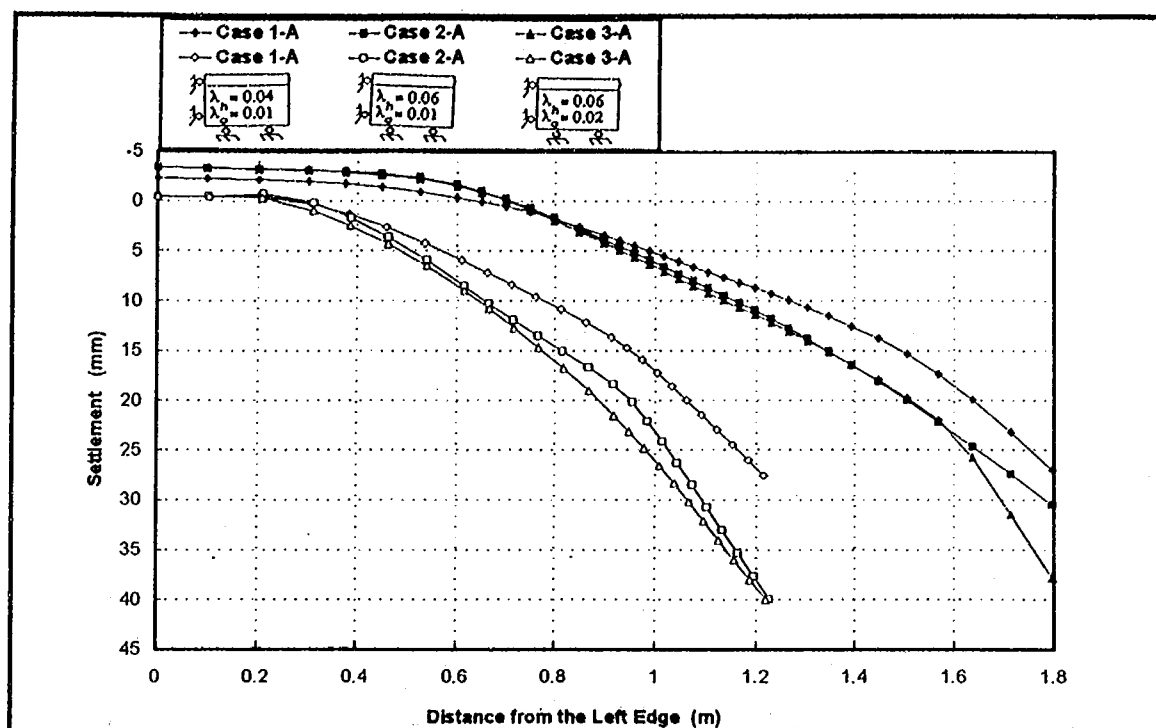
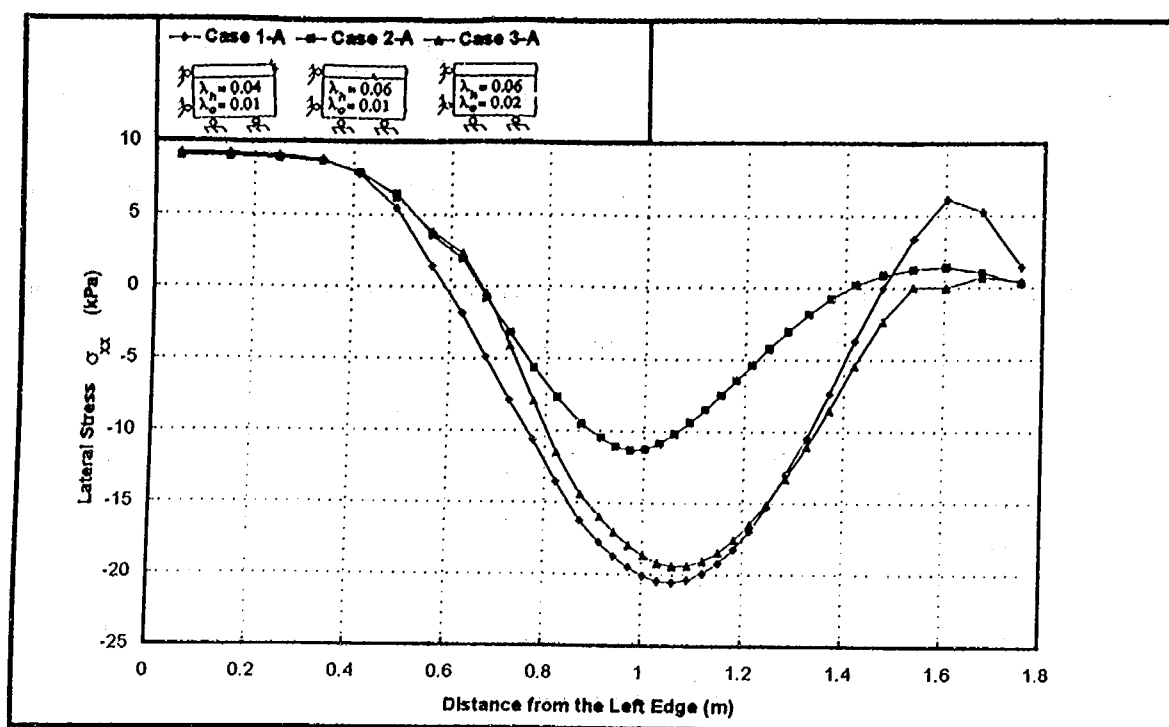
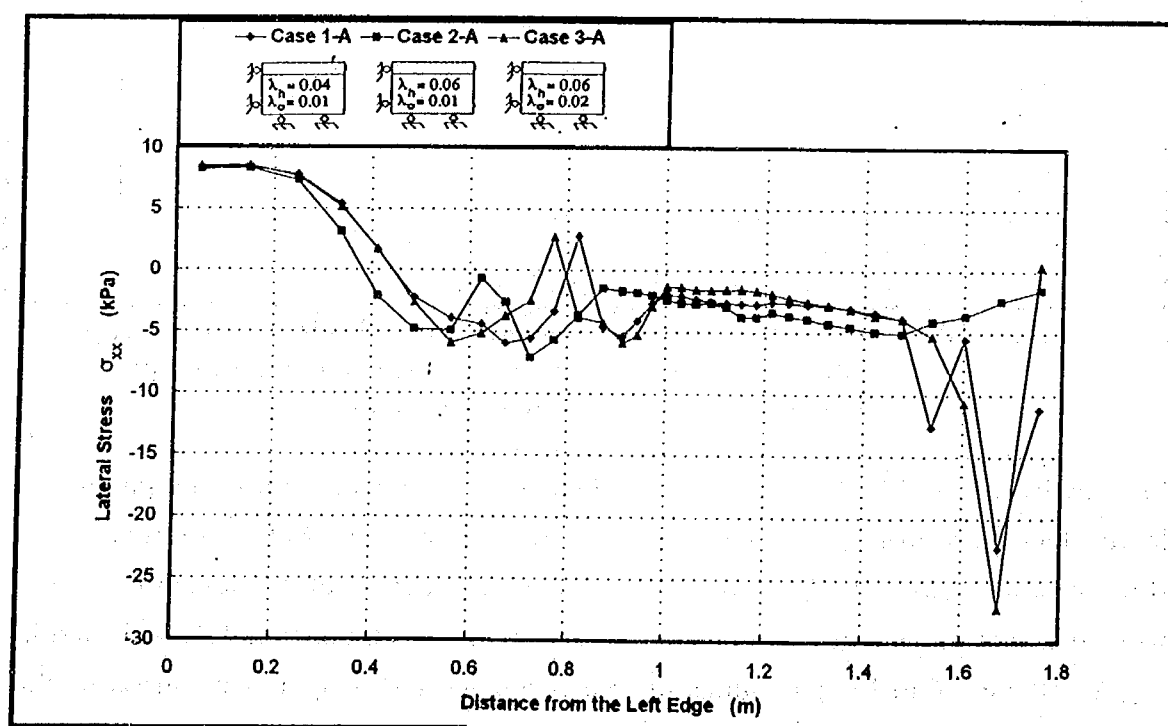


Figure 10.11 Comparative development of subsidence on pavement surface under one and two dimensional drying processes, when subgrade dries out to two months

Figures 10.12 a and b illustrate the lateral stress ' σ_{xx} ' development in the pavement lower face and upper surface when the underlying subgrade undergoes two dimensional drying for a period of two months. The nature of σ_{xx} developed at the top surface of pavement is similar to the cases with one dimensional flow shown in Figure 10.9 a. With higher values of free shrinkage index λ_h , tensile stress in the inner edge as well as compressive stress in the mid width elements are higher than with lower values of λ_h .



(a) At the base of pavement



(b) At the top of pavement

Figure 10.12 Lateral stress σ_{xx} on the pavement under 2-D flow after two months of drying processes

Regarding lateral stress σ_{xx} in the bottom face elements, as depicted from the Figure 10.12 b, the nature of lateral stress σ_{xx} in all cases are similar for the inner elements except near the outer edge. In all cases tensile stress of about 7.5 kPa developed

near the inner edge due to the subgrade's dragging effect. Elements around the mid width showed inconsistent behaviour as those elements reached tension cut-off during the earlier period of drying.

C. Lateral Displacement of the Section

Figure 10.13 shows a comparative plots of lateral displacements of the right edge when the subgrade was dried to two months under one and two dimensional drying processes. The shape of displacement varied considerably with the drying pattern. With two dimensional drying, the whole lower section of subgrade contracted relatively uniformly. But the maximum displacement at the bottom level of subgrade under both one dimensional and two dimensional flow are similar. As for the lateral displacement of the pavement layer, with two dimensional flow the outer edge of pavement is less displaced than with one dimensional flow. Regarding the lateral shrinkage displacement of subgrade with the variation of magnitude of shrinkage indexes, as would be expected, the magnitude of lateral shrinkage was higher with the subgrade having higher free shrinkage index ' λ_h '. But without changing λ_h , the effect of change in the magnitude of shrinkage compression index ' λ_σ ' was very small.

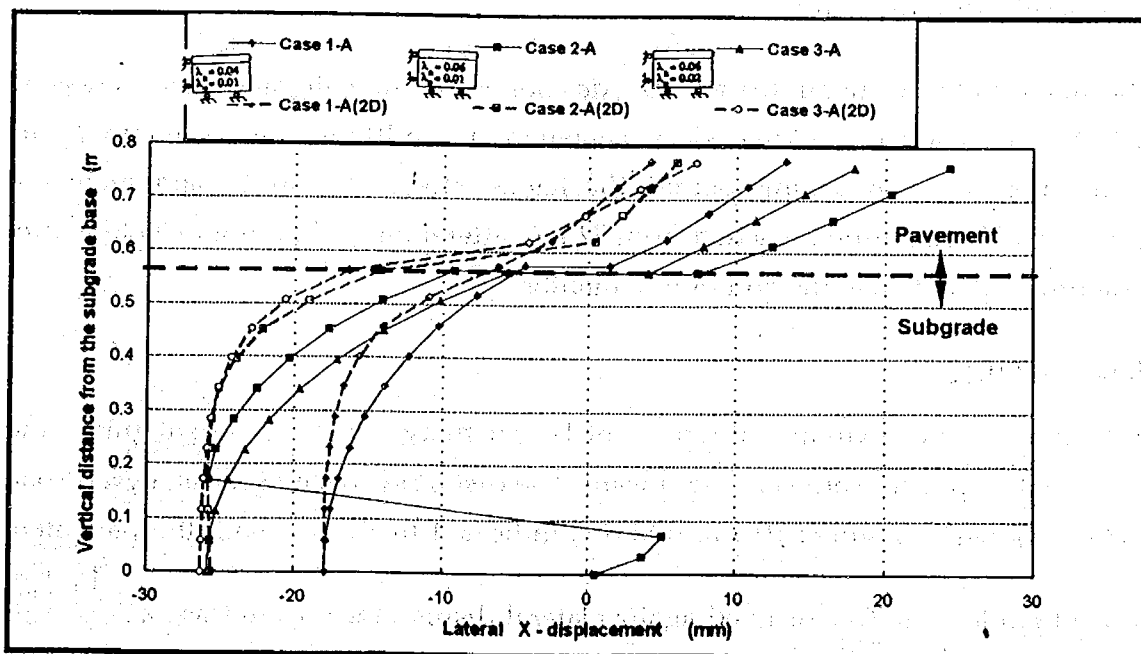


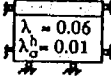
Figure 10.13 Comparative results on lateral displacements under 1-D and 2-D drying fronts

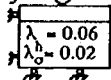
10.3 Review of Parametric Analysis

The discussions presented above on the analysis of results obtained at different drying period for different cases of lateral fixities, different combinations of magnitude of shrinkage parameters, and under two different drying pattern showed that some of the parameters considered have significant effect on the response of subgrade and the overlying pavement when the subgrade dries out and shrinks. In this section, the sensitivity of these parameters are evaluated and discussed.

A. Drying Period

As discussed in Section 10.2.1 for all situations under all types of lateral fixities and magnitude of shrinkage indexes, the extent of subsidence and shrinkage stress in subgrade and consequent response of pavement depended largely on the drying

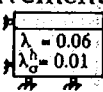
period. For a typical situation, Case 2-B  shown in Figure 10.3, the subsidence on the outer edge of pavement increased from about 15 mm after 10 days of drying to 30 mm after a month, and 38 mm after two months. Similarly, for

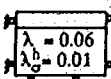
Case 3-A , the inward lateral displacement of the subgrade base at its free right edge increased from 21 mm after one month drying to 28 mm at the end of two months (Figure 10.10).

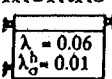
The magnitude of lateral stresses σ_{xx} developed in the subgrade was increasing with the increasing drying period. As shown in Figure 10.1 for subgrade mid depth, with an increase in drying period the lateral stress σ_{xx} in tension increased gradually. Its magnitude reached from 22 kPa after 4 days of drying to 55 kPa after a month, and 65 kPa at the end of two months.

B. Lateral Fixity

With the onset of extreme situations of lateral fixity, its effects in subgrade and pavement response were very significant. The distorted shape of the analysed road section shown in Figures 10.5 a, b and c, indicated that when both the pavement

and subgrade were free or fixed against lateral displacement as in Case 2-A  or,

Case 2-B , subsidence on the pavement surface at the end of two months drying reached to about 40 mm compared to about 30 mm in Case 2-C

 where only the pavement layer was restrained.

Regarding the lateral stress σ_{xx} in pavement, Figures 10.6 a, b and c showed that the elements on the top face of pavement near its outer edge in Case 2-A subjected lower magnitude of compression than in other two cases: Case 2-B and Case 2-C.

C. Effect of Shrinkage Indexes

Evidently, higher magnitude of free shrinkage index ' λ_h ' resulted higher shrinkage stress and subsidence in the subgrade, and consequently higher surface subsidence reflected to the pavement. With higher magnitude of free shrinkage index as in Case 2-A or Case 3-A ($\lambda_h = 0.06$ and $\lambda_o = 0.01$ or 0.02), subsidence of pavement surface after two months of drying (about 40 mm) was significantly higher than the subsidence (28 mm) in Case 1-A ($\lambda_h = 0.04$ and $\lambda_o = 0.01$).

Regarding the lateral displacement of free edge, after two months of drying, outer edge of subgrade with $\lambda_h = 0.06$ moved about 28 mm inward compared to about 18 mm with $\lambda_h = 0.04$.

Without altering the magnitude of free shrinkage index λ_h , an increase in λ_o from 0.01 to 0.02, did not have any significant effect in the subgrade deformation behaviour. This might be due to very small overburden pressure (just the weight of pavement) which caused insignificant shrinkage compression even under higher value of λ_o .

D. Drying Pattern

At the initial stage of drying, the nature of vertical and lateral stresses developed in the subgrade under two dimension flow where moisture exited through the base of pervious shoulder was relatively different than that under one dimensional horizontal drying front. However, after a relatively longer period (say two months), results of response of an over all structure in terms of magnitude of maximum subsidence of the pavement outer edge (Figure 10.11) and the nature of lateral stress development on the pavement (Figures 10.9 a and b versus 10.12 a and b) were almost the same for both types of flow.

Regarding the lateral displacement of free right edge as shown in Figure 10.13, the maximum displacement at the base of subgrade was about of the same magnitude in both cases of flow condition. But the lateral extension of the pavement edge under 1-D flow and under 2-D flow were different. With 2-D flow, this lateral extension was smaller than with 1-D flow.

10.4 Conclusions

In the parametric study conducted above, the nature and magnitude of the stresses and displacements in both the pavement and subgrade were found to be sensitive to several parameters. On the basis of the degree of their effects on subgrade and pavement responses, the following are identified as the critical parameters.

1. **Drying period** is considered as the major parameter that directly controls the magnitude of shrinking and shrinkage pressure development of the subgrade section. The nature and magnitude of stresses and displacements in the pavement are very sensitive to this parameter.
2. **Lateral fixity against displacement** affects the mechanical response of both subgrade and pavement sections. Distorted shape of pavement on its outer edge including the location of potential cracking and crack propagation is different under different types of lateral fixities on its outer edge. From the extreme situations analysed, cases with a free or fixed right edge in both pavement and subgrade produced a higher magnitude of subsidence for all sets of values of shrinkage indexes than the cases with lateral fixity in pavement layer only.
3. **Free shrinkage index** is another major parameter in terms of development of edge subsidence and lateral displacement of both pavement and subgrade. An increase of about 40 percent was observed in vertical subsidence when λ_h was increased from 0.04 to 0.06. However, the effect of increase in the magnitude of shrinkage compression index, λ_v , in the stress/strain response of both subgrade was found to be small in this analysis because of low overburden pressure.
4. **Change in drying pattern** from one dimensional horizontal drying front to two dimensional where moisture exited through the shoulder had relatively less effect when the magnitudes of maximum edge subsidence of pavement and lateral displacement of subgrade.

CHAPTER XI

CONCLUSIONS AND RECOMMENDATIONS

11.1 Conclusions

This research investigated the response of a seal coated road pavement subjected to the volume change of subgrade due to the wetting and drying phenomena. The work covered theoretical and numerical parametric studies on subgrade swell and shrinkage mechanisms and the mechanism of pavement response. Typical results of numerical analyses conducted in Chapter VIII and discussed in Chapters IX and X, concluded that a seal coated thin granular pavement can undergo the following distortion and distress when the outer edge of a road section is free for lateral displacement:

- (i) When the underlying extremely expansive clay subgrade undergoes a monotonic wetting for a period of two months:
 - (a) an edge heave of 57 mm can occur, and
 - (b) a longitudinal cracking of pavement initiates from the base at about 150 mm inside the outer edge and reaches to the pavement surface.
- (ii) When the underlying high shrinking clay subgrade undergoes a monotonic drying for a period of two months:
 - (a) an edge subsidence of 40 mm can occur, and
 - (b) a longitudinal cracking of pavement initiating from the top face of pavement can occur about 100 mm inside the outer edge.

For other soil type and lateral fixities, the type of distortion and the magnitude of distress are presented in detail in Chapters IX and X.

Besides the above conclusive results, the study also developed some important concepts on the mechanisms of subgrade moisture flow and volume change. It identified critical parameters that affect the corresponding response of pavement. These concepts and parameters are concluded below:

1. Characterisation of subgrade moisture and volume change

- (a) An alternative description of non-linear diffusivity relationship (Equation 2.7) resulting a suction dependent moisture conductivity was applied. This alternative

description produced a sharper shape of wetting front in the wetting process, and the results from analyses suggested that a non-linear diffusivity relationship was suitable to describe a monotonic wetting in unsaturated swelling clay subgrades.

The same relationship was extended and applied to the drying process by assuming that with the practical range of suction change, a monotonic drying of a wet soil subjects little effect of hysteresis.

(b) Typical values of Poisson's ratio, and swelling and shrinkage indexes were identified. Moduli of swell suppression and shrinkage compression were evaluated. Both these moduli were distinguished from the load deformation modulus of unwetted stiff dry soils. The load deformation modulus of unwetted soil was found to be at least 10 times higher than the swell suppression modulus and shrinkage compression modulus (Sections 4.9 and 5.3). This important distinction should not be overlooked in modelling the behaviour of reactive soils.

(c) The simple mechanistic models of swell and swell suppression, and shrinkage and compression adopted at this stage were not robust enough to describe the detailed stress and suction path followed during the process of wetting and swelling. They inherited fundamental assumptions of isotropy and homogeneity in the behaviour of soil. Refinements to overcome these limitations should form part of future research work (Sections 4.10 and 5.3).

(d) The result of a set of oedometer swell tests conducted previously on an "undisturbed", highly fissured, extremely expansive clay indicated that a lateral swell pressure of 190 to 240 kPa can develop under a vertical surcharge of 100 kPa. A parametric study conducted using values of swell parameters from the above measurements indicated that the calculated swell suppression modulus for the tested soil defined in the simple mechanistic model should be increased by a factor of 1.5 to 1.75 (Section 4.8).

2. Characterisation of stress/strain, and strength behaviour of a seal coated unbound granular pavement.

(a) The development of stresses and strains in the present situation was due solely to the subgrade volume change. The strain developed in a swelling or shrinking subgrade was significantly larger than the strain at breaking of an unbound granular layer with very limited tensile strength. Therefore a relatively simple elasto-plastic

model with limited tensile strength could be used to represent the stress strain behaviour of such a pavement.

(b) Results of a prototype pavement model test conducted previously on a seal coated unbound gravel base indicated a tensile strength of such pavement section to be about 5.4 kPa (Section 6.5.5).

4. Numerical analysis of a typical situation of subgrade/pavement interaction.

(a) A thermal analogy was applied to analyse the moisture flow and volume change behaviour of subgrade. Application of a transformation variable to relate the change in temperature to an equivalent change in log suction simplified the description of the wetting/swelling processes. It allowed the use of non-linear thermal diffusivity as equivalent to suction dependent moisture diffusivity. The coefficient of thermal expansion was identified as the free swell index (in 3-Dimensions) for swelling process and the free shrinkage index (in 3-Dimensions) for shrinkage process (Sections 8.4.2).

(b) Available finite difference program 'FLAC' is used to the situation. The program is versatile and allows the user to independently program the form of the changing constitutive relationships without the tedium and risks of developing a new suite of finite element analysis programs (Section 7.2.1).

The numerical procedure developed was verified for each process of: moisture flow, stress/strain and failure, volume change. The result of verification tests (Appendix E) indicated that the procedure gave reliable analysis.

(c) The above procedure was applied to investigate a typical situation of soil/pavement interaction for a wide range of reactive soils. The results of analysis indicated that the most critical factors affecting the response of the pavement to subgrade swell and shrinkage were:

- wetting period in swell analysis, and drying period in shrinkage analysis
- lateral fixity (pre-existing cracks in the subgrade, degree of shoulder support, and continuity of pavement across the shoulder, etc.) for both analyses and
- free swell index for swell analysis and free shrinkage index for shrinkage analysis (swell suppression as well as shrinkage compression indexes are not as critical at the relatively low overburden stress level).

(d) The role of interface between subgrade and pavement in the transmission of subgrade effects to the pavement should be significant (Section 9.3.2). In the present analysis, this interface is not applied, and hence its effect is not very well understood.

11.2 Usefulness of Present Work

This report contained a comprehensive study on the response of a seal coated thin granular pavement when subjected to the wetting up and swelling of underlying clay subgrade. As mentioned earlier in Section 11.1, during the course of research work, various parameters related to the unsaturated moisture flow and volume change of subgrade, and the development of lateral stresses and deformations of overlying pavement are identified. These findings are valuable when conducting analysis of subgrade movements and corresponding response of overlying pavement.

The experimental procedure described in this study are useful for a typical lateral swell pressure measurement in the laboratory. With some refinements, the newly developed pavement model test apparatus indicated and discussed in the report is useful for the study of the cracking behaviour seal coated pavement sections subjected to the lateral extension. The apparatus is also capable to measure the strength of such pavements under the direct tension.

The thermal analogy which is developed and applied for the numerical analysis, provides a relatively simple, accessible, powerful tool for the parametric studies on swelling clays behaviour. The work has gone further, and for the first time, deduced (from the literature) the typical ranges of characteristic values of swell and shrinkage indexes for such parametric studies on wide range of reactive soils. Such simplified method of obtaining soil parameters without conducting detailed laboratory measurements, is of great value for field engineers as it reduces the volume of work for an initial analysis.

Overall, the work could supply information to the practicing engineers about the possible distortion and cracking of pavements when thin seal coated roads is constructed over reactive soils. It could also present some simple data to the field engineers to evaluate potential volume change and the associate stress development. Results of this study would be useful for practicing engineer when analyses are to be

carried out for pavements subjected to the moisture flow and volume change behaviour of reactive clay subgrade.

11.3 Recommendations for Further Research

During the course of this research work, some important areas were identified where future research would further refine the description of the different phenomena involved in the problem to enhance the results obtained in the present analysis. Major areas of future research are listed as follows:

1. A further refinement of simple mechanistic models of swell and swell suppression as well as shrinkage and compression developed in this study is necessary to precisely define the volume change and associated stress development in reactive clay subgrades by rigorously following the stress and suction paths.
2. More detailed and precise laboratory measurements on swell pressure, heave, shrinkage stress and subsidence of reactive clays are required to validate the simple mechanistic models. It is important that "undisturbed" soil samples be used. The effect of soil macro-structure, such as fissures and cracks, should be considered carefully, because these have been identified as of paramount importance. A continuous, or at least a series of suction pressure measurements, should be undertaken during the test as this would be useful in the suction and stress path analysis.
3. Necessary refinement to the newly developed laboratory set-up of a prototype pavement model includes improvement to the mechanical loading system and better control of the initial state of the specimen.

The effect of bitumen seal coat on the tensile strength characteristics of pavement is not clear at this stage. There exists a need for further laboratory and theoretical investigation to establish the role of seal coat against cracking of pavement surface.

4. The inclusion of an interface element at the connection between pavement and subgrade will further refine the analysis of interaction between pavement and subgrade. It is recommended to use a well defined interface elements and properties in future numerical analysis.

APPENDIX A

DERIVATION OF SWELL AND SWELL SUPPRESSION EQUATIONS

A.1 Introduction

The swell and swell suppression equations discussed and used in the analytical model in Chapter IV are presented here with detailed step wise development. These equations are based on Wallace and Lytton (1992b). Derivation of equations in both types of material models; uniaxial modular dependence anisotropic material, and stress dependent isotropic material are as follows.

According to Equation (3.2) in Section 3.4, Young's Modulus 'E' in one dimensional oedometer condition can be obtained from the constrained modulus ' $\bar{E}$ ' as

$$E = \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \bar{E} \quad (A.1)$$

When Equation (4.3), which describes swell suppression is considered, Young's Modulus ' E_{s1} ', a tangent modulus describing suppression of swelling of the soil in the vertical direction, varies linearly with the applied stress and is related to σ_1 and Poisson's ratio ' ν ' by the Equation:

$$E_{s1} = \frac{\sigma_1}{\epsilon_v} \times \frac{(1+\nu)(1-2\nu)}{(1-\nu)}$$

$$\text{or} \quad E_{s1} = \frac{\sigma_1}{0.435 \gamma_{\sigma 1}} \times \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \quad (A.2a)$$

$$\text{or} \quad E_{s1} = \beta \sigma_1 \quad (A.2b)$$

This constitutive equation leads to a closed form solution which will be useful in the development of the following two material models.

A.2 Constitutive Relations

A.2.1 Uniaxial Modular Dependence (Anisotropic Case)

If an element of soil as shown in the figure below, undergoes swelling under a stress controlled environment such that the final constrained state is reached in three stages, namely:

1. free swell $-\epsilon_{10} = -\epsilon_{20} = -\epsilon_{30} = -\epsilon_{v0}/3$ (under σ_{ps} up to 7 kPa stress)
2. application of $\sigma_1 = \sigma_2 = \sigma_3$, and
3. increasing $\sigma_3 = \sigma_2$ from σ_1 to $K_s \sigma_1$

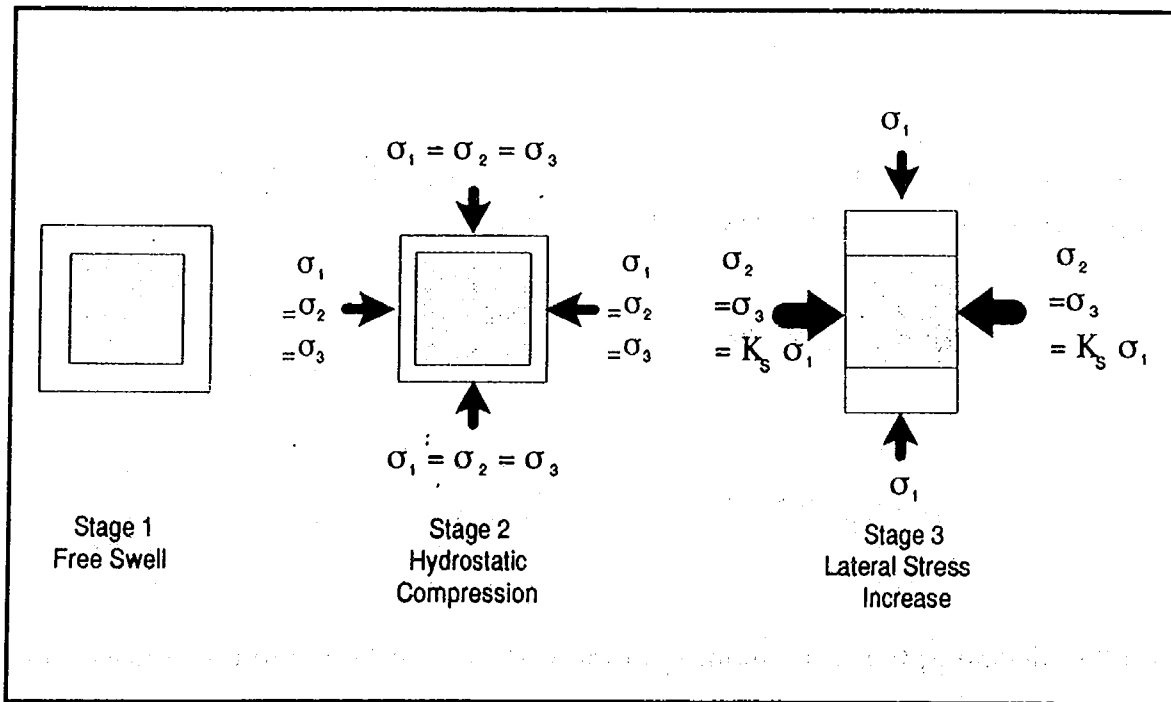


Figure A.1

For stage (2), incremental strain suppression, $\Delta \epsilon_{s1}$ due to $\Delta \sigma_1$ can be obtained by using Young's Modulus of swell suppression as

$$\Delta \epsilon_{s1} = \frac{\Delta \sigma_1}{E_{s1}} \quad (A.3)$$

Substituting Equation(A.2b) and integrating between σ_{ps} and σ_1 , the vertical swell strain due to σ_1 alone is determined as

$$\epsilon_{s1} = \int_{\sigma_{ps}}^{\sigma_1} \frac{1}{\beta \sigma_1} d\sigma_1$$

or
$$\epsilon_{s1} = \frac{1}{\beta} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) \quad (A.4)$$

For simultaneous application of $\sigma_1 = \sigma_2 = \sigma_3$,

$$\epsilon_{s1} = \epsilon_{s2} = \epsilon_{s3} = \frac{1}{\beta} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) - \nu \frac{1}{\beta} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) - \nu \frac{1}{\beta} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right)$$

or
$$\epsilon_{s1} = \epsilon_{s2} = \epsilon_{s3} = \frac{1-2\nu}{\beta} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) \quad (A.5)$$

For stage (3), integrating between σ_1 and $K_s \sigma_1$ yields lateral swell strain suppression due to an increase of σ_3 from σ_1 to $K_s \sigma_1$,

$$\epsilon_{s3} = \int_{\sigma_1}^{K_s \sigma_1} \frac{1}{\beta \sigma_1} d\sigma_1$$

or
$$\epsilon_{s3} = \frac{1}{\beta} \ln K_s \quad (A.6)$$

While calculating the total strain, ϵ_1 in the vertical direction, additional heave due to this lateral confinement in two directions is equal to

$$\epsilon_{s1} = - \frac{2\nu}{\beta} \ln K_s \quad (A.7)$$

Compression in the lateral direction is,

$$\epsilon_{s2} = \epsilon_{s3} = \left(\frac{1-\nu}{\beta} \right) \ln K_s \quad (A.8)$$

Summing up all the three principal strains, the three stages yield the following

$$\varepsilon_1 = -\varepsilon_{10} + \frac{1-2\nu}{\beta} \ln\left(\frac{\sigma_1}{\sigma_{ps}}\right) - \frac{2\nu}{\beta} \ln K_s \quad (A.9)$$

and

$$\varepsilon_3 = -\varepsilon_{30} + \frac{1-2\nu}{\beta} \ln\left(\frac{\sigma_1}{\sigma_{ps}}\right) + \frac{(1-\nu)}{\beta} \ln K_s \quad (A.10)$$

where

$$\beta = \frac{2.3}{\gamma_{\sigma 1}} \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \quad (A.11)$$

Equations (A.9) and (A.10) are the swell suppression equations for uniaxial modular dependence material.

A 2.2 Stress Dependent Isotropic Material

Wallace and Lytton (1992b) assumed that the modulus varies with the 'nth' power of octahedral normal stress such that

$$E_s = E_{s1} = E_{s3} = B \left(\frac{\sigma_1 + 2\sigma_3}{3} \right)^n \quad (A.12)$$

In this model, two cases are prevalent. One with coefficient of lateral swell pressure 'K_s' greater than 1, and the other with K_s less than 1 are considered

(1) K_s > 1 Here also the soil is considered to be swelling in three stages in a similar fashion as described earlier. The first and second stages are similar to the previous model. The relations, Equations (A.4) and (A.5), developed for the second stage in the earlier model is applicable if substitution for swell suppression modulus 'E_s' is made from Equation (A.12) such as

For first stage:

$$\text{free swell } -\varepsilon_{10} = -\varepsilon_{20} = -\varepsilon_{30} = -\varepsilon_{v0}/3 \text{ (under } \sigma_{ps} \text{ up to 7 kPa stress);}$$

and for second stage, $\sigma_1 = \sigma_3$, the deformation is described as

$$\varepsilon_{s1} = \varepsilon_{s2} = \varepsilon_{s3} = \frac{1-2\nu}{B} \ln\left(\frac{\sigma_1}{\sigma_{ps}}\right) \quad (A.13)$$

For stage three, incremental suppression $\Delta\epsilon_3$ in the lateral direction due to $\Delta\sigma_3$ can be obtained by using Young's Modulus of swell suppression ' E_{s1} ' as

$$\Delta\sigma_3 = E_{s1} \Delta\epsilon_3$$

Substituting the value of E_{s1} and rearranging

$$\frac{d\epsilon_3}{d\sigma_3} = \frac{3}{B} \left(\frac{1}{\sigma_1 + 2\sigma_3} \right) \quad (A.14)$$

Thus for this stage, integrating between σ_1 and $K_s\sigma_1$, with σ_1 as constant

$$\epsilon_3 = \int_{\sigma_1}^{K_s\sigma_1} \left\{ \frac{3}{B} \left(\frac{1}{\sigma_1 + 2\sigma_3} \right) \right\} d\sigma_3$$

$$\text{or, } \epsilon_3 = \frac{3}{2B} \ln \left(\frac{1 + 2K_s}{3} \right) \quad (A.15)$$

and the corresponding swell strain in the vertical direction due to lateral confinement is

$$\epsilon_1 = -2 \times \frac{3\nu}{2B} \ln \left(\frac{1 + 2K_s}{3} \right) \quad (A.16)$$

Combining all the strains for the three stages gives, for $K_s > 1$

$$\epsilon_1 = -\epsilon_{10} + \left(\frac{1-2\nu}{B} \right) \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) - \frac{3\nu}{B} \ln \left(\frac{1+2K_s}{3} \right) \quad (A.17)$$

$$\epsilon_3 = -\epsilon_{30} + \left(\frac{1-2\nu}{B} \right) \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) + \frac{3(1-\nu)}{2B} \ln \left(\frac{1+2K_s}{3} \right) \quad (A.18)$$

(2) $K_s < 1$ When $K_s < 1$, the model assumes the second and third stages as:

(ii) application of an all-round confining pressure, $K_s\sigma_1$, and

(iii) increasing σ_3 from $K_s\sigma_1$ to σ_1

For stage (2),

$$\varepsilon_{s3} = \varepsilon_{s1} = \frac{1-2\nu}{B} \ln \left(\frac{K_s \sigma_1}{\sigma_{ps}} \right) \quad (A.19)$$

For stage (3), by increasing σ_1 from $K_s \sigma_1$ to σ_1 and integrating,

$$\varepsilon_1 = \int_{K_s \sigma_1}^{\sigma_1} \left\{ \frac{3}{B} \left(\frac{1}{\sigma_1 + 2\sigma_3} \right) \right\} d\sigma_1$$

or
$$\varepsilon_1 = \frac{3}{B} \ln \left(\frac{1+2K_s}{3K_s} \right) \quad (A.20)$$

and corresponding swell strain in lateral direction is

$$\varepsilon_1 = \frac{3\nu}{B} \ln \left(\frac{1+2K_s}{3K_s} \right) \quad (A.21)$$

Combining all the three strains for the two stages gives, for $K_s < 1$

$$\varepsilon_1 = -\varepsilon_{10} + \frac{1-2\nu}{B} \ln \left(\frac{K_s \sigma_1}{\sigma_{ps}} \right) + \frac{3}{B} \ln \left(\frac{1+2K_s}{3K_s} \right) \quad (A.22)$$

$$\varepsilon_3 = \varepsilon_3 = -\varepsilon_{30} + \frac{1-2\nu}{B} \ln \left(\frac{K_s \sigma_1}{\sigma_{ps}} \right) - \frac{3\nu}{B} \ln \left(\frac{1+2K_s}{3K_s} \right) \quad (A.23)$$

A3.2 Lateral Swell Pressure

When fissures and shrinkage cracks are present in a dry soil, the development of lateral pressure is reduced as a result of release in lateral restraint until such cracks are closed. As discussed earlier in Section 4.5.2, the determination of this effect on swell pressure estimation is very complex. However, Wallace and Lytton (1992b) has indicated that the effect of crack on the swelling pressure can be estimated by introducing a crack closure factor ' ξ ' to Equations (A.14), (A.18), and (A.23), such that the final compressive strain of the soil becomes:

$$\varepsilon_3 = \varepsilon_2 = -\xi \varepsilon_{30} \quad (A.24)$$

This implies that, crack closure factor ' ξ ' can be defined as the ratio of crack closure volume to potential free swell volume. For a soil without any cracks or when crack closure is completely prevented by debris, ξ will be zero.

With the inclusion of the effect of crack, Equation (A.10) can be rearranged to the following form to obtain the lateral swell pressure coefficient ' K_s ' for uniaxial modular dependence material model

$$K_s = \text{EXP} \left[\frac{\left\{ \beta \epsilon_{30} (1 - \xi) - (1 - 2\nu) \ln(\sigma_1 / \sigma_{ps}) \right\}}{(1 - \nu)} \right] \quad (\text{A.25})$$

where β describes the swell suppression at wet limit for the depth being analysed.

Similarly, for isotropic material, when $K_s > 1$, K_s can be obtained from Equation (A.18) as

$$K_s = -0.5 + 1.5 \text{EXP} \left[\frac{\left\{ B \epsilon_{30} (1 - \xi) - (1 - 2\nu) \ln(\sigma_1 / \sigma_{ps}) \right\}}{1.5(1 - \nu)} \right] \quad (\text{A.26})$$

The free swell strain term ' ϵ_{30} ' can be calculated as

$$\epsilon_{30} = \frac{\epsilon_{vo}}{3} = \frac{1 - \nu}{1 + \nu} \gamma_{hl} \Delta(\log h) \quad (\text{A.27})$$

Reviewing the relations developed above, the relation of lateral swell pressure coefficient ' K_s ' is found as function of various factors such as

$$K_s = \phi \left[\nu, \frac{\sigma_1}{\sigma_{ps}}, (1 - \xi), \frac{\gamma_{hl}}{\gamma_{oi}}, \Delta(\log h) \right] \quad (\text{A.28})$$

APPENDIX B

DERIVATION OF SHRINKAGE COMPRESSION EQUATIONS

B.1 Modulus of Shrinkage Compression

The shrinkage compression equations developed in this study and applied in the theoretical and numerical analyses in Chapter V and VIII are derived here. The model of shrinkage compression used in the derivation, follows the concept of suction and stress dependency of the Young's modulus. This is similar to the stress and suction dependency of swell and swell suppression. The shrinkage model presented in three dimension in Figure 5.1, produces a simple stress and suction dependent shrinkage compression modulus as

$$E_{sh} = \frac{2.3 (1 + \nu)(1 - 2\nu)}{\lambda_o (1 - \nu)} \sigma_z \quad (B.1)$$

E_{sh} is a tangent modulus which, when Poisson's ratio ' ν ' remains constant, becomes function of the applied stress σ_z (vertical stress under oedometer test condition), and the shrinkage compression index ' λ_o '. Since the magnitude of λ_o varies with the values of suction, E_{sh} also becomes function of suction. This constitutive relation presents a closed form solution of shrinkage compression, and is useful in the analysis soil shrinkage.

B.2 Strain equations in uniaxial model

In this section equations of lateral shrinkage stresses and the vertical compression are developed. These equations are derived by considering the following four stages of shrinkage compression as illustrated in Figure B.1. By combining all these stages, the final equations for lateral stress and compression are produced.

Stage I *free shrinkage under no confining pressure or up to threshold pressure.*

$$\varepsilon_{1_0} = \varepsilon_{2_0} = \varepsilon_{3_0} = \varepsilon_0 = \frac{\varepsilon_v}{3} = \frac{1 + \nu}{1 - \nu} \cdot \lambda_h \cdot \Delta \log h \quad (B.2)$$

Stage II *application of isotropic compressive stresses $\sigma_1 = \sigma_2 = \sigma_3$ with σ_1 increasing from σ_{ps} to σ_1 .*

$$\Delta \epsilon_1 = \int_{\sigma_{ps}}^{\sigma_1} \frac{d\sigma}{E} = \int_{\sigma_{ps}}^{\sigma_1} \frac{d\sigma_1}{\beta_c \sigma_1} = \frac{1}{\beta_c} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) \quad (B.3)$$

With simultaneous application of $\sigma_1 = \sigma_2 = \sigma_3$

$$\epsilon_1 = \epsilon_3 = \frac{1-2\nu}{\beta_c} \ln \left(\frac{\sigma_1}{\sigma_{ps}} \right) \quad (B.4)$$

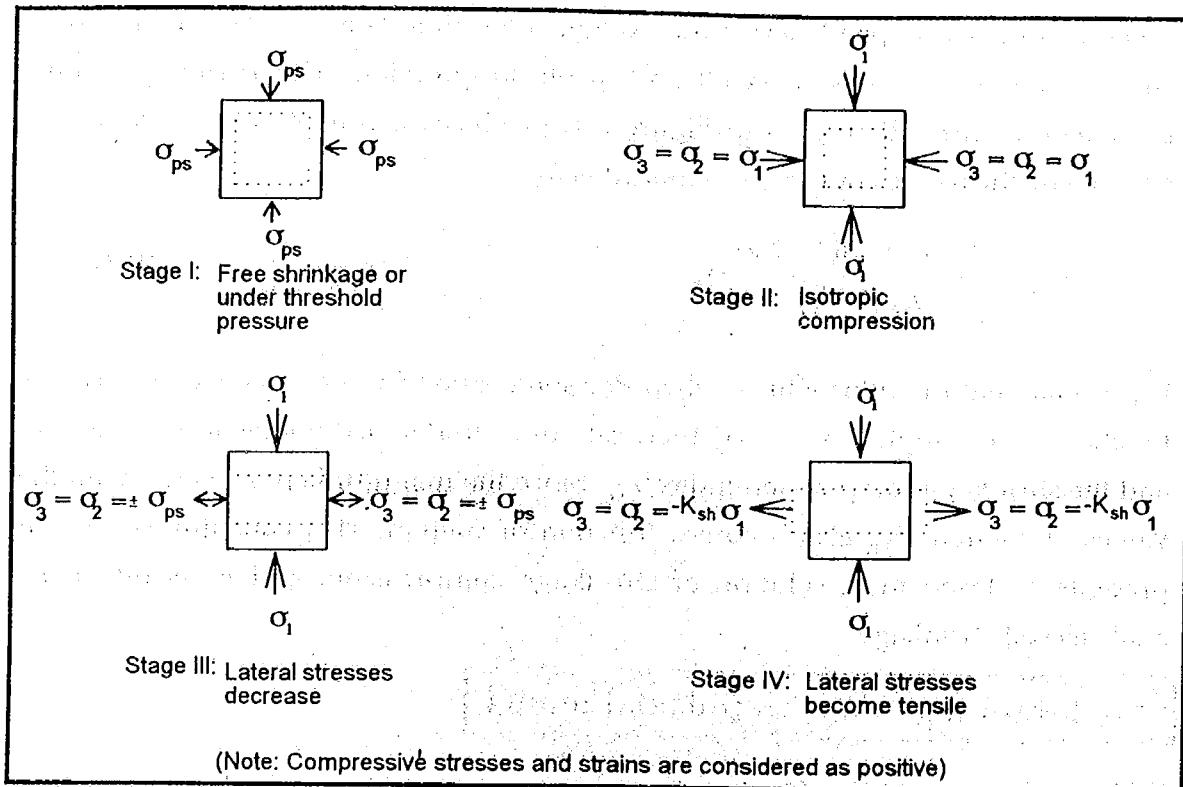


Figure B.1. Stages of shrinkage due to drying under applied vertical stress

Stage III $\sigma_2 = \sigma_3$ decreases from σ_1 to σ_{ps} while σ_1 remains constant.

$$\Delta \epsilon_3 = \int_{\sigma_1}^{\sigma_{ps}} \frac{d\sigma_3}{\beta_c \sigma_3} = \frac{1}{\beta_c} \ln \left(\frac{\sigma_{ps}}{\sigma_1} \right) \quad (B.5)$$

or
$$\epsilon_3 = \frac{1-\nu}{\beta_c} \ln \left(\frac{\sigma_{ps}}{\sigma_1} \right) \quad (B.6)$$
 (positive in form but since $\sigma_{ps} < \sigma_1$, results in negative i.e., extension)

and
$$\epsilon_1 = -\frac{2\nu}{\beta_c} \ln\left(\frac{\sigma_{ps}}{\sigma_1}\right) \quad \text{(the result becomes positive i.e., compression because } \sigma_{ps} < \sigma_1 \text{)} \quad (B.7)$$

Stage IV $\sigma_2 = \sigma_3$ becomes tensile, with value of σ_3 increasing from σ_{ps} to $K_{sh}\sigma_1$, while σ_1 remains constant. K_{sh} is the coefficient of lateral shrinkage stress.

$$\Delta\epsilon_3 = -\int_{\sigma_{ps}}^{K_{sh}\sigma_1} \frac{d\sigma_3}{\beta_c \sigma_3} = -\frac{1}{\beta_c} \ln\left(\frac{K_{sh}\sigma_1}{\sigma_{ps}}\right) \quad (B.8)$$

or,
$$\epsilon_3 = -\frac{1-\nu}{\beta_c} \ln\left(\frac{K_{sh}\sigma_1}{\sigma_{ps}}\right) \quad \text{(tensile)} \quad (B.9)$$

and
$$\epsilon_1 = \frac{2\nu}{\beta_c} \ln\left(\frac{K_{sh}\sigma_1}{\sigma_{ps}}\right) \quad \text{(compression)} \quad (B.10)$$

Summing up these strains at all stages

$$\epsilon_1 = \epsilon_{1_0} + \frac{1}{\beta_c} \ln\left(\frac{\sigma_1}{\sigma_{ps}}\right) + \frac{2\nu}{\beta_c} \ln\left(\frac{K_{sh}\sigma_1}{\sigma_{ps}}\right) \quad (B.11)$$

and
$$\epsilon_3 = \epsilon_{3_0} - \frac{\nu}{\beta_c} \ln\left(\frac{\sigma_1}{\sigma_{ps}}\right) - \frac{1-\nu}{\beta_c} \ln\left(\frac{K_{sh}\sigma_1}{\sigma_{ps}}\right) \quad (B.12)$$

Note that the strain ϵ_1 computed by using Equation (B.11) presents shrinkage compression in the vertical direction under a three dimensional stress/strain environment.

B.3 Lateral Shrinkage Stress

K_{sh} in Equations (B.11) and (B.12) is the ratio of lateral shrinkage stress to applied vertical stress, and determines the magnitude of lateral tensile stress that can develop in a drying soil profile. To determine the value of K_{sh} consider $\epsilon_3 = 0$. Then the expression for K_{sh} becomes

$$K_{sh} = \left(\frac{\sigma_{ps}}{\sigma_1}\right) \cdot \text{Exp}\left[\frac{\beta_c \epsilon_{3_0} - \nu \ln(\sigma_1/\sigma_{ps})}{(1-\nu)}\right] \quad (B.13)$$

With the substitution of β_c and ϵ_{3_0} using Equations (5.7) and (5.9) respectively the relation for K_{sh} becomes

$$K_{sh} = \left(\frac{\sigma_{ps}}{\sigma_1} \right) \cdot \text{Exp} \left[\frac{2.3(1-2\nu)(\Delta \log h)(\lambda_h/\lambda_\sigma) - \nu \ln(\sigma_1/\sigma_{ps})}{(1-\nu)} \right] \quad (B.14)$$

This expression of K_{sh} indicates that by varying λ_h and λ_σ but maintaining their ratio constant, magnitude of K_{sh} remains unchanged.

APPENDIX C

MAGNITUDE OF THERMAL PARAMETERS

C.1 Non-Linear Diffusivity

To apply the thermo-mechanical option of FLAC in the present numerical analysis, a set of values of thermal parameters are required as input parameters. The following section discuss how these parameters are estimated.

The description of non-linear diffusivity $D(\theta)$ discussed in Section 2.4.2C, is given in the relation,

$$D(\theta) = A + B \theta^{2.5} \quad (C.1)$$

Applying its boundary values as

$$D(\theta) = 0.00003 \text{ m}^2/\text{day} \text{ at initial dry state, } \theta = 0.15, \text{ ie., } \log \psi = 3.5 \log \text{ kPa,}$$

$$D(\theta) = 0.0002 \text{ m}^2/\text{day} \text{ at initial dry state, } \theta = 0.45, \text{ ie., } \log \psi = 1.5 \log \text{ kPa,}$$

$$\text{the relation in (C.1) becomes } D(\theta) = 1.8343 \cdot 10^{-5} + 1.3377 \cdot 10^{-3} \theta^{2.5} \quad (C.2)$$

Considering hydraulic diffusivity as analogous to heat diffusion, the diffusivity relation becomes

$$D = \frac{K}{\rho C_h} \quad (C.3)$$

where

ρ = mass density

C_h = specific heat, and

K = thermal conductivity

Using the following SI units for dimensional analysis,

$$D = \text{diffusivity} = \frac{\text{m}^2}{\text{day}}$$

$$K = \text{thermal conductivity} = \frac{\text{kWatts}}{\text{m } ^\circ\text{C}} = \frac{\text{kJ}}{\text{m sec } ^\circ\text{C}}$$

$$\rho = \text{mass density} \frac{\text{ton}}{\text{m}^3} = \frac{\text{k kg}}{\text{m}^3}, \text{ and}$$

$$C_h = \text{specific heat} = \frac{\text{kJ}}{\text{ton } ^\circ\text{C}} = \frac{\text{kJ}}{\text{k kg } ^\circ\text{C}}$$

and dimensional analysis in the above equation (C.3) gives,

$$K = D \rho C_h$$

$$\frac{\times 10^3 \cdot \frac{\text{kg m}}{\text{sec}^2} \cdot \text{m}}{\text{m sec } ^\circ\text{C}} = \frac{\text{m}}{\text{day}} \cdot \frac{\times 10^3 \text{ kg}}{\text{m}^3} \cdot [C_h] \quad (\text{C.4})$$

This yields the unit of $[C_h]$ as $\frac{\text{m}^2 \cdot \text{day}}{\text{sec}^2 \cdot ^\circ\text{C}}$, and in terms of sec as unit of time for compatibility, the unit becomes $86400 \frac{\text{m}^2}{\text{sec}^2 \cdot ^\circ\text{C}}$.

If the conventional unit of thermal heat capacity is reviewed

$$[C_h] = \frac{\text{kJ}}{\text{ton } ^\circ\text{C}}$$

$$= \frac{\times 10^3 \cdot \frac{\text{kg m}}{\text{sec}^2} \cdot \text{m}}{\times 10^3 \cdot \text{kg} \cdot ^\circ\text{C}} = \frac{\text{m}^2}{\text{sec}^2 \cdot ^\circ\text{C}} \quad (\text{C.5})$$

Thus, if $[C_h]$ is defined as in terms of $86400 \frac{\text{m}^2}{\text{sec}^2 \cdot ^\circ\text{C}}$ the unit of diffusivity becomes

$$D = \frac{K}{\rho C_h} = \frac{\times 10^3 \cdot \frac{\text{kg m}}{\text{sec}^2} \cdot \text{m}}{\text{m} \cdot \text{sec } ^\circ\text{C}} \times \frac{\text{m}^3}{\times 10^3 \text{ kg}} \times \frac{\text{sec}^2 \cdot ^\circ\text{C}}{86400 \text{ m}^2}$$

$$= \frac{\text{m}^2}{86400 \text{ sec}} = \frac{\text{m}^2}{\text{day}} \quad (\text{C.6})$$

C.2 Typical Value of Specific Heat

Applying thermal analogy, a typical value of specific heat can be calculated from specific water capacity as will be applied in the thermal analysis.

By definition, specific water capacity is defined in the relation, $C_\theta = \frac{d\theta}{d\psi}$

and although θ versus ψ relation is non linear and shows complex nature, if idealised as log linear an approximate value of specific water capacity can be obtained as

$$\frac{d\theta}{d(\log \psi)} = \frac{0.45 - 0.15}{3.5 - 1.5} = 0.15$$

With T as a transformed variable (Section 7.4.2) as

$$T \text{ (in } ^\circ\text{C)} = 4.5 - \log \psi \text{ (in log kPa)} \quad (\text{C.7})$$

then the specific water capacity considered as analogous to specific heat can be defined in terms of temperature as

$$C_\theta = \frac{d\theta}{d(\log \psi)} = \frac{0.3}{3^\circ\text{C} - 1^\circ\text{C}} = 0.15/^\circ\text{C} \quad (\text{C.8})$$

Thus, for this particular situation, when specific water capacity defined in term of temperature is equated to the specific heat, specific heat becomes

$$\begin{aligned} C_h &= 86400 \frac{\text{m}^2}{\text{sec}^2} \cdot \frac{1}{0.15} C_\theta \\ &= 576000 \frac{\text{m}^2}{\text{sec}^2} \cdot C_\theta \end{aligned} \quad (\text{C.9})$$

This relation shows a typical value of specific heat C_h to define a heat flow between a temperature difference of 1°C and 3°C that is, in term of moisture flow for a suction difference of 3.5 log kPa and 1.5 log kPa. can be considered as

$$C_h = 576000 \frac{\text{kJ}}{\text{ton } ^\circ\text{C}}$$

C.3 Estimation of Conductivity

From the diffusivity relation Equation (C.3), conductivity becomes

$$D = \frac{K}{\rho C_h},$$

$$\begin{aligned} \text{or } K &= D \rho C_h = D * 1.7 * 576000 \\ &= 979200 D \end{aligned}$$

Using the boundary values of D as $0.0002 \text{ m}^2/\text{day}$ at temperature = 3°C (ie., $\log \psi = 1.5 \log \text{kPa}$ or $\theta = 0.45$), and D as $0.00003 \text{ m}^2/\text{day}$ at temperature = 1°C (ie., $\log \psi = 3.5 \log \text{kPa}$ or $\theta = 0.15$), the values of K are determined as

$$K_i = 29.376, \text{ and } K_f = 295.84 \frac{\text{kJ}}{\text{m sec } ^\circ\text{C}} \text{ respectively}$$

These values of conductivity when used in the non linear temperature dependent conductivity relation $K = k_1 + k_2 T^n$, with $n = 2.5$, the values of constants becomes

$k_1 = 17.965$, and $k_2 = 11.411$, and the resulting equation is:

$$K_T = 17.965 + 11.411 k_2 T^n \quad (\text{C.10})$$

The resulting wetting period can be estimated by considering it as analogous to pore pressure release theory used in the one dimensional consolidation process. The concept of estimating consolidation period t , can be applied with the usual relation

$$T_v = \frac{C_v t}{H^2} \quad (\text{C.11})$$

where

T_v = the time factor,

C_v = coefficient of consolidation analogous to diffusivity in m^2/day unit,

H = thickness of layer, ie., the length of moisture flow in m.

For a ninety percent wetting in a 1.2 m wide subgrade, $T_v = 0.85$, and an average diffusivity $D = 0.00012 \text{ m}^2/\text{day}$, the period of wetting becomes

$$t = \frac{0.85 \times (1.2)^2}{0.00012} = 10200 \text{ days}$$

This result indicates a very high value of time in terms of days. If a realistic value of wetting period is selected as 100 days, which represents an order of magnitude of time period commonly observed in the field, the above equation yields a value of diffusivity equal to

$$D = \frac{H^2 T_v}{t} = \frac{1.44 \times 0.85}{100} = 0.0122 \text{ m}^2/\text{day}$$

which is a representative value in a typical case of subgrade wetting.

Using the above defined value of specific heat as $C_h = 576000 \frac{\text{kJ}}{\text{ton } ^\circ\text{C}}$, and mass density $\rho = 1.7 \text{ ton/m}^3$ in Equation (C.4), the value of conductivity can be determined as

$$K_f = D \rho C_h = 11946 \frac{\text{kJ}}{\text{m sec } ^\circ\text{C}}.$$

This value is considered to represent conductivity at temperature 3°C , ie., at wet state having suction pressure of $1.5 \log \text{ kPa}$ (or $\theta = 0.45$). If the ratio of the boundary values of typical diffusivity defined earlier is reconsidered as defined below

$$\frac{D_f}{D_i} = \frac{0.0002}{0.00003} = 6.667$$

then

$$K_i = \frac{0.0122}{6.667} \times 1.7 \times 576000 = 1792 \frac{\text{kJ}}{\text{m sec } ^\circ\text{C}}$$

Using these upper and lower conductivity values, the coefficient involved in the description of non-linear temperature dependent conductivity relation, $K = k_1 + k_2 T^{2.5}$, as $k_1 = 1096$, and $k_2 = 696$. The final form of conductivity relation used is expressed as

$$K = 1096 + 696 T^{2.5}$$

APPENDIX D

PROGRAM FILE FOR FLAC COMMANDS

(An example of swell analysis for Case2-A)

D.1 Initial Equilibrium under Gravity Stresses

- * ONE-DIMENSIONAL WETTING FROM THE RIGHT BOUNDARY CONSIDERED.
- * ANALYSE OF THERMO-MECHANICAL BEHAVIOUR OF 1.2m wide x 0.6m deep
- * SUBGRADE SURFACED WITH A 200 mm THICK PAVEMENT CONDUCTED.
- * MOHR-COULOMB MODEL USED FOR BOTH PAVEMENT AND SUBGRADE.
- * SUBGRADE STRENGTH AND STIFFNESS VARIED WITH SUCTION.

- * This is for case 2-A with horizontal 1-D moisture flow using non linear temperature
- * ie., suction dependent conductivity. Also it uses free deformation boundary
- * condition on right side.
- * NON-LINEAR DIFFUSIVITY $D = K1 + K2 * t^{2.5}$
- * Subgrade Swell Parameters are: $\text{gamah1}=0.07$ $\text{gamasigma1}=0.04$
- * units ton, m, days and kPa.

title

200 mm gravel on 1.2m wide x 0.6m, deep swelling subgrade

config thermal extra 4

* 23 X 14 zones without interfaces

grid 23,14

model Mohr j=1,10

model Mohr j=11,14

.

* Subgrade part

gen 0.0,0.0 0.0,0.6 0.3,0.6 0.3,0.0 i=1,4 j=1,11

gen 0.3,0.0 0.3,0.6 0.6,0.6 0.6,0.0 i=4,8 j=1,11

gen 0.6,0.0 0.6,0.6 0.9,0.6 0.9,0.0 i=8,14 j=1,11

gen 0.9,0.0 0.9,0.6 1.2,0.6 1.2,0.0 i=14,24 j=1,11

* Pavement part

gen 0.0,0.6 0.0,0.8 0.3,0.8 0.3,0.6 i=1,4 j=11,15

gen 0.3,0.6 0.3,0.8 0.6,0.8 0.6,0.6 i=4,8 j=11,15

gen 0.6,0.6 0.6,0.8 0.9,0.8 0.9,0.6 i=8,14 j=11,15

gen 0.9,0.6 0.9,0.8 1.2,0.8 1.2,0.6 i=14,24 j=11,15

* Subgrade Property

- * soil with $E = 5 \text{ MPa}$, Poisson's ratio = 0.3, density = 1.7 ton/m^3 .
- * cohesion=60 kPa, friction=20 degree, dilation=0 degree
- * tensile strength=2*cohesion=120 kPa
- * Subgrade initial properties are selected with high shear and tensile strengths to avoid
- * localised yielding during gravity stress calculation for initial condition.
- * These properties will be updated with the true values
- * after achieving equilibrium under initial stress condition.

prop bu=4.167e3 sh=1.923e3 dens=1.7 i=1,23 j=1,10
 prop coh=200 fric=40 ten=400 dil=0 i=1,23 j=1,10

* Pavement properties

- * Elastic properties of gravel layer are considered the same as of clay
- * with $E=5 \text{ Mpa}$, $\nu=0.3$, density= 2.2 ton/m^3 .

prop bu=4.167e3 sh=1.923e3 dens=2.2 i=1,23 j=11,14
 prop coh=200 fric=40 ten=400 dil=0 i=1,23 j=11,14

- * Boundary conditions- no flow conditions through out the edges.
- * Temperature is increased from the right edge wall
- * For deformation conditions, rollers at bottom and left

fix x i=1
 fix y j=1

- * set converged unbalanced force to less than 0.3% of initial unbalanced force
- set force=3e-03 step=50000

- * Geometric deformation is accounted during the analysis as the deformation
 - * is very large and affects the stress analysis
- set large

- * initialise gravity stresses for plane strain and solve to confirm
- * S_{yy} is calculated as product of density * gravity (as 9.81) * depth at
- * the mid thickness of each grid layer.
- * Note different densities for pavement and soils defined earlier are used.

init $s_{yy} = -13.822$ j=1

init $s_{yy} = -12.822$ j=2

init $s_{yy} = -11.821$ j=3

init $s_{yy} = -10.820$ j=4

init $s_{yy} = -9.820$ j=5

init $s_{yy} = -8.819$ j=6

init $s_{yy} = -7.819$ j=7

init $s_{yy} = -6.818$ j=8

init $s_{yy} = -5.817$ j=9

init $s_{yy} = -4.817$ j=10

init $s_{yy} = -3.777$ j=11

init $s_{yy} = -2.698$ j=12

init $s_{yy} = -1.619$ j=13

init $s_{yy} = -0.540$ j=14

- * S_{zz} is calculated as the product of Poissons ratio and summation of other two
- * stresses as $\nu \cdot (S_{xx} + S_{yy})$

init $s_{zz} = -4.147$ j=1

init $s_{zz} = -3.847$ j=2

init $s_{zz} = -3.546$ j=3

init $s_{zz} = -3.246$ j=4

init $s_{zz} = -2.946$ j=5

init $s_{zz} = -2.646$ j=6

init $s_{zz} = -2.346$ j=7

init $s_{zz} = -2.045$ j=8

init $s_{zz} = -1.745$ j=9

init $s_{zz} = -1.445$ j=10

init $s_{zz} = -1.133$ j=11

init $s_{zz} = -0.809$ j=12

init $s_{zz} = -0.486$ j=13

init $s_{zz} = -0.162$ j=14

- * Assume zero lateral stress

init $s_{xx} = 0.0$ i=1,23 j=1,14

SET MECH ON THERM OFF

solve

set gravity=9.81

solve

SAVE MSUBPV1A.SM1

* Subgrade properties are updated back to actual values of

* shear and tensile strengths such cohesion=60 kPa,

* friction=20 degree, dilation=0 degree

* tensile strength=2*cohesion=120kPa

* Pavement properties are also updated to actual values as

* Cohesion=10 kPa, friction=40 degree, and tensile strength

* = 10 kPa, dilation =0

prop coh=60 fric=20 ten=120 dil=0 i=1,23 j=1,10

prop coh=10 fric=40 ten=10 dil=0 i=1,23 j=11,14

solve

pr sxx syy szz

* Save the analysis up to this stage which cover initialisation of gravity stresses

save initial.sav

* MECHANICAL PART ENDS

* THERMAL PARTS STARTS WITH OTHER FILE

* program calls thermo-mechanical calculation file

ca thermal.pav

D.2 Instantaneous Wetting

* THERMAL ANALYSIS in File named "thermal.pav"

* This is case 2-a with the values of $\text{Gamah1} = 0.07$ $\text{Gamasigma1} = 0.04$

* such that the coefficient of linear expansion "Alpha = 0.038".

* initial displacements in both x and y directions initialised to zero such that

* all displacements account for subgrade swelling/swell suppression effect

init xdis=0 ydis=0

set step 100000000

*THERMAL, NON-LINEAR DIFFUSION

* now commence a thermal analysis for the swelling subgrade

model th_general j=1,10

prop con1=1096 con2=696 n_cond=2.5 j=1,10

prop spec_heat=576000 thexp=0.038 j=1,10

* TEMP IS (4.0-log kPa), wetting from 3.2 to 1.5 log kPa

* is described as heating from 0.8 to 2.5 degrees C.

* initialise to T = 0.8 apart from 1 degree at right face

init temp=0.8 j=1,11

set thdt=0.1

* limit initial time step to maintain stability because the initial step

* set internally for non-linear diffusivity is calculated for the k values

* of the drier initial soil thus overestimating the stable time step

* thermo-mechanical solution with mech sol'n steps through time when boundary

* wets up then at square root time steps up to 50% wetting at 64 days.

his sxx i=10, j=7

his sxx i=10, j=8

his sxx i=10, j=9

* creates some history files to check lateral stress development at different locations

set mech off therm on
fix temp=0.9 i=24 j=1,11
step 1

set therm off mech on
call mod2.fis; calls "mod2.fis" file to modify soil property, "mod2.fis" file is presented later
modulus
solve

set mech off therm on
fix temp=1.0 i=24 j=1,11
step 1

fix temp=1.1 i=24 j=1,11
step 1

fix temp=1.2 i=24 j=1,11
step 1

fix temp=1.3 i=24 j=1,11
step 1

fix temp=1.4 i=24 j=1,11
step 1
set therm off mech on
modulus
solve

set mech off therm on
fix temp=1.6 i=24 j=1,11
step 1

fix temp=1.9 i=24 j=1,11
step 1
set therm off mech on
modulus
solve

set mech off therm on
fix temp=2.1 i=24 j=1,11
step 1

fix temp=2.3 i=24 j=1,11
step 1

fix temp=2.5 i=24 j=1,11
step 1
set therm off mech on
modulus
solve ; end of instantaneous wetting

D.3 Alternate Analysis of Wetting and Swelling

*Now the program conducts alternate analyses of thermal and mechanical equilibrium

* Note: Program saves results at different stages of wetting and swelling

set mech off therm on
solve age=1.5
set therm off mech on
modulus
solve

set mech off therm on
solve age=2.5
set therm off mech on
modulus
solve

set mech off therm on
solve age=4.0
set therm off mech on
modulus
solve

save t4day.sav

* In the same way program runs up to 64 days

D.4 Sub-routine for Strength and Stiffness Modification

```
def modulus
```

```
  loop i (1,IZONES)
```

```
    loop j (1,10)
```

```
      EX_1(i,j)=(temp(i,j)+temp(i+1,j)+temp(i+1,j+1)+temp(i,j+1))/4
```

```
      IF EX_1(I,J)>0.805 THEN
```

```
        TEMP1=EX_1(I,J)
```

```
        EX_2(I,J)=max(7,0.333*(-sxx(i,j)-syy(i,j)-szz(i,j)))
```

```
        TEMP2=EX_2(I,J)
```

```
        Shear_mod(i,j)=((16.751*temp2)/(temp1-0.568))
```

```
        bulk_mod(i,j)=2.17*shear_mod(i,j)
```

```
        cohesion(i,j)=85.88-32.35*temp1
```

```
        tension(i,j)=2*cohesion(i,j)
```

```
      ELSE
```

```
        IF EX_1(I,J)<0.805 THEN
```

```
          Shear_mod(i,j)=1.923e3
```

```
          bulk_mod(i,j)=4.167e3
```

```
          cohesion(i,j)=60.00
```

```
          tension(i,j)=2*cohesion(i,j)
```

```
        END_IF
```

```
      END_IF
```

```
    end_loop
```

```
  end_loop
```

```
end
```

APPENDIX E

VERIFICATION OF NUMERICAL ANALYSIS PROCEDURE

E.1 Verification of Procedure

A detailed discussion on validation of proposed analysis procedure is available in Sapkota (1994), here only the most significant examples are presented.

E.1.1 Moisture Diffusion

As suggested by Richards (1992), the most common example of water flow in geotechnical engineering could be best presented by a simple linear one dimensional consolidation of non-swelling soils or the simple diffusion theory for saturated soils. Since these theories are based on the assumption that the coefficient of consolidation or the diffusivity remains constant during the process, the thermal analogy of heat diffusion used here is verified only under constant diffusivity with the value of conductivity representing the maximum wet condition in the field.

Verification Example:

Consider a subgrade section as shown in Figure E.1 having an initial suction of 3.2 log kPa and final suction of 1.5 log kPa. If the transformed heat flow variable T , equivalent to temperature in $^{\circ}\text{C}$, is considered as $T = 4.0 - \log \psi$, (ψ is in kPa) then the initial temperature becomes 0.8°C , and the final temperature is 2.5°C . This results in a total increase of temperature at source of 1.7°C . Now, a constant diffusivity (equivalent to coefficient of consolidation C_v) $D = 0.012 \text{ m}^2/\text{day}$ is taken, and wetting is carried out under one dimensional horizontal flow condition. It is continued until a 50 percent decrease in suction at the mid-width of the section is obtained. For a 1.2 m width of subgrade, 50 percent wetting occurs after a wetting period of 24 days. The wetting time is calculated from $t = \frac{T_v H^2}{D}$ where time period

T_v is considered as 0.2 which is based with respect to 50 percent equivalent pore pressure release in one dimensional consolidation. H is the width of subgrade as equivalent to the length of flow path for pore water release during the consolidation.

Using the isotropic thermal model of FLAC which employs constant thermal conductivity K , the resulting run for the problem indicates an increase in temperature from 0.8°C to 1.2°C at the far end from the source.

From the standard pore pressure dissipation curve, for $T_v = 0.2$, the dissipation is about 25%. As a result, from theory of pore pressure release, the final temperature at the far end would be

$$T_f = 0.8 + 1.7 * 0.25 = 1.22^\circ\text{C}$$

This indicates that the numerical analysis presents a desirable degree of accuracy and the typical values of parameters selected in the analysis such as specific heat as $C_h = 576000 \frac{\text{kJ}}{\text{ton } ^\circ\text{C}}$, and conductivity at saturated state as $K = 11946 \frac{\text{kJ}}{\text{m sec } ^\circ\text{C}}$ lie within a reasonable order of magnitude.

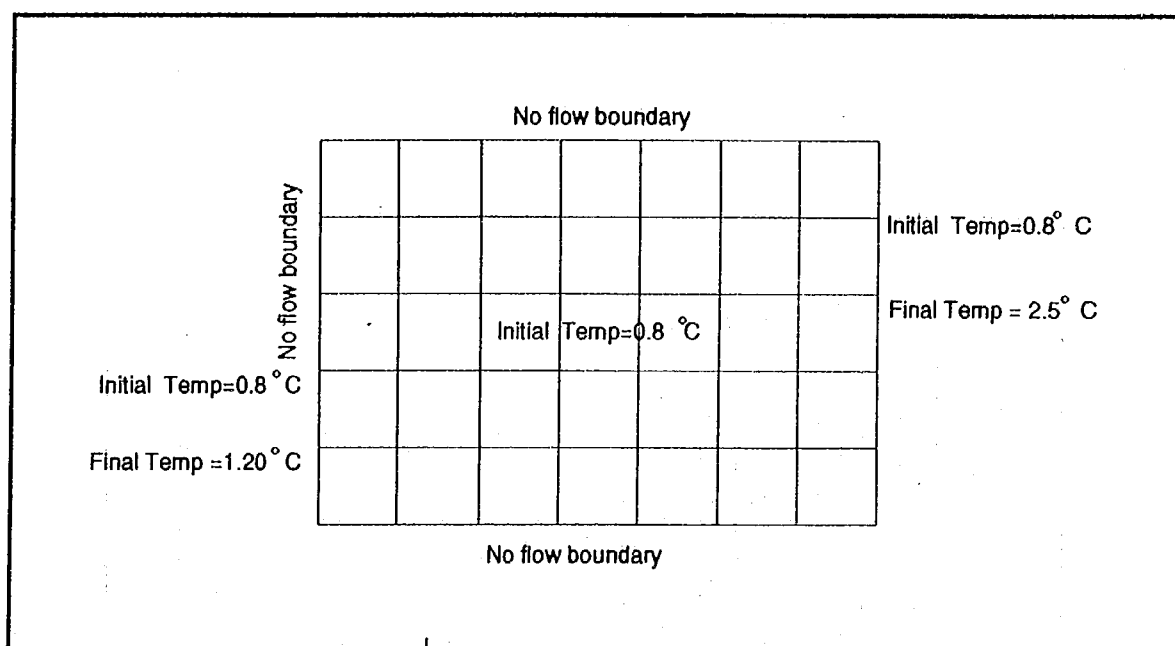


Figure E. 1 Typical grid for verification of result on moisture movement analysis conducted by applying thermal analogy and using the isotropic thermal model of FLAC

As explained in the scope of numerical analysis, the subgrade moisture movement for swell analysis is under monotonic wetting from a uniformly dry stage to a relatively wet stage and under a continuous source of moisture. Thus the effects of hysteresis are not considered for the present analysis and consequently not included in the verification process. The selection of the non-linear diffusivity function is based on the comparison with published results of different authors available in the literature. This was discussed earlier in detail in later part of Chapter II.

E 1.2 Stress, Strain, and Failure

Verification of linear elastic model had been conducted earlier (Sapkota, 1994). An example problem similar to the one presented by Duncan et al (1968) and Richards (1992) was employed. A loading due to strip footing over an elastic subgrade of a finite depth was considered. Comparing the results obtained was from FLAC analysis and with that from closed form elastic solution, it was found that the FLAC could give a reasonable degree of accuracy.

The verification of analysis with Mohr-Coulomb model is presented below.

Most of the analyses in this numerical analysis work employed Mohr-Coulomb plasticity model. It was decided to test the applicability and accuracy of the program with this model. The accuracy of this model was tested with a typical example problem of loading of a flexible friction less footing on a linear elastic soil.

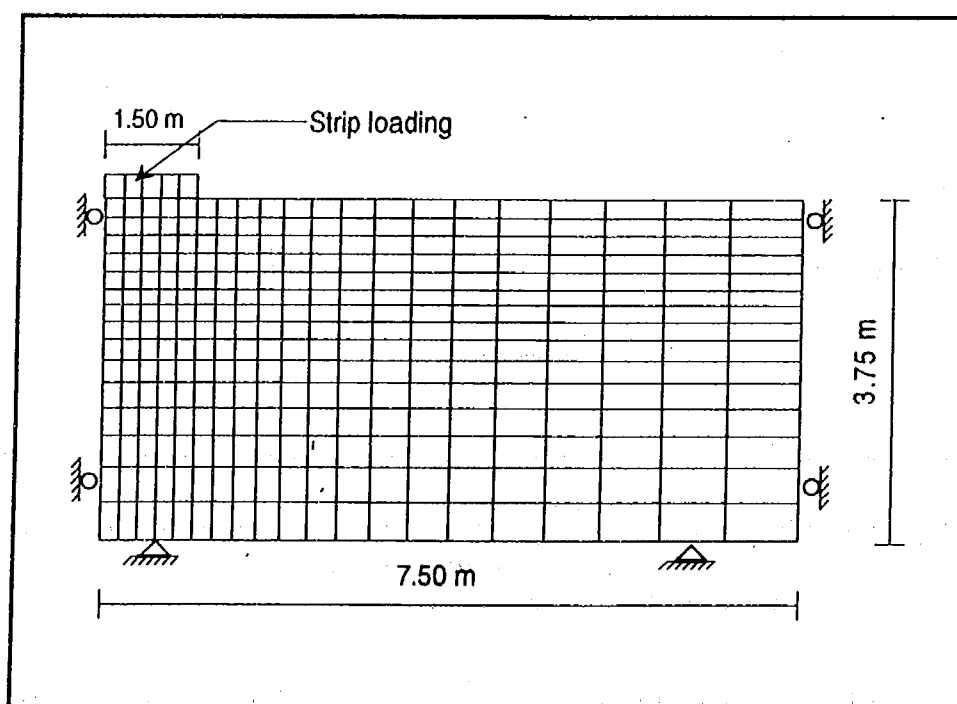


Figure E. 2 Typical geometry to verify FLAC's Mohr-Coulomb model

Figure E.2 shows a typical geometry and grid distribution of the section considered. The advantage of symmetry is considered in describing the displacement boundary conditions. The elastic and Mohr-Coulomb plasticity properties selected for this verification purpose were: Young's modulus $E = 210$ MPa, Poisson's ratio $\nu = 0.3$, cohesion $C = 70$ kPa, friction angle $\phi = 20^\circ$, and tensile strength $T = 192$ kPa. The

dilation angle ψ is assumed to be zero and the media is considered as a weightless soil.

The program was run under a uniform displacement field at a very small rate of 10^{-6} m/sec in downward direction applied to a strip width of 1.5 m at top left corner, close to centre line of the symmetric section shown in the figure. The result of analysis of the program in terms of load/deformation is compared with the solution obtained by Zienkiewicz and Humpheson (1977). Their approach gave a collapse load of about 1030 kPa whereas the present analysis gives a stress displacement history as shown in Figure E.3 with a maximum stress of about 1120 kPa.

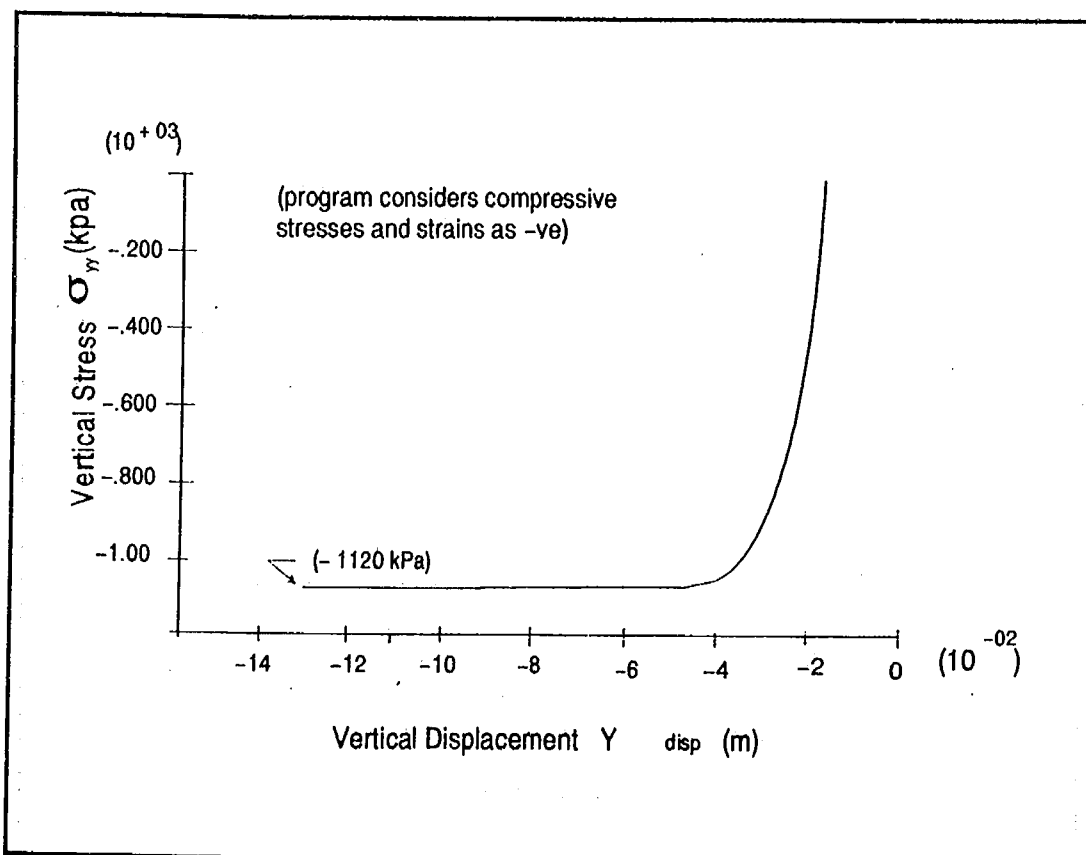


Figure E.3 Result of FLAC verification analysis with Mohr-Coulomb model

E.2. Vertical Heave and Swell Pressure

As already explained in earlier sections, vertical heave and swell pressure were measured in terms of thermo-mechanical behaviour of solid. From basic theory of thermo-physics, the volume of solid increases with an increase in temperature and if this expansion is suppressed by rigid boundary condition or by application of

external opposite pressure, there will develop internal thermal stresses. Using this concept, the analysis adopted the following: free swell was equivalent to thermal expansion of solid under free boundary condition, and swell pressure was equivalent to the development of internal thermal stresses in solid as a result of boundary fixity. The following paragraphs describe a test made to check the accuracy and applicability of the present numerical program in the analysis of development of vertical and swell pressure in a swelling clay profile.

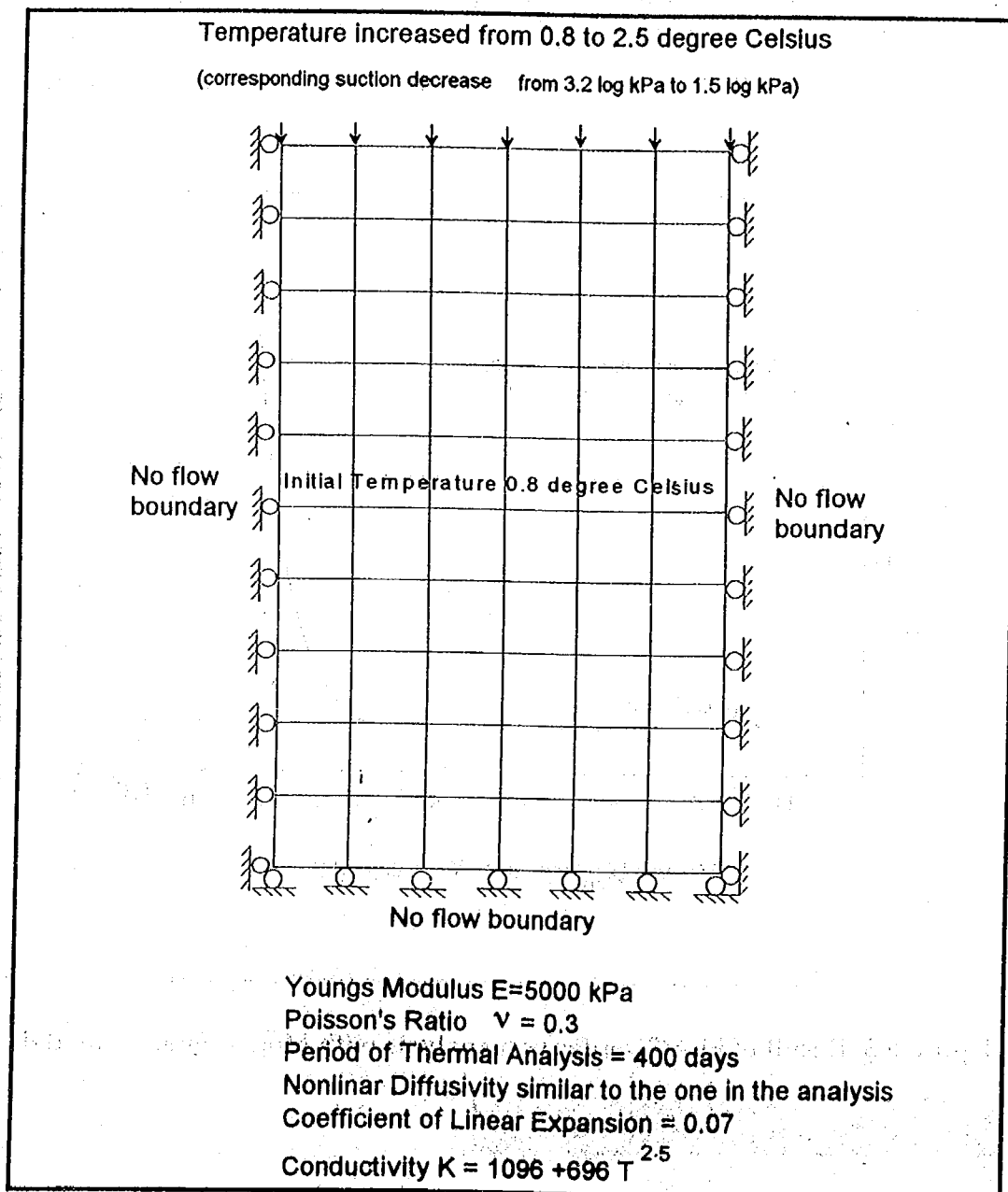


Figure E.4 Typical section considered for verification of thermo-mechanical analysis

A typical section, 3 m wide by 5 m depth, shown in Figure E.4 was considered for analysis. The section behaved as a weightless elastic isotropic soil which allowed non-linear conductivity. The values of elastic and flow properties are indicated in the figure which also indicate the typical displacement and flow boundary conditions.

The program was run in exactly similar way as will be described in Section 8.5.2 and 8.5.3 with instantaneous temperature increased within a short period and an alternate analysis of heat flow in one dimension and mechanical equilibrium in different steps. To achieve a higher range of displacement/stress development, the program was run for a period of 400 days (model time). The profile of temperature is presented in Table E.1.

The analytical solution for thermal stresses and strains under thermo-elastic analysis (Timoshenko and Goodier, 1970) in plane strain analysis can be expressed as

For $\epsilon_{zz} = 0$,

$$\epsilon_{xx} - \alpha T = \frac{1}{E} [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]$$

$$\epsilon_{yy} - \alpha T = \frac{1}{E} [\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})]$$

Considering no lateral displacement in the x-direction such that $(\epsilon_{xx} - \alpha T) = 0$, and free vertical displacement such that $\sigma_{yy} = 0$, thermal stress in lateral direction can be computed analytically as

$$\sigma_{xx} = \frac{E \alpha T}{(1 - \nu)}$$

similarly, in terms of strains, for free expansion in X and Y

$$\epsilon_{xx} = \epsilon_{yy} = \alpha T - \nu \frac{\sigma_{zz}}{E}$$

and for expansion in one axis, eg. in y-axis only,

$$\epsilon_{yy} = \alpha T - \nu \left(\frac{\sigma_{xx} + \sigma_{zz}}{E} \right)$$

Using these equations, the analytical solutions were obtained and used to check the accuracy of the results from the program in terms of vertical heave and swell pressure as shown in Table E.1.

Table E. 1 Result of verification of thermo-mechanical analysis

From FLAC							From Analytical			
Depth from source	Y coord	Temperature	Y displacement	XX stress (multiply values below by 10 ²)	YY stress (multiply values below by 10 ⁻²)	ZZ stress (multiply values below by 10 ²)	xx stress	zz stress	Y displacement of elements	Cumulative Y displacement
(m)	(m)	(°C)	(m)	(kPa)	(kPa)	(kPa)	(kPa)	(kPa)	(m)	(m)
0	5	2.5	0.347							0.347
0.5	4.5	2.275	0.2438	-7.937	-0.049	-7.937	-793.75	-793.75	0.10319	0.2438
1	4	2.005	0.1567	-6.7	-0.27	-6.7	-670	-670	0.0871	0.1567
1.5	3.5	1.688	0.0887	-5.233	-0.677	-5.233	-523.25	-523.25	0.06802	0.0887
2	3	1.344	0.0422	-3.579	-0.876	-3.579	-358	-358	0.04654	0.0422
2.5	2.5	1.052	0.0163	-1.99	-1.117	-1.99	-199	-199	0.02587	0.0163
3	2	0.89	0.0052	-0.855	-1.417	-0.855	-85.5	-85.5	0.01112	0.0052
3.5	1.5	0.826	0.0014	-0.29	-1.501	-0.29	-29	-29	0.00377	0.0014
4	1	0.807	0.0004	-0.083	-1.627	-0.083	-8.25	-8.25	0.00107	0.0003
4.5	0.5	0.802	0.0001	-0.021	-1.733	-0.021	-2.25	-2.25	0.00029	9E-05
5	0	0.801	0	-0.006	-1.767	-0.006	-0.5	-0.75	9E-05	0

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Pavements on Swelling and
Shrinking Clay Subgrades

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Binod Sapkota

Toshiyuki Nakamura

Takaharu Kiriya

要 旨

This is a study on the response of a seal coat surfaced unbound granular pavement when subjected to subgrade volume change. Swelling and shrinkage of Subgrade due to monotonic wetting and drying are considered. In the numerical analysis, thermal analogy is employed in which free swell/shrink is considered as analogous to isotropic thermal expansion/contraction. The result of analysis showed that the numerical procedure is capable to describe the response of pavement to subgrade volume change.

キーワード : Seal Coated Pavements, Swelling Clays, Soil Shrinkage, Expansive Soils,
Thermal Analogy

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建設省土木研究所

道路部舗装研究室 御中

茨城県つくば市大字旭一番地

No. _____

No. _____

配布先氏名

配布先住所



土木研究所資料第 3354 号 部

上記のとおり受領しました。

所 属(住所)

氏 名 印

土木研究所資料第3354号

